Varahamihira Science Forum

Indian Mathematics +Astronomy

18 Siddhantas

R. Gopu
writergopu@yahoo.com
VarahaMihiraGopu.blogspot.com

Ganesha's Timeline of Indian Astronomy

Krita Yuga	Brahma, Vasishta, Kashyapa
Treta, Dvapara Yugas	Surya Siddhanta
Kali Yuga	Parashara Siddhanta
From 500 AD	Aryabhata, Varahamihira, Brahmagupta and others

Ganesha, author of *Brihat Tithi Chintamani*, composed in 1507AD, gave this timeline.
He ignored Lagadha and his Vedanga Jyotisha-s

History of Astronomy

	Vedic period
around 1400 BC	Vedanga jyotisa
500 BC-500 AD	18 Siddhantas
500 - 1700 AD	Classical Period

Globalization: Copernicus, Galileo

18 Rishi Siddhaantas

सूर्यः पितामहो व्यासो वसिष्ठोऽत्रि पराशरः ।
कश्यपो नारदो गर्गो मरीचिर्मनुरङ्गिराः।।
लोमशः पौलिशश्चैव च्यवनो यवनो भृगुः ।
शौनकोऽष्टादशश्चैते ज्योतिःशास्त्र प्रवर्तकः ।।

Surya pitaamaho vyaaso vashishto atri paraasharaH
Kashyapo naarado gargo mareechi-r manu-r angiraaH
lomashaH paulisha-shcaiva chyavano yavano bhrguH
shaunako ashtadasha shchaite jyoti shaastra

Transformational, Enigmatic

The period of the *Eighteen Siddhantas* was

Transformational and Enigmatic

It was transformational because:

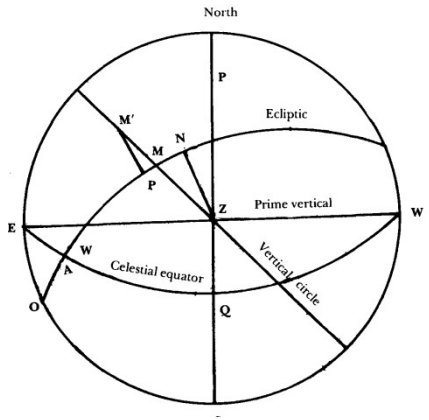
- Astronomy became very mathematical
- Geometry became fundamental to astronomy
- Jyaa (sine) – trigonometry was a new tool
- New concepts were introduced into calendar

But it was enigmatic (mysterious), because almost every book of that era is lost

18 Siddhantas : Inventions and Discoveries

- **Spherical Geometry**
  - Bhugola : Earth as sphere
  - Celestial longitudes and latitudes
  - Geometry – angles, circles, spheres, triangles
  - Ujjain Meridian
- **New Astronomy**
  - Khagola, Bhagola
  - Eclipse as shadow
  - Motions of planets
  - Adoption of Raashi
- **Astrology (Hora shaastra)**
- **Mathematics**
  - Place Value System
  - Zero
  - Birth of Algebra
- **Trigonometry**
  - jyaa (sine)
- **Changes in Calendar**
  - Yuga 43,20,000 years
  - ahargaNa: Day count
  - Seven day Week
  - Redefined tithi
- **Instruments**

  Angular instruments



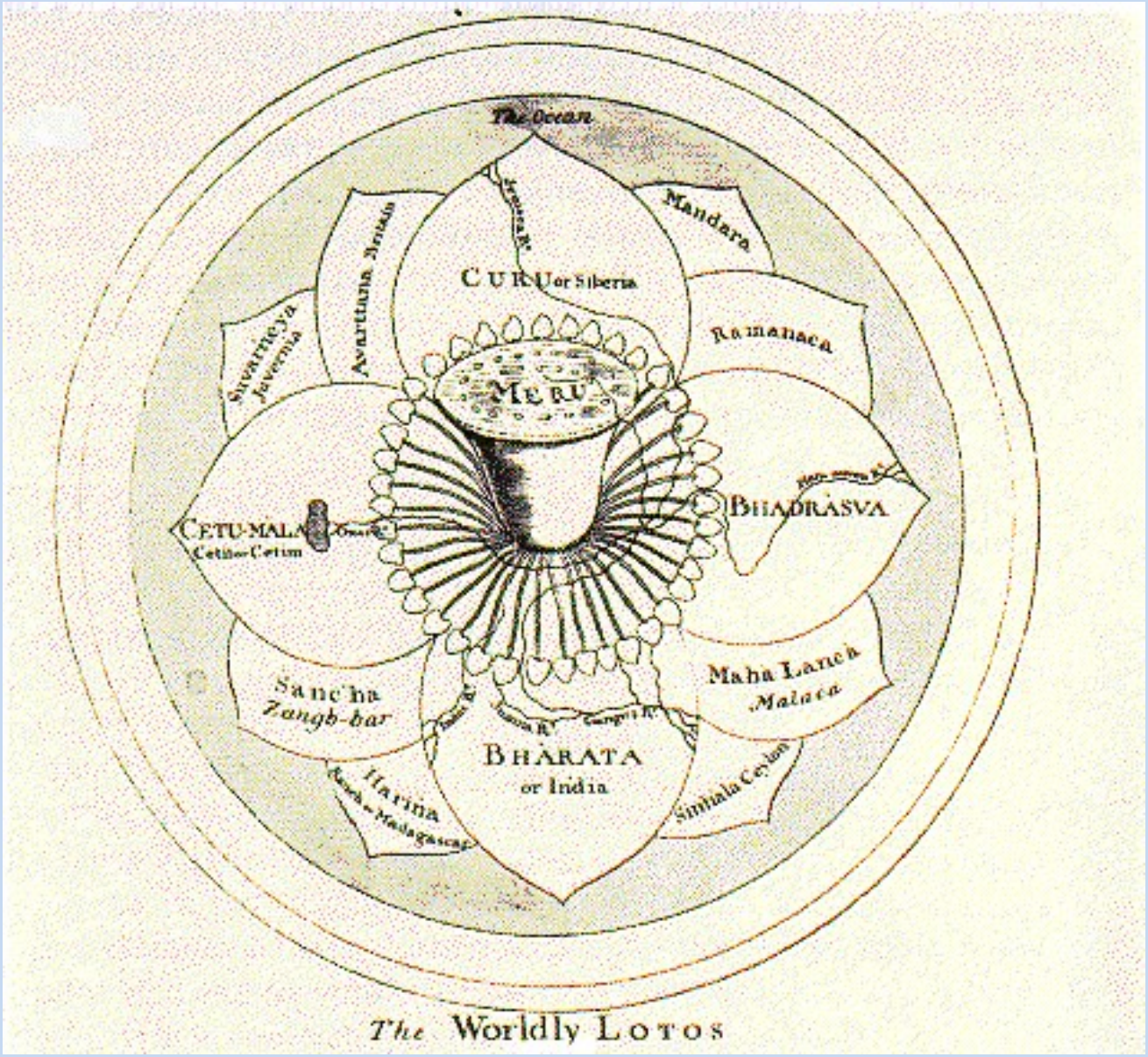
GOLA

गोलः

கோளம்

SPHERICAL GEOMETRY

Vedic Era - Flat Earth



18 Siddhantas : Earth is a globe

From the Puranic view of Flat earth as a lotus
(in the Vedic era)

There was a transition to understanding Earth as a sphere in the Eighteen Siddhantas era

Meru was henceforth considered North Pole
Vadavaamukha was henceforth considered South Pole

Problem: How can humans or trees stand on a curved earth or the lower half of a sphere?

Rishi Siddhantas do NOT explain this

But Varahamihira explains in *Pancha Siddhantika*

Earth as sphere - bhugola

पञ्च महाभूतमयस् तारागण पञ्चरे मही गोलः

खेऽयस्कान्तान्तःस्थो लोह इवावस्थितो वृत्तः ॥ १३-१ ॥

Composed of five elements, among the cage of stars, stands the earth, as a globe

Like a ball of iron between magnets

Varahamihira in *Pancha Siddhaantika* [13-1]

Varahamihira
Nature of Objects

Varahamihira uses *scientific metaphors* to explain why objects stay on Earth, rather than fall off it

Today we call this phenomenon **buoyancy** & **gravity**

But this explanation is not to be confused with Descartes', Newton's or Einstein's concepts of gravity

गगनमुपैति शिखिशिखा क्षिप्तमपि
क्षितिमुपैति गुरु किञ्चित् ।
यद्वदिह मानवानामसुराणां तद्वदेवाधः ॥ १३-४ ॥

The flame of a lamp points skywards and a heavy object thrown skywards falls back to earth; this happens in the lands of men and asuras [*Pancha Siddhantika* 13-4]

Varahamihira - Which is top/bottom?

Varahamihira also makes a joke about Devas and Asuras
Standing on Meru and Vadavamukha respectively
Considering themselves on top, their rivals beneath
implying that a sphere has no Top or Bottom

सलिलतटसङ्गतामामवाङ्मुखी दृश्यते यथा छाया ।
तद्वद् गतिरसुराणां मन्यन्ते नेऽप्यधो विबुधाग्रे ॥ १३-३ ॥

As reflections of men standing on brink of water are seen with faces downwards, so Devas (Vibudhaa) consider the conditions of Asuras to be; and the asuras on their part also [*Pancha Siddhantika* 13-3]

