

VarahaMihira Science Forum

Indian Mathematics +Astronomy

Early Classical Period

R. Gopu

writergopu@yahoo.com

VarahaMihiraGopu.blogspot.com

Caritram

A BRIEF HISTORY

History of Astronomy

1400 BC

Vedic period

500 BC-500 AD

Vedanga jyotisa

500 - 1700 AD

Siddhantas

Classical Period

Copernicus, Galileo, Newton

Classical Period

500 - 800
AD

Early Classical

800 -1300 AD

Medieval

1300 - 1800 AD

Kerala school

Astronomy Books

Siddhanta

Theory & Proof

Bhaashya

Commentary

Karana

Tables for
Astronomers

Panchanga

Calendars
for public

Approximate Geography of Indian Jyotishas



गणितं

MATHEMATICS
IN THE CLASSICAL ERA



आर्यभट्टः

ARYABHATA SINGLE T

Aryabhateeyam

Aryabhata's famous book is Aryabhateeyam

It is 121 stanzas long

It consists of 4 parts

1. Gitika (13 stanzas) – Definitions and calculated constants
2. Ganita – Mathematics (33 stanzas)
3. Kalakriya - Time (25 stanzas)
4. Gola – Celestial Sphere (50 stanzas)

All stanzas are in Arya chandas

Gitika is considered preamble

The other 3 chapters are called Arya

AshtaShatam (108)

Aryabhateeyam is important since it begins the Classical era
It is incredibly short, in comparison to other such books

Aryabhata's Era

षष्ठ्याब्दानां षष्ठ्यर्दा व्यतीतास्त्रयश्च युगपादा ।
त्र्यधीका विंशतिरब्दास्तदेह मम जन्मनोऽतीतः ॥ ३१ ॥
shashTyabdaanam shashTiryadA vyatitAs-
traya-shca yugapAdA
tryadhikA vimshatirabdAs-tadeha mama
3600 after Kali yuga began, my age was 23
janmano-atItAH

shahTi	sixty
abda	years
tryadhika	Plus three
vimshati	Twenty
Janamano	birth
Traya yugapAdA	Three yuga-pAdAs
atItA	passed

Sunday

March 21, 499 AD

So Born in 476 AD

Thrai Raashika - Rule of three

त्रैराशिकफलराशिं तमथेच्छाराशीना हतं कृत्वा ।
लब्धं प्रमाणभजितं तस्मात् इच्छाफलमिदं स्यात् ॥ २६

phala raashi

icchaa phalam = _____ * icchaa raashi
pramaaNa

[Smith's Translation](#)

Fruit

Fruit of desire = _____ * desire
Measure

vipareetam - Inversion

गुणकारा भागहरा भागहरास्ते भवन्ति गुणकाराः ।

guNa-kaaraa bhaaga-haraa bhaaga-haraa-s-tey bhavanti guNa-kaaraaH

यः क्षेपः सोऽपचयोऽपचयः क्षेपश्च विपरीते ॥

ya kshepaH	sa upachaya	upachayaH	kshepa
guNa-kaaraa,			multipliers
citra vipareetey			
bhaaga-haraa			divisors
they		they	
bhavanti		become	
yaH - saH		whichever - that	
kshepa		subtractions	
upachaya		additions	
vipareetam		inversion	

Fractions (chedaaH) - Simplification

छेदा: परस्परहता भवन्ति गुणकारा भागहराणाम् ।

chedaaH paraspara-hataa bhavanti guNa-

kaaraa bhaaga-haraa $\frac{a/b}{c/d} = \frac{a * d}{b * c}$ Naam *

chedaaH	fractions
paraspara	each other
hataa	multiply
bhavanti	become
guNa-kaaraa	multipliers
bhaaga-haraa	divisors
Naam	of

Fractions – Common Denominator

छेदागुणं सच्छेदं परस्परं तत् सवर्गत्वम् ॥

chedaaguNam sacchedam parasparam tat
savargatvam

Multiply numerators and denominators of each fraction

By denominator of each fraction

Thus both have common denominator

$$\frac{a}{b} + \frac{c}{d} = \frac{ad}{bd} + \frac{bc}{bd} = \frac{ad + bc}{bd}$$

Seeds of Algebra (Bija Ganita)

With *Vipareetham* Inversion and
Thrai raashika Rule of three,
you basically have the seeds of algebra

- Finding unknowns
- Expressions using unknowns
- The Equation with implied = sign
- Balancing equations

Roots of quadratic equations, operations of fractions, series summations, square and cube root algorithms clinch this argument

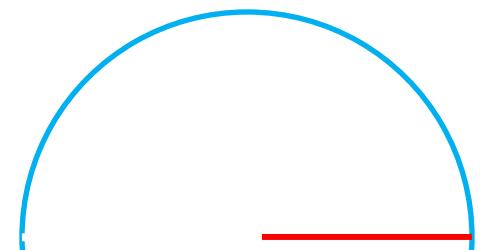
Aryabhata – Area of Circle

समपरिणाहस्यार्ध विष्कम्भार्धहतमेव वृत्तफलम् ।

Sama pariNaahasya ardha vishkamba ardha
hatameva vrtta phalam
Area = perimeter / 2 * diameter / 2

No mention of pi

Sama	equal
pariNaahasya	half Perimeter
ardha vishkamba	half diameter
ardha hatameva	multiply
vrtta	circle
phalam	Result (area)



चतुरधिकं शतमष्टगुणं द्वाषष्टिस्तथा सहस्राणाम् ।

catur-adhikam shatam-ashTa-guNam dvaashashTi-tathA sahasrANAm

अयूतद्वय विष्कम्भस्यासन्नो वृत्तपरिणाहः ॥

a **$104*8 + 62000 = 62832$**

p **For Diameter of 20,000 Circumference is
62,832 approximately**

Catur adhikam

four plus nunarea

shatam

By eight multiply

ashTa guNam

(2+60) * 1000

dvaashashTi

10,000 times 2

sahasrANAm

Diameter's

ayuta dvaya

Approximately

vishkambha-sya

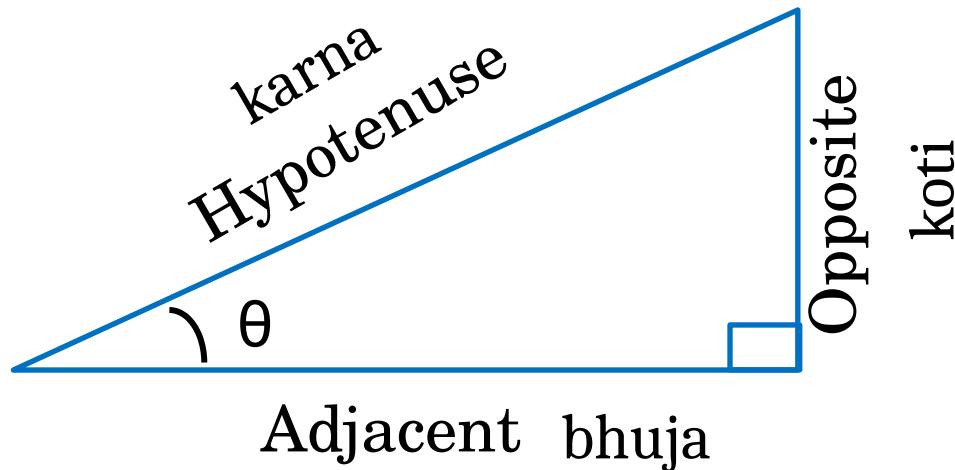
Circle's Circumference

aasannaH

vRtta pariNaaha

An accurate definition for pi

Bhuja koTi karNa nyaaya



Bhuja vargaH koTi vargaH ca karNa vargaH
Side squared and Perpendicular squared
(equal) hypotenuse squared

$$a^2 + b^2 = h^2$$

Arithmetic progression

Sum of a series, increasing by regular interval

- $1 + 2 + 3 + 4 + 5 + 6 \dots N$
- $1 + 4 + 7 + 10 + \dots T$ (gap or interval 3)
- $22, 25, 28, 31, \dots T$ (Begin 22, gap 3)
- $B, B+G, B+2G, B+3G, B+4G, \dots B + (N-1)G$

