Homework 1: Gradient Descent & Friends

```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt

df = pd.read_csv("./real_estate.csv")
```

Question 1 (Pre-processing)

(a) Remove any rows of the data that contain a missing ('NA') value. List the indices of the removed data points. Then, delete all features from the dataset apart from: age, nearestMRT and nConvenience.

```
null_index = df[df.isnull().any(axis=1) == True].index
df.dropna(axis=0, inplace=True)
prices = df['price']
df.drop(columns=['transactiondate', 'latitude', 'longitude', 'price'],
inplace=True)
# print(null_index)
# print(df.head)
print("null indices: " , null_index.values)
```

```
null indices:  [ 19  41 109 144 230 301]
```

(b) normalisation and provide the mean value over your dataset

```
def pre_processing(x_data):
    x_max = x_data.max()
    x_min = x_data.min()
    for _ in range(x_data.size):
        _temp = (x_data[_] - x_min) / (x_max - x_min)
        x_data[_] = _temp
    return x_data

x_new_age = pre_processing(np.array(df['age']))
x_new_mrt = pre_processing(np.array(df['nearestMRT']))
```

```
x_new_nCon = pre_processing(np.array(df['nConvenience']))

x_new_age_mean = x_new_age.mean()
x_new_mrt_mean = x_new_mrt.mean()
x_new_nCon_mean = x_new_nCon.mean()

print("x_new_age_mean: ", x_new_age_mean)
print("x_new_nearestMRT_mean: ", x_new_mrt_mean)
print("x_new_nConvenience_mean: ", x_new_nCon_mean)
```

```
x_new_age_mean:  0.40607932670785213
x_new_nearestMRT_mean:  0.16264267697310722
x_new_nConvenience_mean:  0.4120098039215686
```

Question 2 (Train and Test sets)

first half of observations to create trainning set, remaining half for test set

```
x_new = pd.DataFrame(columns=['age', 'nearestMRT', 'nConvenience'])
x_new['age'] = x_new_age
x_new['nearestMRT'] = x_new_mrt
x_new['nConvenience'] = x_new_nCon
size = x_new.index.size
training_price = prices.values[:int(size / 2)]
test_price = prices.values[int(size / 2):]
training_set = x_new.values[:int(size / 2)]
test_set = x_new.values[int(size / 2):]
```

Print out the first and last rows of both your training and test sets

```
first_training_row = training_set[0]
last_training_row = training_set[-1]
first_test_row = test_set[0]
last_test_row = test_set[-1]

print("first training row: ", first_training_row)
print("last training row: ", last_training_row)
print("first test row: ", first_test_row)
print("last test row: ", last_test_row)
```

```
first training row:  [0.73059361 0.00951267 1.        ]
last training row:  [0.87899543 0.09926012 0.3       ]
first test row:  [0.26255708 0.20677973 0.1       ]
last test row:  [0.14840183 0.0103754  0.9       ]
```

Question 3(Loss Function)

$$L_c(y,\hat{y}) = \frac{1}{n}\sum_{i=1}^{n}\left[\sqrt{\frac{1}{c^2}(y^{(i)} - \langle w^{(t)}, X^{(i)}\rangle)^2 + 1} - 1\right]$$

$$\frac{\partial L_c}{\partial w_k} = \frac{1}{n}\sum_{i=1}^{n}\left[\frac{1}{c^2}(y^{(i)} - \langle w^{(t)}; X^{(i)}\rangle)^2 + 1\right]^{\frac{1}{2}}$$

$$= \frac{\frac{\partial w^{(t)}}{\partial w_k} \cdot (-1) \cdot \frac{1}{c^2} \cdot (y^{(i)} - \langle w^{(t)}; X^{(i)}\rangle) \cdot (2) \cdot (\frac{1}{2})}{\sqrt{\frac{1}{c^2}(y^{(i)} - \langle w^{(t)}, X^{(i)}\rangle)^2 + 1}}$$

$$= \frac{\frac{\partial w^{(t)}}{\partial w_k} \cdot (\langle w^{(t)}, X^{(i)}\rangle - y^{(i)})}{c^2\sqrt{\frac{1}{c^2}(y^{(i)} - \langle w^{(t)}, X^{(i)}\rangle)^2 + 1}}$$

