

**VarahaMihira Science Forum**  
**Indian Mathematics +Astronomy**

**Early Classical Period**

**R. Gopu**

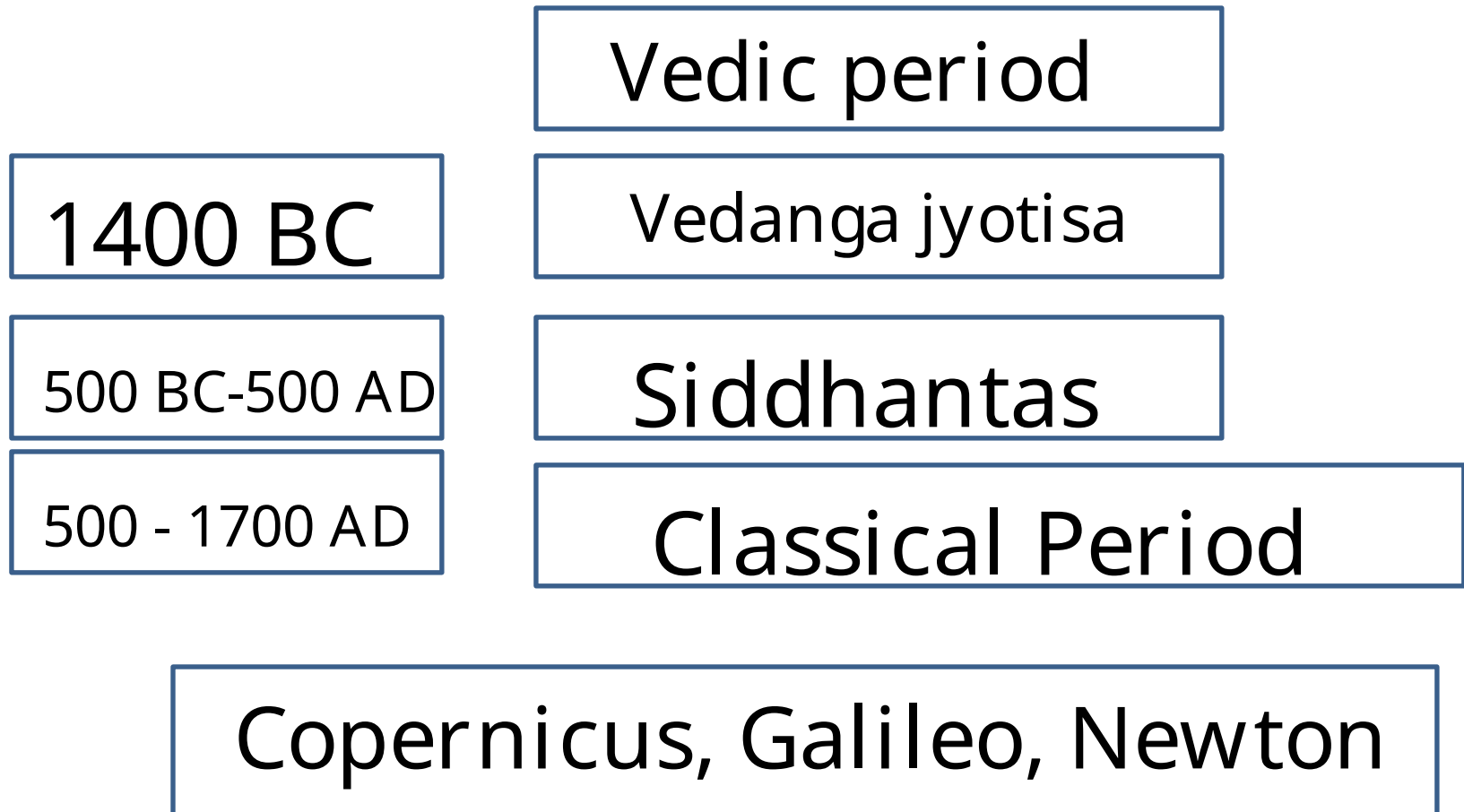
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Caritram

# A BRIEF HISTORY

# History of Astronomy



# Classical Period

500 - 800  
AD

Early Classical

800 -1300 AD

Medieval

1300 - 1800 AD

Kerala school

# Astronomy Books

Siddhanta

Theory & Proof

Bhaashya

Commentary

Karana

Tables for  
Astronomers

Panchanga

Calendars  
for public

## Approximate Geography of Indian Jyotishas



गणितं

**MATHEMATICS**  
**IN THE CLASSICAL ERA**



आर्यभटः

**ARYABHATA**  
**SINGLE T**

# Aryabhateeyam

Aryabhata's famous book is Aryabhateeyam

It is 121 stanzas long

It consists of 4 parts

1. Gitika (13 stanzas) – Definitions and calculated constants
2. Ganita – Mathematics (33 stanzas)
3. Kalakriya - Time (25 stanzas)
4. Gola – Celestial Sphere (50 stanzas)

All stanzas are in Arya chandas

Gitika is considered preamble

The other 3 chapters are called Arya

Ashtashatam (108)

Aryabhateeyam is important since it begins the Classical era  
It is incredibly short, in comparison to other such books

# Aryabhata's Era

षष्ट्याब्दानां षष्टिर्यदा व्यतीतास्त्रयश्च युगपादा ।

त्र्यधीका विंशतिरब्दास्तदेह मम जन्मनोऽतीतः ॥ ३१ ॥

shashTyabdaanam shashTiryadA vyatitAs-  
traya-shca yugapAdA

tryadhika vimshatirabdAs-tadeha mama

3600 after Kali yuga began, my age was 23

janmano-atItAH

shahTi	sixty
abda	years
tryadhika	Plus three
vimshati	Twenty
Janamano	birth
Traya yugapAdA	Three yuga-pAdAs
atItA	passed

Sunday

March 21, 499 AD

So Born in 476 AD

# Thrai Raashika - Rule of three

त्रैराशिकफलराशिं तमथेच्छाराशीना हतं कृत्वा ।  
लब्धं प्रमाणभजितं तस्मात् इच्छाफलमिदं स्यात् ॥ २६

$$\text{icchaa phalam} = \frac{\text{phala raashi}}{\text{pramaaNa}} * \text{icchaa raashi}$$

*Smith's Translation*

$$\text{Fruit of desire} = \frac{\text{Fruit}}{\text{Measure}} * \text{desire}$$

# vipareetam - Inversion

गुणकारा भागहरा भागहरास्ते भवन्ति गुणकाराः ।

guNa-kaaraa bhaaga-haraa bhaaga-haraa-s-  
tey bhavanti guNa-kaaraaH

यः क्षेपः सोऽपचयोऽपचयः क्षेपश्च विपरीते ॥

ya kshepaH sa upachaya upachayaH kshepa	
guNa-kaaraa	multipliers
bhaaga-haraa	divisors
they	they
bhavanti	become
yaH - saH	whichever - that
kshepa	subtractions
upachaya	additions
vipareetam	inversion

# Fractions (chedaaH) - Simplification

छेदाः परस्परहता भवन्ति गुणकारा भागहराणाम् ।

chedaaH paraspara-hataa bhavanti guNa-  
kaaraa bhaaga-haraaNaam

$$\frac{a/b}{c/d} = \frac{a * d}{b * c}$$

chedaaH	fractions
paraspara	each other
hataa	multiply
bhavanti	become
guNa-kaaraa	multipliers
bhaaga-haraa	divisors
Naam	of