T is the Nth term, G is Gap, B is Beginning term

$$\text{Mean} = B + G * (N - 1) / 2$$

$$\text{Sum of series} = \text{Mean} * N$$

Second Formula

$$\text{Sum of series} = (\text{aadi} + \text{antam}) * N / 2$$

citi - Sums of Series of Arithmetic progression

इष्टं व्येकं दलितं सपूर्वमुक्तरगुणं समुखमध्यम् ।

ishtam vyekam dalitam sa-poorvam-uttara-guNam samukha-madhyam

इष्टगुणितमिष्टधनं त्वथवाद्यन्तं पदार्धहतम् ॥ 19 ॥

ishTa-quNitam ishTa-dhanam tvathava aadi-
ishTam Desired result
antam padArDha hatam Decreased by one

uttara guNam Multiplied by interval

sa mukha Plus first term

madhyam Mean (central or average)

According the commentator Bhaskara, (via Ramasubramaniam) this is a **muktaka** (verse that encodes multiple formulae).

First line gives the mean

Second line gives sum, and another formula

citi, varga-citi-varga, ghana-citi-ghana

citi is sum of series of natural numbers

VARGA-citi-ghana is sum of SQUARES of a series

GHANA-citi-ghana is sum of CUBES of a series

- citi $1 + 2 + 3 + 4 + 5 + 6 \dots$

$$c = n(n+1)/2$$

- VARGA-citi-ghana $1^2 + 2^2 + 3^2 + 4^2 + \dots N^2$

$$vc = n(n+1)(2n+1)/6$$

- GHANA-citi-ghana $1^3 + 2^3 + 3^3 + 4^3 + \dots N^3$

$$vc = c^2 = (n(n+1)/2)^2$$

Square Root and Cube Root

Ganita chapter has not only several formulae
But also a few algorithms

Square root, Cube root, and kuTTaka are three
algorithms

Square

वर्गः समचतुरश्रः फलं च सदृशद्वयस्य संवर्गः ।

vargaH sama-chatur-ashraH phalam ca sa-
drsha-dvaya-sya samvargaH

सदृशत्रयसंवर्गो घनस्तथा द्वदशाश्रि स्यात् ॥ ३ ॥

sa-drsha-traya samvargaH ghanaH tathaa

~~vargaH~~
~~dvadasha-ashri~~ ^{varga}
~~sama-chatur-ashraH~~

equal-four-sided

ca

and

sa drsha

same-visible (equal)

dvaya samvargaH

twice multiplied

phalam

result

A varga is
four sided, and,
the same number twice multiplied
Arithmetic



Geometric
 $a*a$

Cube

सदृशत्रयसंवर्गो घनस्तथा द्वदशाश्रि स्यात् ॥ ३ ॥

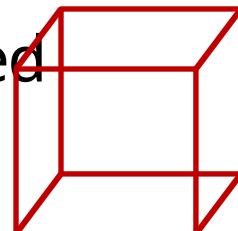
sa-drsha traya samvargaH ghanaH tathaa
dvaadasha-ashri syaat

sa drsha	same-visible (equal)
traya samvargaH	thrice multiplied
ghanaH	ghanaH
tathaa	similarly
dvaadasha ashri	twelve sided

A ghanaH is
the same number thrice multiplied $a*a*a$

Arithmetic

similarly twelve sided



Geometric

KuTTaka

KuTTaka – pulverizer

Breaking down a problem into
smaller, simpler problem

Uses an *iterative* algorithm to solve a problem

Linear Determinate equation

$$47 = 5x + 7$$

Here, x=8. There is ONE solution

kuTTaka - continued

This is easy for small values

$$N = 5x + 2$$

$$N = 8y + 3$$

Very hard for large values

$$N = 1841x + 245$$

$$N = 8205y + 9732$$

Aryabhata's kuTTaka algorithm is a simple method to solve such problems.

A Bit of Mathematics!

- Bhaskara 1st wrote a bhaashya on Aryabhata
- He says
- GanitaPada of Aryabhata is hardly Ganita
- It is only “a little bit of Ganita”

Aryabhata Mathematics

Vipareetam (Inversion)

chedaah (Fractions)

