

## Question 2

For the bishop, there are **k white bishops at the cell  $C(a_i, b_i)$** ,  $(1 \leq a_i, b_i \leq n, 1 < i < k)$

As bishop goes diagonally, we cannot put rooks on those position.

If white bishops' position is  $C(a_i, b_i)$ ,  $(1 \leq a_i, b_i \leq n, 1 < i < k)$ .

We first make a list about area where can be attacked by bishops.

$$C(x_i, y_i) = \{(x_i - i, y_i - i), (x_i + i, y_i - i), (x_i - i, y_i + i), (x_i + i, y_i + i) | i = 1 \dots n\}$$

The left is the row number right is column number, and all area must be in grids.

Now we get a list about where we cannot put the black rooks.

### Making a bipartite

We make a bipartite graph with all columns as vertices on the left-hand side.

Connect each column with the super source by a directed edge and capacity is equal to 1.

Then, make all rows as vertices on the right-hand side.

Connect each row with the super sink by a directed edge and capacity is equal to 1.

Further, each vertex on left side connect to all vertices on right hand side in same direction.

And the edge capacity is also 1.

But we need eliminate some edges, as the left side vertices is column and right side vertices is row. After connecting, the cell represents as  $C(a_m, b_m)$ ,  $(1 \leq a_m, b_m \leq n, )$ .

If the cell is one of the cells from the list which we make at first (bishops attack area), we eliminate the connection(edge).

(for example, if the  $C(2,2)$  is from the list, we eliminate the edge from 2(left side) and 2(right side)).

Further, if the cell is the same position with white bishops, we also eliminate corresponding edge.

(for example, if white bishop is  $C(2,2)$ , we cut edge of 2(left side vertex) to 2(right side vertex))

Therefore, we construct a bipartite graph and turn this question into the flow network.

**Because we set each edge capacity is 1, no two rooks are in the same row or in the same column.**

Using the Edmonds-Karp algorithm to get the max flow

Therefore, we get the largest number of black rooks we can place on the board.

