VarahaMihira Science Forum

Indian Mathematics +Astronomy

Later Classical Period

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Classical Period

500 - 800 AD	Early Classical
800 -1300 AD	Medieval
1300 - 1800 AD	Kerala school

Brahmagupta on Aryabhata

Triprashna Adhyaaya

Arybhatokta yugam khaNdanam
Arybhatokta granthyor matha bedha khaNdanam
Arybhatokta sphuta yuga khaNdanam
Arybhatokta raahu khaNdanam
Arybhatokta matha khaNdanam
Arybhatokta kalpaadi vaarasaya khaNdanam
Arybhatokta vaaraadhi khaNdanam
Arybhatokta bhu vyaasa maana khaNdanam

BrahmaGupta on Aryabhata

Jaanaati ekam api yato na aaryabhato *ganita-kAla-golAnAm*
Na mayaa proktaani tataH prtak prtag dushanAn yeshAm ||
Aryabhata dushanAnam samkhyA vaktum na sakye yasmaat
tasmAd ayam uddesho buddhimatAnyAni yojyAni ||

- BraahmaSphutaSiddhantam XI 43-44

Aryabhata knows nothing of *Math, Time, Spherics*

Khandakaadhyakam
Vakshyami sphutam aacharya Aryabhata tulya phalam
I give corrections equal to Aryabhata's results

Jyotishas/Ganakas of Classical Era & their Siddhanta Books

- Mahavira – Ganita Saara Sangraha (Confluence of Mathematical Rivers)
- Lalla – ShishyaDhiVriddhi("Arithmetic")
- Sridhara – Trishatika ("Three hundred")
- Bhaskara 2 – Siddhanta Sironmani ("Crown Jewel of Siddhanta"), Lilavati
- Narayana Pandita – Siddhanta Kaumudi

Ganita Saara Sangraha - Terminology

Samgya adhikaara	Name section
Kshetra paribhaashaa	Plane technical words
Kaala paribhaashaa	Time technical words
Dhaanya paribhaashaa	Grain " "
Suvarna paribhaashaa	Gold " "
Rajata paribhaashaa	Silver " "
Loha paribhaashaa	Iron " "

धनर्णशून्यसविषयकसामान्यनियमाः

Dhana rNa shoonya vishayaka saamaanya niyamaaH

Ganita Sara Sangraha

Parikarma – Arithmetic operations

PratyutpannaH	Multiplication
Bhaagaa haaraH	Division
VargaH	Square
Varga-moolam	Square root
GhanaH	Cube
Ghana-moolam	Cube root
saMkalitam	Addition
vyutkalitam	Subtraction

Mahavira on Zero operations

ताडितः खेन राशिः खं सोऽविकारी हृतो युतः ।
हीनोऽपि खवधादिः खं योगे कं योज्यरूपकम् ॥

kha * raashi = raashi * kha = kham
kha / raashi = raashi / kha = kham
roopakam – kham = roopakam
roopakam + kham = roopakam

Mahavira on +ves, -ves

ऋणयोर्धनयोर्घाते भजने च फलं धनम्
ऋणं धनर्णयोस्तु स्यात् स्वर्णयोर्विवरं यतौ ॥ 50 ॥

Mulitplied or Divided → result (phalam)
Two RNam give Dhanam
Two Dhanam give Dhanam

RNam and Dhanam give RNam

When added, phalam is their difference

Mahavira on Multiplication

ऋणयोर्धनयोर्योगो यथासङ्ख्यानमृणं धनम् ।
शौध्यं धनमृणं राशेः ऋणं शौध्यं धनं भवेत् ॥ 51 ॥
धनं धनर्णयोर्वर्गो मूले स्वर्णे तयोः क्रमात् ।
ऋणं स्वरूपतोऽवर्गो यतस्तस्मान्न तत्पदम् ॥ 52 ॥

Two rNam added give rNam
Two dhanam give dhanam = Dhanam
a rNam subtracted becomes dhanam
a dhanam subtracted becomes rNam

Ahaaa!