Areas of triangle, circle, trapezium

Accurate value of pi

Bhuja koTi karNa nyaayaa

Two definitions for varga, ghana

Citi, varga citi, ghana citi

Square and Cube roots

Kuttaka

Aryabhata's Instruments

वृत्तं भ्रमेण साध्यं त्रिभुजं च चतुर्भुजं च कण्ठभ्याम् ।

vrTTam bhrameNa saadyam tribhujam ca
chaturbhujam karNAbhyAm

साध्या जलेन समभूरध ऊर्ध्वं लम्बकेनैव ॥ १३ ॥

saadyA jalena samabhuradha urdhvam

lambakena eva Feasible

Bhrama compass

karNa Set square

Sama bhuradha Level earth or surface

lambaka Plumb line

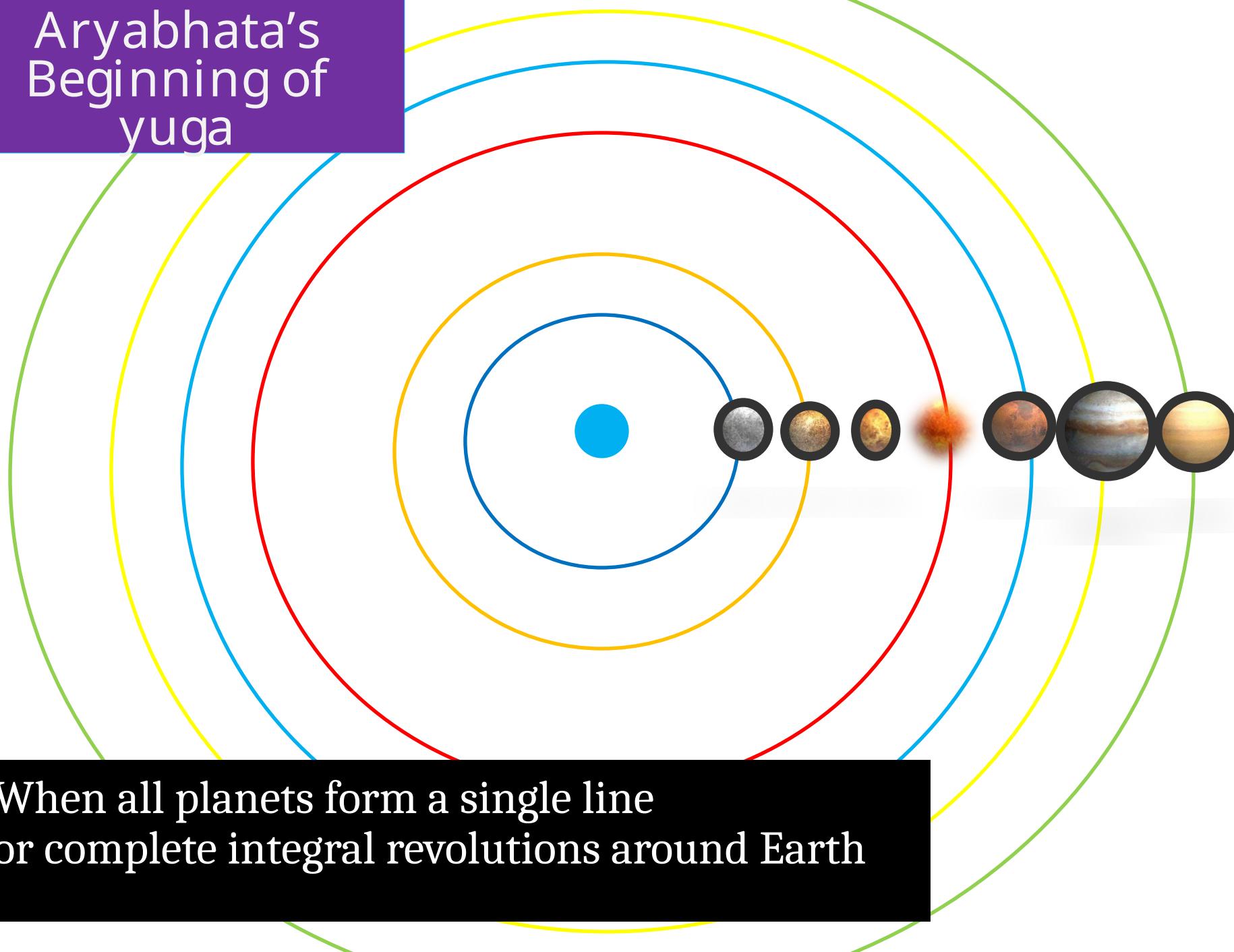
By compass, a circle,

By setsquare, triangle square

By water, a level surface

By plumb line, vertical line ... are feasible

Aryabhata's Beginning of yuga



When all planets form a single line
or complete integral revolutions around Earth

Circle and Time

एवं कालविभागः क्षेत्रविभागस्तथा भगणात् ।

Evam kaala-vibhaaga kshetra-vibhaaga-s-tathaa bhagaNaat

Circle		Time
• Rashi	12	• Maasa
• Kalaa	30	• Dina
• Vikalaa	60	• Naadi
• Liptaa	60	• Vinaadi
• Viliptaa	60	• Guru akshara

Aryabhata

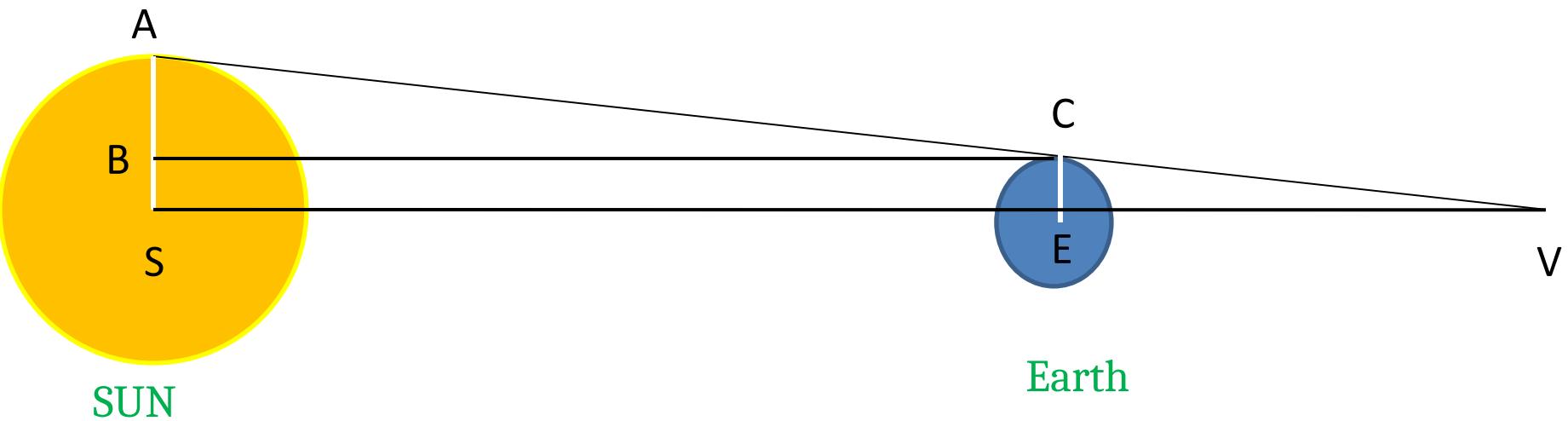
Revolutions in a yuga

Graha / Planet	Revolutions	In days	Modern
Ravi / Sun	432000		
Indu / Moon	53433336	29.53058	29.53059
Kuja / Mars	2023176	779.92125	779.9428
Budha / Mercury	13617020	115.8783	115.8786
Guru / Jupiter	3955776	398.8895	398.8864
Shukra / Venus	2702388	583.8975	583.0000
Shani / Saturn	4173436	378.0859	378.0930

Aryabhata

Length of Earth's shadow

$$\text{Length of Earth's Shadow} = \frac{\text{Sun's Distance} * \text{Earth's diameter}}{\text{Sun's diameter} - \text{Earth's Diameter}}$$
$$EV = \frac{SE * 2 EC}{2 SA - 2 EC}$$



