Question 4

a)

Let M = ⟨1,0, 0, … ,0, 1⟩; N = ⟨1,0, 0, … ,0, 1⟩;

$P_m(x) = 1 + x^{k+1}; P_n(x) = 1 + x^{k+1}$

$$P_m(x) \cdot P_n(x) = (1 + x^{k+1})(1 + x^{k+1})$$

$$= (1 + 2x^{k+1} + (x^{k+1})^2)$$

$$= (1 + 2x^{k+1} + x^{2k+2})$$

$$= \langle 1,\underbrace{0, 0, \dots ,0}_{k},2,\underbrace{0,0, \dots}_{k-1},1\rangle$$

b)

A = ⟨1,0, 0, … ,0, 1⟩ and it has length of k + 2

$P_A(x) = 1 + x^{k+1}$

$$DFT(A) = \langle P_A(\omega_{k+2}^0), P_A(\omega_{k+2}^1), P_A(\omega_{k+3}^2), \dots, P_A(\omega_{k+2}^{k+1})\rangle$$

$$= \langle 1 + \omega_{k+2}^{0\cdot(k+1)}, 1 + \omega_{k+2}^{1\cdot(k+1)}, \dots, 1 + \omega_{k+2}^{(k+1)\cdot(k+1)}\rangle$$

$$= \langle 2, 1 + \omega_{k+2}^{1\cdot(k+1)}, 1 + \omega_{k+2}^{2\cdot(k+1)}, \dots, 1 + \omega_{k+2}^{(k+1)\cdot(k+1)}\rangle$$