How lucky we are to have symbols for

+ - / * =

Mahavira - Squares, Roots

धनं धनर्णयोर्वर्गो मूले स्वर्णे तयोः क्रमात् ।
ऋणं स्वरूपतोऽवर्गो यतस्तस्मान्न तत्पदम् ॥ 52 ॥

Varga of two dhanam or rNam is dhanam
Moolams (roots) are dhanam and rNam respectively
a rNam is never a square number
so it has no square roots

भागश्शेषो मूलकं शेषमूलं स्यातां जाती द्वे द्विरग्रांशमूले ।
भागाभ्यासोऽवोंऽशवर्गोऽथ मूलमिश्रं तस्माद्भिन्नदृश्यं
दशामूः ॥ ३ ॥

Bhaaga shesho moolakam shesha-moolam
Syaataam jaati dve dvir-agra-amsha-moole
Bhaaga-abhyaasa amsha-varga atha
Moola-mishram tasmaat bhinna drshyam
dashaamooH

Mahavira – Velocity Problems

दिवसैसत्रिभिस्सपादैरयोजनषट्कं चतुर्थभागोनम्
गच्छति यः पुरुशोसौ दिनयुतवर्षेण किं कथय ॥ ३ ॥

divasais-tribhis-sa-paadair-yojana-shaTkam caturta-bhaaga-unam
Gacchati yaH purusho-asau dina-yuta-varsheNa kim kathaya

The man (purusha) who (asau) walks (gacchati) quarter (caturtha-bhaaga) less (unam) than six (shaTka) yojanaas in three (tribhi) and quarter (paadai) days (divasau), tell (kataya) how much (kim) he walks in a day (dina) and (yuta) a year (varsha).

Mahavira - Geometry

Triangles – tribhuja

Sama - equilateral
Dvisama - isoceles
Vishama - scalene

Caturashra -quadrilaterals

Sama - square
Dvidvisama - rectangle (two sets of two equal sides)
Trisama – three sides equal
Dvisama – two sides equal
Vishama – random quadrilateral

Mahavira - Geometry

dvidvisa
ma

Caturashra -quadrilaterals

Sama - square
Dvidvisama - rectangle
(two sets of two equal sides)
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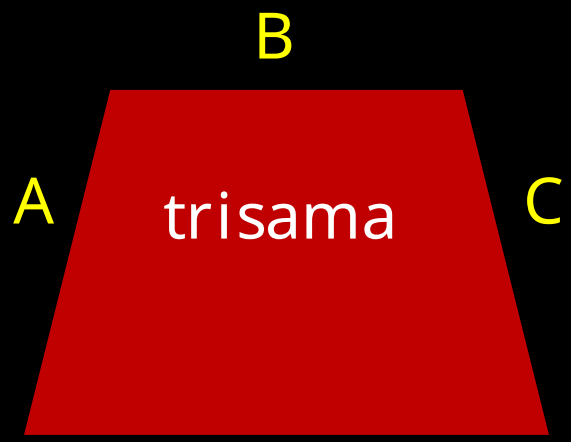
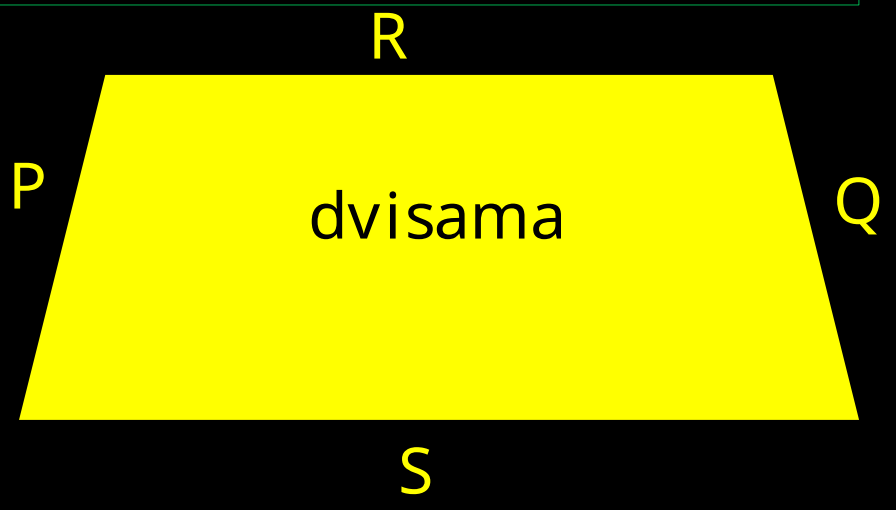
quadrilateral





Mahavira - Geometry

Vrttas

Sama vrtta- circle
Aayatavrtta - ellipse
ardhavrtta – semicircle
Kambuka vrtta – conch shape
Nimnata vrtta – concave
Unnnata vrtta - convex