Triangles CEV and ABC are similar.
The above calculation follows from that

Length of Earth's shadow : Derivation

$$\frac{EV}{EC} = \frac{BC}{AB} = \frac{SE}{SA - EC}$$

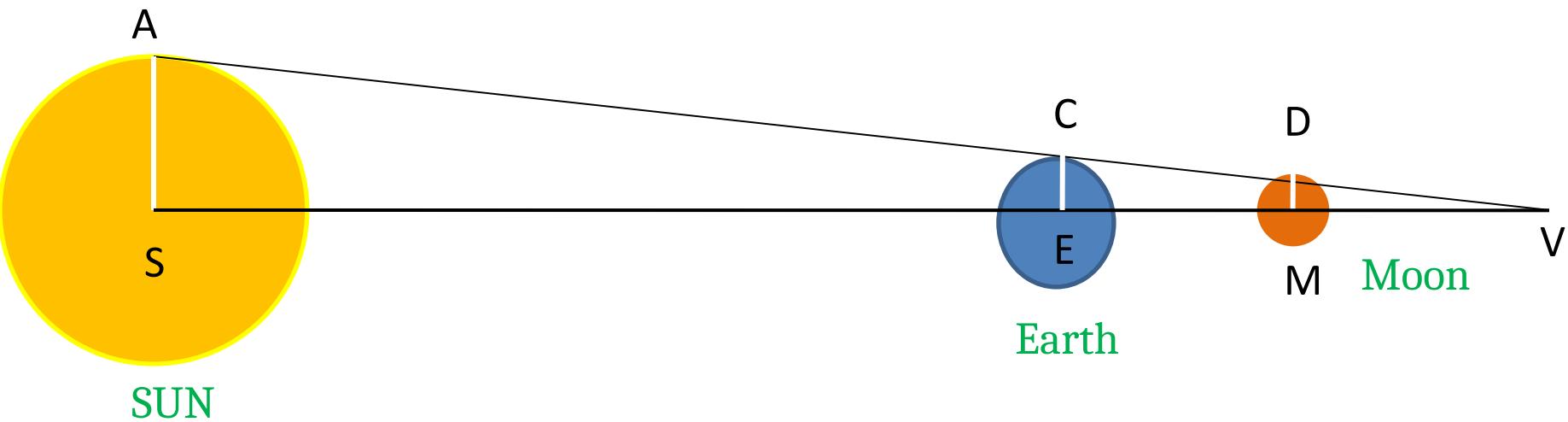
$$\text{Length } EV = \frac{SA * EC}{SA - EC} = \frac{SA * 2 EC}{2 SA - 2 EC}$$

Aryabhata

Diameter of Earth's shadow

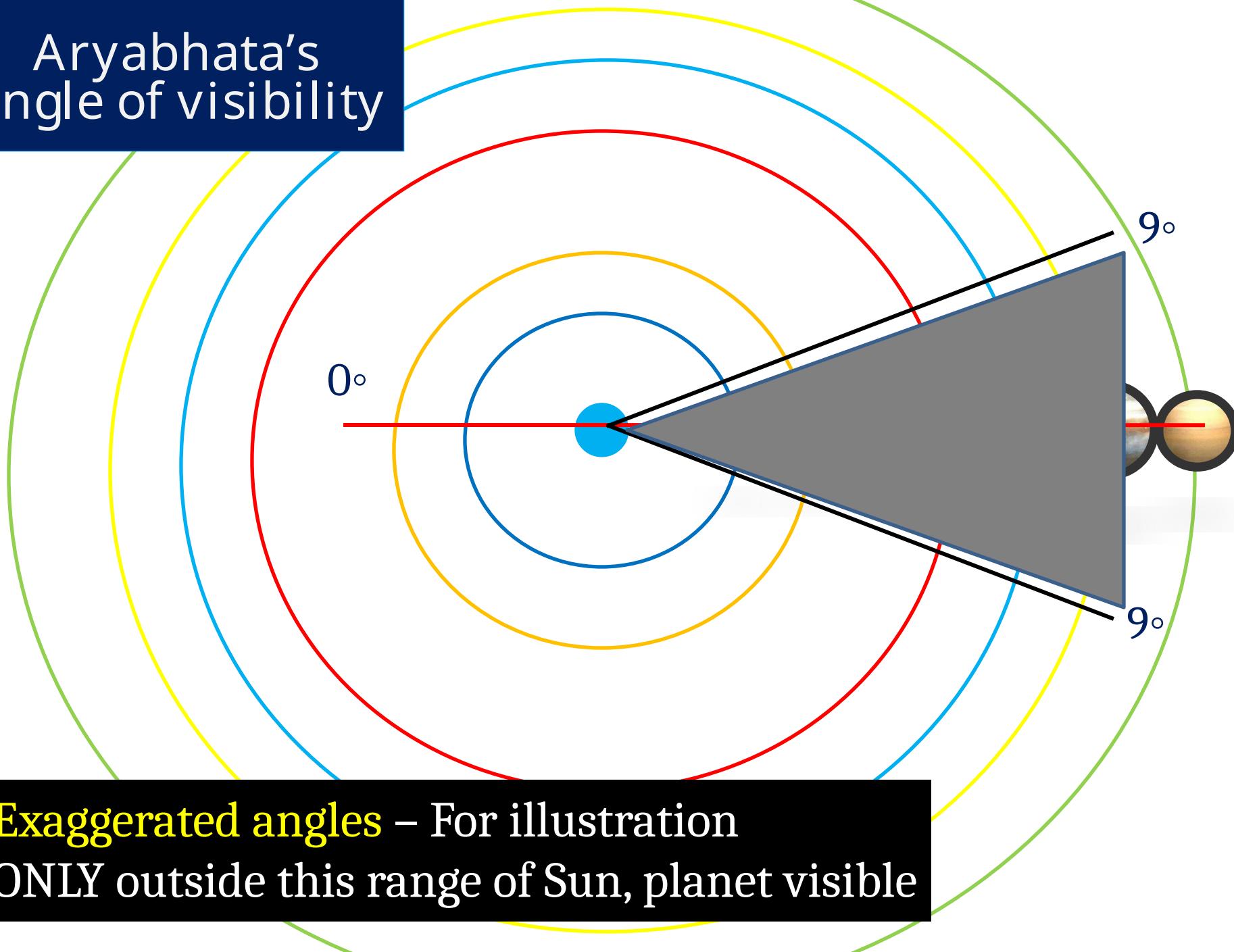
$$\text{Diameter} = \frac{(\text{Length of shadow} - \text{Moon distance}) * \text{Earth's diameter}}{\text{Length of earth's shadow}}$$

$$2DM = \frac{(EV - EM) * 2 * EC}{EV}$$



Triangles CEV and MDV are similar.
The above calculation follows from that

Aryabhatta's Angle of visibility



Exaggerated angles – For illustration
ONLY outside this range of Sun, planet visible

Visibility of Planets

चन्द्रोऽरंशैर्दवादशाभिरविक्षिप्तोऽकर्न्तरस्तिथो दृश्य ।
नवभिर्भूगोस्तैद्वयधिकैद्वयधिकैर्यथाशलक्षणाः ॥ ३१ ॥
candro-amshai-dvAdashA-bhir-avikShipto-
arkAntara-stitho dRshya
navabhir-bhRgo-stair dvar-adhikair-dvar-
adhikair-yathA-lakshNAH

Planet	Visibility angle	
Moon	12	
Venus(bhRgu)	9	
Jupiter	11	+2
Mercury	13	+2
Saturn	15	+2
Mars	17	+2

Rotation of Earth

अनुलोमगतिनैस्तः पश्यत्यचलं विलोमगं यद्वत् ।

अचलानि भानि तद्वत् समपश्चिमगानि लङ्कायाम् ॥ ९ ॥

anulomagati-nau-staH pashyati-acalam
vilomagam yadvat

acalaani bhaani tadvat sama paschima-
gaani lankaayaam
Just as man on a boat thinks trees move backward,
so also stars move westward

anuloma	Forward going
gati	boat
nau	Standers
staH	see
pashyati	Unmoving
acalam	things
vilomaga	backward going
m yadvat	Just as

acalaani	unmoving
bhaani	stars
tadvat	so also
sama	similarly
paschim	westward
a	motion
gaani	Lankans
lankaaya	
am	

