

# Generalized Diffusion Tractography Based on Directional Data Clustering

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**Abstract.** A new methodology to reduce uncertainty in estimating the orientation of neuronal pathways in diffusion magnetic resonance imaging is proposed. The methodology relies on three main features. First, an optimized high angular resolution diffusion imaging reconstruction technique is adopted. For each voxel, the orientation distribution function (ODF) on the unit sphere is reconstructed to extract the principal diffusion directions. Second, directional statistics are used to estimate the principal ODF profile directions from data distributed on the unit sphere. For this purpose, a mixture-model approach to clustering directional data based on von Mises-Fisher distributions is adopted. Third, a modified streamline algorithm able to accommodate multiple fiber tracts and multiple orientations per voxel is used, to exploit the directional information gathered from estimated ODF profiles. The methodology has been tested on synthetic data simulations of crossing fibers and on a real data set.

**Keywords:** von Mises-Fisher Distributions, Generalized q-Sampling Imaging (GQI), High Angular Resolution Diffusion Imaging (HARDI), Fiber Tractography.

## 1 Introduction

Diffusion tensor imaging (DTI) is a widely used method in brain research that models the average diffusion properties of water molecules inside a voxel based on a Gaussian diffusion assumption. Diffusion anisotropy, derived by DTI, has been used to characterize white matter neuronal pathways in the human brain, and infer global connectivity in the central nervous system [1]. White matter fiber tractography is commonly implemented using the principal diffusion direction of the DTI model [2]. Popular fiber tracking approaches, such as the streamline tracking algorithm, uses the DTI model to extract the orientation dependence of the diffusion probability density function of water molecules, and reconstruct the orientation distribution function (ODF) of anisotropic tissues. However, the standard single-tensor DTI model is based on a Gaussian diffusion assumption, thus unable to resolve crossing and splitting of fiber bundles.

High angular resolution diffusion imaging (HARDI) techniques have been proposed in the literature to overcome the limitations of the DTI method, and enable detection of multiple ODF maxima per voxel (see [3] for a review). Several studies have shown that fiber tracking based on HARDI-based techniques is improved and less sensitive to noise errors compared to tensor based tracking [4], [5]. The application of these methods is

based on the assumption that the principal directions extracted from the ODF can be interpreted as principal directions of the underlying fiber architecture. Typically, local maxima of the reconstructed ODF are located simply by selecting a large number of randomly sampled points on the sphere and searching within a fixed radius neighborhood [4]. Some more sophisticated heuristics built on this basic approach have been proposed. For instance, in [6] a Quasi-Newton method is used to refine the position of each local maximum. However, as shown in [7] and [3], the peaks of the ODF profiles identified by these methods do not necessarily match the orientations of the distinct fiber populations. Since uncertainty in tractography arises from uncertainty in estimating the directions of propagation, HARDI reconstructions can still be ambiguous and difficult to interpret in the presence of complex fiber tract configurations. To reduce uncertainty and increase robustness in HARDI reconstructions, one may increase the number of sampling directions, and use higher strengths of diffusion-sensitive gradients (b-values) to attain satisfactory angular resolution [8]. Unfortunately, this solution is impractical in clinical applications. Increasing the number of sampling directions prolongs the scan time, making HARDI reconstructions susceptible to motion-induced errors [9]. Using high b-values in clinical scanners results in low signal-to-noise ratio (SNR) and substantial diffusion-induced signal decay [10]. Poor SNR affects the accuracy of ODF reconstruction, and increases fiber orientation uncertainty.

In this paper, we present a new methodology to reduce uncertainty in estimating the orientation of neuronal pathways in HARDI reconstructions. The methodology may be summarized in the following three aspects. First, an optimized HARDI reconstruction technique based on the generalized q-sampling imaging (GQI) approach [11] is adopted. The ODF profile is reconstructed at each voxel, based on the raw HARDI signal acquired on a grid of q-space, and considering a sampling density of vectors on the unit sphere. Second, directional statistics are used to estimate the principal ODF profile directions from data distributed on the unit sphere. For this purpose, a clustering approach based on mixtures of von Mises-Fisher (vMF) distributions is proposed. As opposed to other approaches where mixture of vMF distributions are used to represent diffusion [12], our method works directly with the sampled ODF distributions. Third, a modified streamline algorithm able to accommodate multiple fiber tracts and multiple orientations per voxel is used to exploit the directional information gathered from estimated ODF profiles. By combining HARDI reconstruction and directional statistics in an integrated framework, the methodology is expected to support more accurate fiber ODF estimation for white matter fiber tractography than other more traditional approaches. The methodology has been tested on synthetic data simulations of crossing fibers and on a real data set. The implementation is integrated in a coherent framework based on the R language [13] with 3D OpenGL visualization capabilities [14].

