Bayesian Network

Definition :

① A Directed Acylic Graph (DAG).

② Nodes : set of random variable

③ Directed Link (Edge) :

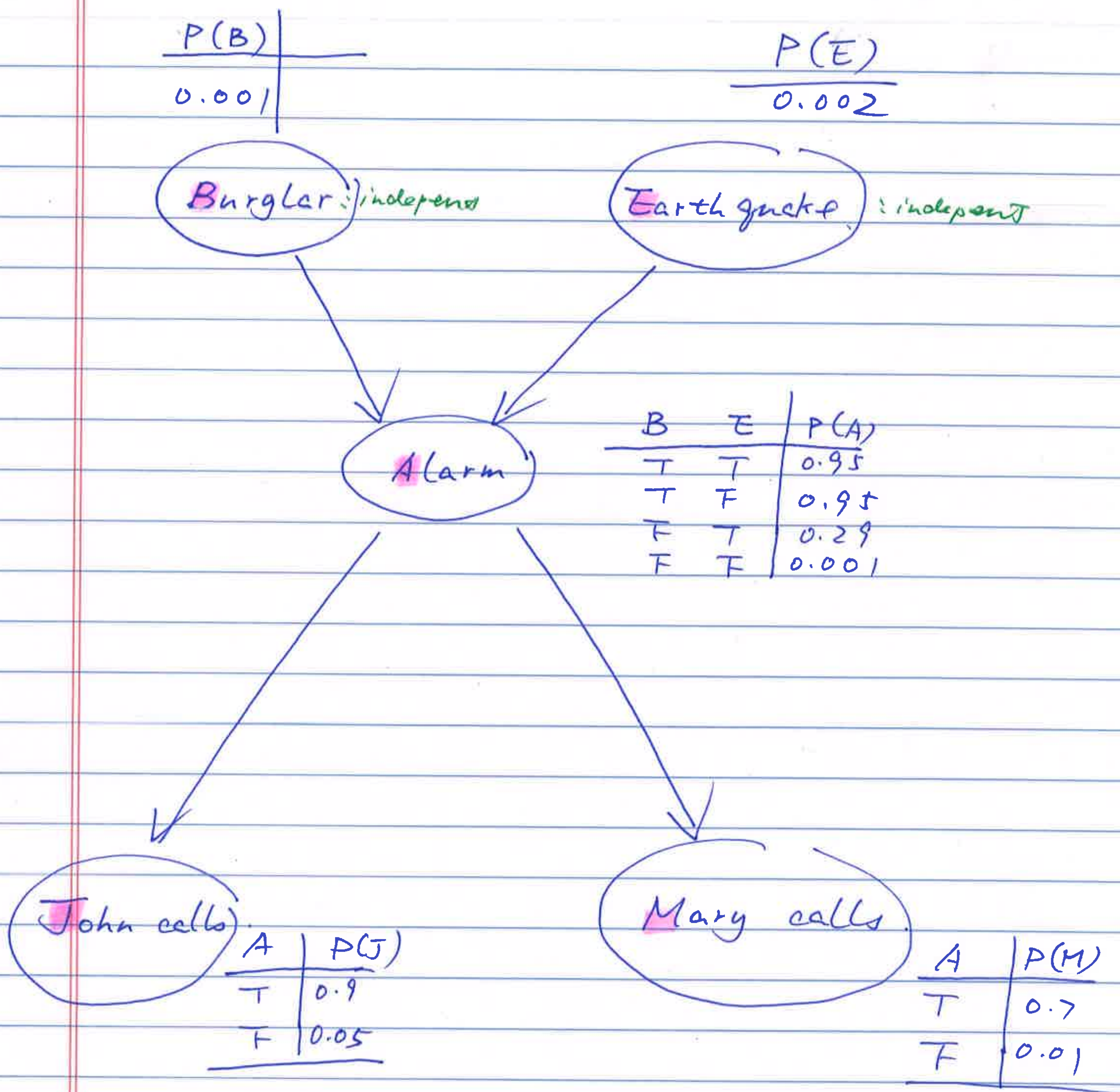
$$X \longrightarrow Y$$

x has direct influence over y

④ Conditional Probability Table (CPD) : each node has it.