Revolution of planets

उद्यास्तमयनिमित्तं नित्यं प्रवहेण वायुना विक्षिप्तः ।

लङ्का समपश्चिमगो भपञ्चरः सग्रहो भ्रमति ॥ १० ॥

Udaya-astamaya-nimittam nityam

pravahENA vAyunA vikshiptaH

lankA sama-pashcima-go bha-pancaraN sa-
graHO bhramati
The cage of stars and planets revolve westward past Lanka
rising and setting daily

Driven by pravahena-vaayu

PravaheNa –Vaayu is a celestial wind

assumed to act across space

moving planets and stars

So Aryabhata did NOT know Gravity

Concluding verse

सदसद्-ज्ञान-समुद्रात् समुद्धृतं ब्रह्मणः प्रसादेन ।

सद्-ज्ञानोत्तमरत्नं मया निमग्नं स्वमतिनवा ॥ ९ ॥

sat-asat-jnaana-samudraat samuddhRtam
brahmaNaH prasAdena

sat-jnAna-uttama-ratnam mayaa nimagnam
From the ocean of true and false knowledge, by delving
I have brought out the great gem of truth with my boat-of-intellect

sat	true
asat	false
jnaana	knowledge
samudraat	ocean-from
samuddhRt	delving
am	by Brahma's
brahmaNa	
H	
prasAdena	grace

sat-	truth
jnAna	best
uttama	gem
ratnam	by me
mayaa	brought
nimagna	out
m sva-	self-
mati	intellect
navaa	boat

A small dose of Aryabhata

- These are only a few items of Aryabhata's Kalakriya and Gola

Deva Vandnam

PRAYERS AND PLAYFULNESS

Aryabhateeyam

प्रणिपत्यैकमनेकं कं सत्यां देवतां परं ब्रह्मा ।
आर्यभटस्त्रीणी गदति गणितं कालक्रियां गोलं ॥
दशगीतिका 1

ப்ரணிபத்யேகமனேகம் கம் ஸத்யாம் தேவதாம் பரம்
ப்ரம்மா
ஆர்யபடஸ்த்ரீஞ் கததி கணிதம் காலக்ரியம் கோலம்.

Bowing to Param **Brahma** who is One, Many, Deity of
Truth,

Aryabhata states Three – Mathematics, Time, Sphere. 1

Brhat Samhita

जयति जगतः प्रसूतिर्विश्वात्मा सहजभूषणं नभसः ।
द्रुतकनकसदृशदशतमयूखमालार्चितः सविताः ॥ १ ॥

ஜயதி ஜகதः ப்ரஸூதிர் விஷ்வாத்மா
ஸஹஜபூஷணம் நபஸः:
த்ருத கனக ஸத்ருஷ்ட சதம் அயுக மாலார்சிதः
ஸவிதா

Glory to the **Sun (Savitr)**, author of Earth and soul of
the World, ornament of Space and all Life, who is
adorned by endless thousand golden rays

Brahma sphuta Siddhantam

जयति प्रणतसुरासुरकिरीटरत्नप्रभाषुरितपादः ।
कर्ता जगदुत्पत्तिस्थितिविलयानां महादेवः ॥
ज्ञयती प्रणेतृ सौराशौर कीर्तरत्नं प्रपासरीत
पातः
कर्ता ज्ञकृत् उत्पत्तिं संस्थिति विलायाणाम्
मण्डोत्तेवः

Glory to **Mahadeva**, Cause of Creation, Preservation and Destruction, whose feet shine by the gems of the crowns of bowing Suras and Asuras

Ganita Sara Sangraha

अलङ्घ्यं त्रिजगत्सारं यस्यानन्तचतुष्टयम् ।
नमस्तस्मै जिनेन्द्राय महावीराय तायिने ॥ १ ॥

alanghyam tri jagat saaram yasyaananta
chatushTayam

Namas tasmai Jinendraaya **Mahaaveera**aya taayinE

அலங்க்யம் த்ரிஜூகத்ஸாரம் யஸ்ய அநந்த
சதுஷ்டயம்
நமஸ்தஸ்மை ஜினேந்த்ராய மகாவீராய தாயினே



Gudhaartha prakaashaka by Ranganatha

ततः स कृष्णो जहंगीर सार्वभौमस्य सर्वाधिगतप्रतिष्ठः ।
श्रीभास्करीयं विवृतं तु बीजं तथा श्रीपतिपद्धतिः सा ॥

tataH sa krushnO **jahaangira** saarvabhaumasya
sarvaadhidigatapratishTa

Sri Bhaskariyam vivrtam tu yena beejaM tathA sripathi
paddhatiH saa

தத ஸ க்ருஷ்ணோ ஜஹங்கீர ஸார்வபெளமஸ்ய
ஸர்வாதிகடப்ரதிஷ்ட |

ஸ்ரீபாஸ்கரீயம் விவ்ருதம் து யேன பீஜம் ததா ஸ்ரீபதி
பத்ததி ஸா ||

Laghu-Bhaskariya Bhaashyam

by Sankaranarayana

आचार्यर्थभटं वराहमिहिरं श्रीमद्भुरु भास्करम्
गोविन्दं हरिदत्तमत्र शिरसा वक्ष्ये प्रणम्य क्रमात् ॥

AcAryAryabhaTam varAhamihiram
srimadguru bhAskaram
govindam haridattamatra shirasA vakshye
praNamya kramAt

ஆசர்யார்யபடம் வராஹமிஹிரம் பூஞ்சத்துரு பாஸ்கரம் |
கோவிந்தம் ஹரிதத்தம் அத்ர ஷிரஸா வச்சுயே
ப்ரணம்ய க்ரமாத் ||

वराहमिहिरः VaraahamihiraH

Brhat Samhita, chapter 76, is about perfumes(gandha)

Gandhya-yukti : Combinatorics

From 16 basic perfumes, choosing any 4, how many combinations are possible?

Varahamihira answers 1820 combinations.

$$16C_4 = 1820$$

षोडशके द्रव्यगुणे चतुर्विकल्पेन भिद्यमानानाम् ।

अश्टदश जायन्ते शतानि सहितानि विंशत्या ॥

shoDashake (16) dravya (perfumes) guNe catur-vikalpena
(choosing four) bhidyamAnAm

ashTdashA (18) jAyante shatAni (100) sahitAni vimshatyA (20)

वराहमिहिरः VarahaMihiraH

Gandhya-yukti : Combinatorics

He then gives a method to construct a Meru (table)

A verse explains how to arrange this as a prastaara

Very similar to the chandas-prastaara of *Pingala*

1	5	15	35	70	126	210	330	495	715	1001	1365	1820		
1	4	10	20	35	56	84	120	165	220	286	364	455	560	
1	3	6	10	15	21	28	36	45	55	66	78	91	105	120
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

$$nCr = n-1Cr-1 + n-2Cr-1 + n-3Cr-1 + \dots n-rCr$$

**BRAHMAGUPTA
GANAKA CHAKRA CHOODAAMANI**

ब्रह्मगुप्तः

BrahmaGupta

For almost every stanza of Aryabhata,
Brahmagupta writes a chapter.

