Question 1

Compute M^n

Write n in binary form,
$n = 2^{k_1} + 2^{k_2} + 2^{k_3} + \cdots + 2^{k_m}$ where $k_1 > k_2 > k_3 > \cdots > k_m$
And $k_1 = \lfloor \log_2 n \rfloor$

Therefore $M^n = M^{2^{k_1}+2^{k_2}+2^{k_3}+\cdots+2^{k_m}} = M^{2^{k_1}} \cdot M^{2^{k_2}} \cdot M^{2^{k_3}} \cdots M^{2^{k_m}}$

It involves at most $\lfloor \log_2 n \rfloor$ multiplications.

So, it can represent as A^{2^k}, $1 \leq k \leq \lfloor \log_2 n \rfloor$ and at most $\lfloor \log_2 n \rfloor$ multiplications

Therefore, compute M^n using $O(\log n)$ multiplications.