# Fractions – Common Denominator

छेदागुणं सच्छेदं परस्परं तत् सवर्गत्वम् ॥

chedaaguNam sacchedam parasparam tat  
savargatvam

Multiply numerators and denominators of each  
fraction

By denominator of each fraction

Thus both have common denominator

$$a / b + c / d = \frac{ad}{bd} + \frac{bc}{bd} = \frac{ad + bc}{bd}$$

# Seeds of Algebra (Bija Ganita)

With *Vipareetham* Inversion and  
*Thrai raashika* Rule of three,  
you basically have the seeds of algebra

- Finding unknowns
- Expressions using unknowns
- The Equation with implied = sign
- Balancing equations

Roots of quadratic equations, operations of  
fractions, series summations, square and  
cube root algorithms clinch this argument

# Aryabhata – Area of Circle

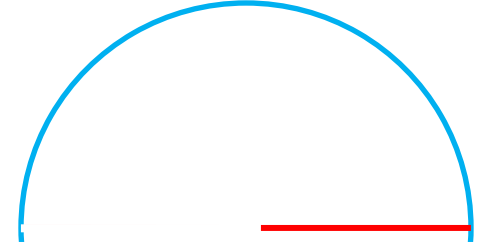
समपरिणाहस्यार्ध विष्कम्भार्धहतमेव वृत्तफलम् ।

Sama pariNaahasya ardha vishkamba ardha  
hatameva vrtta phalam

Area = perimeter/2 \* diameter /2

No mention of pi

Sama	equal
pariNaahasya	half Perimeter
ardha vishkamba	half diameter
ardha hatameva	multiply
vrtta	circle
phalam	Result (area)



चतुरधिकं शतमष्टगुणं द्वाषष्टिस्तथा सहस्राणाम् ।

catur-adhikam shatam-ashTa-guNam dvaashashTi-tathA sahasrANAm

अयूतद्वय विष्कम्भस्यासन्नो वृत्तपरिणाहः ॥

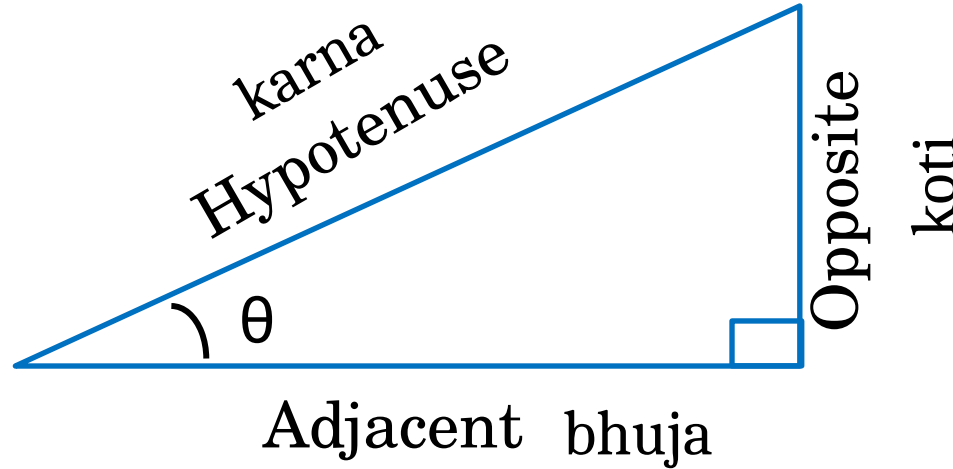
a **104\*8 + 62000 = 62832**

p **For Diameter of 20,000 Circumference is 62,832 approximately**

catur adhikaam	Four plus hundred
shatam	By eight multiply
ashTa guNam	(2+60) * 1000
dvaashashTi	10,000 times 2
sahasrANAm	Diameter's
ayuta dvaya	Approximately
vishkambha-sya	Circle's Circumference
aasannaH	
vRtta pariNaaha	

**An accurate definition for pi**

# Bhujaa koTi karNa nyaaya



Bhujaa vargaH koTi vargaH ca karNa vargaH  
Side squared and Perpendicular squared  
(equal) hypotenuse squared

$$a^2 + b^2 = h^2$$

# Arithmetic progression

Sum of a series, increasing by regular interval

- $1 + 2 + 3 + 4 + 5 + 6 \dots N$
- $1 + 4 + 7 + 10 + \dots T$  (gap or interval 3)
- $22, 25, 28, 31, \dots T$  (Begin 22, gap 3)
- $B, B+G, B+2G, B+3G, B+4G, \dots B + (N-1)G$

T is the Nth term, G is Gap, B is Beginning term

$$\text{Mean} = B + G * (N - 1) / 2$$

$$\text{Sum of series} = \text{Mean} * N$$

## Second Formula

$$\text{Sum of series} = (\text{aadi} + \text{antam}) * N / 2$$

## citi - Sums of Series of Arithmetic progression

इष्टं व्येकं दलितं सपूर्वमुत्तरगुणं समुखमध्यम् ।

ishtam vyekam dalitam sa-poorvam-uttara-  
guNam samukha-madhyam

इष्टगुणितमिष्टधनं त्वथवाद्यन्तं पदार्धहतम् ॥ 19 ॥

ishTa-guNitam	ishTa-dhanam	tvathava aadi-
ishTam	Desired result	
antam	Decreased by one	
ekam dalitam		
uttara guNam	Multiplied by interval	
sa mukha	Plus first term	
madhyam	Mean (central or average)	

According the commentator Bhaskara, (via Ramasubramaniam) this is a **muktaka** (verse that encodes multiple formulae).

First line gives the mean

Second line gives sum, and another formula

# citi, varga-citi-varga, ghana-citi-ghana

citi is sum of series of natural numbers

VARGA-citi-ghana is sum of SQUARES of a series

GHANA-citi-ghana is sum of CUBES of a series

- citi  $1 + 2 + 3 + 4 + 5 + 6 + \dots$

$$c = n(n+1)/2$$

- VARGA-citi-ghana  $1^2 + 2^2 + 3^2 + 4^2 + \dots N^2$

$$vc = n(n+1)(2n+1)/6$$

- GHANA-citi-ghana  $1^3 + 2^3 + 3^3 + 4^3 + \dots N^3$

$$vc = c^2 = (n(n+1)/2)^2$$

# Square Root and Cube Root

Ganita chapter has not only several formulae  
But also a few algorithms

Square root, Cube root, and kuTTaka are three  
algorithms

# Square

वर्गः समचतुरश्रः फलं च सदृशद्वयस्य संवर्गः ।

vargaH sama-chatur-ashraH phalam ca sa-  
drsha-dvaya-sya samvargaH

सदृशत्रयसंवर्गो घनस्तथा द्वदशाश्रि स्यात् ॥ ३ ॥

sa-drsha-traya samvargaH ghanaH tathaa

dvaadasha-ashri syaat

sama-chatur-ashraH	varga equal-four-sided
ca	and
sa drsha	same-visible (equal)
dvaya samvargaH	twice multiplied
phalam	result

A varga is

four sided, and,



the same number twice multiplied

$a*a$

Geometric

Arithmetic

# Cube

सदृशत्रयसंवर्गो घनस्तथा द्वादशाश्रि स्यात् ॥ ३ ॥

sa-drsha traya samvargaH ghanaH tathaa  
dvaadasha-ashri syaat

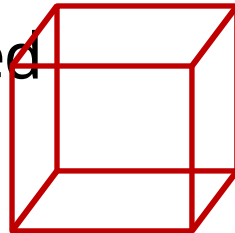
sa drsha	same-visible (equal)
traya samvargaH	thrice multiplied
ghanaH	ghanaH
tathaa	similarly
dvaa dasha ashri	twelve sided

A ghanaH is

the same number thrice multiplied  $a*a*a$

Arithmetic

similarly twelve sided



Geometric

# kuTTaka

KuTTaka – pulverizer

Breaking down a problem into  
smaller, simpler problem

Uses an *iterative* algorithm to solve a problem

**Linear Determinate equation**

$$47 = 5x + 7$$

Here,  $x=8$ . There is ONE solution

# kuTTaka - continued

This is easy for small values

$$N = 5x + 2$$

$$N = 8y + 3$$

Very hard for large values

$$N = 1841x + 245$$

$$N = 8205y + 9732$$

Aryabhata's kuTTaka algorithm is a simple method to solve such problems.

