Space Flight Mechanics - Assignment 1

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1 Nomenclature

r_0	Magnitude of radius vector at t = t_0
r	Radius vector magnitude
v	Velocity vector magnitude
χ	Universal anomaly
$S(z)$	Strumpff function
$C(z)$	Strumpff function
v_0	Magnitude of initial velocity vector
α	Inverse of semi major axis
μ	Gravitational parameter
Δt	Time interval
f, g	Lagrange coefficients

2 Algorithm

2.1 Strumpff Functions

The strumpff functions are expressed in terms of piecewise functions (1). This is programmed easily in matlab.

2.2 Finding universal anomaly

We need to find the universal Kepler's equation for the universal anomaly χ given Δt, r_0, v_{r0} and α. Initial estimate for χ_0 is taken. Newton's method is used to calulate the value of χ as shown in 2.

2.3 Finding Lagrange coefficients

Given radius and velocity vector at t = t_0 we can get r_0, v_0, v_{r0} and α.
We can find the universal anomaly using r_0, α, v_{r0} and Δt.
Lagrange coefficients can be obtained using the formulae in 3.

2.4 Computing Velocity and Position vectors

Position vector is found using f and g that are found in the previous step. Norm of it gives r.
r_0, α, r, χ are used to calculate $\dot{f}$ and $\dot{g}$. This is used to calculate the velocity vector then. 4

3 Link to Repository

GitHub

4 Outputs

4.1 Part 1

- $\vec{r}_0$ = 20,000i -105000j - 19,000k (km).
- $\vec{v}_0$ = 0.9000i -3.4000j -1.5000k (km/s).
- After t = 2hr.
- $\vec{r}$ = (26333.9, -128732, -8095.28) (km).
- $\vec{v}$ = (0.861369, -3.20446, 1.52378) (km/s).

4.2 Part 1 Calculations

- Initial position vector norm (km): r0 = (108563)
- Initial velocity vector norm (km/s): v0 = (3.82361)
- vr0 (km/s) = (3.19168)
- alpha (1/km) = (-1.8256e-05)
- x ($km^{0.5}$) = (37.9686)
- f = (0.993346)
- g (1/s) = (7185.53)
- r (km) = (131647)
- fdot (1/s) = (-1.68462e-06)
- gdot = (0.994513)

4.3 Part 2

- $\vec{r}_0$ = 30,000i -100,000j -20,000k (km).
- $\vec{v}_0$ = 0.8i -3.5j -2k (km/s).
- After t = 2hr.
- $\vec{r}$ = (35544.1, -124468, -34234.8) (km).
- $\vec{v}$ = (0.744827, -3.31158, -1.95529) (km/s).

Algorithm to find universal anamoly (χ)

given Δt, r_0, v_{r_0}, α

(i) use $\chi_0 = \sqrt{\mu}\,|\alpha|\,\Delta t$ as the starting value of χ

(ii) At a given step $\chi = \chi_i$ is obtained from the previous step, calculate

$$f(\chi_i) = \frac{r_0 v_{r_0}}{\sqrt{\mu}} \chi_i^2 C(z_i) + (1 - \alpha r_0)\chi_i^3 S(z_i) + r_0 \chi_i - \sqrt{\mu}\,\Delta t$$

$$f'(\chi_i) = \frac{r_0 v_{r_0}}{\sqrt{\mu}} \chi_i \left[1 - \alpha \chi_i^2 S(z_i)\right] + (1 - \alpha r_0)\chi_i^2 C(z_i) + r_0$$

