Space Flight Mechanics - Assignment 2

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1 Approach to tracing the orbit

Given to us are initial and final position vectors wrt geocentric equatorial frame and time difference of motion. We need to find orbital elements and plot ground track.
We can use $\vec{r_1}, \vec{r_2}, \Delta t$ as inputs for the lambert problem and get $\vec{v_1}, \vec{v_2}$. Then we can get the orbital elements using $\vec{v_1}, \vec{r_1}$. They have been mentioned in 2. Then ground track is plotted by finding Right ascension and Declination wrt rotating earth for small timesteps.

2 Orbital Elements for the given trajectory

- h - 7.6096e+04 km^2/s
- e - 1.2005
- i - 1.0301 rad
- a - 3.2922e+04 km
- θ - 0.6977 rad
- Ω - 2.26904 rad
- ω - 4.5374 rad

3 Ground track

The θ_∞ for the given hyperbolic trajectory was found to be 146.4046 deg. Since the whole trajectory can't be plotted as time period doesn't exist for hyperbolic trajectories, the ground track was plotted from -136.4046 to 136.4046 deg. See Fig 1. The ground track for example 4.12 has been plotted in Fig 2 for 3.25 time periods. Orbital elements for the example problem are below.

- h - 56544 km^2/s
- e - 0.1976
- i - 60 deg
- a - 8350 km
- θ - 230 deg
- Ω - 270 deg
- ω - 45 deg

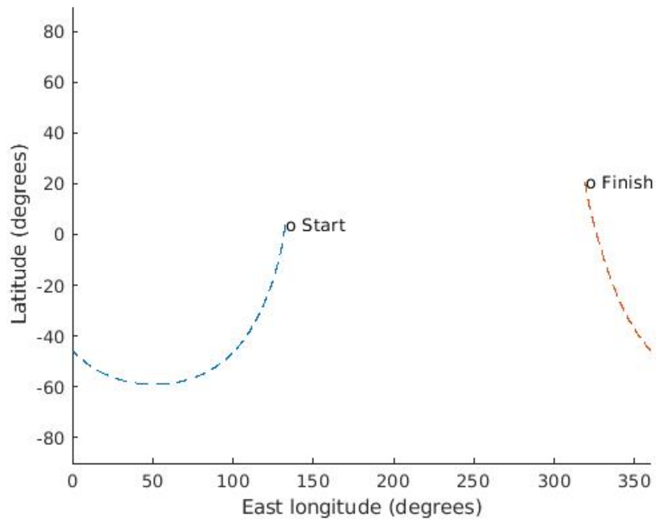

Figure 1: Ground track for the given spacecraft

4 Lunar Transfer Trajectory

4.1 Approach (Fig 3)

- The transfer involves a satellite orbiting earth in the Low Earth Orbit of 2000km initially.
- A delta v is given to the satellite which takes it to the sphere of influence of the moon in an elliptic orbit.
- After that the problem becomes a three body problem.
- The satellite moves from the transfer trajectory to a moon parking orbit (MPO).
- A parking orbit is an orbit that is used to keep the spacecraft temporarily before transferring to another orbit.
- The transfer is modelled from EPO till the sphere of influence of the moon.
- The aim is to calculate the vectors $\vec{R_2}$ and $\vec{R_1}$. After that it is a lambert problem and delta v can be found using $\vec{V_1}$ and $\vec{V_{epo}}$.

4.2 Design of EPO

- The moon's orbit inclination is 5.14 deg wrt ecliptic plane.
- The earth equator is 23.44 deg wrt ecliptic plane.
- Thus, the inclination of moon's orbit wrt geocentric equatorial frame is 28.58 deg.
- The EPO is chosen such that its inclination is close to 28.58 deg. After multiple iterations it was chosen to be 45 deg.
- The aim was to design an EPO such that the satellite moves into the transfer trajectory near the apogee.
- From the orbital elements, the vectors $\vec{R_1} = \vec{R_{epo}}$ and $\vec{V_{epo}}$ are found. While choosing the orbital elements the required position vector of transfer from EPO to transfer orbit was kept in mind.
- The orbital elements of EPO are below.

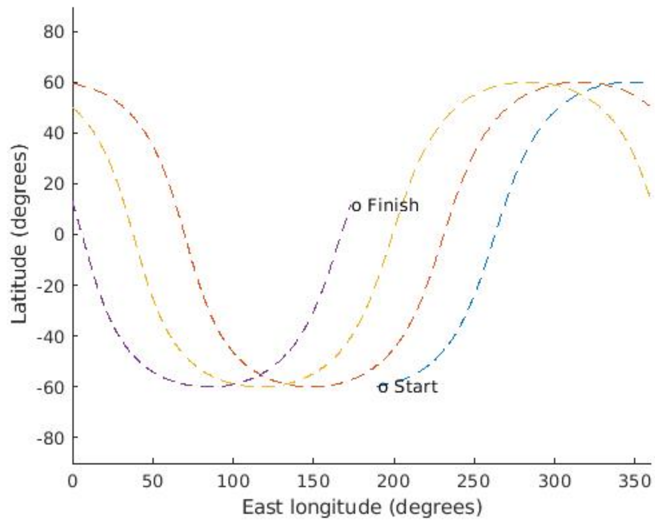

Figure 2: Ground track for example 4.12 - 3.25 Time periods

- h - 7.7531e4 km^2/s
- e - 0.8
- i - 45 deg
- a - 8378 km (LEO)
- θ - 190 deg
- Ω - 0 deg
- ω - 30 deg