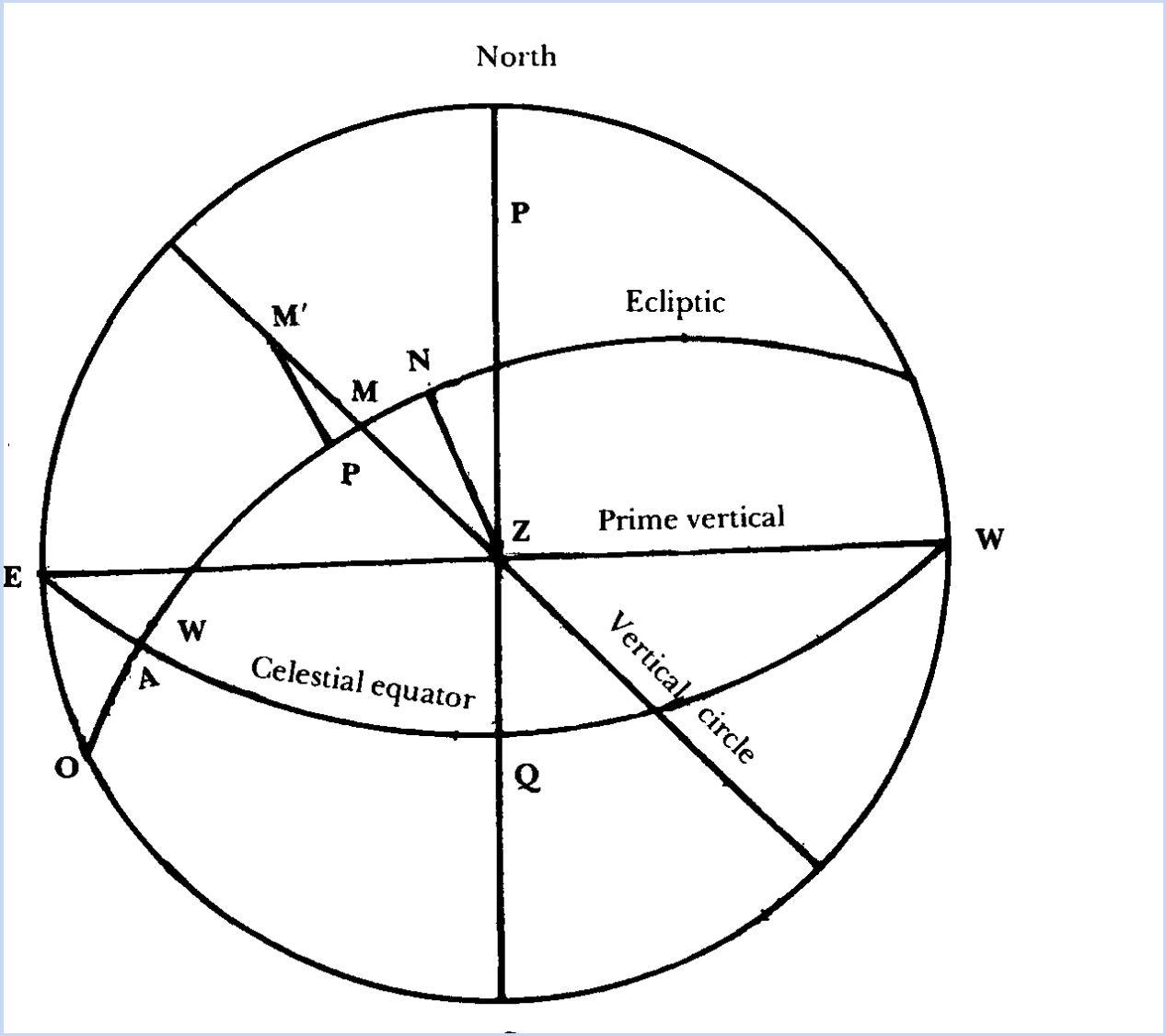
Aryabhata – Spherical Earth

यद्वत् कदम्बपुष्पग्रन्थिः प्रचितः
समन्तः कुसुमैः
तद्वद् सर्वतो वृत्तो भूगोलः

Just as a kadamba flower is
surrounded by blossoms,
So is Earth a sphere, on which live
land creatures and water creatures

Like Varahamihira
Aryabhata also used a
scientific metaphor
to explain why objects stay on Earth,
rather than fall off its lower surface

18 Siddhanta era world - bhugola



Effect of Spherical Model of earth

- In the 18 Siddhantas era, astronomers abandoned flat earth model and adopted spherical (bhugola) model
- Sky (khagola), space (bhagola) also understood as spheres
- This led to
  - Development of Celestial Geometry (Gola)
  - Angle measurements : degrees, minutes, seconds
  - Instruments for measurament of angles
  - Calculation of longitudes, latitudes
  - Ujjain Meridian as Prime or Zero meridian
  - Understanding eclipses as shadows of Moon and Earth
  - Epicylcic model of revolution of gruham-s
  - Trigonometry (jyaa – sine)
  - Trigonometric and Geometric calculation of eclipses(grahanam), conjunctions (*taara graha samyogam*)

Angular measurement

Indian Mathematicians invented a five part angle measurement system, and angular instruments like Dhanur Yantra to measure angles

- Raashi – राशी = 30 degrees
- Degree - अंश amsha (or भाग bhaaga)
- Minute - कला kalaa
- Second – विकला vikalaa
- subsecond – लिप्ता liptaa

Instead of 72° 8’ 25” (three part modern system)

They notated it as 2R 12° 8’ 25’’ 50’’’

(2 raashi 12 amsha 8 kalaa 25 vikalaa 50 liptaa)

Angles – Surya Siddhanta

विकलानां कलाषष्ट्याः तत् षष्ट्या भाग उच्यते ।
तंत्रिंशतां भवेद् राशिः भगणौ द्वादशैच ते ॥
vikalAnAm kalAshashTyAH tatshashTyA bhAga ucyate |
Tam trimShatAm bhaved rAshiH bhagaNau dvAdashaica te ||

Sixty vikalaa-s is a kalaa
sixty kalaa-s is a bhaaga
thirty bhaaga-s is a raashiH
twelve raashis is a bhagaNaH

The full 360˚ zodiac is called a bhagaNaH

Angular measurement

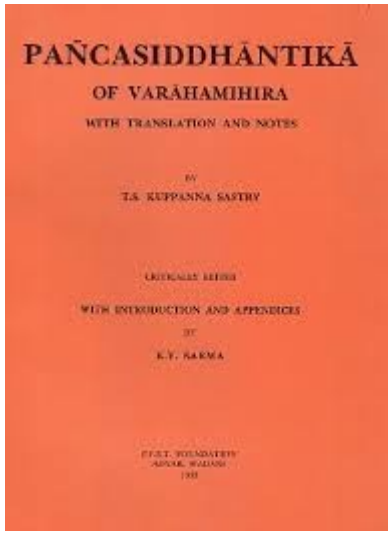
Caution

The terms are not always consistent

Varahamihra uses both kalaa, liptaa for Minute

Angular measurement

- Celestial latitudes and longitudes
- Most of the applied mathematics in jyotisha are these numbers
- Most techniques were developed to calculate rising, setting, angle, position, conjunctions, velocity, of celestial bodies… and predictions



पञ्च सिद्धान्तिकाः

பஞ்ச சித்தாந்திகம்

PANCHA SIDDHAANTIKA

Five Siddhantas

Varahamihira's book *Pancha Siddhantika,* (circa 525 AD) compares FIVE of the 18 Rishi Siddhantas – namely *Surya, Romaka, Paulisha, Paitamaha* and *Vasishta Siddhantas*

It is a unique book in the annals of Indian Astronomy.

Almost all we know about this period is from *Pancha Siddhantika*

While almost every book was lost, *Surya Siddhanta* alone survived. And it is still in use. But it has several interpolations from much later books like Bhaskara's Siddhanta Shiromani.

So, the Surya Siddantha we have is not the same book as the one refered by Varhamihira

Pancha Siddhantika - contents

दिक्–स्थिति–विमर्द–कर्ण–प्रमाणवेला ग्रहाग्रहावेन्दो ।
ताराग्रहसंयोगं देशान्तरसाधनं चास्मिन् ॥ ५ ॥

dik-sthiti-vimarda-karNa-pramANavela grha-agraha-indavo |
tAragraha-samyogam deshAntra-sAdanam casmin ||

Direction, duration, phase, hypotenuse, eclipse and non-eclipse of moon, star-planet conjunctions, longitude variation

Pancha Siddhaantika [1–5]

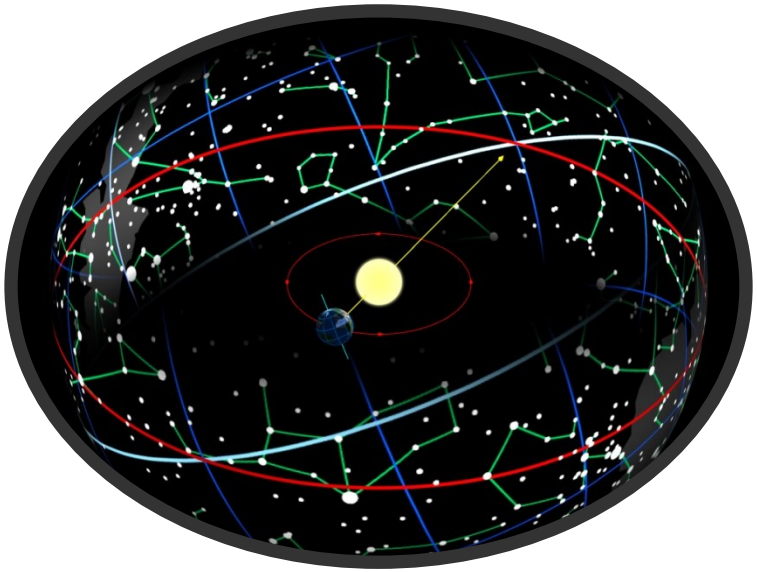
Pancha Siddhantika - contents

सममण्डल–चन्द्रोदय–यंत्र–च्छेद्यानि
शङ्कवच्छायाः
उपकरणाद्यक्षज्यावलम्बकापक्रमाद्यानि ॥ ६ ॥

samamanDala-candrodaya-yantra-
chedyaani shankava-chaayaaH |
upakaraNAd-aksha-jyaa-avalambaka-
apakramaadi-yaani

Prime Vertical, moonrise, instruments, gnomon shadow
Sine of Latitude- colatitude - declination

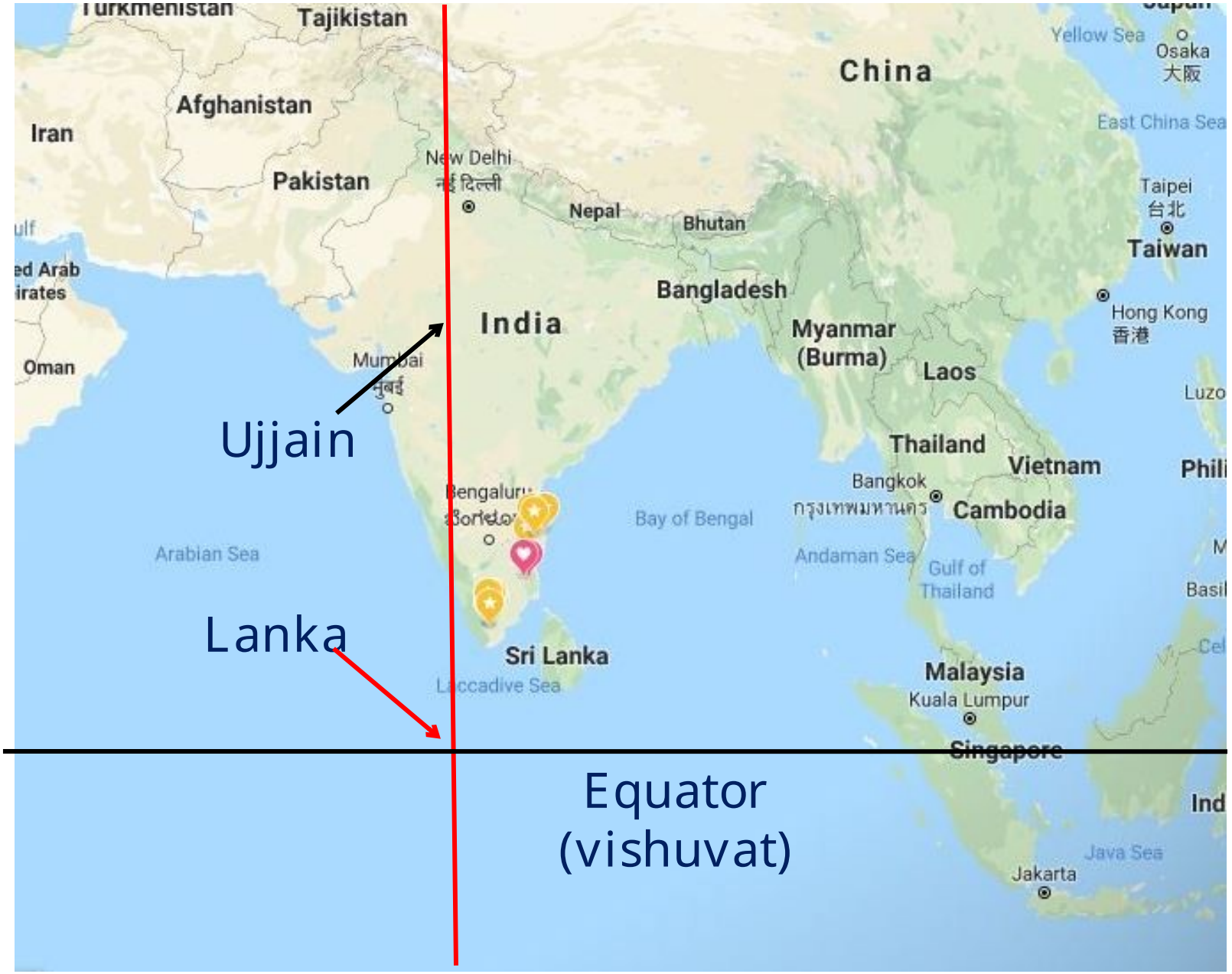
Pancha Siddhaantika [1–6]