## 2 Generalized Diffusion Magnetic Resonance Tractography

### 2.1 GQI Reconstruction

The generalized q-sampling imaging method proposed in [11] is a HARDI approach to estimate the ODF directly from diffusion MR signals. The relation between the acquired diffusion weighted images  $W(\mathbf{r}, \mathbf{q})$  and the measured ODF  $\psi_m(\mathbf{r}, \hat{\mathbf{u}})$  is given by

$$\psi_m(\mathbf{r}, \hat{\mathbf{u}}) = A_q L_\Delta \sum_{\mathbf{q}} W(\mathbf{r}, \mathbf{q}) \text{sinc}(2\pi L_\Delta \mathbf{q} \cdot \hat{\mathbf{u}}), \quad (1)$$

where  $\mathbf{r}$  is the voxel coordinate,  $\hat{\mathbf{u}}$  represents a radial spherical unit direction,  $\mathbf{q}$  is the wave vector in  $q$ -space,  $L_\Delta$  is the diffusion sampling length, and  $A_q$  is a constant area term. The wave vector is given by  $\mathbf{q} = \gamma\delta\mathbf{G}/2\pi$ , where  $\gamma$  is the nuclear gyromagnetic ratio, and  $\mathbf{G}$  and  $\delta$  are the strength and duration of the diffusion-encoding gradient, respectively.

Equation (1) is simple to interpret. The estimated ODF is synthesized from a series of sinc basis functions, weighted by  $W(\mathbf{r}, \mathbf{q})$ . The shape of the basis functions is determined by the value of  $|\mathbf{q}|L_\Delta$ . A higher value of  $|\mathbf{q}|L_\Delta$  represents a sharper contour, and vice versa. Moreover, (1) specifies an operational sampling scheme in  $q$ -space from which the ODF can be estimated. In particular, the number of basis functions used in (1) is not restricted by the shell (or grid) resolution used for MRI signal acquisition. The number of radial sampling directions can be adapted for the purposes of ODF estimation. Typically, sampling densities of  $N = 81$  and  $N = 321$  on the hemisphere are used in ODF profile mapping, corresponding to a third and seventh-order tessellation of the icosahedron, respectively. However, this specification is not imposed a priori by the acquisition resolution on the GQI reconstruction process.

## 2.2 Fiber Mapping Based on Directional Data Clustering

The second main feature of the proposed methodology is concerned with multiple directional mapping. Starting with the raw HARDI signal acquired on a grid of  $q$ -space, the ODF profile is estimated at each voxel, considering a sampling density of unit vectors on a unit  $\mathbb{S}^2$  grid. When a threshold is applied to the estimated ODF at each voxel, the non-thresholded unit vectors provide directional statistics information about the estimated ODF profile. The main ODF orientations at each voxel relevant for fiber tracking may be estimated by clustering the non-thresholded unit vectors. This directional clustering procedure has several advantages compared to traditional approaches for orientation mapping. In fact, current best practices perform multiple maxima extraction based on procedures which are very sensitive to the local modes that appear in the reconstructed ODFs. Signal noise and low sampling resolution yield deformed ODF reconstruction profiles, thus affecting accuracy of the multiple orientation determination. In contrast, estimating orientations from clustered directional data is much less sensitive to local modes in the reconstructed ODF profile. Moreover, the procedure is more robust to noise since it estimates orientations statistically from sampled data.

For directional clustering estimation, we consider a mixture of  $k$  von Mises-Fisher (vMF) distributions [15] that serves as a model for directional ODF profile data, corresponding to multiple fiber orientations. A mixture of  $k$  vMF distributions has a density given by

$$f(\mathbf{x}|\boldsymbol{\Theta}) = \sum_{h=1}^k \alpha_h f_h(\mathbf{x}|\boldsymbol{\theta}_h), \quad (2)$$

where  $f_h(\mathbf{x}|\boldsymbol{\theta}_h)$  denotes a vMF distribution with parameter  $\boldsymbol{\theta}_h = (\boldsymbol{\mu}_h, \boldsymbol{\kappa}_h)$  for  $1 \leq h \leq k$ ,  $\boldsymbol{\Theta} = \{\alpha_1, \dots, \alpha_k, \boldsymbol{\theta}_1, \dots, \boldsymbol{\theta}_k\}$ , and the  $\alpha_h$  are non-negative and sum to 1. A  $d$ -dimensional unit random vector  $\mathbf{x} \in \mathbb{S}^{d-1}$  is said to have  $d$ -variate vMF distribution if its probability density function is given by

