Practical 5 :

To Solve system of ordinary differential equation

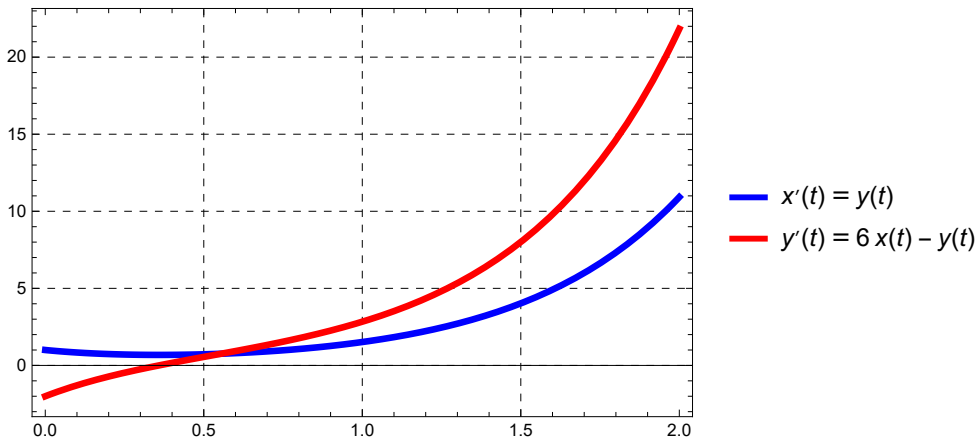
Question 1: x’[t]==y[t]
y’[t]==-y[t] + 6x[t],
x[0]==1, y[0] == -2

In[ ]:=
```
eq1 = {x'[t] == y[t], y'[t] == -y[t] + 6 * x[t], x[0] == 1, y[0] == -2}
DSolve[eq1, {x[t], y[t]}, t]
Plot[Evaluate[{x[t], y[t]} /. %], {t, 0, 2}, PlotLegends → {eq1},
 PlotStyle → {{Blue, Thickness[0.01]}, {Red, Thickness[0.01]}}, Frame → True,
 GridLines → Automatic, GridLinesStyle → Directive[Black, Dashed]]
```

Out[ ]=
$$\{x'[t] == y[t],\ y'[t] == 6\,x[t] - y[t],\ x[0] == 1,\ y[0] == -2\}$$

Out[ ]=
$$\left\{\left\{x[t] \to \frac{1}{5} e^{-3t}\left(4 + e^{5t}\right),\ y[t] \to \frac{2}{5} e^{-3t}\left(-6 + e^{5t}\right)\right\}\right\}$$

Out[ ]=



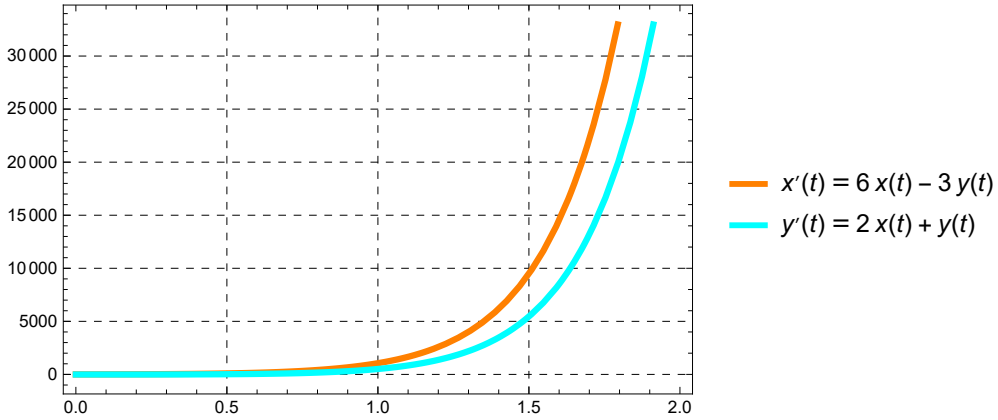
Question 2: x’[t]==6*x[t] - 3*y[t]

y’[t]==2*x[t]+ y[t]

In[ ]:=
```
eq2 := {x'[t] == 6 * x[t] - 3 * y[t], y'[t] == 2 * x[t] + y[t], x[0] == 1, y[0] == -9};
DSolve[eq2, {x[t], y[t]}, t]
Plot[Evaluate[{x[t], y[t]} /. %], {t, 0, 2}, PlotLegends → {eq2},
 PlotStyle → {{Orange, Thickness[0.01]}, {Cyan, Thickness[0.01]}},
 Frame → True, GridLines → Automatic, GridLinesStyle → Directive[Black, Dashed]]
```

Out[ ]=
$$\left\{\left\{x[t] \to e^{3t}\left(-29+30\,e^{t}\right),\ y[t] \to e^{3t}\left(-29+20\,e^{t}\right)\right\}\right\}$$

Out[ ]=



Question 3: x'[t]==6*x'[t] - 3*y[t]
y'[t]==2*x[t]+ y[t]