புதிய கருத்து

NEW CONCEPTS

Ujjain Meridian



Four meridians

उदयो यो लङ्कायां सोऽस्तमयः सवितुरेव सिद्धपुरे ।
मध्यो योऽवन्तीषु स्यात्तत्रैवाधोऽर्धरात्रं

Sunrise in Lanka is Sunset in Siddhapura,

Significance of Ujjain meridian

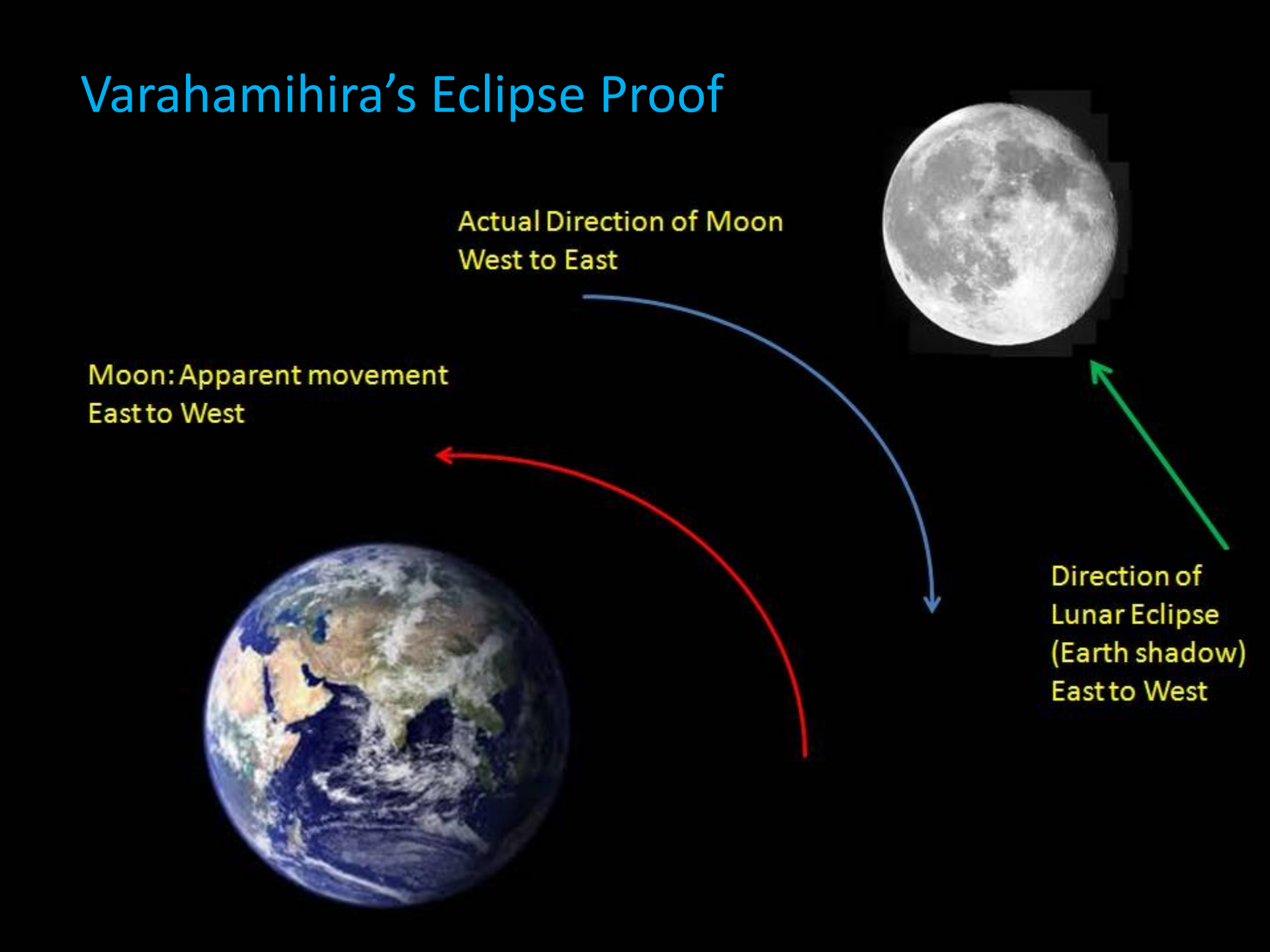
For two thousand years, Indian astronmers used Ujjain meridian as standard meridian

It was also taken as standard in south east Asian countries like Burma, Thailand, Java, Sumatra, Laos, Cambodia etc - wherever Indian astronomy was used.

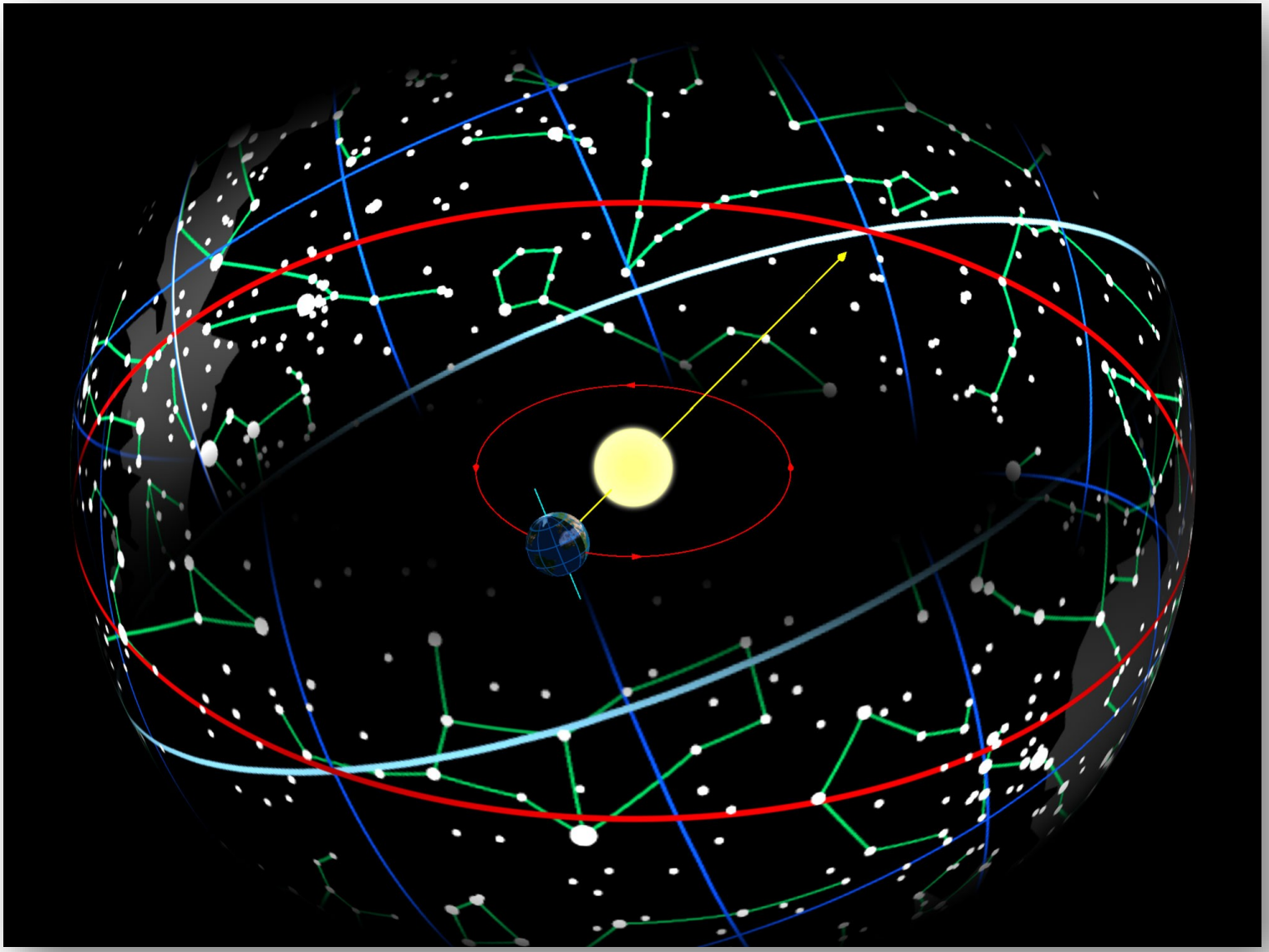
Calendars computed with Ujjain as Zero Longitude, Lanka as Zero Latitude in **Siddhantas**