Example :

Burglar Alarm at Home Problem



P(B)
0.001

P(E)
0.002

B	E	P(A)
T	T	0.95
T	F	0.95
F	T	0.29
F	F	0.001

A	P(J)
T	0.9
F	0.05

A	P(M)
T	0.7
F	0.01

Conditional Probability.

☆ Bayes Rule.

$$P(A \wedge B) = P(A/B) \cdot P(B).$$

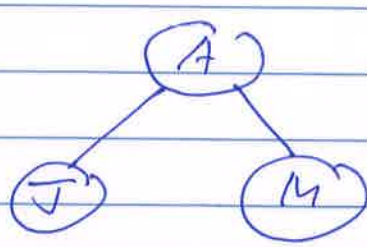
$$P(A \wedge B) = P(B/A) \cdot P(A)$$

$$P(B/A) = \frac{P(A/B) \cdot P(B)}{P(A)}$$

☆ Joint Probability Distribution

$$P(x_1, x_2, \cdots x_n) = \prod_{i=1}^{n} P(x_i / parent(x_i))$$

for example :



$$P(A \wedge J \wedge M) = P(A) * P(J/A) * P(M/A).$$

☆ Marginal Probability

$$P(A) = \sum_i P(A/B_i) \cdot P(B_i).$$

Q1 $P(J \wedge M \wedge A \wedge \sim B \wedge \sim E)$.

$= P(J/A) * P(M/A) * P(A/\sim B, \sim E)$
$* \; P(\sim B) * P(\sim E)$

$= 0.9 * 0.7 * 0.001 * (1-0.001) * (1-0.002)$

$= 0.00628$.

Q2: $P(J) = ?$

Introduce J's all predecessor.

$P(J) = P(J) \cdot (P(A) + P(\sim A))$.

$= P(J/A) \cdot P(A) + P(J/\sim A) \cdot P(\sim A)$.

$= 0.9 \cdot P(A) + 0.05 * P(\sim A)$.

$P(A) = P(A/B, E) \cdot P(BE) +$
$P(A/\sim B, E) \cdot P(\sim BE) +$
$P(A/B, \sim E) \cdot P(B \sim E) +$
$P(A/\bar{B}\bar{E}) \cdot P(\bar{B}\bar{E})$

Q3. What's the Probability Distribution of Burglary given John & Mary call.

$$P(B \mid J, M) = \frac{P(B, J, M)}{P(J M)}$$

$$= \alpha \cdot P(B, J, M)$$

$$= \alpha \cdot \sum_E \sum_A P(B, E, A, J, M)$$

$$= \alpha \cdot \sum_E \sum_A \cdot P(B) \cdot P(E) \cdot P(A/BE) \cdot P(J/A) \cdot P(M/A)$$

=

E	A	
F	F	$P(B) \cdot P(E=f)\ P(A=f)\ P(B) \cdot P(E=f)$.
F	T	$P(B) \cdot P(\bar{E}) \cdot P(A/B\bar{E}) \cdot P(J/A) \cdot P(M/A)$
T	F	$P(B) \cdot P(E) \cdot P(\bar{A}/BE) \cdot P(J/\bar{A}) \cdot P(M/\bar{A})$
T	T	$P(B) \cdot P(E) \cdot P(A/BE) \cdot P(J/A) \cdot P(M/A)$

$$= \alpha \cdot \underbrace{P(B)}_{B} \cdot \sum_E \underbrace{P(E)}_{E} \cdot \sum_A \underbrace{P(A/BE)}_{A} \cdot \underbrace{P(J/A)}_{J} \cdot \underbrace{P(M/A)}_{M}$$