# A Bit of Mathematics!

- Bhaskara 1<sup>st</sup> wrote a bhaashya on Aryabhata
- He says
- GanitaPada of Aryabhata is hardly Ganita
- It is only “a little bit of Ganita”

# Aryabhata Mathematics

Vipareetam (Inversion)

chedaah (Fractions)

Areas of triangle, circle, trapezium

Accurate value of pi

Bhuja koTi karNa nyaayaa

Two definitions for varga, ghana

Citi, varga citi, ghana citi

Square and Cube roots

Kuttaka

# Aryabhata's Instruments

वृत्तं भ्रमेण साध्यं त्रिभुजं च चतुर्भुजं च कर्णाभ्याम् ।

vrttam bhrameNa saadyam tribhujam ca  
chaturbhujam karNAbhyAm

साध्या जलेन समभूरध ऊर्ध्वं लम्बकेनैव ॥ १३ ॥

saadyA jalena samabhuradha urdhvam

saadyam	Feasible
lambakena eva	
Bhrama	compass
karNa	Set square
Sama bhuradha	Level earth or surface
lambaka	Plumb line

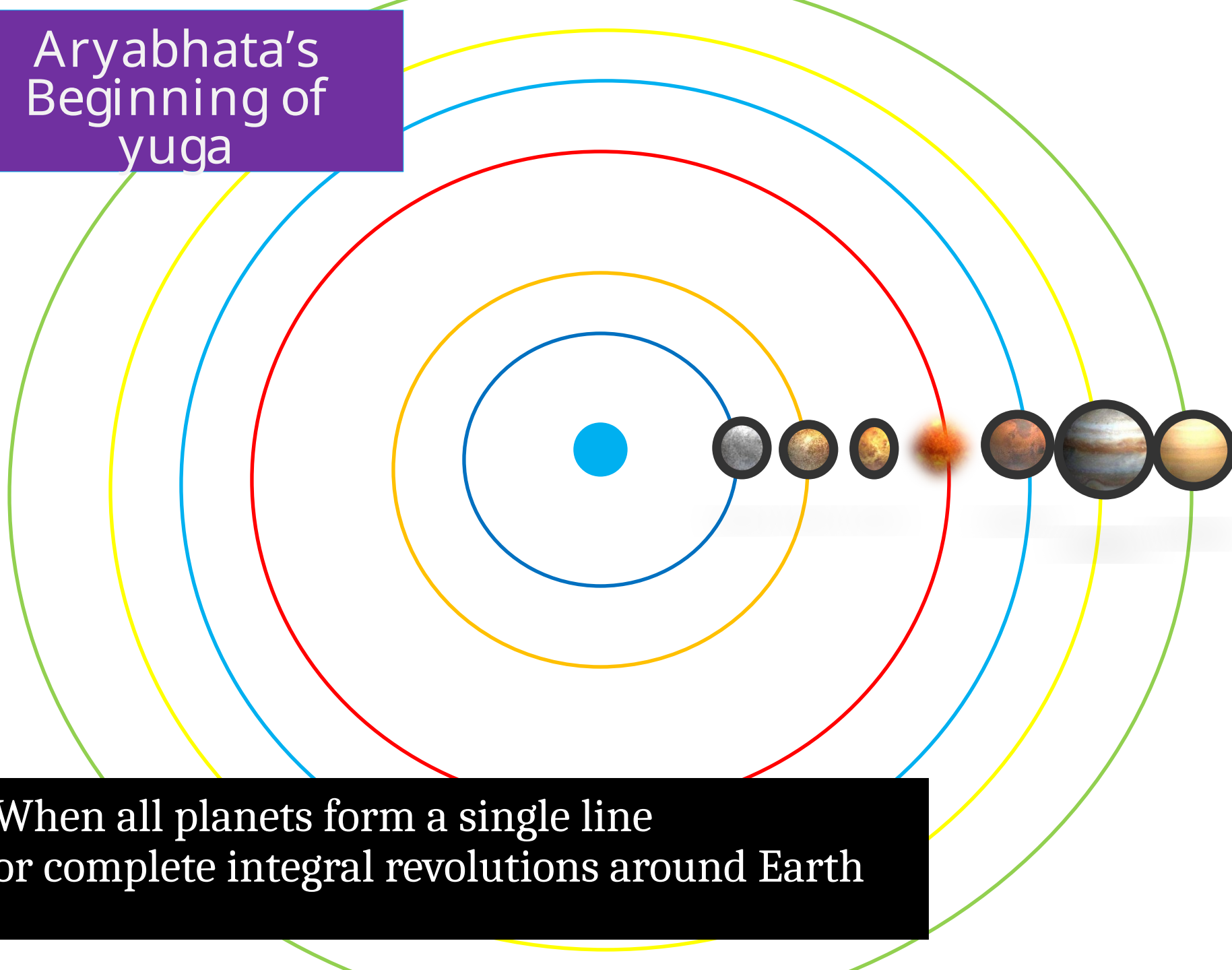
By **compass**, a circle,

By **setsquare**, triangle square

By water, a level surface

By **plumb line**, vertical line ... are feasible

Aryabhata's  
Beginning of  
yuga



When all planets form a single line  
or complete integral revolutions around Earth

# Circle and Time

एवं कालविभागः क्षेत्रविभागस्तथा भगणात् ।

Evam kaala-vibhaaga kshetra-vibhaaga-s-tathaa bhagaNaat

## Circle

- Rashi 12
- Kalaa 30
- Vikalaa 60
- Liptaa 60
- Viliptaa 60

## Time

- Maasa
- Dina
- Naadi
- Vinaadi
- Guru akshara

# Aryabhata Revolutions in a yuga

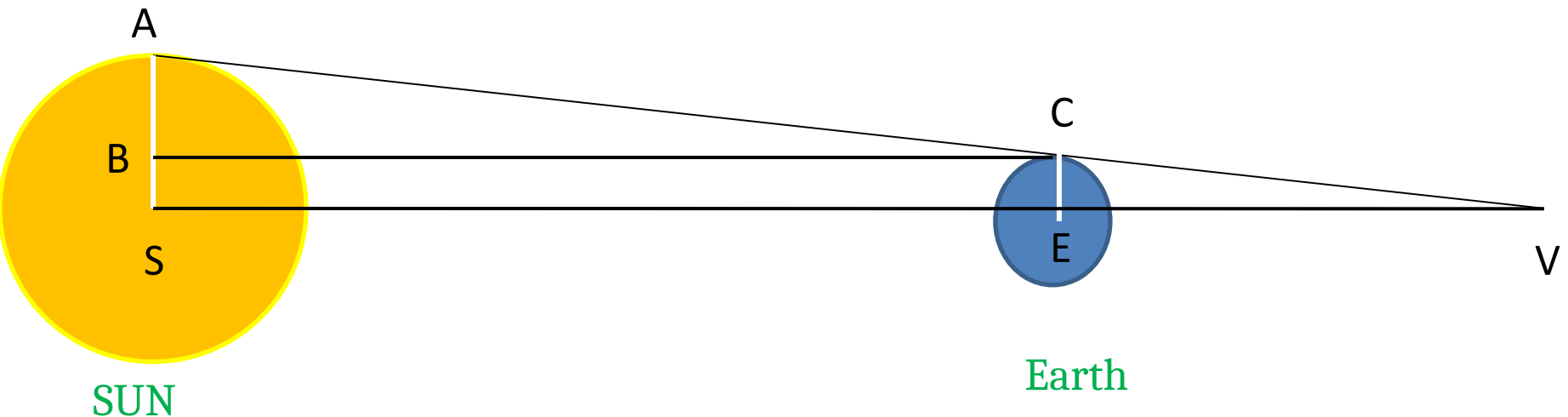
Graha / Planet	Revolutions	In days	Modern
Ravi / Sun	432000		
Indu / Moon	53433336	29.53058	29.53059
Kuja / Mars	2023176	779.92125	779.9428
Budha / Mercury	13617020	115.8783	115.8786
Guru / Jupiter	3955776	398.8895	398.8864
Shukra / Venus	2702388	583.8975	583.0000
Shani / Saturn	4173436	378.0859	378.0930