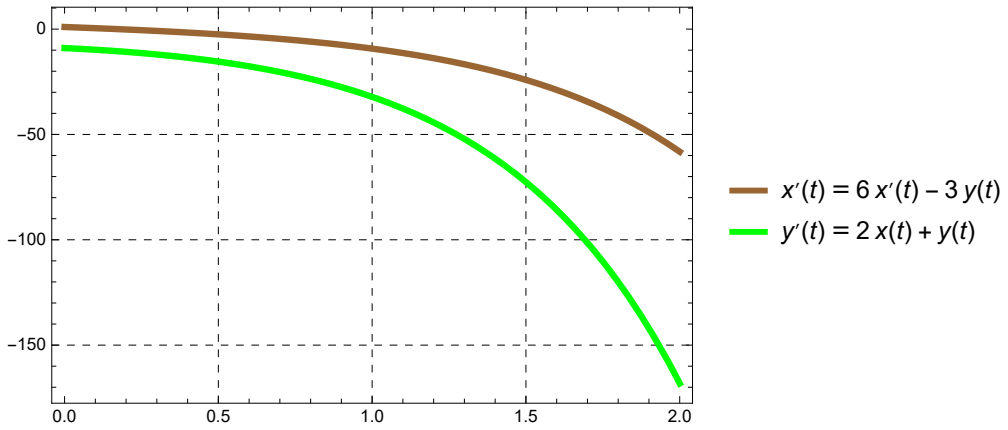
In[ ]:=
```
eq3 := {x'[t] == 6 * x'[t] - 3 * y[t], y'[t] == 2 * x[t] + y[t], x[0] == 1, y[0] == -9};
DSolve[eq3, {x[t], y[t]}, t]
Plot[Evaluate[{x[t], y[t]} /. %], {t, 0, 2}, PlotLegends → {eq3},
 PlotStyle → {{Brown, Thickness[0.01]}, {Green, Thickness[0.01]}},
 Frame → True, GridLines → Automatic, GridLinesStyle → Directive[Black, Dashed]]
```

Out[ ]=

$$\left\{\left\{x[t] \to -\frac{1}{290} e^{\frac{t}{2}-\frac{1}{2}\sqrt{\frac{29}{5}}\,t}\left(-145-59\sqrt{145}-145\,e^{\sqrt{\frac{29}{5}}\,t}+59\sqrt{145}\,e^{\sqrt{\frac{29}{5}}\,t}\right),\right.\right.$$

$$\left.\left.y[t] \to -\frac{1}{58} e^{\frac{t}{2}-\frac{1}{2}\sqrt{\frac{29}{5}}\,t}\left(261-5\sqrt{145}+261\,e^{\sqrt{\frac{29}{5}}\,t}+5\sqrt{145}\,e^{\sqrt{\frac{29}{5}}\,t}\right)\right\}\right\}$$

Out[ ]=

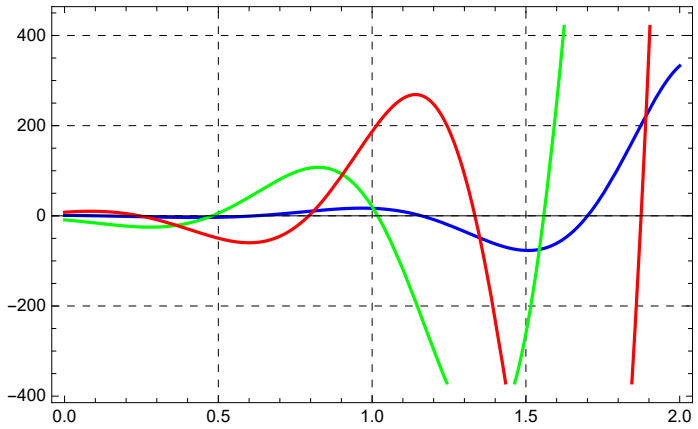


Question 4 : x'[t]==6*x'[t] - 3*y[t]+ z[t]
y'[t]==2*x[t]+ y[t]-6*z[t]
y[t]+56*x[t]==z'[t]

In[•]:=
```
eq4 := {x'[t] == 6 * x'[t] - 3 * y[t] + z[t], y'[t] == 2 * x[t] + y[t] - 6 * z[t],
   y[t] + 56 * x[t] == z'[t], x[0] == 1, y[0] == -9, z[0] == 8};
DSolve[eq4, {x[t], y[t], z[t]}, t]
Plot[Evaluate[{x[t], y[t], z[t]} /. %], {t, 0, 2}, PlotLegends → {eq4}, PlotStyle →
  {{Blue, Thickness[0.005]}, {Green, Thickness[0.005]}, {Red, Thickness[0.005]}},
 Frame → True, GridLines → Automatic, GridLinesStyle → Directive[Black, Dashed]]
```

