Question3

Setup

There are N days, 3 activities.

For each day $i \leq N$ and each job $j \leq 3$

We assume $F(i,j)$ represent the amount of enjoyment of i_{th} day and j_{th} activity.

Subproblem

The subproblem is the maximum amount of enjoyment form first day to i_{th} day.

$opt(i,j)$ represents the maximum of the sum of the enjoyment.

Using backtrack method, finding total enjoyment on last day.

Recursion:

$$opt(i,j) = \max : \begin{cases} opt(i-1,2) + F(i,j), opt(i-1,3) + F(i,j) : if\ j = 1 \\ opt(i-1,1) + F(i,j), opt(i-1,3) + F(i,j) : if\ j = 2 \\ opt(i-1,1) + F(i,j), opt(i-1,2) + F(i,j) : if\ j = 3 \end{cases} \}$$

$$MAX(N,j) = \max \{ \begin{cases} opt(N,1) \\ opt(N,2) \\ opt(N,3) \end{cases}$$

Using backtrack to find the sum of enjoyment on the last day. Then the max one is final solution.

Base case

The base case $opt(1,j) = F(1,j)$

Final solution

$$MAX(N,j) = \max \{ \begin{cases} opt(N,1) \\ opt(N,2) \\ opt(N,3) \end{cases}$$

Time complexity

$O(n^2)$

Example Case

There are 3 days, N = 3

	Day1	Day2	Day3
Activity 1	5	2	9
Activity 2	10	1	3
Activity 3	9	3	15

Using the recursion function and using back track method.

Compare total employment on last day for each activity

We first find:

$opt(3,1) = \max\{opt(2,2) + F(3,1), opt(2,3) + F(3,1)\}$

Then after the recursion we can get the answer for opt(3,1)

Do the same on opt(3,2) and opt(3,3)

We can get the sum of enjoyment for each steps.

	Day1	Day2	Day3
Activity 1	5	12 From(1,2)	22 From(2,3)
Activity 2	10	10 From(1,3)	16 From(2,3)
Activity 3	9	13 From(1,2)	27 From(2,1)

And the final answer is the maximum of them.

In this case the maximum total enjoyment is 27.