# Aryabhata

## Length of Earth's shadow

Length of Earth's Shadow =  $\frac{\text{Sun's Distance} * \text{Earth's diameter}}{\text{Sun's diameter} - \text{Earth's Diameter}}$

$$EV = \frac{SE * 2 EC}{2 SA - 2 EC}$$



Triangles CEV and ABC are similar.

The above calculation follows from that

# Length of Earth's shadow : Derivation

$$\frac{EV}{EC} = \frac{BC}{AB} = \frac{SE}{SA - EC}$$

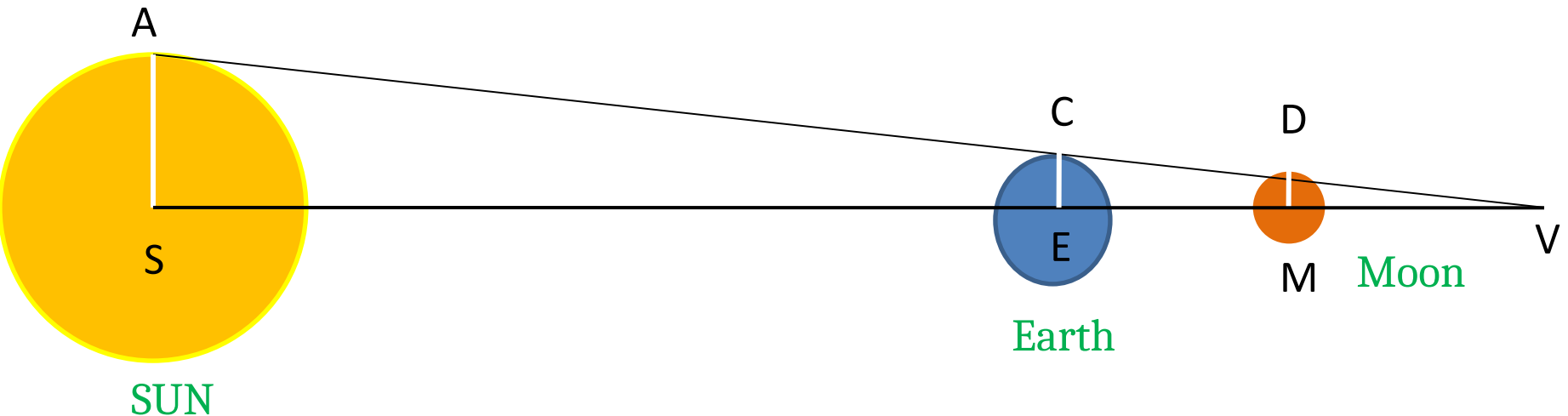
$$\text{Length } EV = \frac{SA * EC}{SA - EC} = \frac{SA * 2 EC}{2 SA - 2 EC}$$

# Aryabhata

## Diameter of Earth's shadow

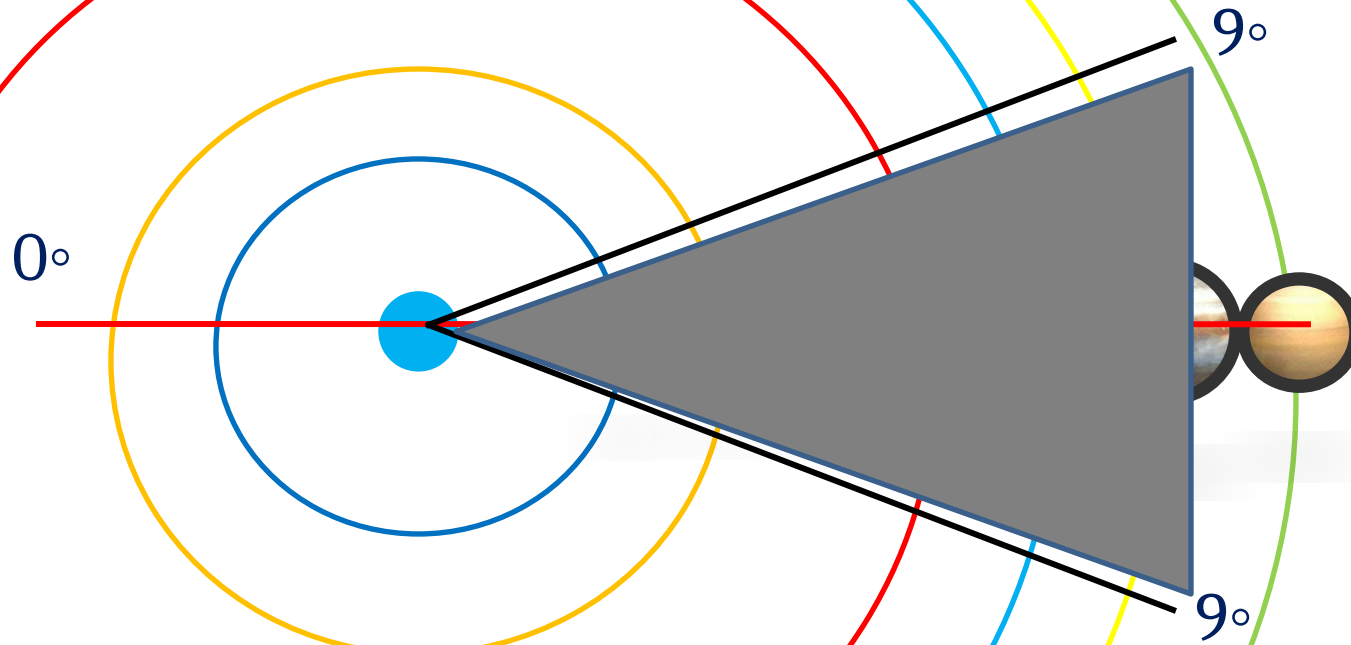
Diameter =  $\frac{(\text{Length of shadow} - \text{Moon distance}) * \text{Earth's diameter}}{\text{Length of earth's shadow}}$

$$2DM = \frac{(EV - EM) * 2 * EC}{EV}$$



Triangles CEV and MDV are similar.  
The above calculation follows from that

# Aryabhata's Angle of visibility



**Exaggerated angles** – For illustration  
ONLY outside this range of Sun, planet visible

# Visibility of Planets

चन्द्रोऽरंशैर्द्वादशाभिरविक्षिप्तोऽर्कान्तरस्तिथो दृश्य ।

नवभिर्भृगोस्तैर्द्वयधिकैर्द्वयधिकैर्यथाशलक्षणाः ॥ ३१ ॥

candro-amshai-dvAdashA-bhir-avikShipto-  
arkAntara-stitho dRshya

navabhir-bhRgo-stair dvar-adhikair-dvar-  
adhikair-yathA-lakshNAH

<u>Planet</u>	<u>Visibility angle</u>	
Moon	12	
Venus(bhRgu)	9	
Jupiter	11	+2
Mercury	13	+2
Saturn	15	+2
Mars	17	+2