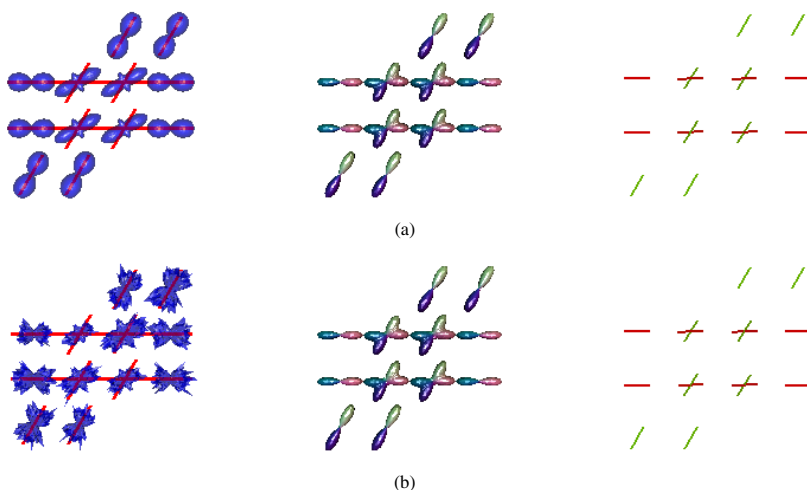
$$f_h(\mathbf{x}|\boldsymbol{\mu}, \boldsymbol{\kappa}) = c_d(\boldsymbol{\kappa}) e^{\boldsymbol{\kappa} \boldsymbol{\mu}^T \mathbf{x}}, \quad (3)$$

where  $\|\boldsymbol{\mu}\| = 1$ ,  $\boldsymbol{\kappa} \geq 0$ ,  $d \geq 2$ , and  $c_d(\boldsymbol{\kappa})$  is a normalizing constant [16]. The density  $f_h(\mathbf{x}|\boldsymbol{\mu}, \boldsymbol{\kappa})$  is parameterized by the mean direction  $\boldsymbol{\mu}$ , and the concentration parameter  $\boldsymbol{\kappa}$ . The  $\boldsymbol{\kappa}$  parameter characterizes how strongly the unit vectors drawn according to  $f_h(\mathbf{x}|\boldsymbol{\mu}, \boldsymbol{\kappa})$  are concentrated about the mean direction  $\boldsymbol{\mu}$ . In this work, we used the procedure for clustering directional data outlined in [15], and implemented in [17].

The principal ODF profile directions are extracted directly from the estimated clusters. The number of fibers in each voxel is automatically estimated from the reconstructed ODF profile by the vMF approach using the BIC criterion. To decide on the number of components to select we apply the Bayesian information criterion (BIC) [18]. All relevant statistical information about the ODF orientation and multiple fiber components may then be extracted from this fitting process.

### 2.3 Tractography

The ultimate goal of fiber orientation mapping procedures is to be able to delineate accurate white matter fiber pathways between cortical and sub-cortical brain structures. The network of fiber tract connections and its density provide valuable information in medical applications and diagnoses. Using the voxel directional information estimated by the vMF approach outlined in Section 2.2, we implemented a streamline tractographic algorithm to represent and visualize fiber tracts. The algorithm is a modified version of the fiber tracking algorithm described in [2]. The modifications were implemented in order to deal with multiple directional orientations and multiple fiber tracts per voxel. Fiber tracts are initiated in every voxel within a specified user defined region-of-interest (ROI) using one of the estimated main voxel ODF directions, and are extended bi-directionally in steps less than half of the voxel dimension. The tracts are then propagated a step parallel to the selected direction. For each new voxel in the path front, one specific direction among the estimated voxel ODF directions is selected. The voxel ODF direction that produces least curvature with the incoming path is selected for propagation. Multiple tracts per voxel are accommodated by initializing the tracts with random real values within the seed voxel. The number of initializing tracts may be specified by the user, enabling him to strike a balance between fiber bundle density and running time. The usual criteria for line keeping and line termination have been adopted. In particular, the following criteria have been specified: minimum fiber length (50 mm), maximum fiber length (600 mm), maximum admissible fiber deviation angle ( $60^\circ$ ), and generalized fractional anisotropy threshold (0.4).