मनुज ग्रन्ताः

LATER CLASSICAL - MEDIEVAL

Sanskrit Bhashyas on Aryabhateeyam

Prabhakara	unknown	?
Bhaskara I	Gujarat	7
Someshvara	unknown	10-12
Bhutivishnu	Tamilnadu	11
Suryadeva Yajvan	Tamilnadu	12
Parameshvara	Kerala	14
Nilakantha	Kerala	15
Raghunatha Raja	Andhra	16
Madhava	Andhra	?
Ghatigopa	Kerala	17-18
Kodandarama	Andhra	19

Siddhantas – Classical era

Maha Bhaaskariya	Bhaskara I	630
Laghu Bhasakariya	Bhaskara I	630
Dhavala	Virasena	816
PaaTi Ganita	Sridhara	850
Trishatika	Sridhara	850
Ganita Sara Sangraha	Mahavira	850
Laghumaanasa	Munjala	930
Siddhaanta Shekhara	Sripati	1040
Siddhanta Siromani	Bhaskara II	1150
Bijaganita	Bhaskara II	1150
Ganita Kaumudi	Narayana	1350

Bhaskara and Infinity

Bhaskara Acharya (1150 AD) in *Siddhanta Sironmani*

explores the Arithmetic of **infinity**

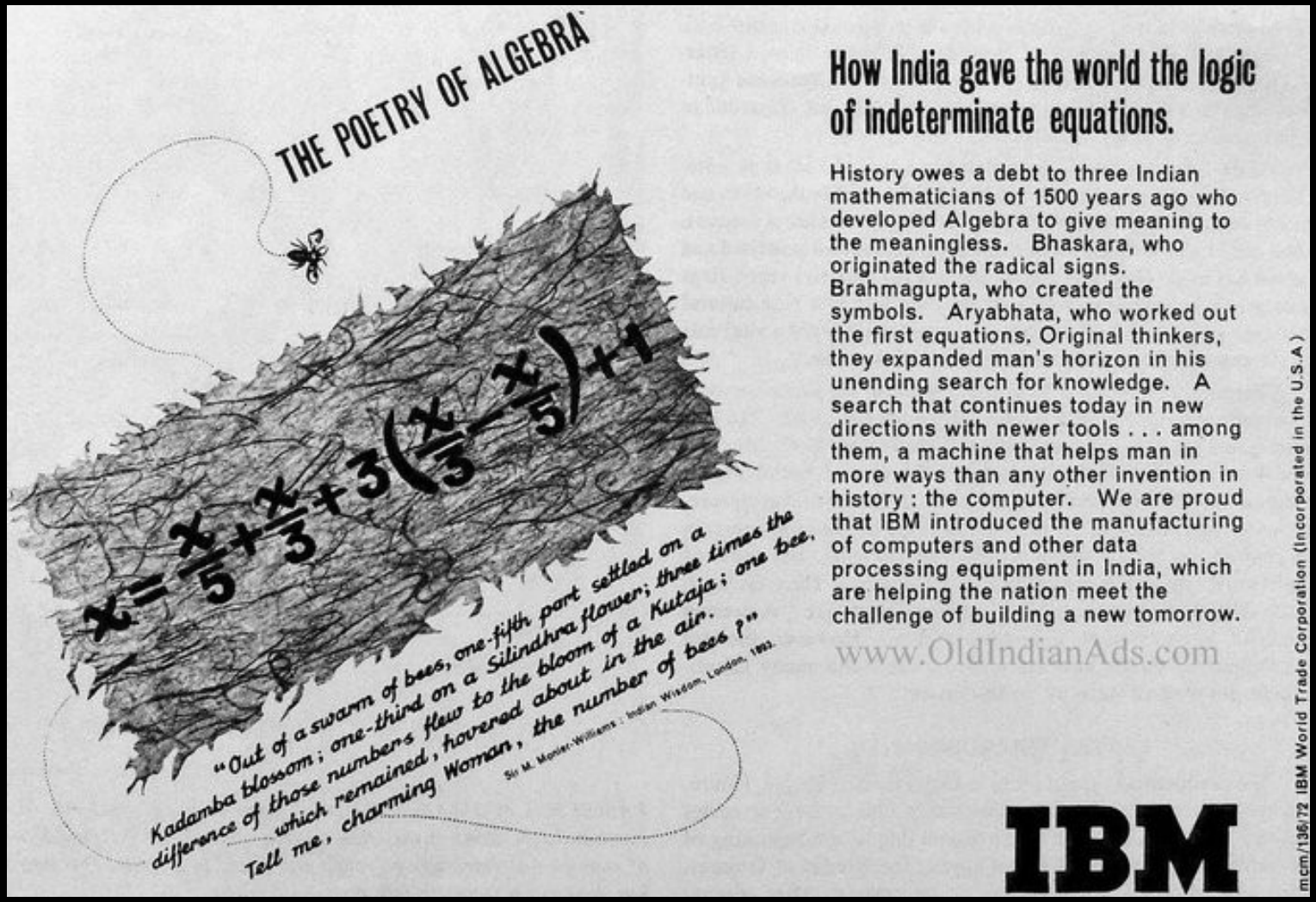
He calls it kha-hara

Kha – zero

hara – division

khahara - Number divided by zero

IBM honours
Aryabhata Brahmagupta Bhaskara



Corrections to Aryabhata

Aryabhata stated

A. Area of triangle = ½ * base * perpendicular
B. Volume of tetrahedron = ½ * Area of triangle * height

C. Area of circle = Diameter * Circumference
D. Volume of sphere = Area of circle * sqrt (area of circle)

A and C are correct. B and D are wrong.
Yes, even Aryabhata made mistakes in mathematics!

Brahmagupta (625AD) corrected B
Volume of tetrahedron= 1/3 * * Area of triangle * height

Bhaskara Acharya (1150 AD) corrected D
Volume of sphere = Circumference * (diameter)^2 / 6

Volume of Sphere – Case Study