Deshantara – longitude adjustment –

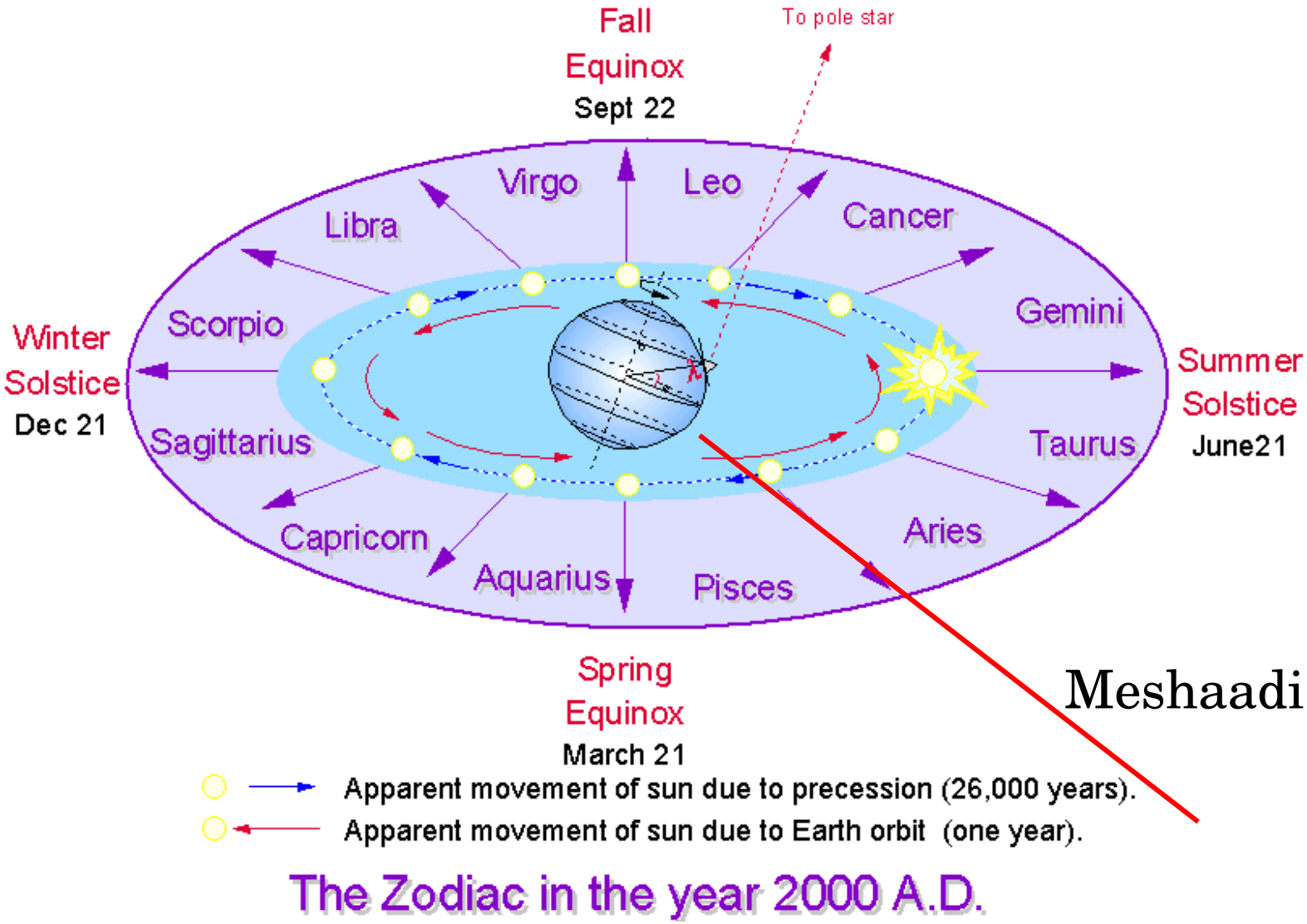
Varahamihira's Eclipse Proof



Sun’s seeming path in Zodiac



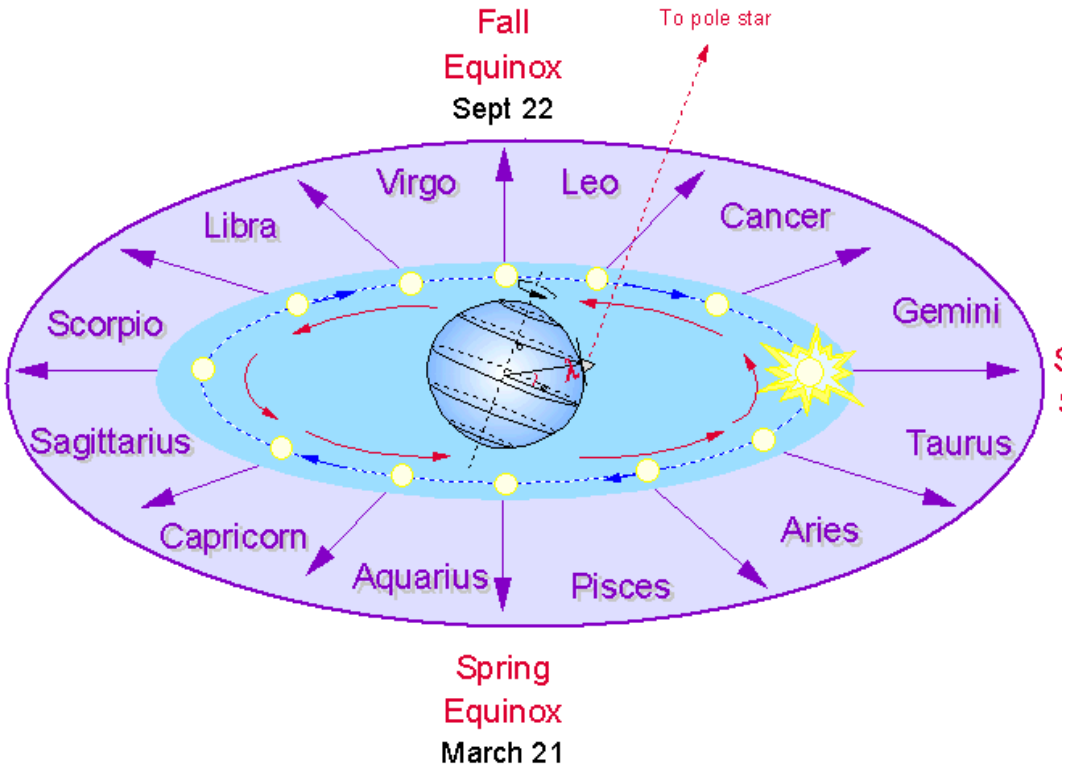
Twelve raashis



The Zodiac in the year 2000 A.D.

Days in Solar Months - Surya Siddhaanta

No.	Name.	Duration.				
		d	n	v	′′′	′′′′
1	Vâiçâkha,	30	55	32	2	39
2	Jyâishṭha,	31	24	12	2	41
3	Âshâḍha,	31	36	38	2	44
4	Çrâvaṇa,	31	28	12	2	42
5	Bhâdrapada,	31	2	10	2	40
6	Âçvina,	30	27	22	2	38
7	Kârttika,	29	54	7	2	35
8	Mârgaçîrsha,	29	30	24	2	33
9	Pâusha,	29	20	53	2	31
10	Mâgha,	29	27	16	2	32
11	Phâlguna,	29	48	24	2	33
12	Châitra,	30	20	21	2	36



A **solar month** is measured as the time the sun spends in one raashi (constellation)

<u>Reminder</u> : Lunar month is pournami to pournami

MODERN Astronomy

- Declination – Celestial Latitude
  - Measured in Degrees
  - Sun's declination goes from 23N to 23 S
  - Other planets, stars declination also expressed in degrees
- Right Ascension – Celestial Longitude
  - Expressed in hours and minutes
  - 0 degrees is 0 hours
  - Every 15 degrees is ONE hour

INDIAN Astronomy

- Celestial Latitude is parallel to Earth's Equator
- Celestial Longitude sets zero as Meshaadi
  - Mesha (Aries) Aadi (beginning)
  - Each raashi is 30 degrees
  - Times calculated based on occurrence at Ujjain meridian
  - Local Longitude correction (deshaantara) applied
  - Same way local latitude correction is applied

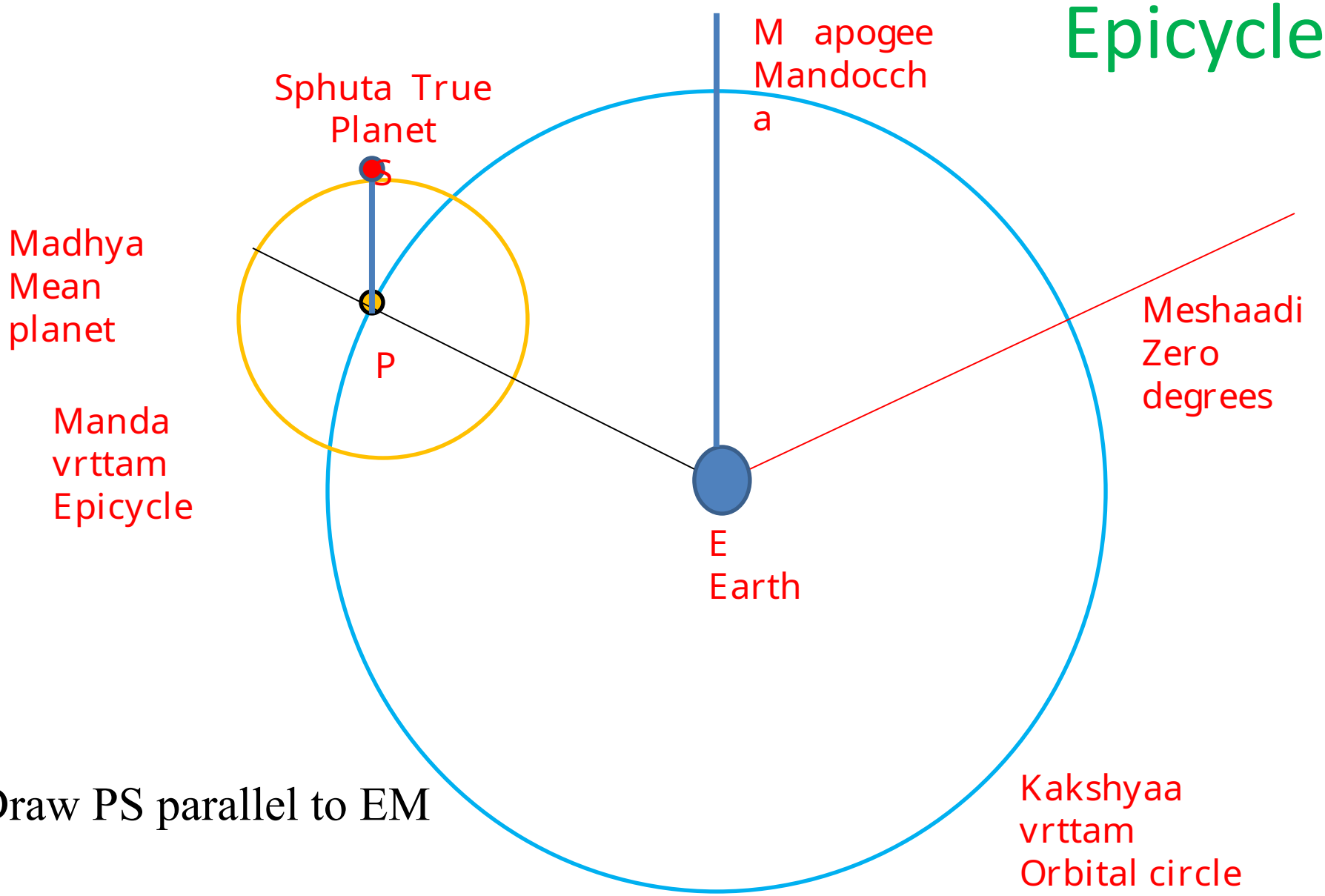
Epicyle – Manda vrtta
मन्द वृततम् மந்த விருத்தம்

- Celestial Latitude is parallel to Earth's Equator
- Celestial Longitude sets zero as Meshaadi
  - Mesha (Aries) Aadi (beginning)
  - Each raashi is 30 degrees
  - Times calculated based on occurrence at Ujjain meridian
  - Local Longitude correction (deshaantara) applied
  - Same way local latitude correction is applied
- Each gruham observed to revolve on some latitude – a uniform circle
- Gruhams travel at varying speed, NOT uniform speed
- This was explained with an Epicycle Theory