**Fig. 1.** (a) Simulated noise free field of diffusion profiles, reconstructed field of ODF glyphs, and estimated ODF directions, from left-to-right respectively. (b) Simulation as in (a) with Rician noise level SNR=30.

## 3 Experiments

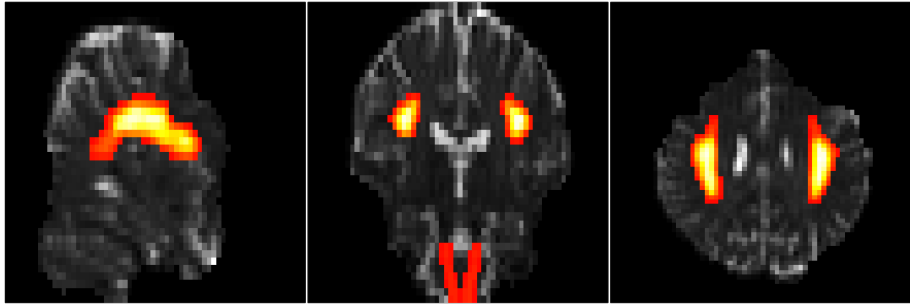
### 3.1 Simulated Field of Diffusion Profiles

To illustrate the methodology described in Section 2.1, we generated a field of simulated diffusion profiles as depicted in Fig. 1. The field simulates crossing fibers with an angle of  $60^\circ$ , and  $b=4500$ . Fig. 1(a) illustrates the simulated noise free field, the reconstructed field of ODF glyphs using the GQI method, and the estimated ODF directions based on the vMF mixture approach. A similar profile simulation with added Rician noise, and a signal-to-noise (SNR) value of 30 is shown in Fig. 1(b). As illustrated, the vMF approach correctly identifies the underlying fiber orientations in both cases.

### 3.2 Real Data Experiment

We report on experiments using a DICOM data set provided by the “Advanced Biomedical MRI Lab, National Taiwan University Hospital”. Specifically, we have used the data set “DSI 203-point 3mm” which is included in the “DSI Studio” package, publicly available from the NITRC repository (<http://www.nitrc.org>). This data set is from a normal 24-year-old male volunteer, and has been provided as a demonstration data set in connection with the “DSI Studio” software for diffusion MR images analysis [11]. The data set was obtained with an echo planar imaging diffusion sequence with twice-refocused echo, dimension  $64 \times 64 \times 40$ , and slice thickness 2.9 mm. Further details on the data set specification are available from the NITRC repository.

We have tested our model with the two b-tables that accompanies the data set. One is a b-table for a  $S^2$  grid denoted by “dsi203\_bmax4000.txt”. The other is the b-table



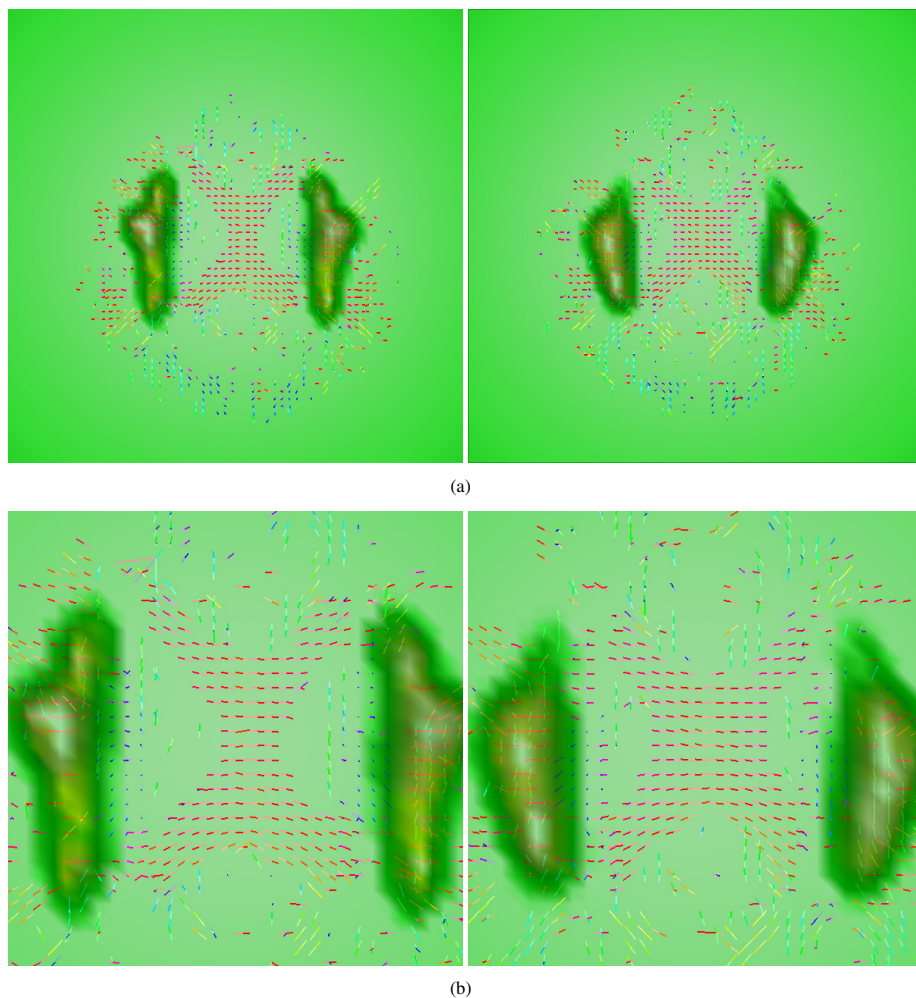
**Fig. 2.** Sagittal, coronal and axial views (from left-to-right) for slices  $[X, Y, Z] = [22, 29, 24]$  of the selected ROI volume overlayed on the original data set with non-brain tissue removed. The sagittal view has the front brain facing right; coronal and axial views have the right hemisphere on the left of the image (radiological convention).

