Practical 7

To plot the characteristics plot for a first order partial differential equation

Question 1:

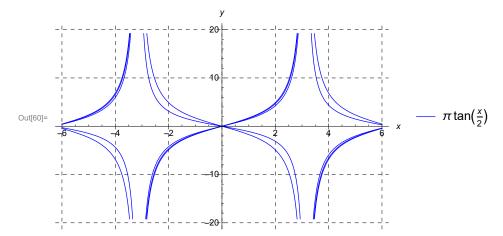
```
 \begin{aligned} &\inf\{4S\} = & \text{ eqn1} = 3 \star y \,' \, [x] \, - 2 = \theta \\ & \text{ sol1} = & \text{ DSolve}[\text{ eqn1}, y[x], \, x] \\ & \text{ part1} = & y[x] \, / \, \, \text{ sol1} \, / \, \, C[1] \, \rightarrow \, \{1, \, -2, \, -1, \, 3, \, 4\} \\ & \text{ Plot}[\text{part1}, \, \{x, \, -5, \, 5\}, \, \text{ AxesLabel} \, \rightarrow \, \{x, \, y\}, \, \text{ PlotStyle} \, \rightarrow \, \{\text{Thickness}[0.004], \, \text{Cyan}\}, \\ & \text{ PlotLegends} \, \rightarrow \, \{\text{part1}\}, \, \text{ GridLines} \, \rightarrow \, \text{ Automatic, GridLinesStyle} \, \rightarrow \, \text{ Directive}[\text{Black, Dashed}]] \\ & \text{Oul}[46] = & \left\{ \left\{ y[x] \, \rightarrow \, \frac{2x}{3} \, + \, c_1 \right\} \right\} \\ & \text{Oul}[47] = & \left\{ \left\{ 1 + \frac{2x}{3}, \, -2 + \frac{2x}{3}, \, -1 + \frac{2x}{3}, \, 3 + \frac{2x}{3}, \, 4 + \frac{2x}{3} \right\} \right\} \end{aligned}
```

Question 2:

$$\begin{aligned} & \text{In}[57] = & \text{eqn2} := \text{Sin}[x] * y'[x] - y[x] == 0 \\ & \text{sol2} = & \text{DSolve}[\text{eqn2}, \ y[x], \ x] \\ & \text{part2} = & y[x] \ /. \ \text{sol2} \ /. \ \text{C}[1] \rightarrow \{\pi, \, \text{e}, \, \text{i}, \, -2, \, 3, \, -\pi\} \\ & \text{Plot}[\text{part2}, \{x, \, -6, \, 6\}, \, \text{AxesLabel} \rightarrow \{x, \, y\}, \, \text{PlotStyle} \rightarrow \{\text{Thickness}[0.002], \, \text{Blue}\}, \\ & \text{PlotLegends} \rightarrow \{\text{part2}\}, \, \text{GridLines} \rightarrow \text{Automatic}, \, \text{GridLinesStyle} \rightarrow \text{Directive}[\text{Black}, \, \text{Dashed}]] \end{aligned}$$

Out[58]=
$$\left\{ \left\{ y \left[x \right] \rightarrow \mathbb{C}_1 \operatorname{Tan} \left[\frac{x}{2} \right] \right\} \right\}$$

$$\text{Out[59]= } \left\{ \left\{ \pi \, \mathsf{Tan} \left[\, \frac{\mathsf{x}}{2} \, \right] \, \text{, } \, \mathbb{e} \, \mathsf{Tan} \left[\, \frac{\mathsf{x}}{2} \, \right] \, \text{, } \, \mathbb{i} \, \mathsf{Tan} \left[\, \frac{\mathsf{x}}{2} \, \right] \, \text{, } \, -2 \, \mathsf{Tan} \left[\, \frac{\mathsf{x}}{2} \, \right] \, \text{, } \, 3 \, \mathsf{Tan} \left[\, \frac{\mathsf{x}}{2} \, \right] \, \text{, } \, -\pi \, \mathsf{Tan} \left[\, \frac{\mathsf{x}}{2} \, \right] \right\} \right\}$$