Madhya and Sphuta

- Madhya is Mean Planet (gruham)
  - The average path of the planet over the year
- Sphuta is True Planet (gruham)
  - But NO planet moves with even velocity
  - A epicyclic correction is calculated for each planet
  - To find its true position at any instant
  - This is based on the manda vrttam and mandoccha angle

Epicycle



Draw PS parallel to EM

Vakra gati
Retrograde motion of Planets

- Mars, Jupiter and Saturn make larger orbits around the Sun
- When they are in same section of orbit as the earth, they seem to move backwards (retrograde) for a short period
- Similar to a train which your train overtakes

  seems to move backward

Vakra gati
Retrograde motion of Mars

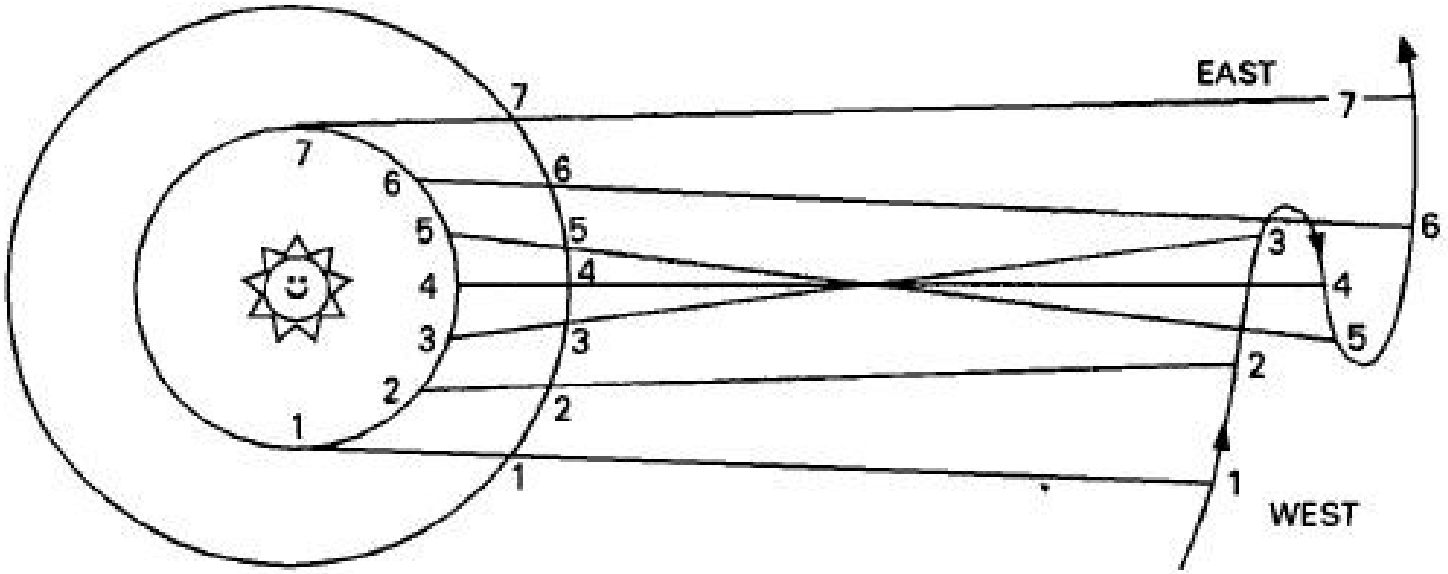
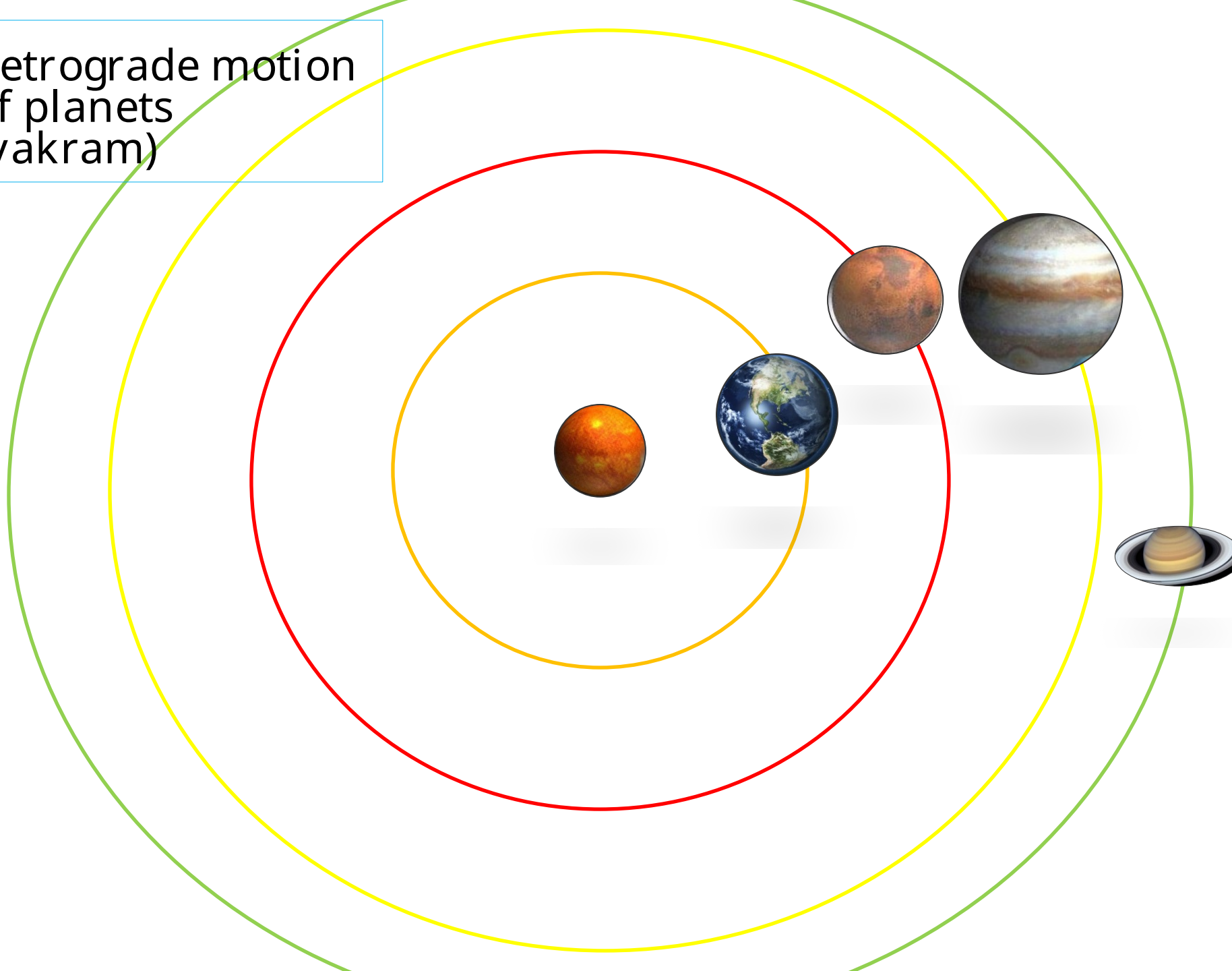


Figure 9.2 Retrograde motion of *Kuja*

Picture from S Balachandra Rao's book Indian Astronomy – An Introduction

Retrograde motion of planets (vakram)



पञ्चाङ्गम्

பஞ்சாங்கம்

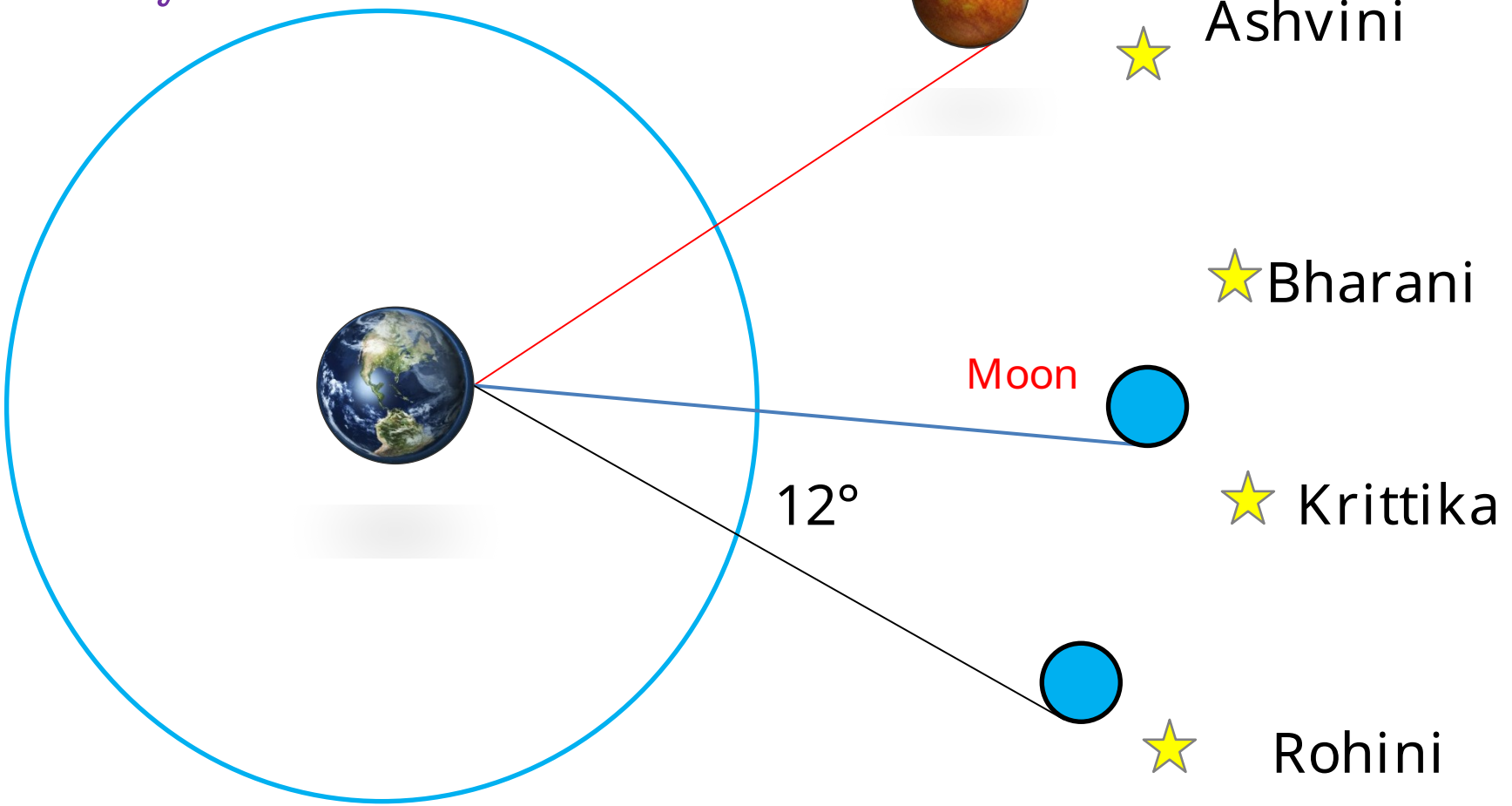
PANCHAANGAM - CALENDAR

Tithi, Nakshatram – redefined

- Tithi,Nakshatram redefined as angle differences
- The tithi was re-defined as a difference of 12 degrees between Sun and Moon
  - Within those 12 degrees, it is the same tithi
  - When it crosses 12 degrees, it is the next tithi