# Rotation of Earth

अनुलोमगतिर्नौस्तः पश्यत्यचलं विलोमगं यद्वत् ।  
अचलानि भानि तद्वत् समपश्चिमगानि लङ्कायाम् ॥ ९ ॥  
anulomagati-nau-staH pashyati-acalam  
vilomagam yadvat

acalaani bhaani tadvat sama-paschima-  
gaani lankaayam  
Just as man on a boat thinks trees move backward,  
so people in Lanka think stars move westward

anuloma	Forward going
gati	boat
nau	Standers
staH	see
pashyati	Unmoving
acalam	things
vilomaga	backward going
m yadvat	Just as

acalaani	unmoving
bhaani	stars
tadvat	so also
sama	similarly
paschim	westward
a	
gaani	motion
lankaaya	Lankans
am	

# Revolution of planets

उदयास्तमयनिमित्तं नित्यं प्रवहेण वायुना विक्षिप्तः ।

लङ्का समपश्चिमगो भपञ्चरः सग्रहो भ्रमति ॥ १० ॥

Udaya-astamaya-nimittam nityam

pravahENa vAyunA vikshiptaH

lankA sama-pashcima-go bha-pancaraH sa-

graho bhramati  
Rising and setting daily

Driven by pravahena-vaayu

PravaheNa –Vaayu is a celestial wind  
assumed to act across space  
moving planets and stars

So Aryabhata did NOT know Gravity

## Concluding verse

सदसद्-ज्ञान-समुद्रात् समुद्धृतं ब्रह्मणः प्रसादेन ।  
सद्-ज्ञानोत्तमरत्नं मया निमग्नं स्वमतिनवा ॥ ९ ॥

sat-asat-jnaana-samudraat samuddhRtam  
brahmaNaH prasAdena

sat-jnAna-uttama-ratnam mayaa nimagnam

From the ocean of true and false knowledge, by delving

I have brought out the great gem of truth with my boat-of-intellect

sat	true
asat	false
jnaana	knowledge
samudraat	ocean-from
samuddhRt	delving
am	
brahmaNa	by Brahma's
H	
prasAdena	grace

sat-	truth
jnAna	best
uttama	gem
ratnam	by me
mayaa	brought
nimagna	out
m sva-	self-
mati	intellect
navaa	boat

# A small dose of Aryabhata

- These are only a few items of Aryabhata's Kalakriya and Gola

Deva Vandanam

# PRAYERS AND PLAYFULNESS

# Aryabhateeyam

प्रणिपत्यैकमनेकं कं सत्यां देवतां परं ब्रह्मा ।  
आर्यभटस्त्रीणी गदति गणितं कालक्रियां गोलं । ।  
दशगीतिका 1

ப்ரணிபத்யேகமனேகம் கம் ஸத்யாம் தேவதாம் பரம்  
ப்ரம்மா  
ஆர்யபடஸ்த்ரீணீ கததி கணிதம் காலக்ரியம் கோலம்.

Bowing to Param **Brahma** who is One, Many, Deity of  
Truth,

Aryabhata states Three – Mathematics, Time, Sphere. 1

# Brhat Samhita

जयति जगतः प्रसूतिर्विश्वात्मा सहजभूषणं नभसः ।  
द्रुतकनकसदृशदशतमयूखमालार्चितः सविताः ॥ १ ॥

ஐயதி ஐகத: ப்ரஸுதிர் விஷ்வாத்மா  
ஸஹஜபூஷணம் நபஸ:  
த்ருத கனக ஸத்ருஷத சதம் அயூக மாலார்சித:  
ஸவிதா

Glory to the **Sun (Savitr)**, author of Earth and soul of the World, ornament of Space and all Life, who is adorned by endless thousand golden rays

# Brahma sphuta Siddhantam

जयति प्रणतसुरासुरकिरीटरत्नप्रभाछुरितपादः ।

कर्ता जगदुत्पत्तिस्थितिविलयानां महादेवः ॥

ஜயதி ப்ரணத ஸுராஸுர கிரீடரத்ன ப்ரபாக்ரித  
பாத:

கர்தா ஜகத் உத்பத்தி ஸ்திதி விலாயானாம்  
மஹாதேவ:

Glory to **Mahadeva**, Cause of Creation, Preservation  
and Destruction, whose feet shine by the gems of  
the crowns of bowing Suras and Asuras

# Ganita Sara Sangraha

अलङ्घ्यं त्रिजगत्सारं यस्यानन्तचतुष्टयम् ।

नमस्तस्मै जिनेन्द्राय महावीराय तायिने ॥ १ ॥

alanghyaM tri jagat saaram yasyaananta  
chatusTayam

Namas tasmai Jinendraaya **Mahaaveera**aya taayinE

அலங்க்யம் த்ரிஜகத்ஸாரம் யஸ்ய் அநந்த  
சதுஷ்டயம்

நமஸ்தஸ்மை ஜிணேந்த்ராய மகாவீராய தாயிணே



# Gudhaartha prakaashaka by Ranganatha

ततः स कृष्णो जहंगीर सार्वभौमस्य सर्वाधिगतप्रतिष्ठः ।  
श्रीभास्करीयं विवृतं तु बीजं तथा श्रीपतिपद्धतिः सा ॥

tataH sa krushnO **jahaangira** saarvabhaumasya  
sarvaadhigatapratishTa  
Sri Bhaskariyam vivrtam tu yena beejaM tathA sripathi  
paddhatiH saa

தத ஸ க்ருஷ்ணோ ஜஹங்கீர் ஸார்வபௌமஸ்ய  
ஸர்வாதிகடப்ரதிஷ்ட |  
ஸ்ரீபாஸ்கரீயம் விவ்ருதம் து யேன பீஜம் ததா ஸ்ரீபதி  
பத்ததி ஸா ||

# Laghu-Bhaskariya Bhaashyam by Sankaranarayana

आचार्यार्यभटं वराहमिहिरं श्रीमद्गुरु भास्करम्  
गोविन्दं हरिदत्तमत्र शिरसा वक्ष्ये प्रणम्य क्रमात् ॥

AcAryAryabhaTam varAhamihiram  
srimadguru bhAskaram  
govindam haridattamatra shirasA vakshye  
praNamya kramAt

ஆசர்யார்யபடம் வராஹமிஹிரம் ஸ்ரீமத்குரு பாஸ்கரம் |  
கோவிந்தம் ஹரிதத்தம் அத்ர வ்ஷிரஸா வக்ஷ்யே  
ப்ரணம்ய க்ரமாத் ||

# वराहमिहिरः VaraahamihiraH

Brhat Samhita, chapter 76, is about perfumes(gandha)

## Gandhya-yukti : Combinatorics

From 16 basic perfumes, choosing any 4, how many combinations are possible?