Question 3:

```
ln[77] = eqn2 := Sin[x] * y'[x] - Cos[x] y[x] == 0
          sol2 = DSolve[eqn2, y[x], x]
          part2 = y[x] /. sol2 /. C[1] \rightarrow {\pi, e, i, -2, 3, -\pi}
          Plot[part2, \{x, -6, 6\}, AxesLabel \rightarrow \{x, y\}, PlotStyle \rightarrow \{Thickness[0.002], Red\},
            PlotLegends → {part2}, GridLines → Automatic, GridLinesStyle → Directive[Black, Dashed]]
\text{Out} [78] \text{= } \left\{ \, \left\{ \, y \, [\, x \, ] \, \rightarrow \, \mathbb{C}_{\mathbf{1}} \, \mathsf{Sin} \, [\, x \, ] \, \, \right\} \, \right\}
\texttt{Out[79]=} \ \{ \{ \pi \, \texttt{Sin}[x] \, , \, \texttt{e} \, \texttt{Sin}[x] \, , \, \texttt{i} \, \texttt{Sin}[x] \, , \, -2 \, \texttt{Sin}[x] \, , \, 3 \, \texttt{Sin}[x] \, , \, -\pi \, \texttt{Sin}[x] \, \} \}
Out[80]=
                                                                                                                    -\pi\sin(x)
```

Question 4:

$$\label{eq:constraints} \begin{split} & \text{ln}[\texttt{B1}] = \text{ eqn2 := e * y '}[\texttt{x}] - \text{Sec}[\texttt{x}] \ y[\texttt{x}] \ == 0 \\ & \text{sol2 = DSolve}[\text{eqn2, y}[\texttt{x}], \texttt{x}] \\ & \text{part2 = y}[\texttt{x}] \ /. \ \text{sol2 /. C}[\texttt{1}] \rightarrow \{\texttt{1, -2, -\pi, Pi}\} \\ & \text{Plot}[\text{part2, } \{\texttt{x, -10, 10}\}, \ \text{AxesLabel} \rightarrow \{\texttt{x, y}\}, \ \text{PlotStyle} \rightarrow \{\text{Thickness}[\texttt{0.003}], \ \text{Purple}\}, \\ & \text{PlotLegends} \rightarrow \{\text{part2}\}, \ \text{GridLines} \rightarrow \text{Automatic, GridLinesStyle} \rightarrow \text{Directive}[\text{Black, Dashed}]] \end{split}$$

$$\text{Out}[82] = \left\{ \left\{ \mathbf{y} \left[\mathbf{X} \right] \rightarrow \mathbf{e}^{\frac{2 \operatorname{ArcTanh} \left[\operatorname{Tan} \left[\frac{x}{2} \right] \right]}{\mathbf{e}}}, -2 \right. \mathbf{e}^{\frac{2 \operatorname{ArcTanh} \left[\operatorname{Tan} \left[\frac{x}{2} \right] \right]}{\mathbf{e}}}, -\mathbf{e}^{\frac{2 \operatorname{ArcTanh} \left[\operatorname{Tan} \left[\frac{x}{2} \right] \right]}{\mathbf{e}}}, -\mathbf{e}^{\frac{2 \operatorname{ArcTanh} \left[\operatorname{Tan} \left[\frac{x}{2} \right] \right]}{\mathbf{e}}}, \mathbf{\pi}, \mathbf{e}^{\frac{2 \operatorname{ArcTanh} \left[\operatorname{Tan} \left[\frac{x}{2} \right] \right]}{\mathbf{e}}} \right\} \right\}$$

$$\text{Out}[84] = \begin{bmatrix} \mathbf{y} \\ \mathbf{e} \end{bmatrix}$$

Question 5:

```
ln[89]:= eqn2 := y'[x] + x^2 * y[x] == 0
     sol2 = DSolve[eqn2, y[x], x]
     part2 = y[x] /. sol2 /. C[1] \rightarrow \{1, -2, -\pi, Pi\}
     Plot[part2, \{x, -3, 3\}, AxesLabel \rightarrow \{x, y\}, PlotStyle \rightarrow \{Thickness[0.004], Green\},
       PlotLegends → {part2}, GridLines → Automatic, GridLinesStyle → Directive[Black, Dashed]]
```

$$\text{Out}[90] = \; \left\{ \left. \left\{ y \left[\, x \, \right] \right. \right. \right. \rightarrow \left. e^{-\frac{x^3}{3}} \right. \left. \mathbb{C}_1 \right. \right\} \right\}$$

$$\text{Out}[91] = \left\{ \left\{ e^{-\frac{x^3}{3}} \text{, } -2 e^{-\frac{x^3}{3}} \text{, } -e^{-\frac{x^3}{3}} \pi \text{, } e^{-\frac{x^3}{3}} \pi \right\} \right\}$$

