Question2

Setup

$R \times \ N$ grid of squares

Only move to the right and below. (from question definition)

From the question, we need find the minimum number of moves from lower elevation to higher elevation.

In other words, we only find the minimum times of gaining elevation to reach square$(C, 1)$ from square$(1, R)$

Subproblem

Find the minimum times of gaining elevation to reach square $(\boldsymbol{i}, \boldsymbol{j})$

Let $E(i, j)$ represents a number of the elevation of the terrain at that square.

find each square's minimum numbers of gaining elevation

$F[(i,j),(m,n)]$ uses to judge if gaining elevation, return 1 if right and down square's elevation is smaller than the square ($E(i,j) < E(i+1,j)$ or $E(i,j) < E(i,j+1)$)
Return 0 if it is bigger.
However, in this question we use backtrack method so in the recursion, comparing elevation with left and up square. (($E(i,j) > E(i,j+1), E(i,j) > E(i-1,j)$ return 1)

Then define a function to trace the square which is visited.

Recursion

$$opt(i,j) = \min\{opt(i,j+1) \ + F[(i,j),(i,j+1)], opt(i-1,j) + F[(i,j),(i-1,j)] : i \le C, j \le R, j+1 \le R\}$$

$$F[(i,j),(m,n)] = \begin{cases} 1 & , if \ E(i,j) > E(m,n) \\ 0 & , if \ E(i,j) \le E(m,n) \end{cases}$$

In order to get the path, define the function to record which square is visited.

$$From(i,j) = \arg\{opt(i,j)\}$$

This function returns the index (m,n) which produce the minimum value of opt(i,j)
For example, if opt(i,j) is making from opt(i,j+1), the function returns (i,j+1)

Final solution

To get the result, we backtrack from square (C,1) then using function From(i,j) to backtrack the path until we reach (1,R)

Find $opt(C, 1)$ and after recursion and the function $From(i, j)$ we can get the path.

Time Complexity

There are N*R grids, therefore time complexity is $O(n^2)$

Example Case

There are 3*3 grids

7	2	14
8	1	12
8	12	15

Using back track, find $opt(3,1)$, and the function $From(3,1)$

Then after recursion we can get the table

	1	2	3
3	0 Start point	0 From(2,3)=(1,3)	1 From(3,3)=(2,3)
2	1 From(1,2)=(1,3)	0 From(2,2)=(2,3)	1 From(3,2)=(3,3),(2,2)
1	1 From(1,1)=(1,2)	1 From(2,1)=(2,2)	2 From(3,1)=(2,1),(3,2)

Backtrack the path we get (3,1)(2,1)(2,2)(2,3)(1,3) or (3,1)(3,2)(3,3)(2,3)(1,3) or (3,1)(3,2)(2,2)(2,3)(1,3)

Reverse above we can get the path.