Lesson 0A : Probability & Statistic.
(Yu Liang)

Probability Rules

① Complement Rule

$$P(A) = 1 - P(\bar{A}) \quad (\bar{A} = A^c = \Omega \setminus A).$$

② Conditional Probability

$$P(A/B) = P(AB) / P(B).$$

$\Downarrow$

③ Conjunction Rule

$$P(AB) = P(A/B) \cdot P(B)$$
$$= P(B/A) \cdot P(A).$$

$P(AB) = P(A) \cdot P(B)$ if A is not correlated with B.

$\Downarrow$

④ Bayes Rule

$$P(A/B) = \frac{P(AB)}{P(B)} = \frac{\overset{\text{posterior}}{P(B/A)} \cdot \overset{\text{priori}}{P(A)}}{P(B)}.$$

⑤ Marginal Rule

$$P(A) = \sum_{\forall y} P(A, B=y)$$

⑥ Chain Rule

$$P(A_1 A_2 \cdots A_n) = \prod_{i=1}^{n} P(A_i / \text{parents}(A_i))$$

vehicle \ age	young	middle	senior ~~old~~	
SUV	0.1	0.2	0.1	
CAR	0.05	0.1	0.05	
pickup.	0.05	0.2	0.15 .	
	0.2 .			

$$P(age='young') = \sum_{\forall vehicle} P(vehicle, age='young')$$

$$= P(vehicle=SUV, age='young') + P(vehicle=CAR, age='young') + P(vehicle=pickup, age='young')$$

$$= 0.1 + 0.05 + 0.05$$

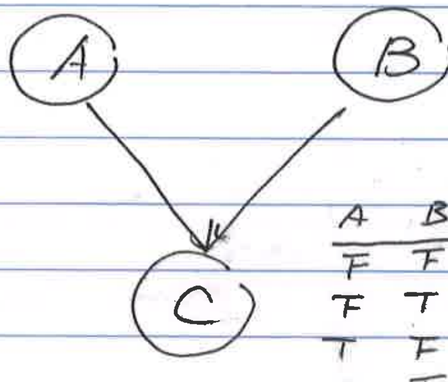
$$= 0.2 .$$

$$P(vehicle=SUV / age=middle) = P(v=SUV, age=mid) / P(age=mid)$$

Applications of Chain Rule

Example 1 :

$P(A) : 0.8.$ $P(B) : 0.3$

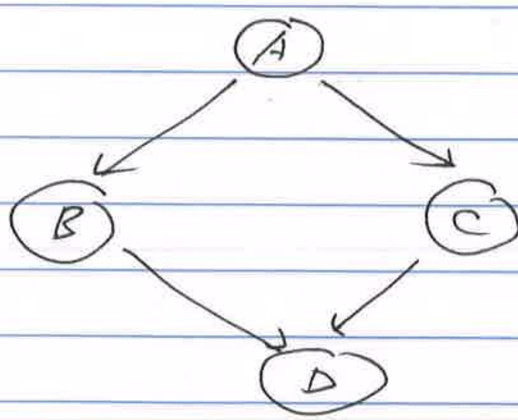


A	B	C
F	F	0.2
F	T	0.2
T	F	0.1
T	T	0.5

$$P(ABC) = P(A/\text{parents}(A)) \cdot P(B/\text{parents}(B))$$
$$P(C/\text{parents}(C))$$
$$= P(A) \cdot P(B) \cdot P(C/AB)$$
$$= 0.8 * 0.3 * 0.5$$

$$P(\bar{A} B C) = P(\bar{A}) \cdot P(B) \cdot P(C/\bar{A} B).$$
$$= 0.2 * 0.3 * 0.2.$$

Example 2 :



Statistics : Concepts :

N : # of Observations.

μ : mean of population

$\overline{x}$: mean of Observations.

$\mathcal{N}$: $(0, 1)$: Normal Distribution (standard)

$\mathcal{N}$: (μ, σ) : Normal Distribution.