for the 3D, Diffusion Spectrum Magnetic Resonance Imaging (DSI) sampling scheme used in the DICOM data acquisition. This b-table has 203 points uniformly distributed on a 3D grid limited to the volume of the unit sphere. In both tables, the b-values range from 0 to 4000.

Fig. 2 shows the views sagittal, coronal and axial for slices  $[X, Y, Z] = [22, 29, 24]$  of a region of interest (ROI) overlayed on the original data set with non-brain tissue removed. The ROI image depicts brain regions where anatomic white matter fiber crossings are known to exist, forming multiple pathway bundles connected to the cerebral cortex. The ROI was formed by extracting the superior longitudinal fasciculus (SLF) and corticospinal tract (CT) regions based on the “ICBM-DTI-81 White-Matter” atlas included in the FSL toolbox [19]. The extracted regions were registered to the DSI data set using the FSL/FLIRT tool. Using the procedure outlined in Section 2.1, we estimated for each voxel of the DSI data set the main ODF directions. This information enables us to draw linemaps showing the estimated orientations, and number of fibers for each voxel.

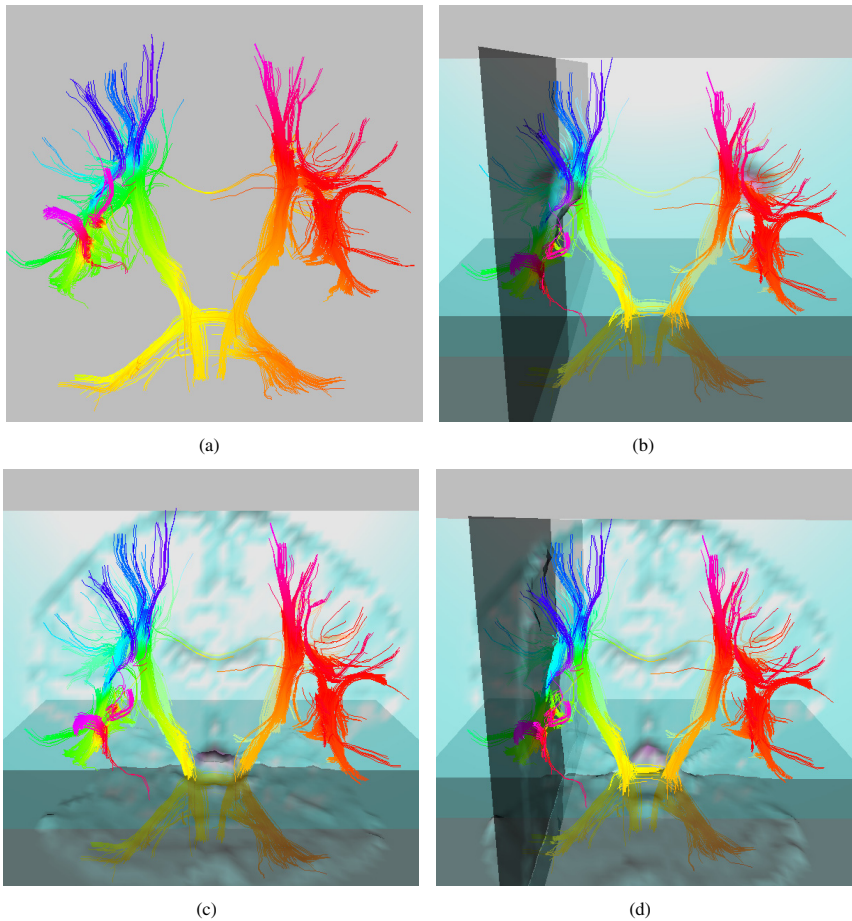
We show in Fig. 3 linemaps for the field of profiles estimated from the ODFs, for voxels in axial slices 23 (left), and 24 (right). The selected ROI image is overlayed on Fig. 3, in order to pinpoint the location of the SLF and CT regions on the linemaps. The panels also depict the ROI (SLF and CT regions as dark hues) overlayed on the central regions of slices 23 and 24. Fig. 3(b) depicts zoomed-in central-region images of the panels shown in Fig. 3(a). A large number of voxels with crossing fibers is clearly visible in these figures (see right SLF for slice 23, and left SLF for slice 24). The central area in the panels of Fig. 3 is typical of horizontal fibers associated with the corpus callosum.