- Each Nakshatram was redefined as segment of 13°20’ of the celestial circle of stars

A new **tithi** starts when
The angle between sun & moon
increases by 12°



Nakshatram was redefined as
13°20′ between two lunar
mansions

Panchaangam

- Introduction of zodiac, solar months, weekday, spherical geometry modified Indian calendar
- Earlier calendar : Dina-maasa-rtu-ayana only
- New calendar had five (pancha) major elements (angam) :
  - tithi
  - nakshatra
  - vaara (weekday)
  - karaNa (half a tithi)
  - yoga

Yuga – Surya Siddhanta

- A mahayuga is 43,20,000 years
- A mahayuga consists of four yuga-paada
  - Krta yuga 17,28,000
  - Treta yuga 12,96,000
  - Dvapara yuga 8,64,000
  - Kali yuga 4,32,000 years
- Kali yuga begins with end of Mahabharata
  - 18/17 February, 3101 BC
  - ahargaNa (aha = day, gaNa = count)

Yuga – other Siddhantas

- Paitamaha Siddhanta : yuga is FIVE years
  - Seems to have followed Lagadha
  - Indicates it is very ancient
  - No mention of zodiac, angles etc
- Vasishta Siddhanta: Not mentioned in PS
  - Angles, zodiac included. So more recent than Paitamaha
- Romaka Siddhanta: 2850 years
- Paulisa Siddhanta : 120 years

अहर्गणः ahargaNa

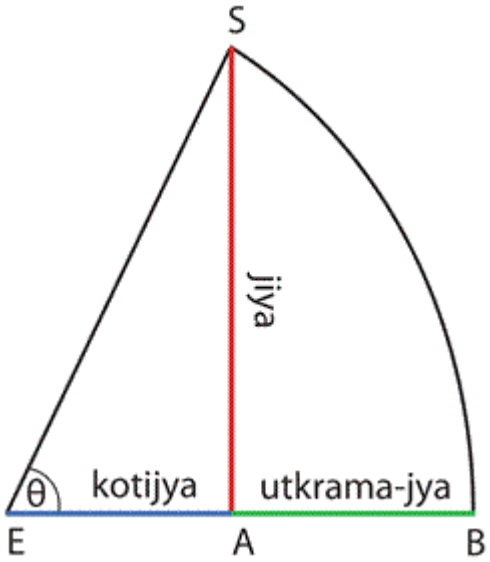
- ahargaNa is number of days from start of Kali yuga
  - ahaH अहः = day (sunrise to sunrise)
  - gaNa = count
- Calculation
  - Add 3101 to Saka year (78 AD)
  - Multiply years by 12 to get elapsed months
  - Subtract adhika maasas
  - Multiply months by 30 to get elapsed days
  - Add elapsed days since beginning of month
  - Caution: This is an approximate method. For actual method please read the corresponding Siddhaanta

- ahargaNa 16,80,548 is March 22, 1500 AD

Tables of Astronomical Data

4. Cycles, years, months and days.

—		Jyotisha Vedānga.	Romaka Siddhānta.	Old Sūrya Siddhānta	Āryabhata and Puliśa.	Brahmagupta.	Sūrya Siddhānta.
Years in cycle .	Y	5	2,850	180,000	4,320,000	4,320,000,000	4,320,000
Intercalary months	M_i	2	1,050	66,389	1,593,336	1,593,300,000	1,593,336
Omitted tithis .	D_o	30	16,547	1,045,095	25,082,280	25,082,550,000	25,082,252
Solar months .	M_s	60	34,200	2,160,000	51,840,000	51,840,000,000	51,840,000
Synodic months .	M_l	62	35,250	2,226,389	53,433,336	53,433,300,000	53,433,336
Sidereal months .	M_*	67	38,100	2,406,389	57,753,336	57,753,300,000	57,753,336
Solar days . .	D_s	1,800	1,026,000	64,800,000	1,555,200,000	1,555,200,000,000	1,555,200,000
Natural days . .	D	1,830	1,040,953	65,746,575	1,577,917,500	1,577,916,450,000	1,577,917,828
Tithis .	D_l	1,860	1,057,500	66,791,670	1,603,000,080	1,602,999,000,000	1,603,000,080
Sidereal days .	D_*	1,835	1,043,843	65,926,575	1,582,237,500	1,582,236,450,000	1,582,237,828



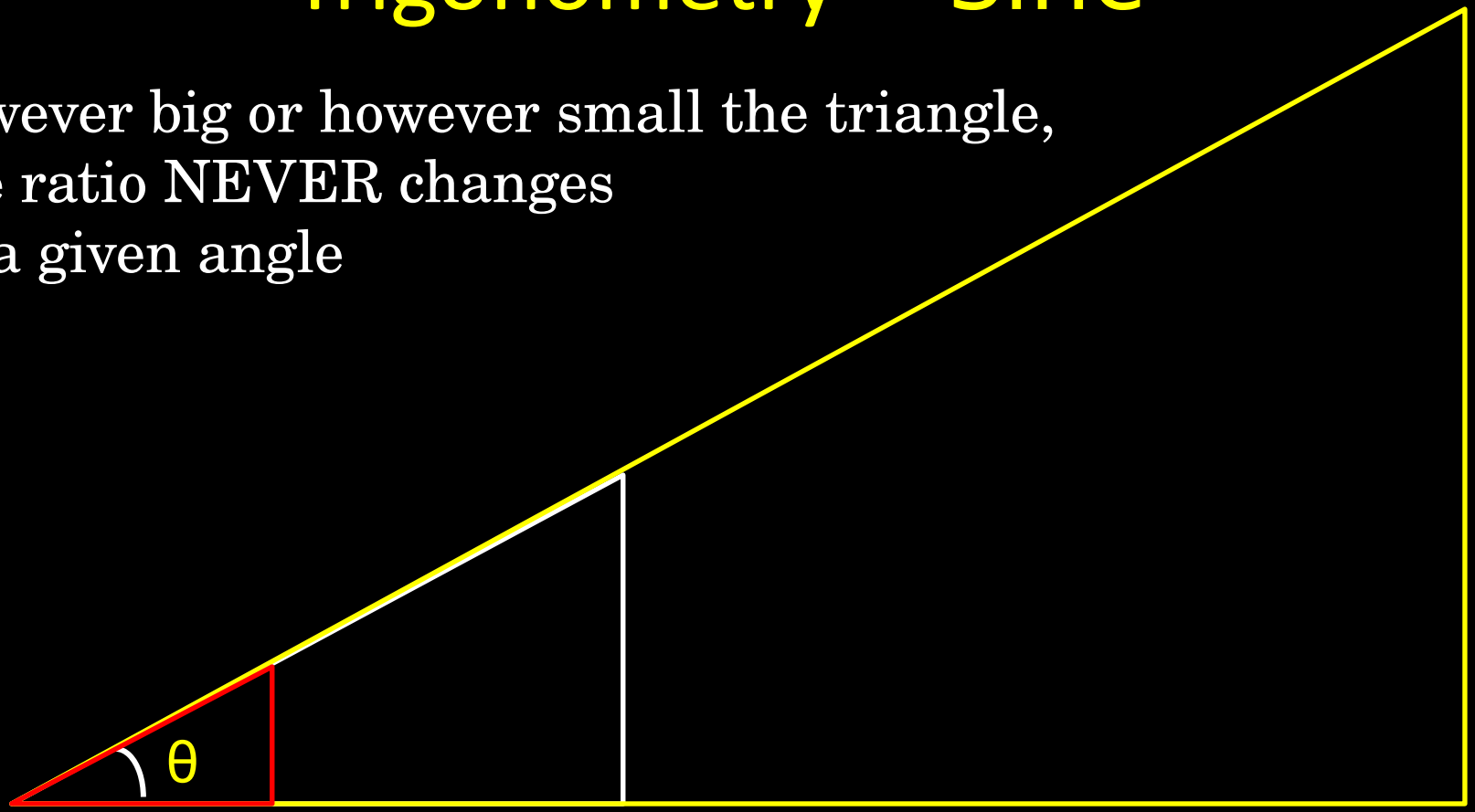
ज्या

ஜ்யா

TRIGONOMETRY

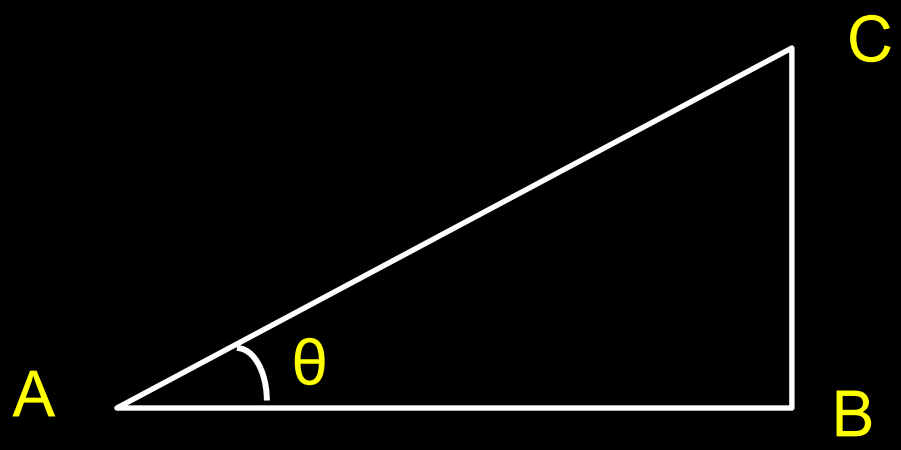
Trigonometry - Sine

However big or however small the triangle,
The ratio NEVER changes
for a given angle



Sin θ = Opposite / Hypotenuse

Usefulness of Sine (jyaa)