- D = diameter of circle; C circumference; R = Radius; A = area
- Jyotishas ignored Aryabhata's Π=3.1416. They used √(10) instead
- Aryabhata's formula (500AD)
  - Area of circle A = C/2 * D/2 = Π * (r^2)
  - Volume of Sphere V = A* √(A) = Π * √Π * (r^3)
- Mahavira's formula (850 AD) – uses 3 instead of pi
  - A = 3 * D
  - Approximate V = (D^3)/2 + (9/2)
  - Accurate V = (D^3)/2 + (9/2) * (9/10) = 81/40 * (D^3)
- Sridhara's formula (850 AD)
  - V = (D^3)/2 + (1/18)*(D^3 / 2) =19/6* 4/3 * (r^3)
  - 19/6 = √(10) : so, quite close to correct value
- Bhaskara 2nd's formula (1150 AD)
  - A = C*D/4
  - Surface Area S = C*D = 4 Π * (r^2)
  - V = S*D/6 = 4Π/3 * (r^3)
  - Correct Formula, but 650 years for correct value

Narayana Pandita - Vaarasamkalita

Vaarasamkalita = Sum of powers of a series

एकादिकवारमिताः पदादिरूपोत्तर पृतक् तेंऽशाः ।
एकाद्येकचयहरास्तद्धातो वारसङ्कलितम् ॥

Eka-aadika-vaaramitaaH pada-aadi-roopa-uttaraa prthak te-amshaaH |
Ekaadi-ekacaya-haraas-tad-dhaato vaarasamkalitam ||

Eka-adika	One more than
vaaramitaa	Number of sums
Pada-aadi	From first term
Roopa uttara	
amshaa	numerators
haraa	denominators
Ekaadi eka caya	From one, increasing by one

Narayana Pandita

Bhadra Ganita – Magic Square

sarveshaam bhadrANAm shredirItyA bhaved ganitam ।
yeshAm ganitamabhishTam saadhyau teshAm mukhapracayau ।

Bhadra – magic square
shredirItya – arithmentic progression
Mukha – first term
praca - interval

A **Bhadra or magic square** is
an arrangement of numbers, where
The sum of each ROW
The sum of each COLUMN
The sum of DIAGONALS
Are all the same