$$= \frac{\frac{\partial w^{(t)}}{\partial w_k} \cdot (\langle w^{(t)}, X^{(i)}\rangle - y^{(i)})}{\sqrt{c^2(y^{(i)} - \langle w^{(t)}, X^{(i)}\rangle)^2 + c^4}}$$

$$= \frac{\boxed{\frac{\partial w^{(t)}}{\partial w_k}} \cdot (\langle w^{(t)}, X^{(i)}\rangle - y^{(i)})}{c^2\sqrt{\frac{1}{c^2}(y^{(i)} - \langle w^{(t)}, X^{(i)}\rangle)^2 + 1}}$$

for $\frac{\partial w^{(t)}}{\partial w_k} = \frac{\partial(w_0 + w_1 x_1^{(i)} + w_2 x_2^{(i)} + w_3 x_3^{(i)})}{\partial w_k}$

$$= \underbrace{1 + x_1^{(i)} + x_2^{(i)} + x_3^{(i)}}_{\text{let } = X_k^{(i)}}$$

$$\frac{X_k^{(i)} \cdot (\langle w^{(t)}, X^{(i)} \rangle - y^{(i)})}{c^2 \cdot \sqrt{\frac{1}{c^2} (y^{(i)} - \langle w^{(t)}, X^{(i)} \rangle)^2 + 1}}$$

Question 4 (Gradient Descent Psuedocode)

gradient descent updates

```
for i <- 0 to nIteration:
    derivate_sum <- 0
    for j <- 0 to tranning_set_size:
        derivate_sum += loss_function_derivate
    derivate_loss_mean <- derivate_sum / tranning_set_size

    w <- w - learning_rate * derivate_loss_mean
```

stochastic gradient descent updates

```
for i <- 0 to nIteration:
    for epoch <- 0 to epoch_times:
        w = w - learning_rate * loss_function_derivate
```

Question 5 (Gradient Descent Implementation)