σ^2 : variance

σ : standard deviation

cov_{xy} : co-variance of x & y

	Random variable is discrete		Random variable is continuous.
	population	Sample	
expected value.	$\mu = \sum_{i=1}^{N} x_i \cdot P(x_i)$.	$\bar{x} = \sum_{i=1}^{N} x_i / N$.	$E(x) = \int_a^b x \cdot \underline{f(x)} \cdot dx$. probability density function
variance	$\frac{\sum_{i=1}^{N} (x_i - \mu)}{N} = \sigma^2$	$\frac{\sum_{i=1}^{N} (x_i - \bar{x})}{N-1} = s^2$	$Var(x) = E(x - E(x))^2$. $= \int_a^b (x - E(x))^2 \cdot f(x) . dx$.
standard deviation	$\sigma = \sqrt{\frac{\sum_{i=1}^{N} (x_i - \mu)}{N}}$	$S = \sqrt{\frac{\sum (x_i - \bar{x})^2}{N-1}}$	$SD(X) = \sqrt{Var(X)}$.
co-variance	$Cov(x,y) = \frac{\sum_{i=1}^{N} (x_i - \mu_x)(y_i - \mu_y)}{N}$.	$cov(x,y) = \frac{\sum_{i=1}^{N} (x_i - \bar{x})(y_i - \bar{y})}{N-1}$	$cov(x,y) = E((x - E(x)) * (y - E(y)))$.
correlation	$Corr(x,y) = \frac{Cov(x,y)}{\sigma_x \cdot \sigma_y}$.	$Corr(x,y) = \frac{cov(x,y)}{S_x \cdot S_y}$	$Corr(x,y) = \frac{cov(x,y)}{SD(x) \cdot SD(Y)}$.

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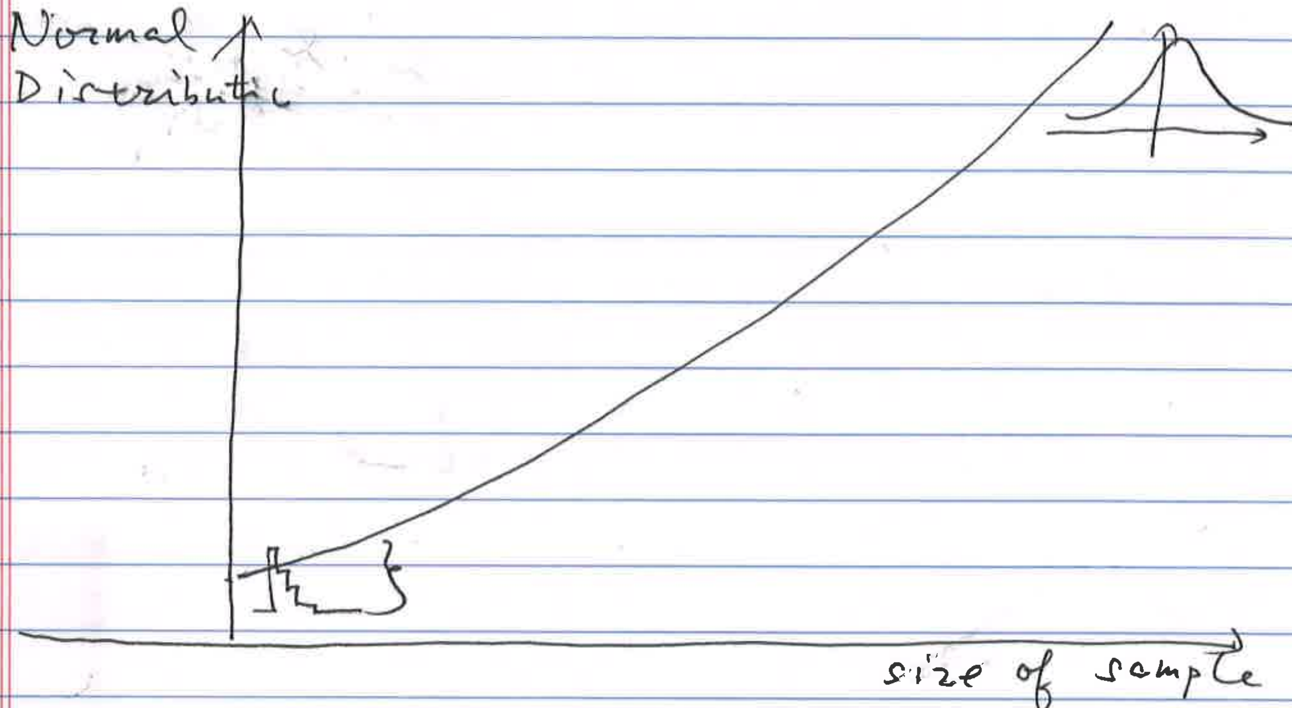
2 concepts in Statistic.