Sum = 64

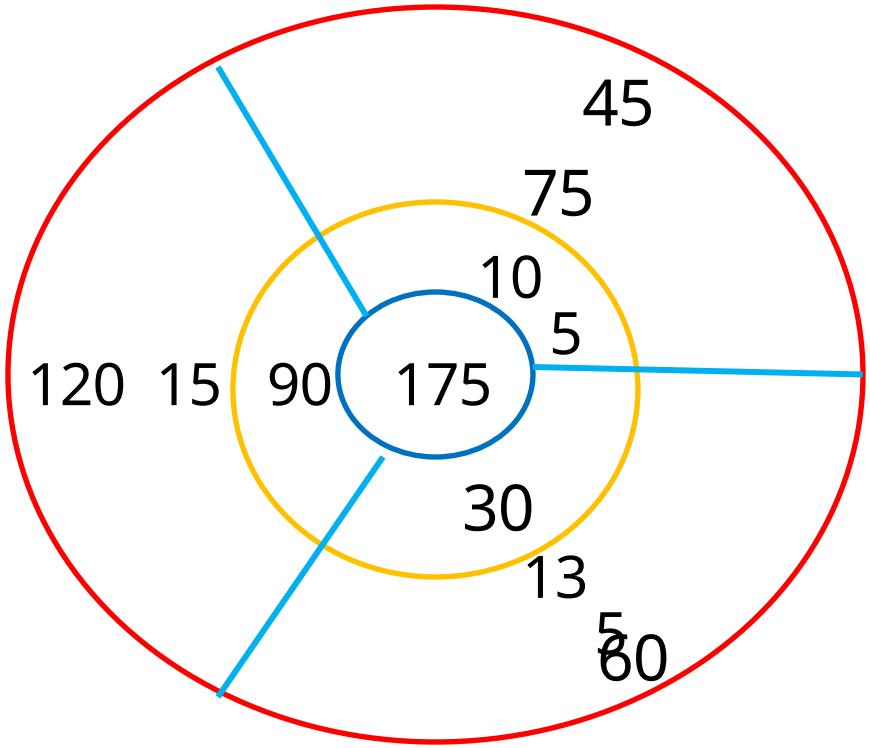
1	15	25	23
27	21	3	13
7	9	31	17
29	19	5	11

Narayana Pandita – Magic squares

1	15	25	23
27	21	3	13
7	9	31	17
29	19	5	11

Sum = 64

-14	14	34	30
38	26	-10	10
-2	2	46	18
42	22	-6	6



Sum = 400

Nagarjuna 100 BCE

kacchapuTha

Arka indunidhAnAri tena lagna vinAsanam ||

अर्क इन्दुनिधानारी तेन लग्ना विनासनम् ॥

In **kaTapayaadi** notation, this forms a magic square

अ र् क			
	इ न् दु		
ति	धा		ना
	ल री्	ना	
वि	ना		स

0	1	0	8
0	9	0	2
6	0	3	0
4	0	7	0

केरल परंपरा

KERALA SCHOOL

Undisturbed development

- From Madhava 1350 to Rajaraja Varma1850
- Madhava considered founder of Kerala school
- Some Achievements of Kerala School
  - Drk Ganitam (observational corrections)
  - Infinite series
  - Infinitesimals → Calculus
  - Upapattis (Proofs)
  - Quasi Helio centric theory

Kerala school - Guru Parampara

	Govinda Bhattatiri (1237-96)
Grandson	Parameshvara (1360-1430)
son	Damodara
shishya	Nilakantha Somayaaji(1443-1555)
shishya	Jyeshtadeva(1500-1600)
shishya	Acytua PisaraaTi (1550-1621)
shishya	Trippanikaara Poduvaal (15??)
shishya	Naavaaykulaatu AaalaaDi (16??)
shishya	Pulimugattu PoTTi (1686-1785)
shishya	Raman aasaan(17??)
shishya	Krishnadaasan (1756-1812)
shishya	Mangalari Dakshinaamurthy Mussathu (17??-18??)
shishya	NaalekaaTTil Balaraman Pillai (18??)
shishya	Prince Rajaraja Varma(1812-1846)

Madhava series for pi (π)

व्यासे वारिधिनिहते रूपहृते व्याससागरभिहते।
Vyaase vaaridhi nihate rupa hrte vyAsa saagarabhihite
त्रिशरादिविषमसङ्ख्याभक्तमृणं स्वं पृथक् क्रमात् कुर्यात् ॥
Tri-sharAdi-vishama-sankhyA-bhaktam-rNam svam prtak kramAt kuryAt

Tri-sharaa	Three five
aadi	beginning
Vishama sankhya	Uneven (odd) sum
bhaktam	dividing
rNam	Negative
Roopa hrte	Subtract from one (rupa)
Vyaasa saagara	Diameter - four
hatE	multiply

Paridhi = 4 * vyaasa * (1 – 1/3 + 1/5 – 1/7 + 1/9….)