Bhaskara II does the same.

Aryabhata has 108 stanzas. The others have a 1008.

Bhaskara calls Brahmagupta *Ganaka Chakra Chudamani*

Crown-jewel in Ocean of Mathematicians

Multiplication Methods

Brahmagupta gives FOUR methods of multiplication

1. Gomutrika (cow's urine – zig zag method)
2. khanDa (parts multiplication)
3. bheda
4. ishTa (desired or assumed number)

A cow's urine falls in a zig-zag manner.
Multiply like it

For example: $1254 * 231$

$$\begin{array}{r} 2 \ 1254 & 2508 \\ 3 \ 1254 & 3762 \\ 1 \ 1254 & 1254 \\ \hline & \end{array}$$

Multiplication - khanDa and ishTa

khanDa (multiplication by parts)

$$13 * 158 = (6 + 7) * 158 = 6 * 158 + 7 * 158 = 948 + 1106 \\ = \dots$$

$$32 * 751 = (2 * 4 * 4) * 751 = 2 * 751 * 4 * 4 = \dots$$

ishta-guNana (algebraic)

$$105 * 17 = (100 + 5) * 17 = 100 * 17 + 85 = 1785$$

$$94 * 13 = (94 + 6) * 13 - (6 * 13) = 100 * 13 - 78 = 1300 - 78 = 1222$$

We learn this but as associative, commutative
property etc

With no mention of BrahmaGupta

Integer Addition - Brahmagupta

धनयोः धनं ऋणं ऋणयोः धन ऋणयोः अन्तरं सम ऐक्यं
खं ।

ऋणं ऐक्यं च धनं ऋणं धन शून्योः शून्ययोः शून्यं ॥

Dhanam : Positive number

RNam : Negative number

Kham or Shunyam : Zero

Dhanam + Dhanam = Dhanam

RNam + RNam = RNam

Dhanam + RNam = Dhanam or RNam

Dhanam + RNam = kham (if samam)

Dhanam + Shunyam= Dhanam

RNam + Shunyam = RNam

Shunyam + Shunyam = Shunyam

$$X + Y = Z$$

$$(-X) + (-Y) = (-Z)$$

$$X + (-Y) = W \text{ or } -Z$$

$$X + -X = 0$$

$$X + 0 = X$$

$$-X + 0 = -X$$

$$0 + 0 = 0$$

Integer Subtraction - Brahmagupta

Dhanam - Dhanam = Dhanam

RNam - RNam = RNam

Dhanam - Shunyam= Dhanam

RNam - Shunyam = RNam

Shunyam - Shunyam = Shunyam

Dhanam - RNam = Dhanam

RNam - Dhanam = RNam

Integer Multiplication - Brahmagupta

ऋणं ऋणधनयोः धातो धनमृणयोः धनवधोः धनं भवति

|

शून्यं ऋणयोः ख धनयोः ख शून्योः वा वधः शून्यं ॥

$$RNam * Dhanam = RNam$$

$$RNam * RNam = Dhanam$$

$$Dhanam * Dhanam = Dhanam$$

$$Shunyam * RNam = Shunyam$$

$$kha * Dhanam = Shunyam$$

$$kha * Shunyam = Shunyam$$

$$(-X) * Y = (-Z)$$

$$(-X) * (-Y) = Z$$

$$X + Y = Z$$

$$0 * (-X) = 0$$

$$0 * X = 0$$

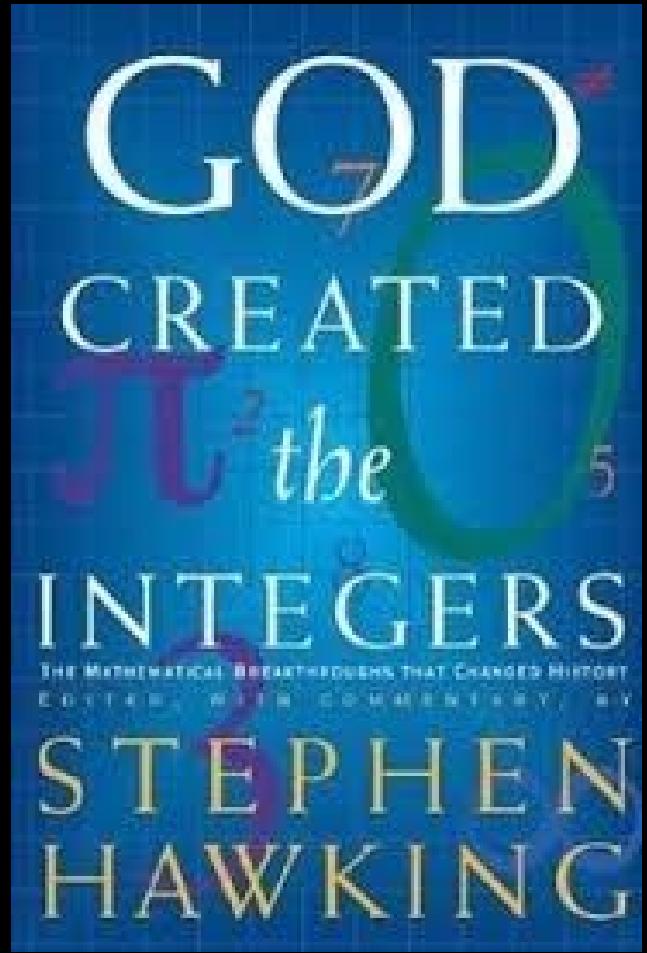
$$0 * 0 = 0$$



Brahmagupta

Maybe God **created** integers.

Brahmagupta **explained** them!!



A bija, by any name

Names given to unknown quantities (variables)

x, y, z	English modern
yaavat taavat	Jaina ~ 300 BC
ishta, iccha	Aryabhata
avyakta	Brahmagupta
bija	Bhaskara

Brahmagupta's Equation (samikaraNa)

Example

$$5x^2 + 10x - 8 = 2x^2 + 1$$

was written as:

itara pakshaa
apara pakshaa

yaa va 5 yaa 10 ru 8
yaa va 2 yaa 0 ru 1



yaa va = yaavat varga (square of unknown)

yaa = yaavat taavat (unknown)

ru = rupa (constant)