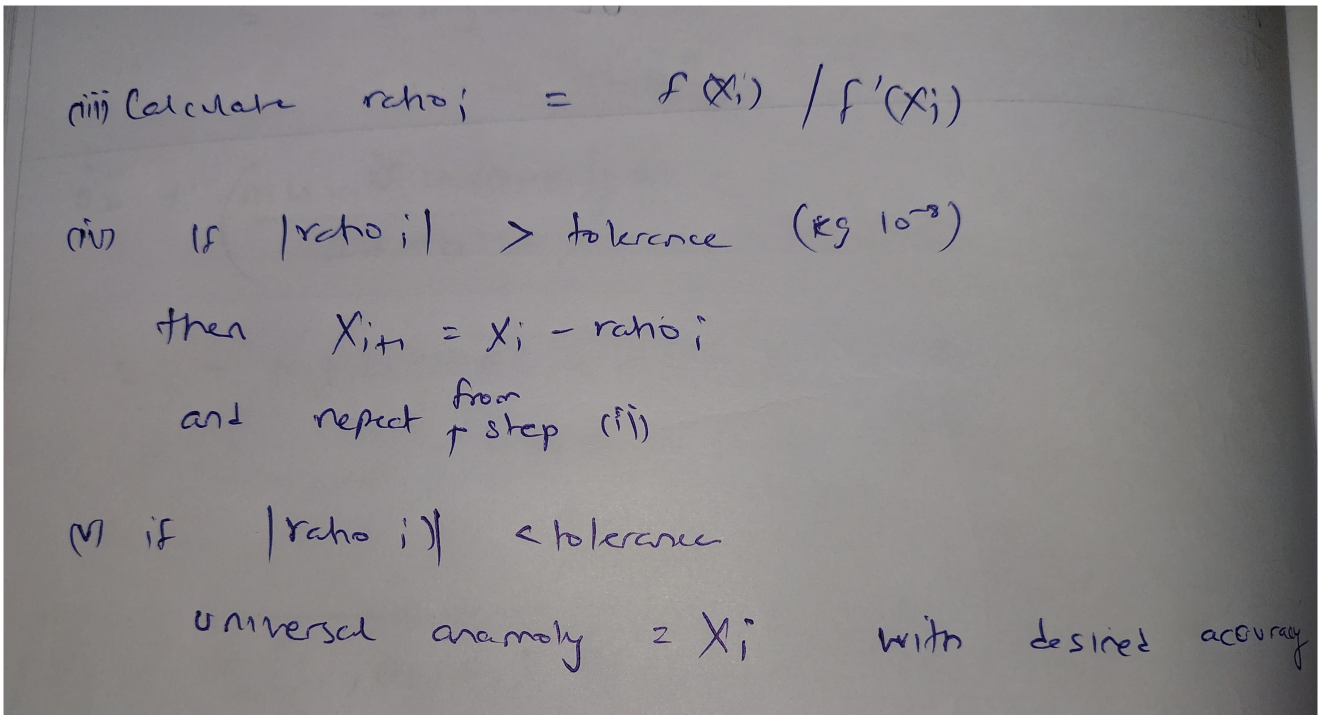
where $z_i = \alpha \chi_i^2$

where $C(z_i)$ is a strumpff function

$$C(z_i) = \begin{cases} \dfrac{1 - \cos\sqrt{z}}{z} & z > 0 \\ \dfrac{\cosh\sqrt{-z} - 1}{-z} & z < 0 \\ 0.5 & z = 0 \end{cases}$$

$$S(z_i) = \begin{cases} \dfrac{\sqrt{z} - \sin\sqrt{z}}{(\sqrt{z})^3} & z > 0 \\ \dfrac{\sinh(\sqrt{-z}) - (\sqrt{-z})}{(\sqrt{-z})^3} & z < 0 \\ 1/6 & z = 0 \end{cases}$$

Figure 1: Finding Universal Anomaly



(iii) Calculate $ratio_i = f(X_i) / f'(X_i)$

(iv) If $|ratio_i| >$ tolerance (eg 10^{-8})

then $X_{i+1} = X_i - ratio_i$

and repeat from step (ii)

(v) if $|ratio_i| <$ tolerance

universal anamoly $= X_i$ with desired accuracy

Figure 2: Finding Universal Anomaly

Given position, velocity vector $\vec{r}_0, \vec{v}_0$ at time t_0, we can find $\vec{r}, \vec{v}$ at time t.

$$\vec{r} = f\vec{r}_0 + g\vec{v}_0$$

$$\vec{v} = \dot{f}\vec{r}_0 + \dot{g}\vec{v}_0$$

$$f = 1 - \frac{\chi^2}{r_0} C(\alpha \chi^2)$$

$$g = \Delta t - \frac{1}{\sqrt{\mu}} \chi^3 S(\alpha \chi^2)$$

$$\dot{f} = \frac{\sqrt{\mu}}{r r_0} \left[\alpha \chi^3 S(\alpha \chi^2) - \chi\right]$$

$$\dot{g} = 1 - \frac{\chi^2}{r} C(\alpha \chi^2)$$

Algorithm to find $\vec{r}, \vec{v}$.

(i) use $\vec{r}_0, \vec{v}_0$ to find $|\vec{r}_0|, |\vec{v}_0|$

$$|\vec{r}_0| = \sqrt{\vec{r}_0 \cdot \vec{r}_0} \qquad |\vec{v}_0| = \sqrt{\vec{v}_0 \cdot \vec{v}_0}$$

(ii) $$v_{r_0} = \frac{\vec{r}_0 \cdot \vec{v}_0}{r_0}$$

(iii) $$\alpha = \frac{2}{r_0} - \frac{v_0^2}{\mu}$$

if $\alpha > 0$ ellipse, $\alpha < 0$ hyperbola, $\alpha = 0$ parabola.

(iv) with $r_0, v_{r_0}, \Delta t, \alpha$ use previous algorithm to find universal anomaly χ.

(v) Use $\alpha, r_0, \chi, \Delta t$ to find f, g and subsequently $\vec{r}$.

Figure 3: Finding Lagrange Coefficients and Final Velocity and Radius vectors

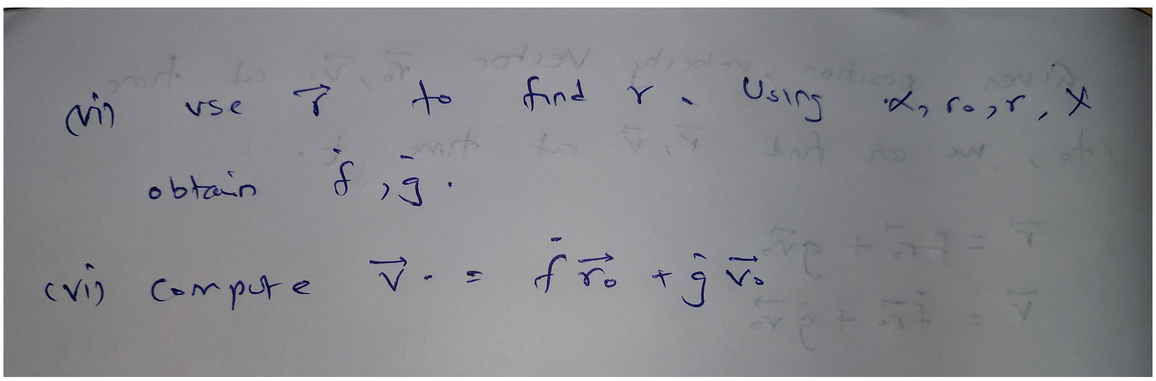


Figure 4: Finding Lagrange Coefficients and Final Velocity and Radius vectors