Varahamihira answers 1820 combinations.

$${}^{16}C_4 = 1820$$

षोडशके द्रव्यगुणे चतुर्विकल्पेन भिद्यमानानाम् ।

अष्टदश जायन्ते शतानि सहितानि विंशत्या ॥

shoDashake (16) dravya (perfumes) guNe catur-vikalpena  
(choosing four) bhidyamAnAm

ashTdasha (18) jAyante shatAni (100) sahitAni vimshatyA (20)

# वराहमिहिरः VaraahamihiraH

## Gandhya-yukti : Combinatorics

He then gives a method to construct a Meru (table)

A verse explains how to arrange this as a prastaara

Very similar to the chandas-prastaara of Pingala

1	5	15	35	70	126	210	330	495	715	1001	1365	1820			
1	4	10	20	35	56	84	120	165	220	286	364	455	560		
1	3	6	10	15	21	28	36	45	55	66	78	91	105	120	
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

$$nC_r = n-1C_{r-1} + n-2C_{r-1} + n-3C_{r-1} + \dots n-rC_r$$

ब्रह्मगुप्तः

**BRAHMAGUPTA**

**GANAKA CHAKRA CHOODAAMANI**

# BrahmaGupta

For almost every stanza of Aryabhata,  
Brahmagupta writes a chapter.

Bhaskara II does the same.

Aryabhata has 108 stanzas. The others have a 1008.

Bhaskara calls Brahmagupta *Ganaka Chakra Chudamani*

Crown-jewel in Ocean of Mathematicians

# Multiplication Methods

Brahmagupta gives FOUR methods of multiplication

1. Gomutrika (cow's urine – zig zag method)
2. khanDa (parts multiplication)
3. bheda
4. ishTa (desired or assumed number)

A cow's urine falls in a zig-zag manner.  
Multiply like it

For example:  $1254 * 231$

2	1254	2508
3	1254	3762
1	1254	1254

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299654

# Multiplication - khanDa and ishTa

## khanDa (multiplication by parts)

$$13 * 158 = (6 + 7) * 158 = 6 * 158 + 7 * 158 = 948 + 1106 \\ = \dots$$

$$32 * 751 = (2 * 4 * 4) * 751 = 2 * 751 * 4 * 4 = \dots$$

## ishta-guNana (algebraic)

$$105 * 17 = (100 + 5) * 17 = 100 * 17 + 85 = 1785$$

$$94 * 13 = (94 + 6) * 13 - (6 * 13) = 100 * 13 - 78 = 1300 - 78 = 1222$$

We learn this but as associative, commutative property etc

With no mention of BrahmaGupta

# Integer Addition - Brahmagupta

धनयोः धनं ऋणं ऋणयोः धन ऋणयोः अन्तरं सम ऐक्यं खं ।

ऋणं ऐक्यं च धनं ऋणं धन शून्योः शून्ययोः शून्यं ॥

Dhanam : Positive number

RNam : Negative number

Kham or Shunyam : Zero

Dhanam + Dhanam = Dhanam

RNam + RNam = RNam

Dhanam + RNam = Dhanam or RNam

Dhanam + RNam = kham (if samam)

Dhanam + Shunyam = Dhanam

RNam + Shunyam = RNam

Shunyam + Shunyam = Shunyam

$$X + Y = Z$$

$$(-X) + (-Y) = (-Z)$$

$$X + (-Y) = W \text{ or } -Z$$

$$X + -X = 0$$

$$X + 0 = X$$

$$-X + 0 = -X$$

$$0 + 0 = 0$$

# Integer Subtraction - Brahmagupta

Dhanam - Dhanam = Dhanam

RNam - RNam = RNam

Dhanam - Shunyam = Dhanam

RNam - Shunyam = RNam

Shunyam - Shunyam = Shunyam

Dhanam - RNam = Dhanam

RNam - Dhanam = RNam

# Integer Multiplication - Brahmagupta

ऋणं ऋणधनयोः घातो धनमृणयोः धनवधोः धनं भवति

|

शून्य ऋणयोः ख धनयोः ख शून्योः वा वधः शून्यं ॥

RNam \* Dhanam = RNam  
RNome \* RNome = Dhanam  
Dhanam \* Dhanam = Dhanam

Shunyam \* RNome = Shunyam

kha \* Dhanam = Shunyam

kha \* Shunyam = Shunyam

$$(-X) * Y = (-Z)$$

$$(-X)*(-Y) = Z$$

$$X + Y = Z$$

$$0 * (-X) = 0$$

$$0 * X = 0$$

$$0 * 0 = 0$$

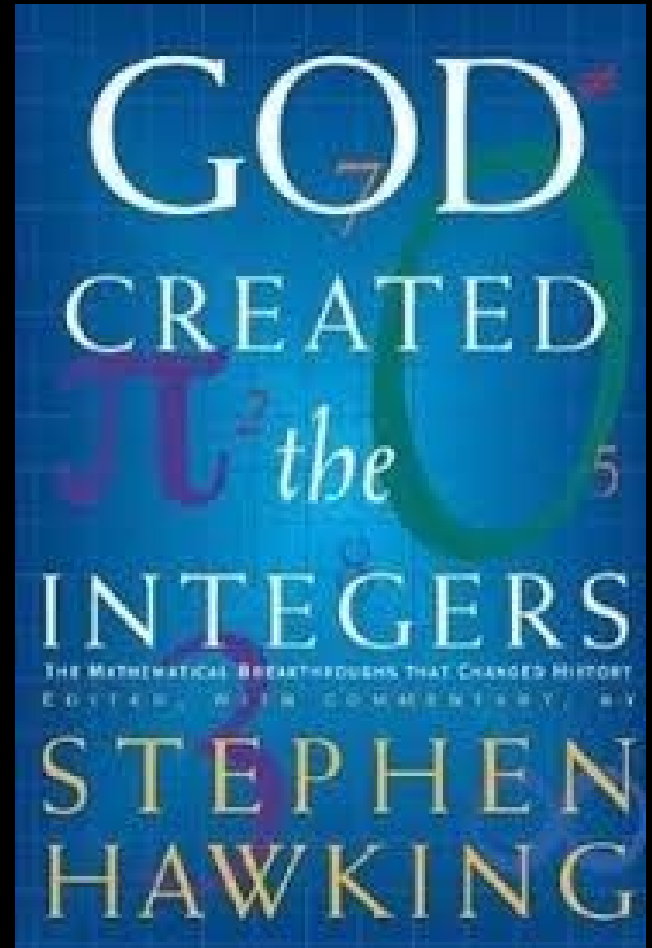
<https://VarahaMihiraGopu.blogspot.com>

# Brahmagupta



Maybe God created integers.

Brahmagupta explained them!!



<https://VarahaMihiraGopu.blogspot.com>

# A bija, by any name

Names given to unknown quantities (variables)

x, y, z	English modern
yaavat taavat	Jaina ~ 300 BC
ishta, iccha	Aryabhata
avyakta	Brahmagupta
bija	Bhaskara

# Brahmagupta's Equation (samikaraNa)

Example

$$5x^2 + 10x - 8 = 2x^2 + 1$$

was written as:

itara pakshaa	yaa va 5 yaa 10 ru 8
apara pakshaa	yaa va 2 yaa 0 ru 1

yaa va = yaavat varga (square of unknown)

yaa = yaavat taavat (unknown)

ru = rupa (constant)