The **dot** over 8 indicates it is a negative number

samikaraNa is what we call an equation

itara paksha and apara paksha are
left and right hand sides of equations

Brahmagupta Exponents , Coefficients

अव्यक्त वर्ग घन वर्गवर्ग पञ्चगत षड्गतादीनाम् ।

Avyakta varga ghana vargavarga pancagata
shadgata aadinaam

तुल्यानां संकलितं व्यवकलिते पृथुगतुल्यानाम् ॥ 18-41 ॥

Tulyaanaam samkalitam vyavakalite

avyakta tulyaanaam unknown

tulyaanaam Equal values

samkalitam Addition

vyavakalita Subtraction

corresponding
ly

varga	square
ghana	cube
vargavar ga	4 th power
pancagat a	5 th power
shadgata	6 th power

tulyaanaam here means coefficients of equal powers

Brahmagupta - Roots of Quadratic Equation

वर्गचतुर्गुणितानां रूपाणां मध्यवर्गसहितानाम् ।

varga-catur-guNitAnAm rUpANAm madhya-varga-sahitAnAm

मूलं मध्येनोनं वर्गद्विगुणोद्धतं मध्यः ॥

$$mUlam \text{ n } dhrtam \text{ n } x = \frac{\sqrt{(4AC + B^2)} - B}{2A} \text{ juNod-}$$

varga \rightarrow A → Coefficient of varga (x^2)

catur-guNitAnAm \rightarrow Multiplied by 4

rUpANAm \rightarrow C → of Constant

madhya-varga-sahitAnAm \rightarrow B² → Middle-square-added

mUlam \rightarrow x → Root of all this above

madhyena-Unam \rightarrow Minus middle-term (B)

vargA-dviguNod-dhRtam \rightarrow 2 * coeff(varga) : divided by /2A

Brahmagupta - Bhaavita

भावितकरूपगुणन सा अव्यक्तवधा इष्टभाजिता इष्ट-आप्त्योः ।

bhaavitaka-rUpa-guNaa sA avyaktavadhA ishTabhAjitA
ishTaptyoH

अल्पेऽधिकोऽधिकेऽल्पः क्षेपः भावितहृतौ व्यस्तम् ॥

Alpe-adhiko-adhike-alpaH kShepaH bhaavita-hrtau vyastam ॥ 60
॥

$$Ax + Cy = D$$

Choose m , such that $q = (AD + BC) / m$

$$x = (C + m) / A \quad ; \quad y = (B + q) / A$$

bhaavitaka	Coefficient of xy (A)
rupa	Constant (D)
avyakta	Coefficients of x,y (B,C)
iShTa	desired number (m)
aapti	Obtained quotient (q)
bhaavtiaka rupa	$A * D$
guNana	$B * C$
avyakta-vadhA	

Varga Prakriti

KuTTaka solves linear, indeterminate

Varga Prakriti solves quadratic, indeterminate
equations

Quadratic indeterminate

$$Nx^2 + 1 = y^2$$

Given a known N,

a series of numbers x and y can be calculated

Bhaavana

Bhavana means ***composition***

Brahmagupta discovered that if (m,n) solves $Ax^2 + J = y^2$ and (p,q) solves $Ax^2 + K = y^2$

Then $(mq+pn, nq+Amp)$ solves $Ax^2 + J^*K = y^2$

Also $(mq-pn, nq-Amp)$ solves $Ax^2 + J^*K = y^2$

Basically, Brahmagupta discovered that
composing two solutions to one equation
produced a third solution the same equation

Mumford on Brahmagupta

It seems clear to me that Brahmagupta is the key person in the creation of Algebra as we know it

David Mumford, Fields Medalist

Email to Prof Amartya Kumar Datta 8.2.2013

Brahmagupta – Area of Triangle and Quadrilateral

स्थूलफलं त्रिचतुर्भुजबाहुप्रतिबाहुयोगदलघातः ।

भुजयोगार्द्धचतुष्यभुजोनघातात् पदं सूक्षमम् ॥

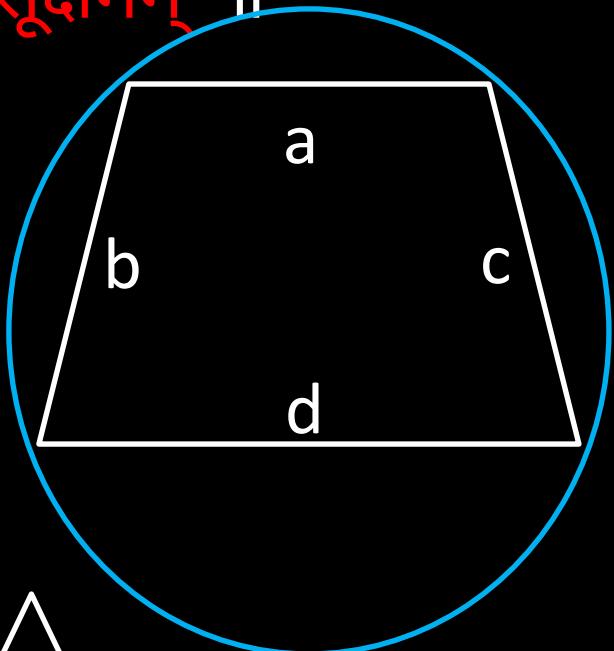
12-21 ॥

Cyclic Quadrilateral

Half perimeter $s = (a + b + c + d) / 2$

Gross Area = $[(a + d) / 2] * [(b + c) / 2]$

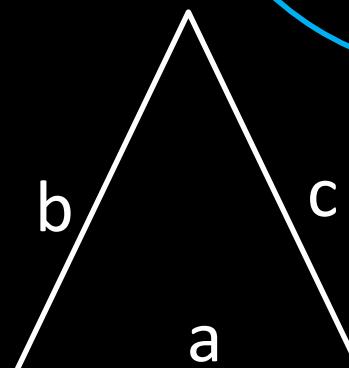
Exact Area = $\sqrt{[(s-a)(s-b)(s-c)(s-d)]}$



Triangle

Half perimeter $s = (a + b + c) / 2$

Exact Area = $\sqrt{[(s-a)(s-b)(s-c)]}$



Brahmagupta – Diagonals of Quadrilateral

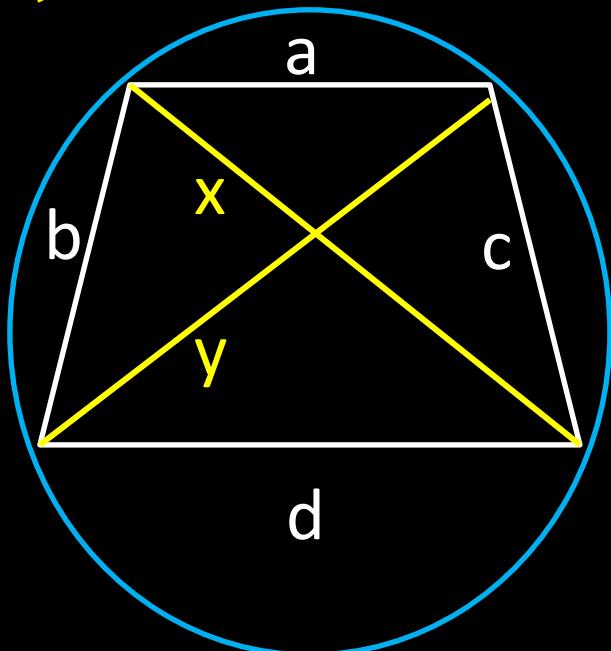
कर्णाश्रितभुजगातैक्यमुभयान्योन्यभाजितं
गुणयेत् ।

