Question 2

Solution:

$P(x) = A_0 + A_1x^{100} + A_2x^{200}$

Let $m = x^{100}$, $A_1x^{100} = A_1m$; $A_2x^{200} = A_2(x^{100})^2 = A_2m^2$

Therefore, $P(x) = A_0 + A_1m + A_2m^2$

$[P(x)]^2 = (A_0 + A_1m + A_2m^2)(A_0 + A_1m + A_2m^2)$

$= A_0^2 + A_0A_1m + A_0A_2m^2 + A_0A_1m + A_1^2m^2 + A_1A_2m^3 + A_0A_2m^2 + A_1A_2m^3 + A_2m^4$

$= A_0^2 + A_1^2m^2 + A_2^2m^4 + 2A_0A_1m + 2A_0A_2m^2 + 2A_1A_2m^3$

$= A_0^2 + 2A_0A_1m + (A_1^2 + 2A_0A_2)m^2 + 2A_1A_2m^3 + A_2^2m^4$

As $A_1^2 + 2A_0A_2 = (A_1 + 2A_0)(A_1 + A_2) - A_1A_2 - 2A_0A_1$

$= A_0^2 + 2A_0A_1m + \big((A_1 + 2A_0)(A_1 + A_2) - A_1A_2 - 2A_0A_1\big)m^2 + 2A_1A_2m^3 + A_2^2m^4$

$= A_0^2 + 2A_0A_1x^{100} + \big((A_1 + 2A_0)(A_1 + A_2) - A_1A_2 - 2A_0A_1\big)x^{200} + 2A_1A_2x^{300} + A_2^2x^{400}$

So we need only 5 large integer multiplications:

A_0^2 , A_0A_1 , A_1A_2 , A_2^2 , $(A_1 + 2A_0)(A_1 + A_2)$