5

Q4. Consider the Query

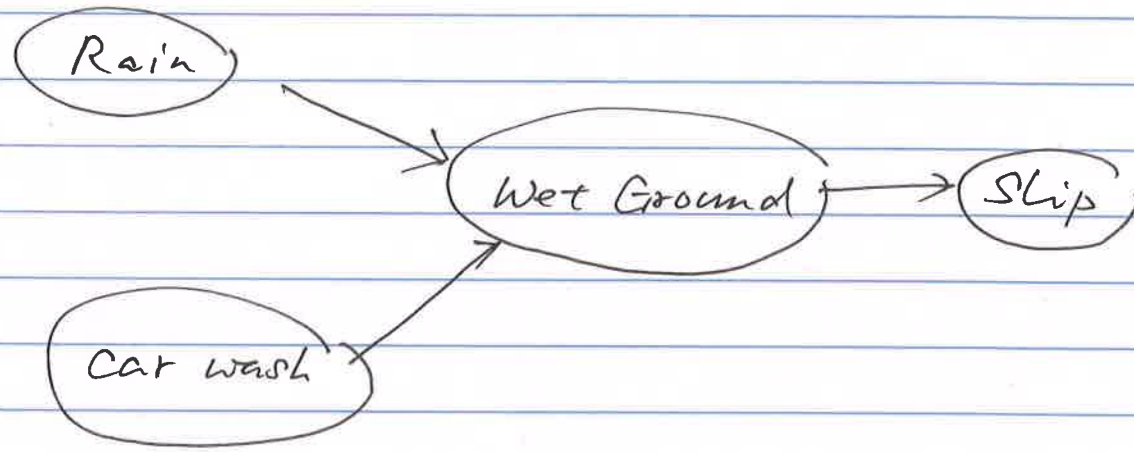
$P\,(\text{John Calls} / \text{Burglary} = \text{TRUE})$.

$$P(J/B) = \frac{P(J\,B)}{P(B)}$$

$$= \alpha .\ P(B) \sum_{E} P(E) \cdot \sum_{A} P(A/B,E) \cdot P(J/A) .$$

$$\cdot \sum_{M} P(M/A) .$$

Another Bayesian Network



$$P(R, W, S, C) = P(R) \cdot P(C) \boxed{\cdot P(W/RC)} \cdot P(S/W)$$

$$P(X/\text{parents}(X))$$

$$P(R/S) = \frac{P(RS)}{P(S)}$$

$$= \sum_{W} \sum_{C} P(R, W, S, C) / P(S)$$

marginalization

$$= \frac{1}{P(S)} \cdot \sum_{W} \sum_{C} P(R) \cdot P(C) \cdot P(W/RC) \cdot P(S/W)$$

$$= \frac{1}{P(S)} P(R) \sum_{W} P(S/W) \cdot \sum_{C} P(C) \cdot P(W/CR).$$

Variable Elimination.

$$P(R/S) \propto \sum_W \sum_C P(R) \cdot \boxed{P(C)\, P(W/CR)} \cdot P(S/W)$$

Let: $f_C(W) = \sum_C P(C) \cdot P(W/CR)$.

$$\Rightarrow \propto \sum_W P(R)\, P(S/W) \cdot f_C(W).$$

$$W \to X \to Y \to Z$$

Another Example

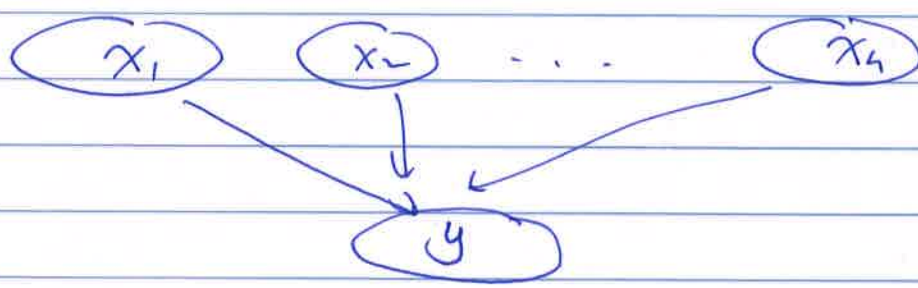
$$P(W, X, Y, Z) = P(W) \cdot P(X/W) \cdot P(Y/X)\, P(Z/Y)$$

$$P(Y) = \sum_W \sum_X \sum_Z P(W)\, P(X/W) \cdot P(Y/X)\, P(Z/Y).$$

Naive Bayes

$$P(x, y_1, y_2 \dots y_n) = P(x) \cdot \prod_{i=1}^{n} \left(P(y_i/x)\right)$$

Logistic Regression.



$$P(x_1, x_2, \cdot\cdot\, x_n, y) = \prod_{i=1}^{n} P(x_i) \cdot P(y/x_i)$$