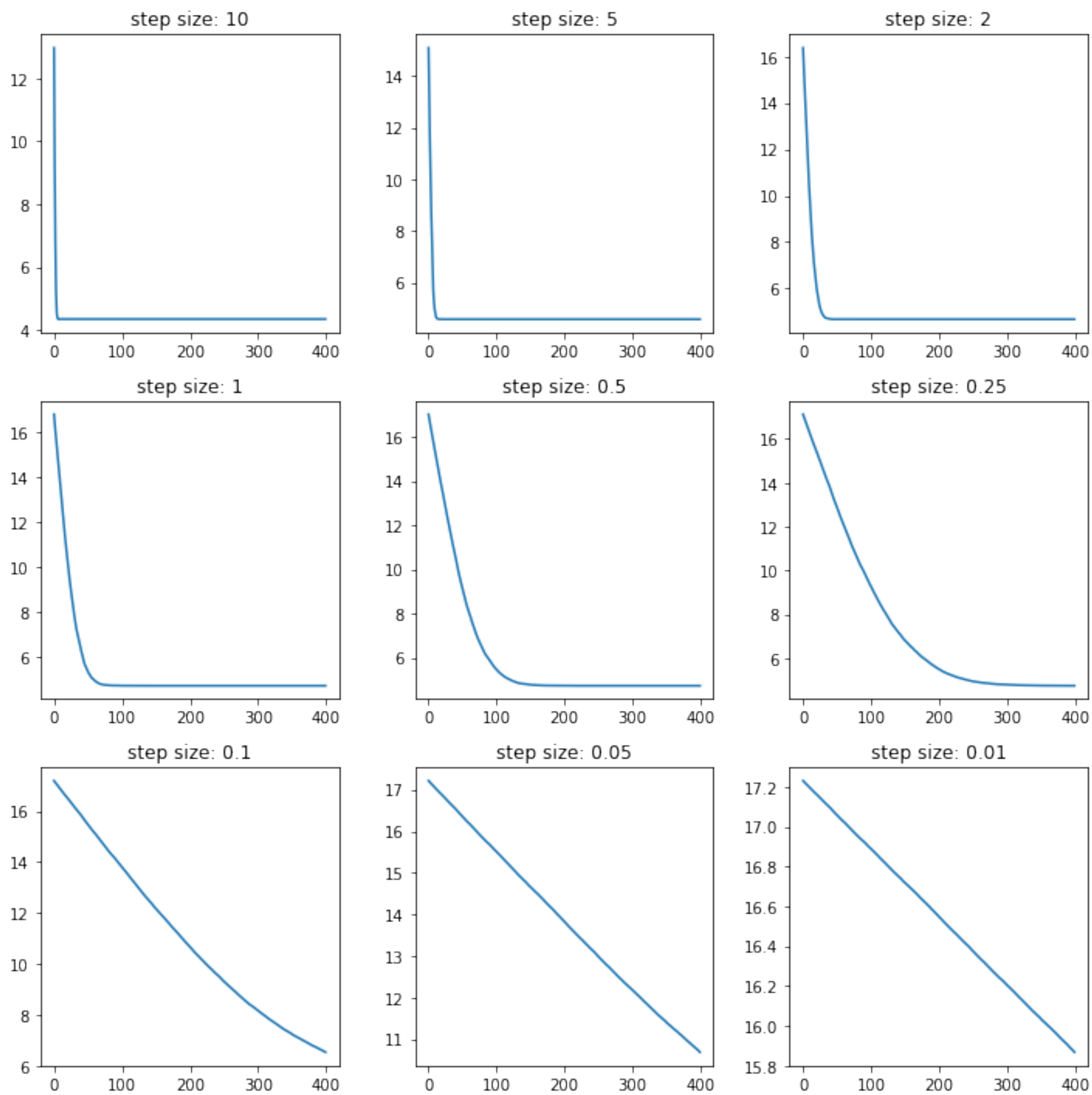
helper function

```
def gradient_update(_X_k_i, w_t, training_values, prices_values, _size):
    _sum_gd = 0
    for _i in range(_size):
        wX = np.dot(w_t, training_values[_i])
        _sum_gd += ((_X_k_i * (wX - prices_values[_i])) / (2 * np.sqrt((wX - prices_values[_i]) ** 2 + 1)))
    return _sum_gd / _size


def loss_achieved(w_t, training_values, prices_values, _size, hyper):
    _sum_loss = 0
    for _i in range(_size):
        wX = np.dot(w_t.T, training_values[_i])
        _sum_loss += (np.sqrt(1 / (hyper ** 2) * (prices_values[_i] - wX) ** 2 + 1) - 1)
    return _sum_loss / _size
```

(a) Generate a 3×3 grid of plots showing performance for each step-size.

```
fig, ax = plt.subplots(3, 3, figsize=(10, 10))
nIter = 400
alphas = [10, 5, 2, 1, 0.5, 0.25, 0.1, 0.05, 0.01]
losses = []
training_size = training_set.shape[0]
training_gd_set = np.insert(training_set, 0, 1, axis=1)
c = 2
loss = []
w_plot = []
for i, ax in enumerate(ax.flat):
    w = np.ones(4)
    for index in range(nIter):
        temp = index
        if temp >= training_size:
            temp -= training_size
```

```
        w = w - gradient_update(training_gd_set[temp], w, training_gd_set, training_price, training_size) * alphas[i]
        loss_mean = loss_achieved(w, training_gd_set, training_price, training_size, c)
        loss.append(loss_mean)
    losses.append(loss)
    ax.plot(losses[i])
    loss.clear()
    ax.set_title(f"step size: {alphas[i]}")

plt.tight_layout()
plt.show()
```

