Assignment 2

Due date Tuesday Jun 30, 2020 at 9am

You have **five problems**, marked out of a total of 100 marks.

NOTE: Your solutions must be typed, machine readable .pdf files. **All submissions will be checked for plagiarism!**

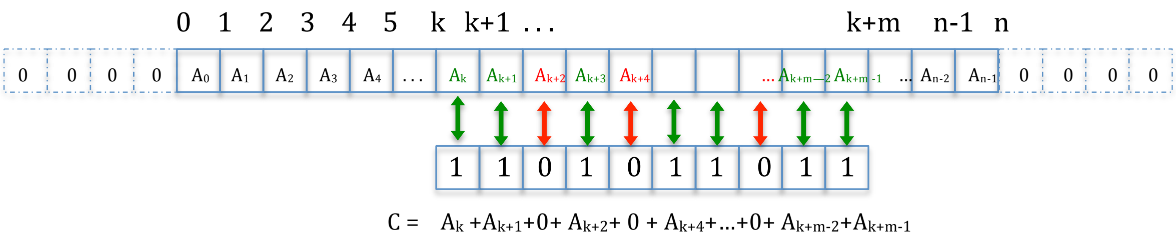
1. Given positive integers M and n compute M^n using only $O(\log n)$ many multiplications. (15 pts)

   Hint: You might want to write n in binary, i.e., as $n = 2^{k_1} + 2^{k_2} + 2^{k_m}$ where $k_1 > k_2 > \ldots k_m$ and $k_1 = \lfloor \log_2 n \rfloor$; then $M^n = M^{2^{k_1}} M^{2^{k_2}} \ldots M^{2^{k_m}}$. This involves at most $\lfloor \log_2 n \rfloor$ multiplications. So it is enough to compute all of m^{2^j} for all $1 \leq j \leq \lfloor \log_2 n \rfloor$ with at most $\lfloor \log_2 n \rfloor$ multiplications. To do that, use repeated squaring.

2. You are given a polynomial $P(x) = A_0 + A_1 x^{100} + A_2 x^{200}$ where A_0, A_1, A_2 can be arbitrarily large integers. Design an algorithm which squares $P(x)$ using only 5 large integer multiplications. (15 pts)

   Hint: Use substitution $y = x^{100}$, square the resulting polynomial and then substitute back y with x^{100}.

3. Assume you are given a map of a straight sea shore of length $100n$ meters as a sequence on $100n$ numbers such that A_i is the number of fish between i^{th} meter of the shore and $(i+1)^{th}$ meter, $0 \leq i \leq 100n - 1$. You also have a net of length n meters but unfortunately it has holes in it. Such a net is described as a sequence N of n ones and zeros, where 0's denote where the holes are. If you throw such a net starting at meter k and ending at meter $k + n$, then you will catch only the fish in one meter stretches of the shore where the corresponding bit of the net is 1. Find the spot where you should place the left end of your net in order to catch the largest possible number of fish using an algorithm which runs in time $O(n \log n)$. (30 pts)



   Hint: Let N' be the net sequence N in the reverse order; compute the sequence $A * B'$; see the figure and then look where the largest value of this sequence is.

4. (a) Compute the convolution $\langle 1, \underbrace{0, 0, \ldots, 0}_{k}, 1\rangle * \langle 1, \underbrace{0, 0, \ldots, 0}_{k}, 1\rangle$ (10 pts)

   (b) Compute the DFT of the sequence $\langle 1, \underbrace{0, 0, \ldots, 0}_{k}, 1\rangle$ (10 pts)

   Hint: Find the polynomial associated with the sequence $\langle 1, \underbrace{0, 0, \ldots, 0}_{k}, 1\rangle$ *and then square it to get (a) and evaluate it at all roots of unity of order* $k+2$ *to get (b).*

5. Find the sequence x satisfying $x * \langle 1, 1, -1\rangle = \langle 1, 0, -1, 2, -1\rangle$.(20 pts)

   Hint: Clearly, x is a sequence of length 5+1-3=3; write it as $\langle a, b, c\rangle$. Find the polynomials which correspond to sequences x and $\langle 1, 1, -1\rangle$ and multiply them. Equate the coefficients of the product polynomial with the terms of the sequence $\langle 1, 0, -1, 2, -1\rangle$.