Out[•]=
$$\left\{\left\{x[t] \to -8\,\text{RootSum}\left[23\,850+400\,\#1-5\,\#1^2+\#1^3\,\&,\ \frac{85\,e^{\frac{t\,\#1}{5}}+e^{\frac{t\,\#1}{5}}\,\#1}{400-10\,\#1+3\,\#1^2}\,\&\right]-\right.\right.$$
$$9\,\text{RootSum}\left[23\,850+400\,\#1-5\,\#1^2+\#1^3\,\&,\ \frac{-5\,e^{\frac{t\,\#1}{5}}+3\,e^{\frac{t\,\#1}{5}}\,\#1}{400-10\,\#1+3\,\#1^2}\,\&\right]+$$
$$\text{RootSum}\left[23\,850+400\,\#1-5\,\#1^2+\#1^3\,\&,\ \frac{150\,e^{\frac{t\,\#1}{5}}-5\,e^{\frac{t\,\#1}{5}}\,\#1+e^{\frac{t\,\#1}{5}}\,\#1^2}{400-10\,\#1+3\,\#1^2}\,\&\right],$$
$$y[t] \to 10\,\text{RootSum}\left[23\,850+400\,\#1-5\,\#1^2+\#1^3\,\&,\ \frac{-840\,e^{\frac{t\,\#1}{5}}+e^{\frac{t\,\#1}{5}}\,\#1}{400-10\,\#1+3\,\#1^2}\,\&\right]-$$
$$80\,\text{RootSum}\left[23\,850+400\,\#1-5\,\#1^2+\#1^3\,\&,\ \frac{e^{\frac{t\,\#1}{5}}+3\,e^{\frac{t\,\#1}{5}}\,\#1}{400-10\,\#1+3\,\#1^2}\,\&\right]-$$
$$9\,\text{RootSum}\left[23\,850+400\,\#1-5\,\#1^2+\#1^3\,\&,\ \frac{280\,e^{\frac{t\,\#1}{5}}+e^{\frac{t\,\#1}{5}}\,\#1^2}{400-10\,\#1+3\,\#1^2}\,\&\right],$$
$$z[t] \to -45\,\text{RootSum}\left[23\,850+400\,\#1-5\,\#1^2+\#1^3\,\&,\ \frac{168\,e^{\frac{t\,\#1}{5}}+e^{\frac{t\,\#1}{5}}\,\#1}{400-10\,\#1+3\,\#1^2}\,\&\right]+$$
$$10\,\text{RootSum}\left[23\,850+400\,\#1-5\,\#1^2+\#1^3\,\&,\ \frac{-135\,e^{\frac{t\,\#1}{5}}+28\,e^{\frac{t\,\#1}{5}}\,\#1}{400-10\,\#1+3\,\#1^2}\,\&\right]+$$
$$\left.\left.8\,\text{RootSum}\left[23\,850+400\,\#1-5\,\#1^2+\#1^3\,\&,\ \frac{-30\,e^{\frac{t\,\#1}{5}}-5\,e^{\frac{t\,\#1}{5}}\,\#1+e^{\frac{t\,\#1}{5}}\,\#1^2}{400-10\,\#1+3\,\#1^2}\,\&\right]\right\}\right\}$$

Out[•]=



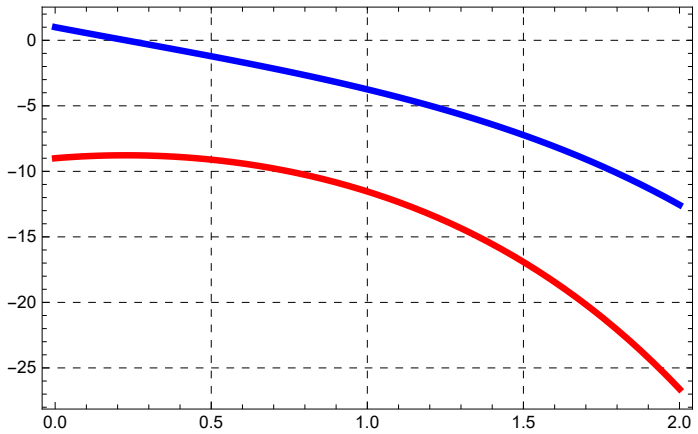
$x'(t) = 6\,x'(t) - 3\,y(t) + z(t)$
$y'(t) = 2\,x(t) + y(t) - 6\,z(t)$
$56\,x(t) + y(t) = z'(t)$

Question 5: x'[t]==7*x'[t]- 3*y[t], y'[t]==2*x[t]

In[•]:=
```
eq3 := {x'[t] == 7 * x'[t] - 3 * y[t], y'[t] == 2 * x[t], x[0] == 1, y[0] == -9};
DSolve[eq3, {x[t], y[t]}, t]
Plot[Evaluate[{x[t], y[t]} /. %], {t, 0, 2}, PlotLegends → {eq3},
 PlotStyle → {{Blue, Thickness[0.01]}, {Red, Thickness[0.01]}}, Frame → True,
 GridLines → Automatic, GridLinesStyle → Directive[Black, Dashed]]
```

Out[•]=
$$\left\{\left\{x[t] \to -\frac{1}{4} e^{-t}\left(-11 + 7\,e^{2t}\right),\ y[t] \to -\frac{1}{2} e^{-t}\left(11 + 7\,e^{2t}\right)\right\}\right\}$$

Out[•]=



$x'(t) = 7\,x'(t) - 3\,y(t)$
$y'(t) = 2\,x(t)$