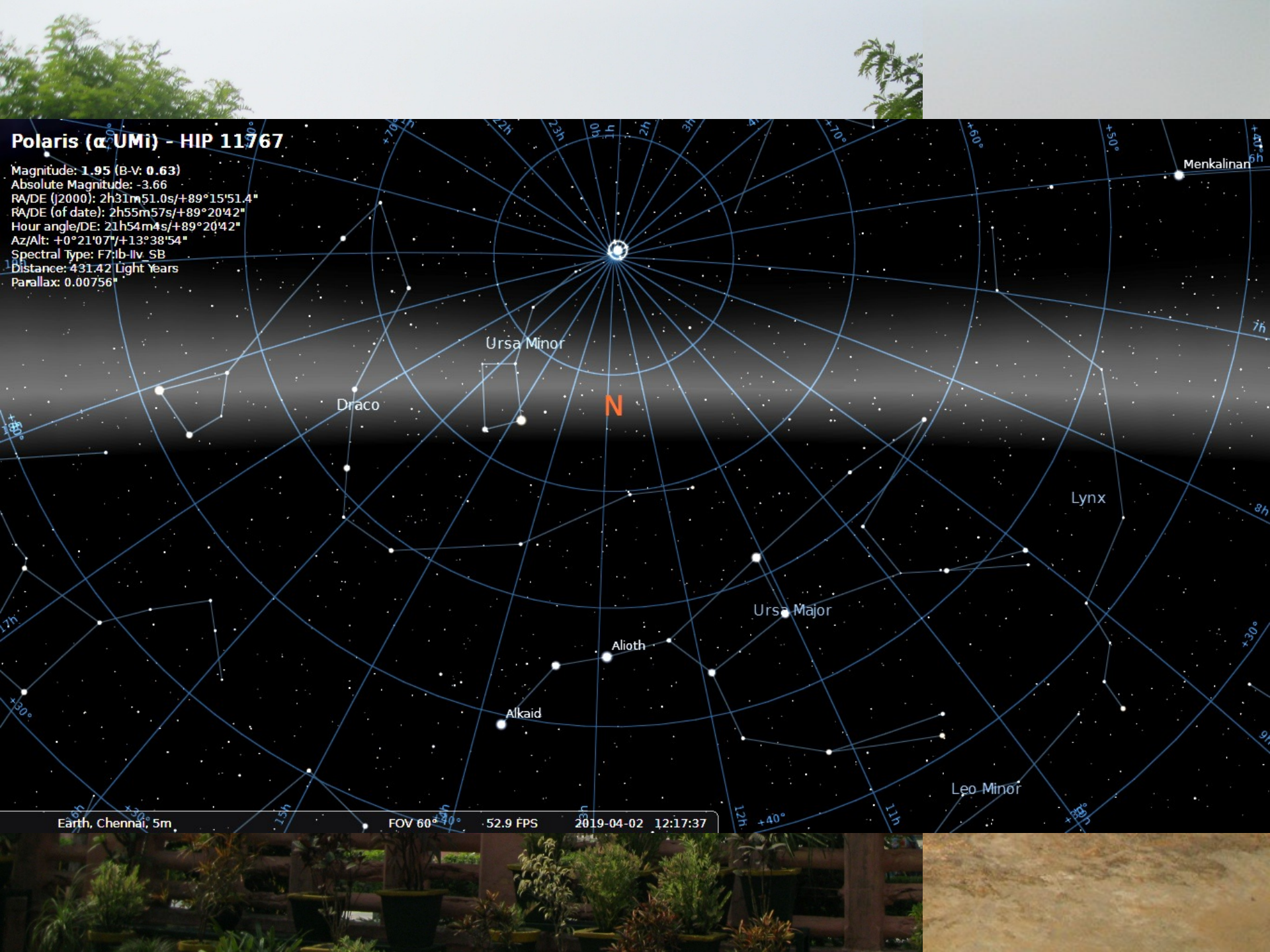
Sin θ = BC / AC

AC * Sin θ = BC

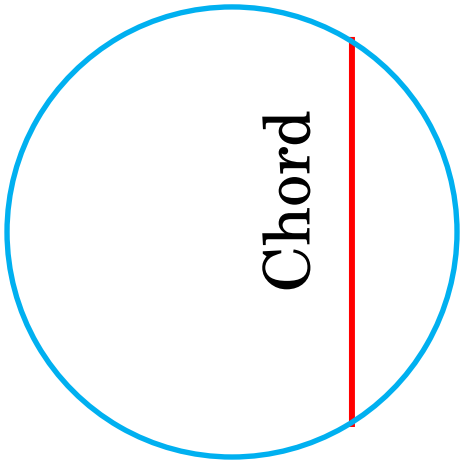
AC = BC / Sin θ

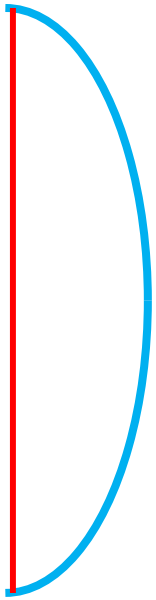
Sine is very useful for calculating heights and distances
Using only angles and sine itself

Hence sine (jyaa) is very useful in Astronomy – to calculate distances of planets and stars


Polaris (α UMi) - HIP 11767
Magnitude: 1.95 (B-V: 0.63)
Absolute Magnitude: -3.66
RA/DE (J2000): 2h31m51.0s/+89°15'51.4"
RA/DE (of date): 2h55m57s/+89°20'42"
Hour angle/DE: 21h54m4s/+89°20'42"
Az/Alt: +0°21'07"/+13°38'54"
Spectral Type: F7:Ib-IIv_SB
Distance: 431.42 Light Years
Parallax: 0.00756"
Menkalinan
Ursa Minor
Draco
N
Lynx
Ursa Major
Alioth
Alkaid
Leo Minor
Earth, Chennai, 5m
FOV 60°
52.9 FPS
2019-04-02 12:17:37

jyaa



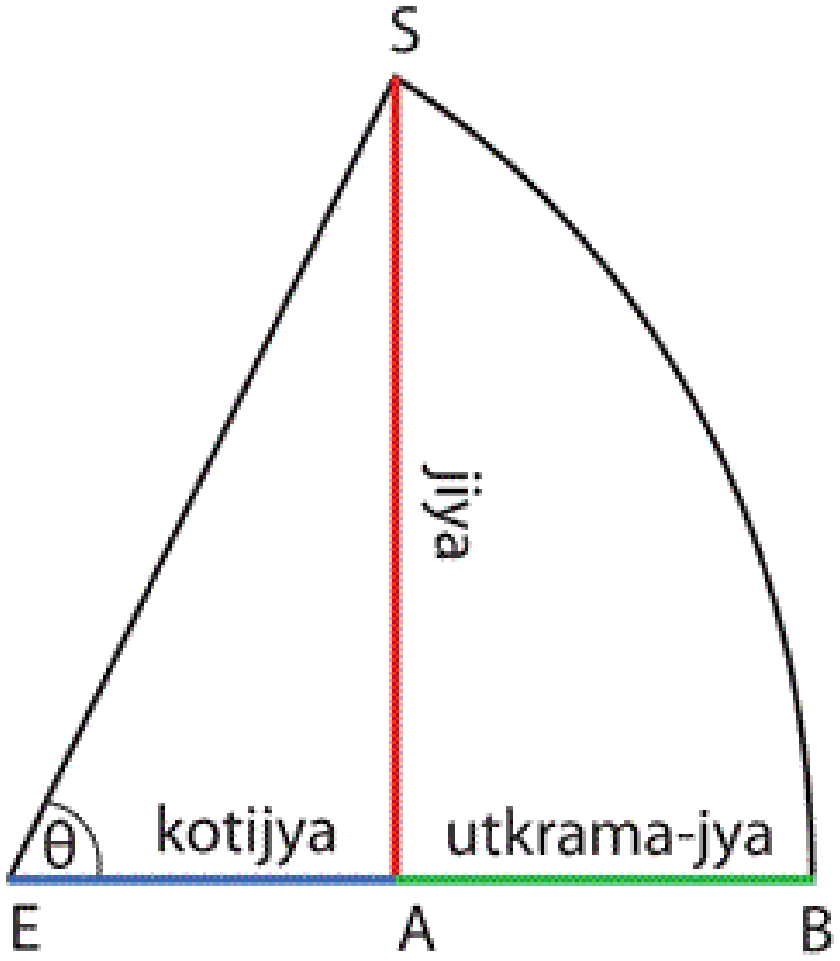


Chaapa = Arc

jyaa = bowstring
= chord

Ardha-jyaa = sine

koti-jyaa = cos



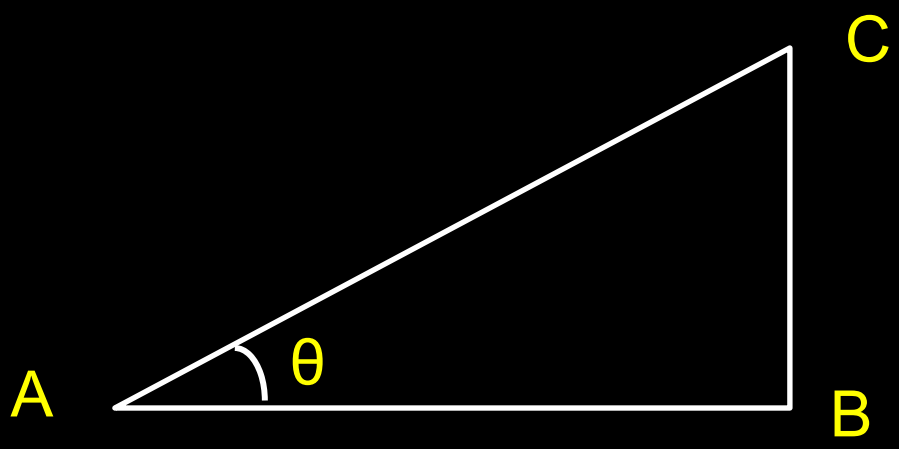
Jyaa (SINE)

In Indian mathematicians Sine was
Circle based; not triangle based

Sine was the measure of the half-chord
in a circle's quadrant,
and a measure of the Arc of the angle

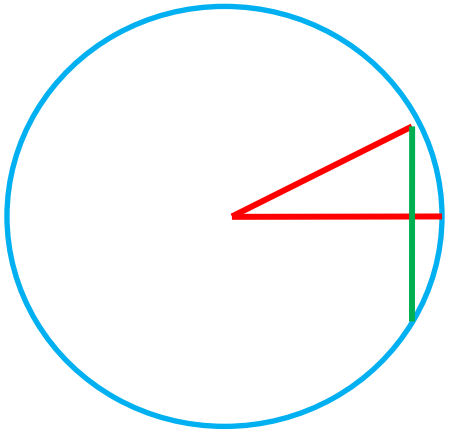
not merely
opposite/hypotenuse of triangle

Triangle vs Circle Sine

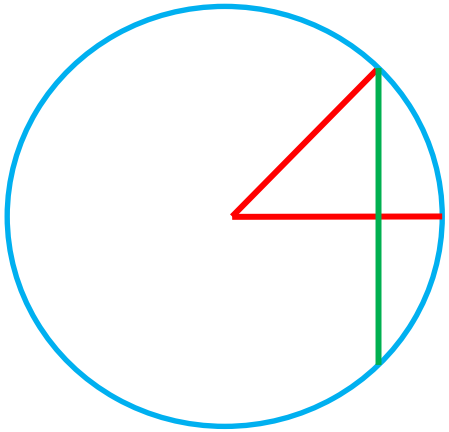


No triangle has 0 or 180 degrees angle
But with circle you can have sine
for 0 to 360
and higher angles
And only India actually had the number Zero
from 1st to 9th or 12th centuries

