Dimensional analysis - Problem Set

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September 16, 2021

1 Thermal transfer in a building

In order to reduce global warming, we are progressively trying to improve the thermal properties of our houses, particularly their insulation. The insulating ability of the various materials usually found in the walls is measured by a physical variable k [W/(m.K)] called thermal conductivity. This factor is supposed to relate the flux of energy that crosses the walls Q [W/m^2] (power per unit of wall surface) to the difference of temperature $\Delta\theta$ between indoor and outdoor.

(a) Find the scaling relation between Q, $\Delta\theta$ and k. The thickness of the walls e should also appear in your formula.

(b) Deduce the power required to heat a house which total surface of the walls is S.

(c) Consider a house of length 10m, width 10m and height 6m. Both the walls, the roof and the ground are made of wood panels [$k = 0.04$W/(m.K)] of thickness $e = 10$cm. It is observed that, when the outdoor temperature is 17.6° F (265K), the indoor temperature is 65° F (291K), and the ground temperature 32° F (273K), the power used to heat the house is 4256W. Estimate the power required to maintain the house in the same thermal conditions when the wall thickness is 30cm instead of 10cm.

(d) Practically, the flux of heat through the walls does not follow instantaneously the daily variations of the outdoor temperature. Indeed, the walls have a thermal inertia; they are able to store energy and restitute it later. The inertia coefficient ρC is proportional to the specific heat capacity of the material; for the wood, $\rho C = 288000$J/(m^3.K). Find the scaling law for the time shift τ between the external and internal temperature variations as a function of ρC, k, e and $\Delta\theta$. Estimate the time that this wooden house takes to cool from 291K to 265K in absence of heating. Usually, we want some extra cool in summer and some extra heat in winter. In other words, we want this time shift to be about 6 months. What should be the thickness of the walls to satisfy this time shift ? Does it seem reasonable? If not, what else could we improve in order to meet this criterion (one idea, in a few words)?

2 Stride frequency

Enjambée

In Fig. 1, you can see the stride frequency of various mammals as they switch from trot to gallop, represented as a function of their mass. Measure graphically, then explain this scaling law by involving both the kinetic and the elastic energy of the legs (length L, radius r). Remember that the elastic energy of bending is Er^4/L, where E is the Young's modulus of the material, measured in N/m^2. Use the Kleiber's law of elastic similarity to relate r and L to the mass M of the animal.

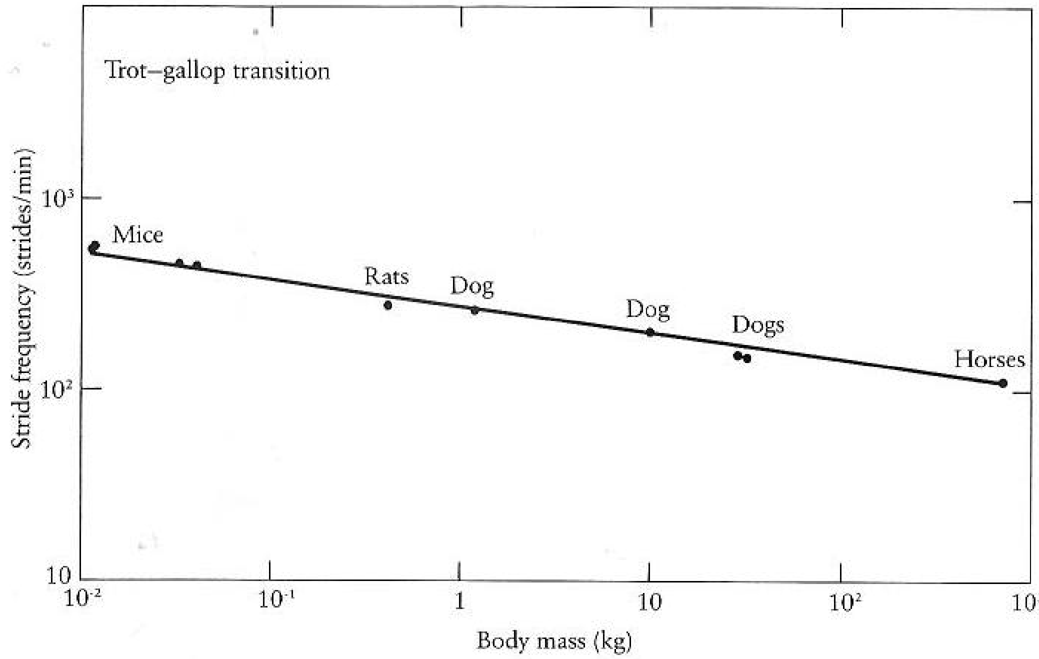

Figure 1: Stride frequency of various mammals as they switch from trot to gallop.

3 Flying animals

We have seen that objects the size of a soccer ball flying in the air experience an aerodynamic force that scales as $\rho_a U^2 S$, where ρ_a is the air density, U the speed and S the surface of the object. In steady flight (horizontal trajectory and constant velocity), this force is decomposed into two parts: the lift that balances the weight, and the drag that must be balanced by thrust.

(a) As birds are made to fly and not to rest on the ground like buckling columns, they do not obey the elastic similarity. Instead, they are relatively isometric ($S \sim M^{2/3}$). Assuming that, find a scaling law relating their speed U to their mass M.

(b) Find the scaling for the muscular power P that the bird must supply in order to balance drag. Use the scaling you obtained previously to get rid of U in your answer.

(c) By comparing this power to the metabolic rate, explain why hummingbirds are able to hover as helicopters while condors prefer to soar.

4 Can you detect the Coriolis effect in your sink?

It is sometimes said that the direction of the vortex observed in our sinks depends on the hemisphere in which we live: counter-clockwise in the northern hemisphere, and clockwise in the southern hemisphere. As you know, the inertial force exerted by the moving liquid scales as $\rho L^2 U^2$, where L is the dimension of the sink, U the velocity of the water and ρ its density. Estimate the Coriolis force on this motion. The ratio between inertia and Coriolis is called the Rossby number. Can you interpret this number as the ratio between two characteristic times? Conclude about the impact of the Coriolis force on the vortex you observe in your sink. Is the 'legend' true?

5 Brownian motion

As discovered by R. Brown, very small particles (less than 10μm) are observed to move randomly in a liquid. This motion is characterized by a coefficient of mass diffusion D [m^2/s]. Some decades later, Einstein proposed a formula to relate this coefficient to the size of the particle R, the viscosity of the liquid μ [kg/(m.s)] and the thermal energy $k_B T$[J]. Could you find this relation by only using dimensional analysis?