Madhava series for pi (π)

This was rediscovered by Gregory and Leibniz in 1674

And called Leibniz series or Gregory series for π

$\pi/4 = 1 - 1/3 + 1/5 - 1/7 + 1/9....$

But this is the same as the Madhava series (1350-1425)

So, now it is called Madhava-Leibniz series

Madhava and Infinite series

Madhava also discovered infinite series for sine, cosine and arctangent

The discovery of sums of infinite series began a new era in mathematics

Summation of infinitesimals is the basis for integral calculus as discovered by Newton and Leibniz

Parameshvara

Author of Drk Ganitha द्रक् गणित

When one's calculations disagree
with planetary positions,
one must revise calculations,
since planets wont adjust paths & speeds!!

Principle of observation based corrections

Nilakanta Somayyaji

Nilakanta Somayyaji followed Madhava and Parameshvara

He was a polymath like Varahamihira

Authored TantraSangraha, Arybhatiya bhasha

Continued Drk Ganita

New insights and proofs on infine series of sums

Developed Quasi-Helio-centric theory

Nilakanta Somayajji

Virasena (circa 816 AD) in his commentary *Dhavala* gave this sum of a infinite series

$$¼ + (¼)^2 + (¼)^3 + \ldots + (¼)^n = 1/3$$

Nilakantha in his bhashya on Aryabhateeya asks

"The entire series of powers of ¼ adds up to 1/3. How is it that the sum of a series only increases upto that **limiting** value?

Does it increase until that value?"

Nilakanta Somayaaji

$$¼ + (¼)^2 + (¼)^3 + \ldots + (¼)^n = 1/3$$

He reasoned and explained it by deriving the following sequence of results

$$1/3 = 1/4 + 1/(4*3)$$
$$1/(4*3) = 1/(4*4) + 1/(4*4*3)$$
$$1/(4*4*3) = 1/(4*4*4) + 1/(4*4*4*3)$$

As we add more terms, argued Nilakantha, the difference between 1/3 and the powers of 1/4 become extremely small…

but never zero, until we add upto **infinity**.

Nilakanta Somayaaji

What we see

- Approach to infinite series
- Understanding of Limits
- Intergration as a sum of an infinite series

- These are the basic elements of Integral Calculus
- No differential calculus is seen

