Question4

Set up

G = (V,E)
There are n vertexes
i is node, $1 \leq i \leq n$
k is length, $1 \leq k \leq K$

Subproblem:

The subproblem is the maximum amount of the total weight when it travels k length.

$opt(i, k)$ be the maximum number of the sum of maximum weight when it travels k length.

$W(j, i)$ is the weight of the edge form vertex j to i;

j node represents all possible value that can connect from j to i ($\mathbf{1 \leq \boldsymbol{j} \leq \boldsymbol{i}}$)

Recursion

$$\boldsymbol{opt(i, k) = \max\{opt(j, k-1) + W(j, i), \dots\}}$$

(for example:
if $i = 1$, there are vertex 2,3,5 can connect to vertex 1, the recursion would be
$opt(1, k) = \max\{opt(2, k-1) + W(2,1), opt(3, k-1) + W(3,1), opt(5, k-1) + W(5,1)\}$
)

Define a function to find the path (which vertex has been visited):

$$From(i, k) = \arg\big(opt(i, k)\big)$$

This function returns the vertex that produce the maximum weight.
(Like above example, if 3 can make the total maximum weight, this function returns 3)

Base case

Let base case $\boldsymbol{opt(i, 0) = 0}$

Final solution

$$\max\{opt(1, k), opt(2, k), opt(3, k) \dots opt(n, k)\}$$

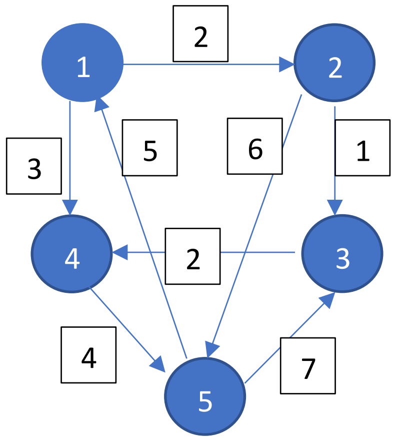
if find the maximum optimal solution, using defined function $From(i, k)$ to get the path for the solution.

Time Complexity

Runs in $|V|^3$

Example Case

There is a directed weighted graph.
In this case, let K = 3



There are 5 vertexes.

Using opt() function to find every vertex's maximum solution.

Vertex 1:
$opt(1,3) = \max\{opt(5,2) + W(5,1)\}$
Using recursion function:
$opt(5,2) = \max\{opt(2,1) + W(2,5), opt(4,1) + W(4,5)\}$

$opt(2,1) = \max\{opt(1,0) + W(1,2)\} = 2$
$opt(4,1) = \max\{opt(1,0) + W(1,4), opt(3,0) + W(3,4)\} = \max\{3,2\} = 3$

Therefore $opt(5,2) = \max\{2 + 6, 3 + 4\} = 8$
So, $opt(1,3) = 8 + 5 = 13$

Do the same on $opt(2,3), opt(3,3), opt(4,3), opt(5,3)$

Then, the maximum of them is the solution.

In this case, opt(3,3) and opt(4,3) are the solution which are 15.

By using $From()$ function, we can find the path.

For $opt(3,3)$ 1->2->5->3
For $opt(4,3)$ 2->5->3->4