

## Question 1

### **Making a bipartite graph.**

Left vertices represent population of the corresponding cities and the right vertices represent the number of pods in the corresponding of cities.

### **Connecting**

Connect each left vertex city with super source by a directed edge of population of this city as  $p_i$ .

Connect right vertices with the super sink  $t$  which represent the maximum flow and at initial it is 0.

The capacity with the edge is the number of pods which are constructed by each city.

Connecting left hand vertices and right hand vertices with directed edge if two cities can travel by each.

And the capacity of the edge is the number of pods.

And in the process of finding max flow, we also add another edge weighted as  $t(i, j)$  which is the number of days transfer from  $i$  to  $j$ . If the sum of path travelled exceed  $X$ , that is not feasible path, we need choose another path. (Assume the path is  $1 \rightarrow 2 \rightarrow 4 \rightarrow t$ , if  $t(1,2) + t(2,4) > X$ , in this situation, this path is not valid.)

Therefore, we construct a bipartite graph, and turn the question into the flow network.

Using Edmonds-Karp algorithm to find the max flow.

Therefore, we can get the largest number of invaders.