The **dot** over 8 indicates it is a negative number

samikaraNa is what we call an equation

itara paksha and apara paksha are  
left and right hand sides of equations

## Brahmagupta Exponents , Coefficients

अव्यक्त वर्ग घन वर्गवर्ग पञ्चगत षड्गतादीनाम् ।

Avyakta varga ghana vargavarga pancagata  
shadgata aadinaam

तुल्यानां संकलितं व्यवकलिते पृथुत्तुल्यानाम् ॥ 18-41 ॥

Tulyaanaam samkalaitam vyavakalite

avyakta tulyaanaam  
Unknown

tulyaanaam Equal values

samkalitam Addition

vyavakalita Subtraction

corresponding  
pृथुत्

varga square

ghana cube

vargavar  
ga 4<sup>th</sup> power

pancagat 5<sup>th</sup> power

a 6<sup>th</sup> power

shadgata

*tulyaanaam* here means coefficients of equal powers

# Brahmagupta - Roots of Quadratic Equation

वर्गचतुर्गुणितानां रूपाणां मध्यवर्गसहितानाम् ।

varga-catur-guNitAnAm rUpANAm madhya-  
varga-sahitAnAm

मूलं मध्येनोनं वर्गद्विगुणोद्धृतं मध्यः ॥

mUlam r  
dhrtam r  $x = \frac{[\sqrt{(4AC + B^2)} - B]}{2A}$  yuNod-

varga	$A \rightarrow$ Coefficient of varga ( $x^2$ )
catur-guNitAnAm	Multiplied by 4
rUpANAm	$C \rightarrow$ of Constant
madhya-varga-	$B^2 \rightarrow$ Middle-square-added
sahitAnAm	$x \rightarrow$ Root of all this above
mUlam	Minus middle-term (B)
madhyena-Unam	$2 * \text{coeff}(\text{varga}) : \text{divided by } /2A$
vargA-dviguNod-	
dhRtam	

# Brahmagupta - Bhaavita

भावितकरूपगुणन सा अव्यक्तवधा इष्टभाजिता इष्ट-आप्त्योः ।

bhaavitaka-rUpa-guNaa sA avyaktavadhA ishTabhAjitA  
ishTapyoH

अल्पेऽधिकोऽधिकेऽल्पः क्षेपः भावितहृतौ व्यस्तम् ॥

Alpe-adhiko-adhike-alpaH kShepaH bhaavita-hrtau vyastam ॥ 60  
॥

$$Axy = Bx + Cy + D$$

Choose  $m$ , such that  $q = (AD + BC) / m$

$$x = (C + m) / A \quad ; \quad y = (B + q) / A$$

bhaavitaka	Coefficient of xy ( $A$ )
rupa	Constant ( $D$ )
avyakta	Coefficients of x,y ( $B,C$ )
iShTa	desired number ( $m$ )
aapti	Obtained quotient ( $q$ )
bhaavtiaka rupa	$A * D$
guNana	$B * C$
avyakta-vadhA	

# Varga Prakriti

KuTTaka solves linear, indeterminate

Varga Prakriti solves quadratic, indeterminate equations

## Quadratic indeterminate

$$Nx^2 + 1 = y^2$$

Given a known N,

a series of numbers x and y can be calculated

# Bhaavana

Bhavana means ***composition***

Brahmagupta discovered that if  $(m,n)$  solves  $Ax^2 + J = y^2$  and  $(p, q)$  solves  $Ax^2 + K = y^2$

Then  $(mq+pn, nq+Ap)$  solves  $Ax^2 + J*K = y^2$

Also  $(mq-pn, nq-Ap)$  solves  $Ax^2 + J*K = y^2$

Basically, Brahmagupta discovered that **composing** two solutions to one equation produced a third solution the same equation

# Mumford on Brahmagupta

It seems clear to me that Brahmagupta is the key person in the creation of Algebra as we know it

***David Mumford, Fields Medalist***

*Email to Prof Amartya Kumar Datta 8.2.2013*

# Brahmagupta – Area of Triangle and Quadrilateral

स्थूलफलं त्रिचतुर्भुजबाहुप्रतिबाहुयोगदलघातः ।

भुजयोगार्द्धचतुष्टयभुजोनघातात् पदं सूक्ष्मम् ॥

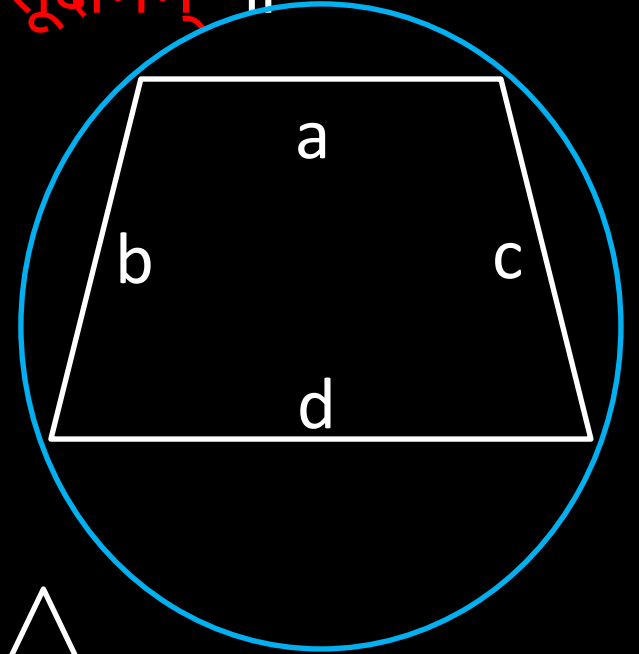
12-21 ॥

Cyclic Quadrilateral

Half perimeter  $s = (a + b + c + d) / 2$

Gross Area =  $[(a + d) / 2] * [(b + c) / 2]$

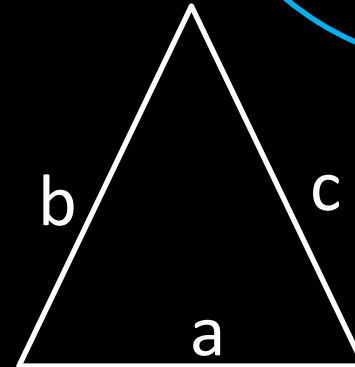
Exact Area =  $\sqrt{(s-a)(s-b)(s-c)(s-d)}$



Triangle

Half perimeter  $s = (a + b + c) / 2$

Exact Area =  $\sqrt{(s-a)(s-b)(s-c)}$



# Brahmagupta – Diagonals of Quadrilateral

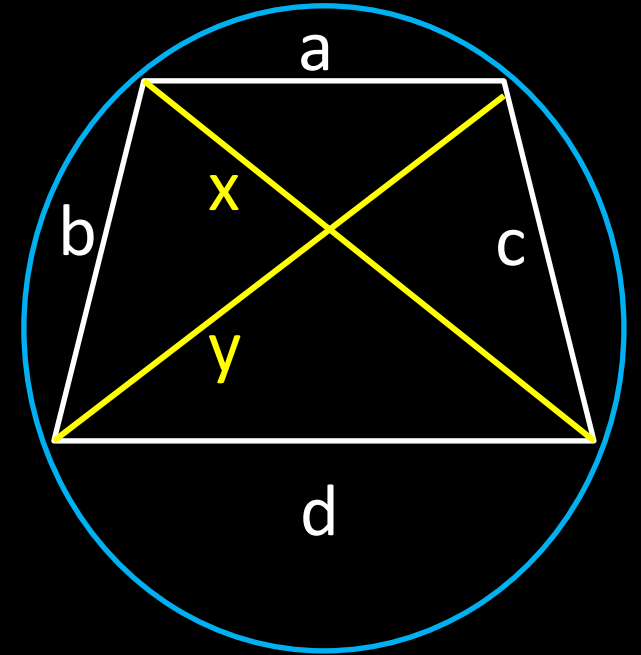
कर्णाश्रितभुजगातैक्यमुभयान्योन्यभाजितं  
गुणयेत् ।