- In *GanitaYuktiBhaasha* Jyeshtadeva expanded this

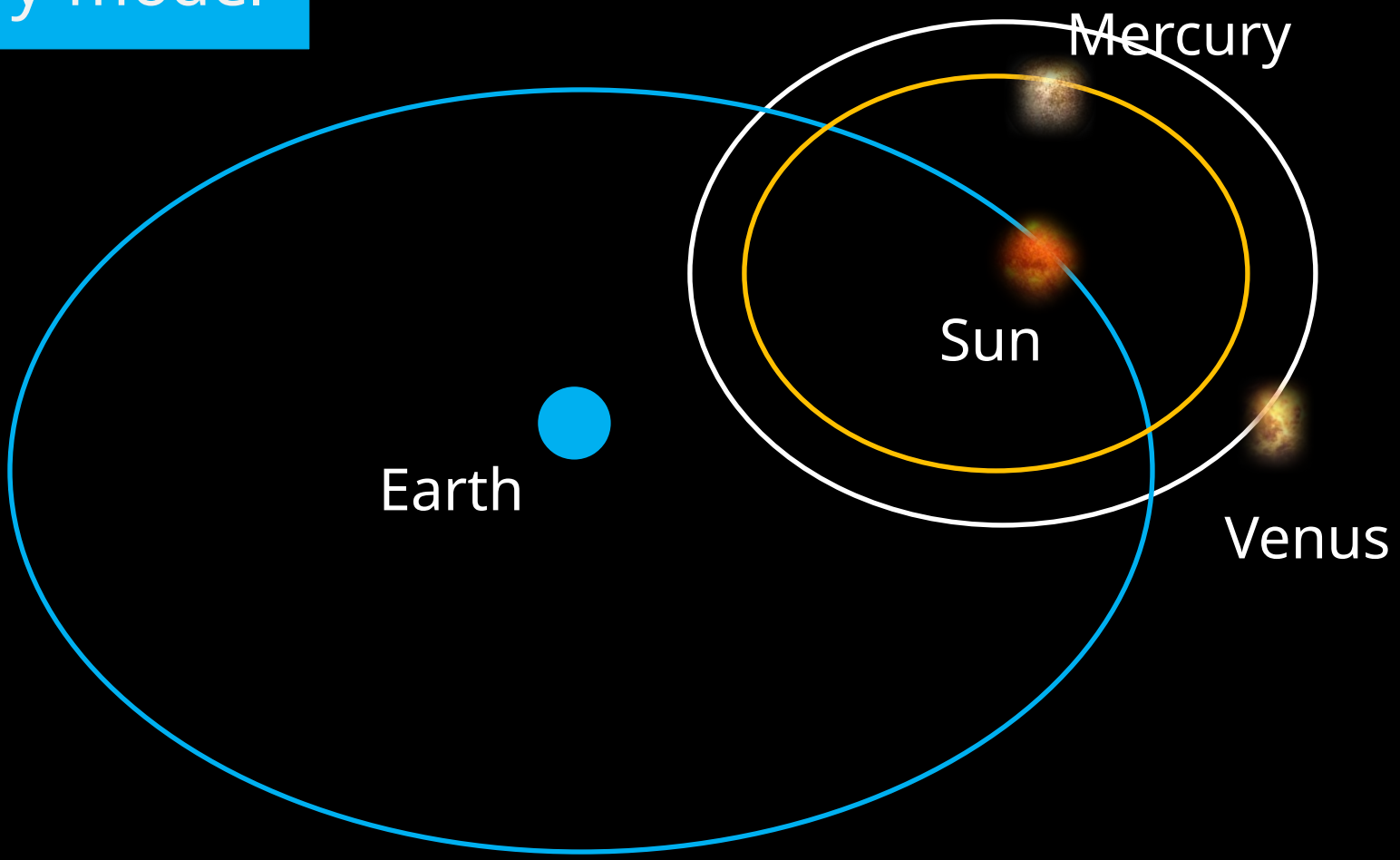
Nilakanta Somayyaji
Quasi helio centric model

Five planets revolve around the sun

And sun revolves around the Earth

Proposed model in Tantra Sangraha
Explained details in Aryabhatiya bhashya

Nilakantha's Planetary model



Tantra Sangraha

हे विष्णो निहितं कृत्स्नं जगत् त्वय्येव कारणे |
ज्योतिषां ज्योतिषे तस्मै नमो नारायणाय ते ||

Hey vishnO nihitam krtsnam

क त ह न ण व ह
1 6 8 0 5 4 8

No of days from Kali = March 22, 1500
Date of composition!!

Jyeshtadeva

Jyeshtadeva, a contemporary of Nilakantha Somayaji, composed YuktiBhasha, a Malayalam book on mathematics

Jyesthadeva provides several proofs (upapatti) for many of the infinite series derived by Madhava and Nilakantha

Some historians consider this the first book of Calculus

What did these INDIANS discover?

Pythagoras	Aryabhata
Copernicus	Varahamihira
Isaac Newton	Brahmagupta
Charles Darwin	Madhava
Mendeleev	Nilakanta Somayaji

