statistically independent. In the numerical example presented below, it turns out that decreasing the number N of trajectories from 1000 to 100 has no sensible effects on the residual of the difference between the true states and the means of the inferred probabilities.The second drawback is the need of large computing times, since the complexity of the resampling procedure is $O(N\log(N))$ [20].

3. Application to a literature example

The MC simulation approach to model-based fault diagnosis has been applied to a simple problem taken from the literature [2,10–15]. This case study has been used frequently for benchmarking dynamic reliability methodologies and fault diagnostic techniques.

The system, sketched in Fig. 1, consists of a tank containing a fluid whose level is controlled based on the signals of suitable detectors which command actions on units 1, 2, 3 to regulate the fluid inlet/outlet flow. A thermal power source W heats uniformly the fluid in the tank under adiabatic conditions (no heat is released to the outside). The objective of the control is to maintain the fluid level x_1 in the range (HLV, HLP).

At any time t, the (hidden) system states are the position parameters $\alpha_i(t)$ of the three units which can only assume the values 1 or 0 according to whether the unit is on ($\alpha_i = 1$) or off ($\alpha_i = 0$); the measured process variables are the fluid level $x_1(t)$ (m) and temperature $x_2(t)$ (°C). The unknown states $\alpha_i(t)$ of the three units depend on the fluid level value $x_1(t)$ according to the following control laws:

$$\alpha_1(x_1) = \begin{cases} 1 & \text{if } x_1 < \text{HLV} \\ 0 & \text{if } x_1 > \text{HLP} \\ 0 \text{ or } 1 & \text{depending on previous switching} \end{cases}$$

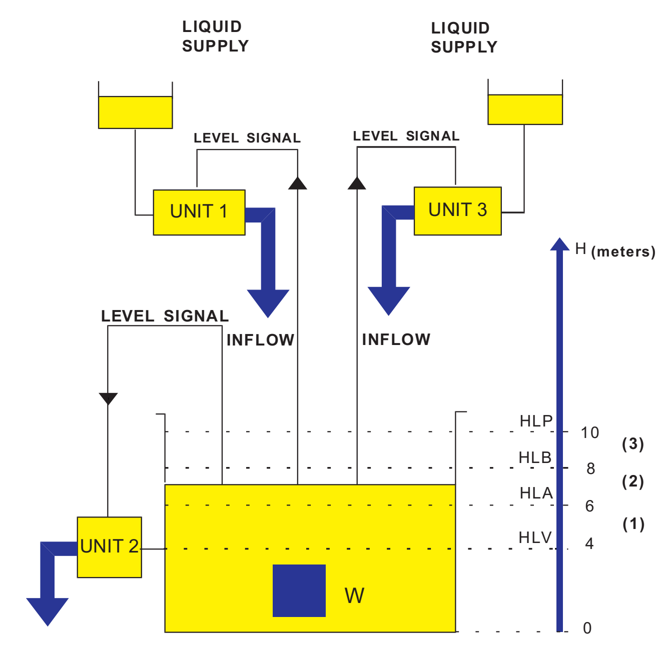

Fig. 1. Sketch of the tank control system [2,14].

$$\alpha_2(x_1) = \begin{cases} 1 & \text{if } x_1 > \text{HLV} \\ 0 & \text{otherwise} \end{cases}$$

$$\alpha_3(x_1) = \begin{cases} 1 & \text{if } x_1 < \text{HLV} \\ 0 & \text{if } x_1 > \text{HLP} \\ 0 \text{ or } 1 & \text{depending on previous switching} \end{cases}$$

unless the units are in a failed state, which can be either on or off, in which case the corresponding α_i states are constant, independent of the fluid level.

The analysis aims at inferring the position of the three units from the measured $x_1(t)$ and $x_2(t)$ profiles. The following simplifying physical assumptions are made, as they do not influence the focus of the analysis: (1) the fluid input in the tank by units 1 and 3 mixes instantaneously; (2) the flow rate through the outlet unit 2 is independent of the fluid head. With these assumptions, the time evolution of the measured variables $x_1(t)$ and $x_2(t)$ can be described by a pair of first-order nonlinear differential equations determined by the mass and energy conservation laws [2]. Discretization of these equations yields ($t = 0,1,2,..$)

$$\begin{aligned} x_1(t) &= x_1(t-1) + dt[\alpha_1(x_1(t-1))Q_1(t-1) + \alpha_3 Q_3(t-1) \\ &\quad - \alpha_2(x_1(t-1))Q_2(t-1)] + v_1(t) \\ x_2(t) &= x_2(t-1) + \frac{dt}{x_1(t-1)}\{[\alpha_1(x_1(t-1))Q_1(t-1) \\ &\quad + \alpha_3 Q_3(t-1)](\vartheta_m - x_2(t-1)) + 23.88915\} + v_2(t) \end{aligned} \tag{9}$$

where the $Q_i(t)$ ($i = 1,2,3$) are the fluid flow rates (m/h) through the units, ϑ_m is the assigned inlet fluid temperature, $v_1(t)$ and $v_2(t)$ are the measurement noises.

Note that the Q_i's are actually random quantities, since the amount of fluid entering or exiting the tank during dt is always affected by (small) fluctuations.

In spite of its simple structure, the system considered is representative of the operation of nonlinear control systems and possesses mathematical features that pose difficulties to the application of conventional model-based estimation techniques. For instance, the linearization of the original differential equations required by the extended-Kalman filter approach is not applicable because of the stepwise dependence of the parameters α_i on the system variable x_1.

The scenario considered in this application is the same of [2]. The relevant data are summarized in Table 1.

Unit 3 is supposed to fail on ($\alpha_3 = 1$) upon demand at time $t = 0$ and to remain in this condition throughout the time of interest. The control thresholds on the level are set at HLV = 4 m and HLP = 10 m. The fluid level and the temperature measurements are supposed to be affected by a 2% and 0.1% Gaussian noise, respectively. The analysis is performed in the time interval (0, 74.5 h) discretized with a dt = 0.5 h. A Gaussian noise of 0.5% is added to the Q_i flow rates to model their fluctuations.