योगेन भुजप्रतिभुजवदयोः करणौ पदे विषमे ॥

12-28 ॥

$$\text{Diagonal } x = \frac{(ac+bd) * (ad+bc)}{(ab+cd)}$$

$$\text{Diagonal } y = \frac{(ab+cd) * (ad+bc)}{(ac+bd)}$$



## Yantra - Braahma Sphuta Siddhanta

सप्तदश कालयन्त्राण्यतो धनुस्तुर्यगोलकंचक्रम् ।  
यष्टिः शंकुर्घटिका कपालकं कर्तरी पीठम् ॥  
सलिलं भ्रमिऽवलम्बः कर्णश्छाया दिनार्धमर्कोऽक्षः ।  
नतकालज्ञानार्थं तेषां संसाधनान्यष्टौ ॥ 23-5,6 ॥

saptadasha kAla-yantraANy-ato dhanu-turyagolakam-cakram  
yaShTiH shankur-ghaTikA kapAlakam kartari pITham ||  
salilam bhrami-avalambaH karNa-sh-chAyA dinArkam-  
arkokshaH  
nata-kAla-jnAnArtha teShAm saMsAdhanA-anyashTau ||

# Instruments - Braahma Sphuta Siddhanta

## Angle Measurement

- Dhanur yantra - bow
- Turyagolaka yantra - Quadrant ( $1/4$  circle)
- Cakra yantra – wheel
- Peetha yantra – Pedestal or seat
- Kapaala yantra – bowl or potsherd
- Dinaardha yantra – Midday measure instrument
- Arka yantra – Sun instrument
- Aksha or palaansha yantra – small degree arc measure instrument

## Geometrical Drawing instruments

- Bhrama or saana yantra – for drawing circles (compass)
- Karna or chaaya karna - Set of squares or diagonals

# Instruments - Braahma Sphuta Siddhanta

## **Shadow Measurement**

- Yashti yantra – pole or staff
- Shanku yantra – gnomon

## **Length Measurement**

- Rajju– rope
- danDa – measuring rod

## **Time Measurement**

- Ghatika yantra – pot or clock
- Chaaya or shanku chaaya – Sundial

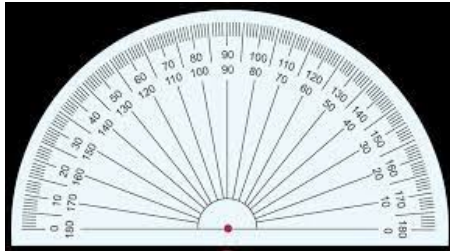
## **Level instruments**

- Salila yantra – water leveller
- Avalamba sutra – Plumbline

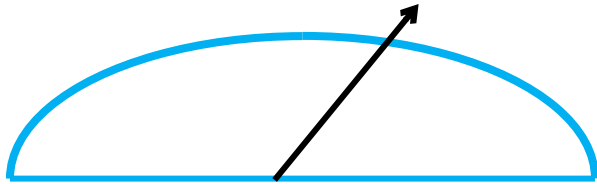
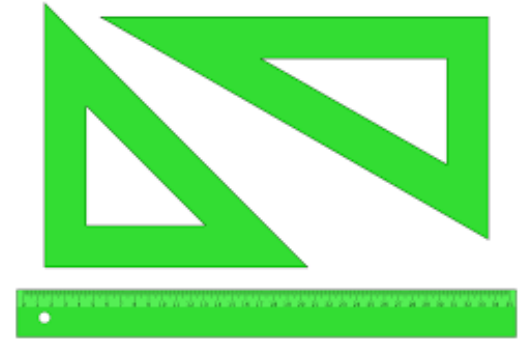
# यन्त्रम् YANTRAM

## *Astronomical Instruments*

कर्ण यन्त्र  
Set square



गोल यन्त्र  
Armillary  
sphere



धनुः यन्त्र  
Protractor

Most Indian instruments were made of cheap and perishable material like mud, clay, rope, string, wood, water

Most were also not very large or based on complex mechanical arrangements

Hence very few of them except some bronze devices have survived as artifacts

# Instruments - Europe

- 1600 Galileo used a handheld telescope
  - A few feet long
- 1670 Newton made larger reflecting telescopes
- 1790 Herschel telescope 1.2 m lens diameter, 40 feet focal length, metal tube
  - Weighed 450 KG
- Large observatories
- 1950 Radio telescopes cover several acres

# Instruments - India

Era	Yantras
1400BC Vedanga Jyotisha	Water clock (ghati)
800-500 BC Sulba Sutras	Rajju (rope), ghati
500BC-500 AD Eighteen Siddhantas	Shanku (gnomon), danda (stick)
400 BC Arthashastra	Naalika (waterclock), chaaya (shadow)
500 AD Aryabahata	Bhrama (compass), jala (water), lambaka (plumb line), karNa (set square)
625 Brahmagupta	Dhanush (protractor), peeTa (seat), yashTi(shadow), bhrama(compass), cakra(wheel), Gola Yantra (Armillary sphere), ghatika(pot), avalamba
850 Lalla	Bhagana, salaaka, shakaTa

# Yantra-prakaasha

तनुनेत्रैर्न्यूना नृपतिरहिता राजनगरी  
सरस्य निष्पद्मा युवतिरपि कान्तेन रहिता ।  
निशा निःशीतांशुः सरिदपि यथा चक्ररहिता  
तथा ज्योतिर्विद्या भवति विफला यन्त्ररहिता ॥

*tanur netrair nyūnā nṛpatirahitā rājanagarī  
sarasyo niṣpadmā yuvatir api kāntena rahitā /  
niśā niḥśītāṃśuḥ sarid api yathā cakrarahitā  
tathā jyotirvidyā bhavati viphalā yantrarahitā //*

Like body without eyes, royal capital minus the king,  
lakes devoid of lotus flowers, a young woman without a lover,  
the night without the moon, a river bereft of *Cakravāka* birds,  
even so astronomical science is fruitless without instruments.

— Rāmacandra Vājapeyin, 1428, *Yantra-prakāśa*

# Brahmagupta innovations

Arithmetic of Zero

Arithmetic of Integers

Unknowns – avyakta –  $x, y, z$

Equations : sama, samikarana

Exponents  $> 3$ : pancagata, shadgata

Solution of quadratic equation

Fractions : five types

Cyclic quadrilaterals

Varga Prakriti  $Nx^2+1=y^2$

Bhaavita : equations with two unknowns  $x, y$

Bhaavana : composition of functions  $f(x) \bullet f(y) = g(x, y)$

This is only a sample of Brahmagupta's innovations

# Brahmagupta Becoming a Master

कुट्टक ख ऋण धन अव्यक्त मध्यहरण एकवर्ण  
भावितकैः ।

आचार्यः तन्त्रविदां ज्ञातैः वर्गप्रकृत्या च ॥

kuTTaka– solve kuTTaka problem

kha rNa dhana– operations with zero, -ve and +ve  
quantities

avyaktha – doing mathematical operation with  
unknowns

madhya haraNa – elimination of middle term in a  
quadratic

eka varNa – solving equations with single unknown

bhaavita – solving equations with products of

# Indian Numerals

Note that before 5th century  
ONLY India developed  
Zero and place value system

Most of these **arithmetic and algebraic  
procedures, algorithms etc**  
are NOT applicable to or inventable in  
Egyptian, Greek, Sumerian, Chinese number  
systems

# Mathematics in School

Nearly 80 EIGHTY percent  
of school mathematics is **Indian**

But we think it is **Greek!**

Hence, this course:  
**Indian Astronomy and Mathematics**

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Thank you