Ignorance is

Scientific Illiteracy Normal

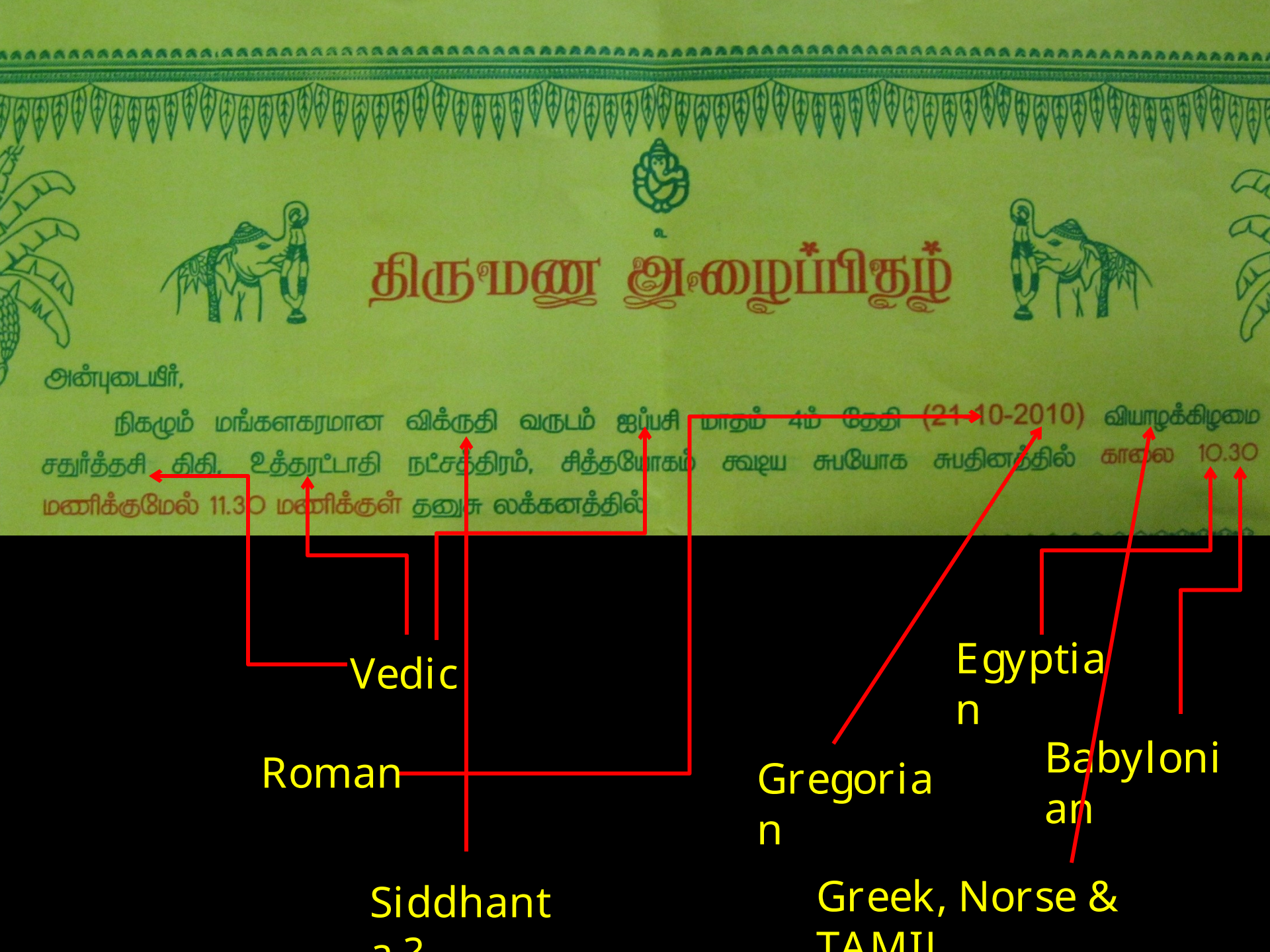
Aryabhata

Square root, Cube root
citi, varga-citi-ghana, ghana-citi-ghana
Area of triangle, circle, trapezium
Vipareetam (inversion)
Fractions – addition, division
Sine table
kuTTaka

Brahmagupta

Arithmetic of Integers
Arithmetic of Zero
Many methods of Multiplication
Cyclic Quadrilateral
Equations : *sama*, *samikaraNa*
Roots of Quadratic Equation
Varga Prakriti (Quadratic indeterminate equations)
Bhavana (composition of functions)

VarahaMihira	Eclipse proof
	Comparative astronomy
	Combinatorics (gandha yukti)
Mahavira	Fractions – bhinna
	Areas of plane geometric figures
	Distance-Velocity problems
Bhaskara	Geometry, Infinity (khahara)
	Volume of Sphere
Madhava	Infinite series
	Infinitesimals
Narayana Pandita	Magic squares, Sums of Series
Parameshvara	Drk Ganitam
Nilakantha	Quasi Helio centric theory
	Sums of infinite series


திருமண அழைப்பிதழ்
அன்புடையீர்,
நிகழும் மங்களகரமான விக்ருதி வருடம் ஐப்பசி மாதம் 4ம் தேதி (21-10-2010) வியாழக்கிழமை
சதுர்த்தசி திதி, உத்தரட்டாதி நட்சத்திரம், சித்தயோகம் கூடிய சுபயோக சுபதினத்தில் காலை 10.30
மணிக்குமேல் 11.30 மணிக்குள் தனுசு லக்கனத்தில்
Vedic
Roman
Siddhanta
Gregorian
Egyptian
Babylonian
Greek, Norse &
TAMIL

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Thank you