Using the estimated voxel directional information, we tested the streamline tractographic implementation outlined in Section 2.3, to represent and visualize the fiber tracts. In Fig. 4 we show several panels representing the 3D OpenGL [20] interface used to visualize the estimated fiber pattern in the context of the brain anatomy. Fig. 4(a) illustrates the pattern of interconnections using the voxels in SLF and CT regions as seeds, and bundles with 10 randomly initialized tracts per voxel. Fig. 4(b) maps the



**Fig. 3.** (a) Linemaps for the field of profiles estimated from the ODFs, for voxels in axial slices 23 (left), and 24 (right). The panels also depict the ROI (SLF and CT regions as dark hues) overlayed on the central regions of slices 23 and 24; (b) zoomed-in images of the linemap panels depicted in (a).

fiber tracts overlayed on image slices of the selected ROI region. Fig. 4(c) and Fig. 4(d) map the fiber tracts overlayed on image slices of the brain.



**Fig. 4.** Panels representing the 3D OpenGL interface used to visualize the estimated fiber pattern in the context of the brain anatomy: (a) Estimated fiber pattern using multiple directional orientation and multiple fiber tracts per voxel; In (b) fiber tracts are overlaid on image slices of the selected ROI region; In (c) and (d) fiber tracts are overlaid on image slices of the brain

## 4 Conclusions

We have presented a generalized diffusion imaging approach incorporating directional statistics information to support in vivo fiber tractography. Based on experiments, the proposed approach was found to be more accurate in estimating local fiber orientations than traditional deterministic techniques based on multiple maxima extraction. Directional accuracy impacts strongly on the quality of the reconstructed fiber maps, and subsequent interpretation of fiber tract anatomy for use in clinical imaging. An extended tractographic procedure able to accommodate multiple pathways and crossing fibers was outlined to profit from the richer directional information gathered at each voxel.



The directional statistics procedure applies the BIC model selection criterion to automatically select the number of mixing components, i.e., the number of fibers per voxel to be used for tractography purposes. This selection procedure was found to be robust to noise in discriminating crossing-fiber configurations. The reported experiments used the GQI method for ODF estimation. However, the proposed technique remains valid when other HARDI reconstruction methods are used, such as Q-ball imaging or DSI. The reconstructed ODF profiles do not depend on the type of grid used in the diffusion data acquisition process. The user may specify different grid types and resolutions for ODF reconstruction and fiber directional estimation.

We believe that the directional statistics technique proposed in this work offers significant increases in sensitivity for anatomical analysis over traditional approaches. We intend to build on the quantitative and qualitative information provided by the proposed directional statistics approach to support the study of fiber tract architecture in the brain. In particular, this information may be explored to build robust probabilistic tractographic algorithms for complex fiber configurations.

The analyses and figures described in this work were performed using software programmed entirely in **R** [13]. The **R**-package **gdimap** [14] implements the reconstruction and vMF estimation methodology described in this work, and is freely available from the CRAN repository (<http://CRAN.R-project.org>).

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