योगेन भुजप्रतिभुजवदयोः करणौ पदे विषमे ॥

12-28 ॥

$$\text{Diagonal } x = \frac{(ac+bd) * (ad+bc)}{(ab+cd)}$$

$$\text{Diagonal } y = \frac{(ab+cd) * (ad+bc)}{(ac+bd)}$$



Yantra - Brahma Sphuta Siddhanta

सप्तदश कालयन्त्राण्यतो धनुस्तुर्यगोलकंचक्रम् ।

यष्टिः शंकुर्धटिका कपालकं कर्त्तरी पीठम् ॥

सलिलं भ्रमिऽवलम्बः कर्णश्छाया दिनार्धमर्कोऽक्षः ।

नतकालज्ञानार्थं तेषां संसाधनान्यष्टौ ॥ 23-5,6 ॥

saptadasha kAla-yantraANy-ato dhanu-turyagolakam-cakram
yaShTiH shankur-ghaTikA kapAlakam kartari pITham ||
salilam bhrami-avalambaH karNa-sh-chAyA dinArkam-
arkokshaH

nata-kAla-jnAnArtha teShAm saMsAdhanA-anyashTau ||

Instruments - Brahma Sphuta Siddhanta

Angle Measurement

- Dhanur yantra - bow
- Turyagolaka yantra - Quadrant (1/4 circle)
- Cakra yantra – wheel
- Peetha yantra – Pedestal or seat
- Kapaala yantra – bowl or potsherd
- Dinaardha yantra – Midday measure instrument
- Arka yantra – Sun instrument
- Aksha or palaansha yantra – small degree arc measure insrument

Geometrical Drawing instruments

- Bhrama or saana yantra – for drawing circles (compass)
- Karna or chaaya karna - Set of squares or diagonals

Instruments - Brahma Sphuta Siddhanta

Shadow Measurement

- Yashti yantra – pole or staff
- Shanku yantra – gnomon

Length Measurement

- Rajju- rope
- danDa – measuring rod

Time Measurement

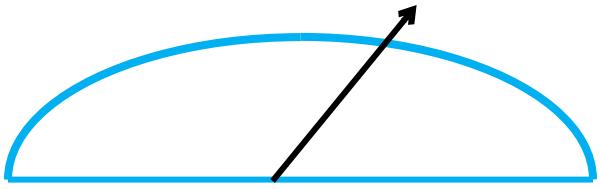
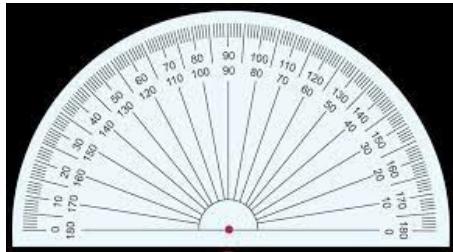
- Ghatika yantra – pot or clock
- Chaaya or shanku chaaya – Sundial

Level instruments

- Salila yantra – water leveller
- Avalamba sutra – Plumbline

यन्त्रम् YANTRAM

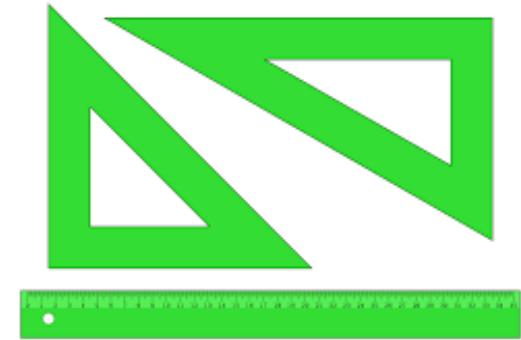
Astronomical Instruments



धनुः यन्त्र
Protractor



गोल यन्त्र
Armillary
sphere



कर्ण यन्त्र
Set square

Most Indian instruments were made of cheap and perishable material like mud, clay, rope, string, wood, water

Most were also not very large or based on complex mechanical arrangements

Hence very few of them except some bronze devices have survived as artifacts

Instruments - Europe

- 1600 Galileo used a handheld telescope
 - A few feet long
- 1670 Newton made larger reflecting telescopes
- 1790 Herschel telescope 1.2 m lens diameter, 40 feet focal length, metal tube
 - Weighed 450 KG
- Large observatories
- 1950 Radio telescopes cover several acres

Instruments - India

Era	Yantras
1400BC Vedanga Jyotisha	Water clock (ghati)
800-500 BC Sulba Sutras	Rajju (rope), ghati
500BC-500 AD Eighteen Siddhantas	Shanku (gnomon), danda (stick)
400 BC Arthashastra	Naalika (waterclock), chaaya (shadow)
500 AD Aryabahata	Bhrama (compass), jala (water), lambaka (plumb line), karNa (set square)
625 Brahmagupta	Dhanush (protractor), peeTa (seat), yashTi(shadow), bhrama(compass), cakra(wheel), Gola Yantra (Armillary sphere), ghatika(pot), avalamba
850 Lalla	Bhagana, salaaka, shakaTa

Yantra-prakaasha

तनुनेत्रैर्न्यूना नृपतिरहिता राजनगरी
सरस्य निष्पद्मा युवतिरपि कान्तेन रहिता ।
निशा निःशीतांशुः सरिदपि यथा चक्ररहिता
तथा ज्योतिर्विद्या भवति विफला यन्त्ररहिता ॥

*tanur netrair nyūnā nr̥patirahitā rājanagarī
sarasyo niśpadmā yuvatir api kāntena rahitā /
niśā niḥśītāṁśuḥ sarid api yathā cakrarahitā
tathā jyotirvidyā bhavati viphalā yantrarahitā //*

Like body without eyes, royal capital minus the king,
lakes devoid of lotus flowers, a young woman without a lover,
the night without the moon, a river bereft of *Cakravāka birds*,
even so astronomical science is fruitless without instruments.

— Rāmacandra Vājapeyin, 1428, *Yantra-prakāśa*

Brahmagupta innovations

Arithmetical of Zero

Arithmetical of Integers

Unknowns – avyakta – x, y, z

Equations : sama, samikarana

Exponents > 3: pancagata, shadgata

Solution of quadratic equation

Fractions : five types

Cyclic quadrilaterals

Varga Prakriti $Nx^2+1=y^2$

Bhaavita : equations with two unknowns x,y

Bhaavana : composition of functions $f(x) \bullet f(y) = g(x,y)$

This is only a sample of Brahmagupta's innovations

Brahmagupta

Becoming a Master

कुट्टक ख ऋण धन अव्यक्त मध्यहरण एकवर्ण
भावितकैः ।

आचार्यः तन्त्रविदां ज्ञातैः वर्गप्रकृत्या च ॥

kuTTaka- solve kuTTaka problem

kha rNa dhana- operations with zero, -ve and +ve
qualities

avyaktha - doing mathematical operation with
unknowns

madhya haraNa – eliminationn of middle term in a
quadratic

eka varNa – solving equations with single unknown
bhaavita – solving equations with products of

Indian Numerals

Note that before 5th century
ONLY India developed
Zero and place value system

Most of these **arithmetic and algebraic**
procedures, algorithms etc
are NOT applicable to or inventable in
Egyptian, Greek, Sumerian, Chinese number
systems

Mathematics in School

Nearly 80 EIGHTY percent
of school mathematics is Indian

But we think it is Greek!

Hence, this course:
Indian Astronomy and Mathematics

R. Gopu

writergopu@yahoo.com

VarahaMihiraGopu.blogspot.com

Thank you