4.3 Design of Transfer Trajectory

- The vector from earth's centre to moon - R_{moon} in code, is found using the magnitude 386378km, Right ascension which was taken as 30 deg and Declination of 28.58 deg.
- $\vec{R_{SOI}}$ is the vector from moon's centre to the point 2 at the sphere of influence as shown in the Fig 3.

$$\vec{R_2} = \vec{R_{SOI}} + \vec{R_{moon}} \tag{1}$$

$$\Delta v = \quad \bmod \vec{V_1} - \vec{V_{epo}} \tag{2}$$

- Using vector addition, $\vec{R_2}$ is found. Thus, the start and end points of the transfer trajectory were found.
- Using $\vec{R_1}$ and $\vec{V_1}$ the orbital parameters were found.

h km2̂/s	e	RA rad	inc rad	w rad	TA rad	a km
228209.0733	1.3266	6.1690	0.7007	4.8109	5.3962	171963.5340

Table 1: Orbital Parameters for the transfer trajectory

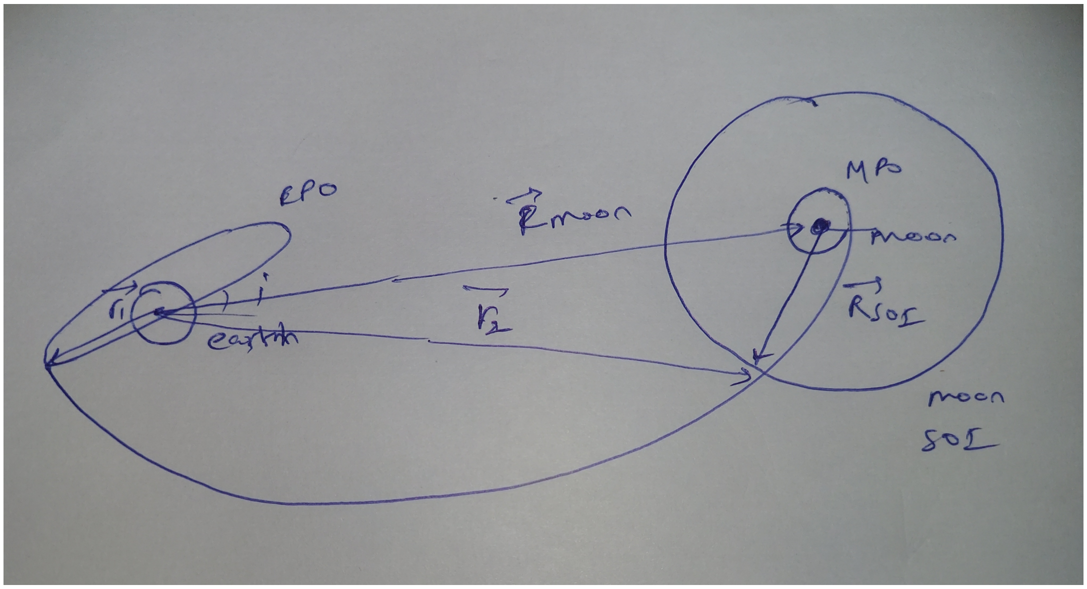

Figure 3: Schematic of the Lunar Transfer Trajectory

4.4 Numbers

- dt = 3 days
- $\vec{R_2}$ = 3.2893I + 1.3265J + 1.4277K 10^5 km
- $\vec{R_1}$ = -5.4452I + -3.2308J + -3.2308K 10^4 km
- $\vec{R_{SOI}}$ = 3.5085I + -3.6997J + -4.2064K 10^4 km
- $\vec{R_{moon}}$ = 2.9384I + 1.6965J + 1.8484K 10^5 km
- delta v = 2.06 km/s

4.5 Observations

- As the dt is increased, the delta v requirement decreases. For ex when time is decreased to 2 days, the delta v increases to 2.389 km/s.
- The inclination of initial EPO plays an important role in the delta v requirements.
- In general, the orbital elements decide the velocity and radius vectors, so they have to be chosen carefully.
- On looking at available literature for trans lunar injections, it is found that delta v requirements are around 3km/s for such transfers.

4.6 Shortcomings

- The inclination of moons orbit varies from minimum to maximum. This has to be taken into consideration.
- The moon's orbit is not circular. It has an e = 0.054.
- Dates of transfer from EPO to transfer trajectory can be analyzed for a more efficient synchronized lunar transfer.
- In order to reduce delta v even more, the satellite can be moved into multiple successive larger EPO's and finally transferred to an MPO. This is a possible improvement.