☆ Central Limit Theorem



☆ How to standardrize Normal Distribution

$$N(\mu, \sigma) \longrightarrow N(0, 1)$$

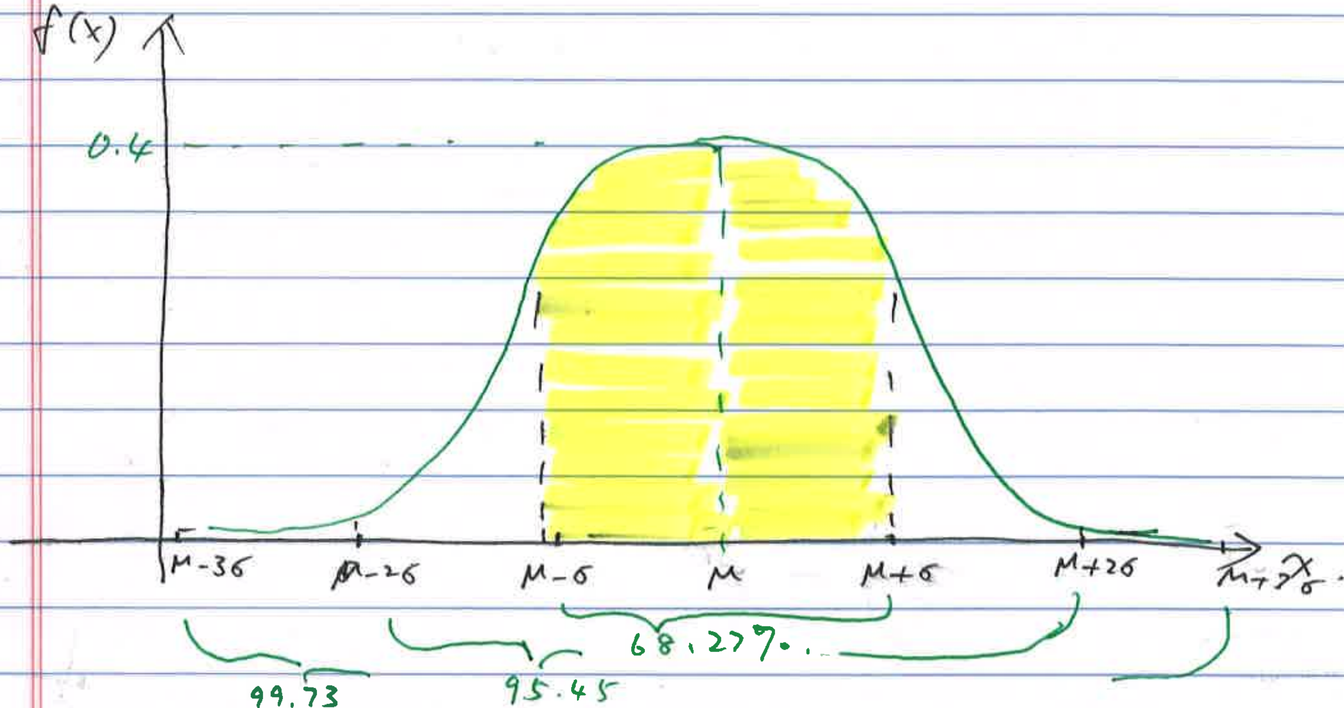
① shift.
② scale.

$$N(\mu, \sigma): \; f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}.$$

$$N(0, 1): \; f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}},$$

Normal Distribution

$$N(\mu, \sigma): f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$



$\boxed{68 - 95 - 99.7 \text{ rules}}$