Question 5

a)

f(n) = $(\log_2(n))^2$

g(n) = $\log_2(n^{\log_2 n})^2$ = $2 * \log_2 n * \log_2 n = 2 * (\log_2(n))^2$

g(n) > f(n) for all n > 0;

So f(n) = O(g(n))

b)

f(n) = n^{10}

g(n) = $2^{\sqrt[10]{n}}$

then $\log_2 f(n) \leq \log_2 C * 2^{n^{\frac{1}{10}}}$

then $10 * \log_2 n \leq \log_2 c + \log_2 2^{n^{\frac{1}{10}}})$

then $10 * \log_2 n \leq \log_2 c + (\mathrm{n}^{\frac{1}{10}})\log_2 2$

then we take c = 2 so 10 * $\log_2 n \leq n^{\frac{1}{10}}$

then $\lim_{n\to\infty} \frac{n^{\frac{1}{10}}}{10*\log_2 n}$ using L'Hˆopital's to compute the limit

we can get $\lim_{n\to\infty} \frac{\frac{1}{10}*n^{-\frac{9}{10}}}{10*n*\ln 2}$ = 0

Since it is 0, for sufficiently large n we will have $\frac{n^{\frac{1}{10}}}{10*\log_2 n}$ < 1

We said g(n) = O(f(n)).

C)

It is easy to prove there are $0 \le c_1 * g(n) \le\ f(n) \le c_2 * g(n)$

So $0 \le c_1 * n \le n^{1+(-1)^n} \le c_2 * g(n)$

When n is odd $n^{1+(-1)^n} \le c_2 * n$ when $c_2 \geq 1$ for all n > 0.

When n is even $c_1 * n \le n^{1+(-1)^n}$ when $c_1 \geq 1$ for all n > 0.

Therefore we can said that f(n) = Θ(g(n)).