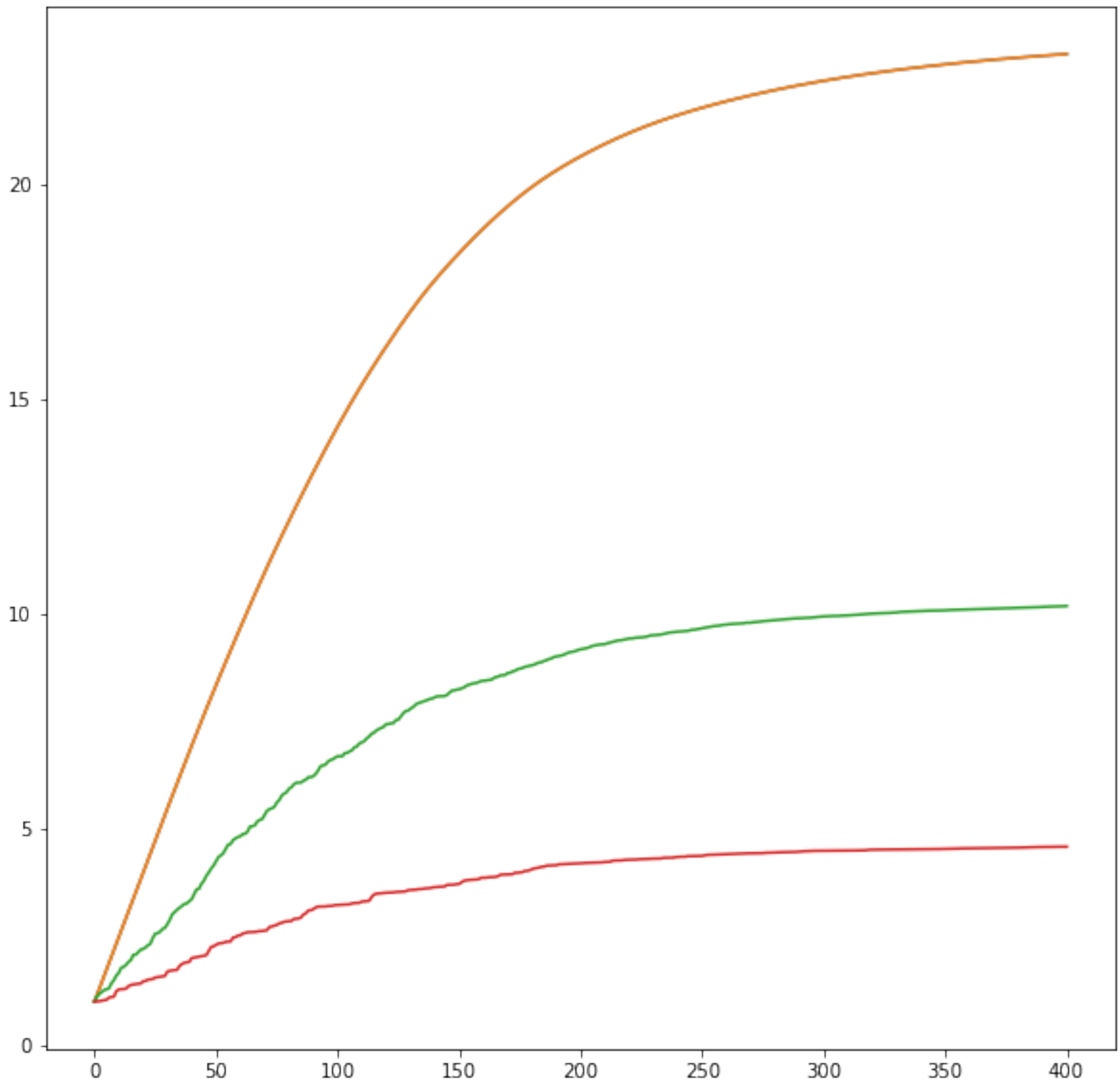


(b) choose an appropriate step size (and state your choice), and explain why you made this choice.

I will choose **step size = 0.5**, for `0.1`, it is more stable and it does not fast to get the lowest loss.

(c) plot the progression of each of the four weights over the iterations, run your model on the train and test set, and print the achieved losses.

```
w_plot.clear()
fig, ax = plt.subplots(figsize=(10, 10))
w = np.ones(4)
w_plot.append(w)
for index in range(nIter):
    temp = index
    if temp >= training_size:
        temp -= training_size
    Xki = np.array([1, training_gd_set[temp][0], training_gd_set[temp][1], training_gd_set[temp][2]])
    w = w - gradient_update(Xki, w, training_gd_set, training_price, training_size) * 0.3
    w_plot.append(w)
ax.plot(w_plot)
plt.show()

print("w0: ", w[0])
print("w1: ", w[1])
print("w2: ", w[2])
print("w3: ", w[3])

test_size = test_set.shape[0]
test_set_gd_set = np.insert(test_set, 0, 1, axis=1)
test_loss = loss_achieved(w, test_set_gd_set, test_price, test_size, c)
train_loss = loss_achieved(w, training_gd_set, training_price, training_size, c)
print("batch training loss: ", train_loss)
print("batch test_loss: ", test_loss)
```

