Assignment 5 hints

NOTE: Due date August 7, 2020 at 8 PM sharp!

You have **three problems**, marked out of a total of 60 marks.

NOTE: Your solutions must be typed, machine readable .pdf files. **All submissions will be checked for plagiarism!**

1. Today was just a regular day for everyone in Krypton until a news flashed that a meteor is going to destroy Krypton in X days. Krypton has N cities, some of which are connected by bidirectional roads. You are given a road map of Krypton; for every two cities C_i and C_j which are connected by a (direct) road from C_i straight to C_j you are given the value $t(i, j)$ which is the number of days to travel from city C_i to city C_j. (You can of course also go from a city C_m to city C_k without a direct road from C_m to C_k by going through a sequence of intermediate cities connected by direct roads.)

   In each city C_i the Krypton Government built q_i pods to carry inhabitants in case of any calamity, which will transport them to Earth. City C_i has population p_i. As soon as the people hear this news they try to save themselves by acquiring these pods either at their own city or in other city before the meteor destroys everything. Note that a pod can carry only one person. Find the largest number of invaders the Earth will have to deal with. (20 pts)

   Hint: this is a typical Max Flow problem. Each inhabitant of Krypton can access the pods which are either in their own city or in cities which are less than X day's trip away from their city, so start by determining for each city C_i the set of cities you can reach within X days (what algorithm can you use for this task?). To make the representation compact, make a bipartite graph with vertices corresponding to all cities both on the left and on the right side but with different interpretation: on the left vertices represent populations of the corresponding cities; on the right the vertices represent the set of ponds in the corresponding cities. How do you turn this into a (precisely described) flow network?

2. You are given a usual $n \times n$ chess board with k white bishops on the board at the given cells (a_i, b_i), $(1 \leq a_i, b_i \leq n$, $1 \leq i \leq k)$. You have to determine the largest number of black rooks you can place on the board so that no two rooks are in the same row or in the same column and are not under the attack of any of the k bishops (recall that bishops go diagonally).(20 pts)

Hints: Make a bipartite graph with all the columns as vertices on the left hand side and all the rows as vertices on the right hand side. Each square s_{ij} can now be represented by and edge from column j to row i. Think which edges you allow.

3. There are N computers in a network, labelled $\{1, 2, 3, \ldots, N\}$. There are M one-directional links which connect pairs of computers. Computer 1 is trying to send a virus to computer N. This can happen as long as there is a path of links from computer 1 to computer N. To prevent this, you've decided to remove some of the links from the network so that the two computers are no longer connected. For each link, you've calculated the cost of removing it. What is the minimum total cost to disconnect the computers as required, and which edges should be removed to achieve this minimum cost? (20 pts)

   Hint: Again, a typical Max Flow - Min Cut problem. What should be the capacities of the edges be set to?