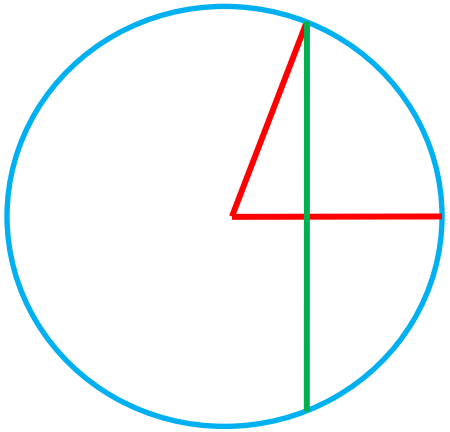
Jyaa



30°



45°



80°

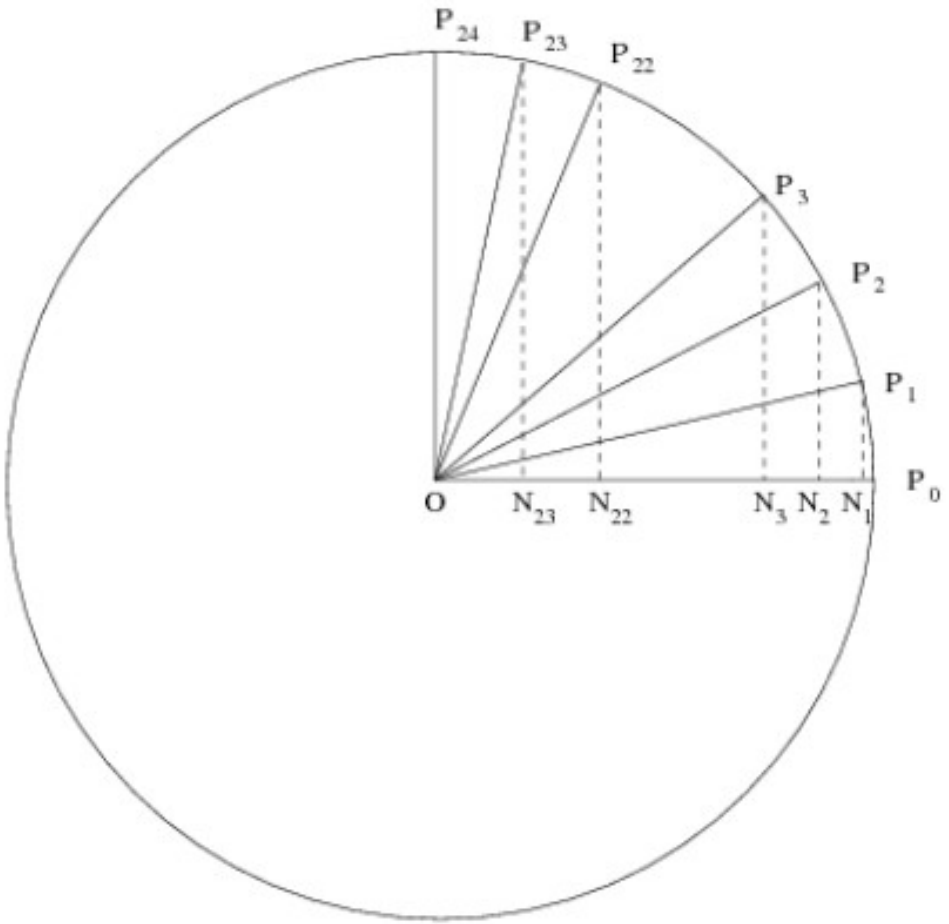
In each case, the hypotenuse is the radius
In each case, the opposite side is half chord (ardha jyaa)

The jyotishas called it both **ardha-jyaa** and simply **jyaa**
Or sometimes called it **jiyaa**

Jyotisha Siddhanthas divided circle into 24 equal parts
and gave a table of 24 sines of the first quadrant
Not sine 0,30,45,60, 90 as in modern mathematics

Jyaa

24 sines



Length (Nx to Px)
is proportional to
Angle Nx-O-Px

$\text{Sine} < N_x O P_x = N_x P_x \,/\, O P_x$

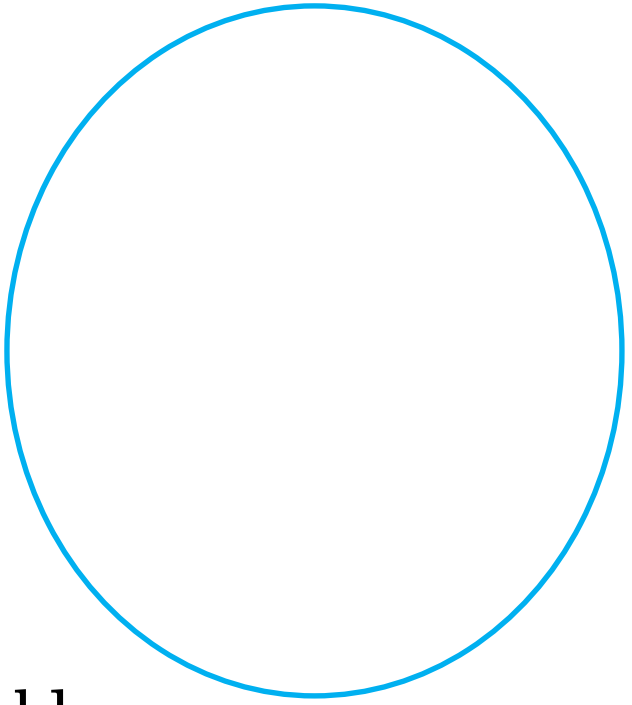
$OP_1 = OP_2 = OP_3 = Op_n$
Since every OPx is the radius

Hence Indians used OPx
i.e. the height of the cord as the
jyaa or sine measure,
ignoring the denominator

Modern mathematicians hence call this the R-Sine
That is, Radius multiplied by Sine ratio

Jyaa

360 degrees is 21600 minutes
A circle of radius 3438 units has
circumference 21600 units



21600 is divisible by:
360 → 1 degree = 60 minutes
12 → 1 Raashi = 30 degrees
27 → 1 Nakshatram = 800 minutes

Indian Astronomers calculated sine table
using chords, diameter, triangle sides
all measured in minutes (kalaa)
for a circle of circumference 21600 units

Aryabhata - Circle Chord Radius

परिधेः षड्भागज्या विष्कम्भारधेन सा तुल्या ॥ 9 ॥

paridhE shad-bhaaga-jyaa vishkambha-ardhena saa tulyaa

paridhi	circumference
shad-bhaaga-jyaa	One sixth of jyaa
vishkambha ardha	Half diameter
saa	that
tulyaa	equals

The chord which divides the cirumference by a sixth is equal to the radius of the circle in length

Using this hexagon theorem, Aryabhata calculated all 24 sines

Presumably, earlier Indian mathematicians – the authors of Rishi Siddhantas - used similar tables to calculate the sines also

Calculating jyaa for 60°

Length of Radius = Length of hexagon side

AC=BC = AB

Jyaa of angle <ACN = Length AN

Circumference of circle = 21600

Radius CB =CA = 3438

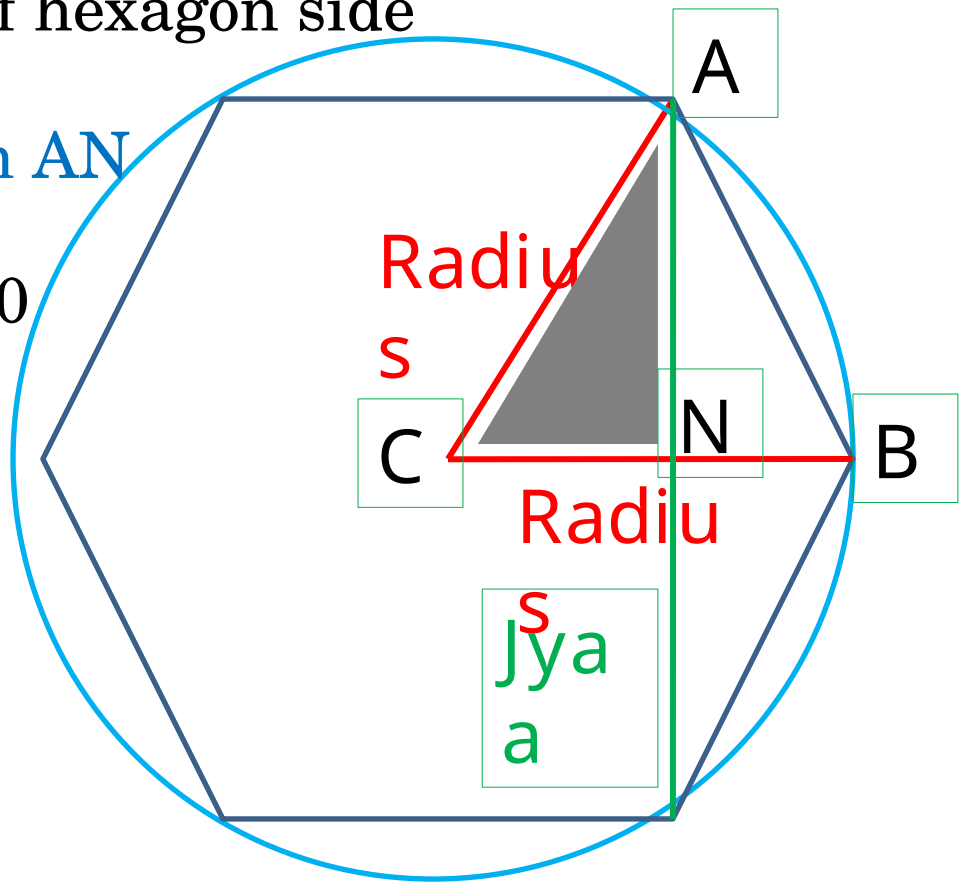
AN = sqrt(AC^2 – CN^2)

CN = CB/2

AN = sqrt(3438^2 –1719^2)

= 2977

So, jyaa of 60° = 2977 units



Once sine 60 was calculated,
Indian Jyotishas calculated all other sines from this,
using smaller triangles and **bhuja-koTi-karNa nyaaya**

How Jyaa became Sine

Jyaa – jyab – sinus – sine
Sanskrit – Arabic –Latin – English

The Sanskrit word **jyaa** (bowstring)
was mispronounced and miswritten as **jyaab** (pocket)
in Arabic, when Arybhata and Brahmagupta's books
were translated by al-Jazari

The Europeans translated jyaab into **sinus** (pocket)
and the English reduced it to sine

So **Sanskrit jyaa** came back India to as **English sine**

Some notes on an unusual astronomer

VARAHAMIHIRA

On VarahaMihira

Along "a wide range of scholarship" VM writes with "chaste language, brevity and linguistic elegance"

A master not only of expression, but also metre

A sample - poems on Agasthya udaya in BrhatSamhita

Gochara adhyaaya (Transits of planets) in BrhatSamhita

Uses different chandas for his 64 stanzas

Weaves in the names of the chandas in each stanza

Glittering with slesha (puns)

Commentator Utpala "Surya himself incarnated

VarahaMihira's Books

Three major works

Pancha Siddhantika – Comparative astronomy

Brhat Jataka – Horoscopy

Brhat Samhita

– Encyclopedia

– Mundane astronomy and astrology

Minor works

For each of these, he wrote abdridged versions

Also Military astrology – Mahayatra, Svalpayatra, Yogayatra

Marital horoscopy – Vivaaha patala

Next →
Classical Period