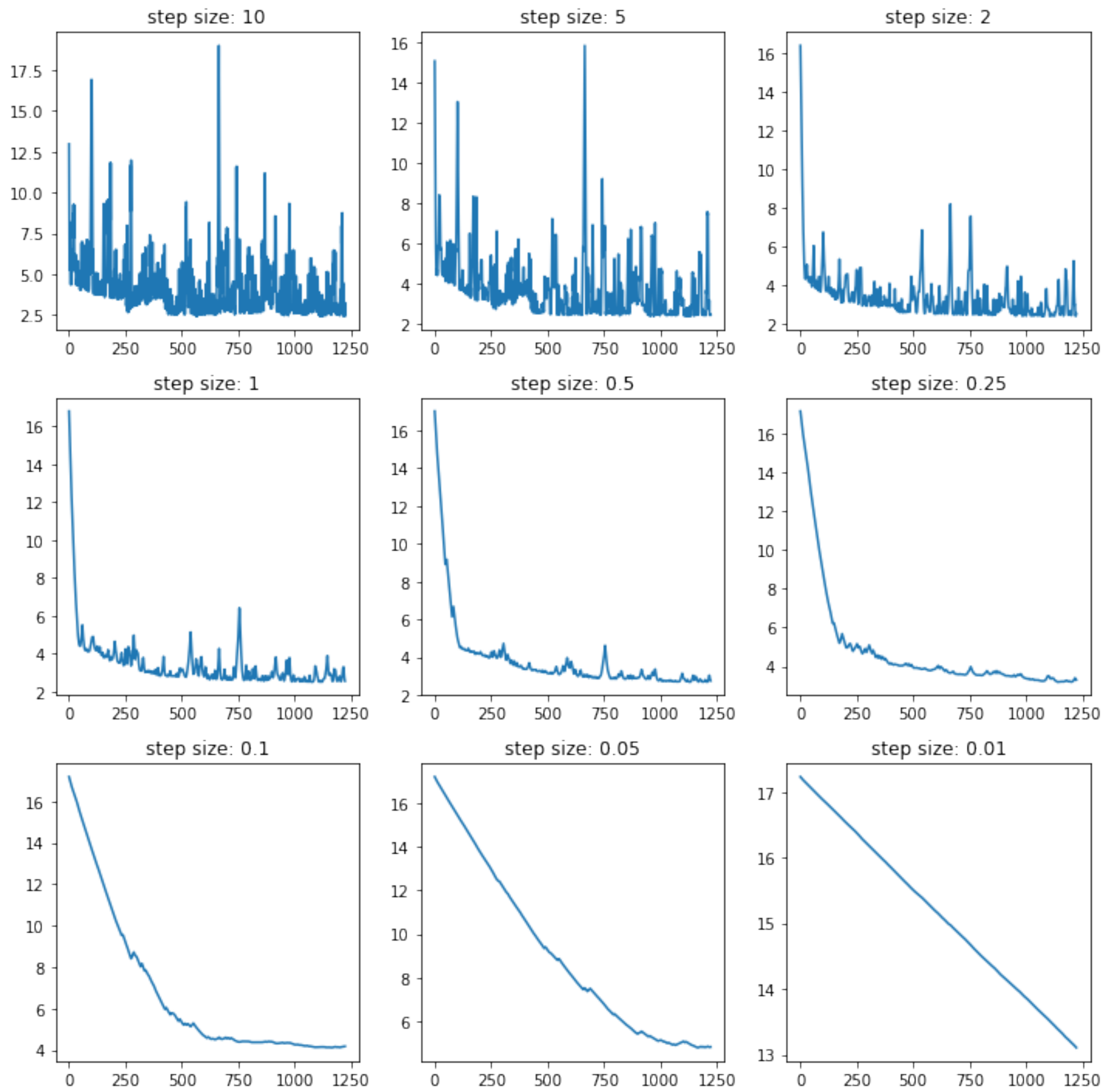



```
w0:  23.022722678965167
w1:  23.022722678965167
w2:  10.192369050047924
w3:  4.597963020259536
batch training loss:  6.097840919924557
batch test_loss:  5.516429121702068
```

Question 6

(a) plot Stochastic Gradient Descent Implementation

```
fig, ax = plt.subplots(3, 3, figsize=(10, 10))
losses.clear()
loss.clear()
epoch_times = 6
for i, ax in enumerate(ax.flat):
    w = np.ones(4)
    for index in range(training_size):
        for _ in range(epoch_times):
            # Xki = np.array([1, training_gd_set[index][0], training_gd_set[index][1], training_gd_set[index][2]])
            Xki = np.array(training_gd_set[index])
            wX = np.dot(w, training_gd_set[index].T)
            derivative_loss = (wX - training_price[index]) / (2 * np.sqrt((wX - training_price[index]) ** 2 + 4))
            w = w - alphas[i] * derivative_loss * Xki
            loss_mean = loss_achieved(w, training_gd_set, training_price, training_size, c)
            loss.append(loss_mean)

    losses.append(loss)
    ax.plot(losses[i])
    loss.clear()
    ax.set_title(f"step size: {alphas[i]}")

plt.tight_layout()
plt.show()
```



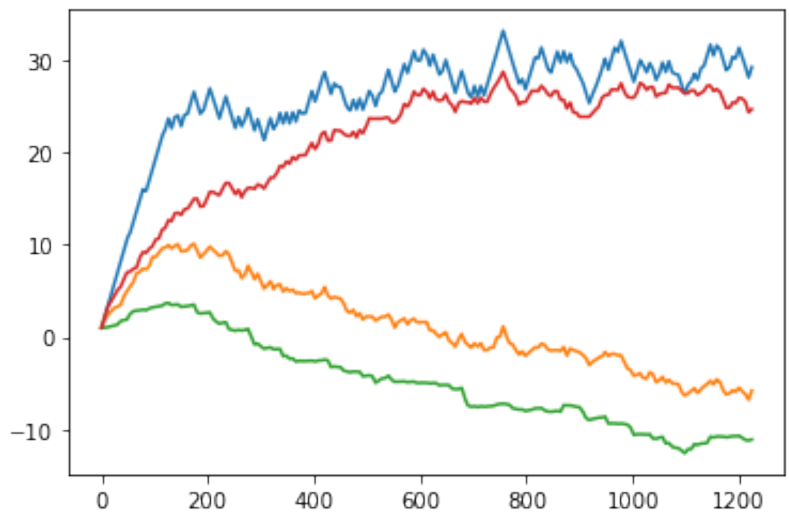
(b) choose an appropriate step size, explain why

I will choose **step size = 0.25**, it is more stable and it can achieve lower loss.

(c) plot the progression of each of the four weights over the iterations, run your model on the train and test set, and record the achieved losses.

```
helper = []
w = np.ones(4)
helper.append(w)
for index in range(training_size):
    for _ in range(epoch_times):
        Xki = np.array(training_gd_set[index])
```

```
        wX = np.dot(w, training_gd_set[index].T)
        derivative_loss = (wX - training_price[index]) / (2 * np.sqrt((wX - training_price[index]) ** 2 + 4))
        w = w - 0.4 * derivative_loss * Xki
        helper.append(w)

plt.plot(helper)
plt.show()

print("w0: ", w[0])
print("w1: ", w[1])
print("w2: ", w[2])
print("w3: ", w[3])

test_loss = loss_achieved(w, test_set_gd_set, test_price, test_size, c)
train_loss = loss_achieved(w, training_gd_set, training_price, training_size, c)
print("SGD train_loss: ", train_loss)
print("SGD test_loss: ", test_loss)
```




```
w0:  29.209580076958723
w1:  -5.802603909995271
w2:  -11.093131590341907
w3:  24.654321874675617
SGD train_loss:  2.8777859585228605
SGD test_loss:  2.8711540128158446
```

Question7. Results Analysis

By adjusting different learning rate, we can get more stable and lower loss model. If we use a big learning rate,
we may get a poor loss, however if we use a small learning rate, we need take more time to wait it to get the good loss.

Since GD need to get all sum and get the mean, it takes lots of time, however, sgd fix this problem.

SGD will frequently update gradient with a high varience causing fluctuation, therefore, GD has a smoother loss path than SGD.