Practical - 2

To solve 2nd order differential equation and plotting its solutions
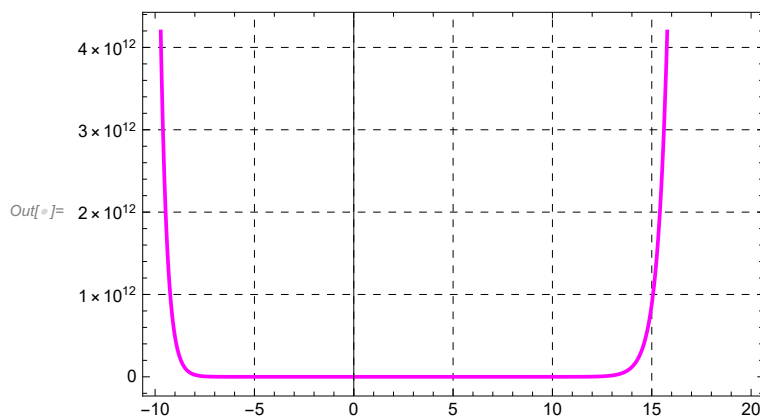

Question - 1: $y'' + y' - 6y = 0$

```
In[ ]:= eq1 = y''[x] + y'[x] - 6 * y[x] == 0
       DSolve[{y''[x] + y'[x] - 6 * y[x] == 0}, y[x], x]
       sol1 = DSolve[{y''[x] + y'[x] - 6 * y[x] == 0, y[0] == 1, y'[2] == 9}, y[x], x]
       Plot[y[x] /. sol1, {x, -10, 20}, PlotLegends → {eq1},
        PlotStyle → {{Magenta, Thickness[0.006]}, {Red, Thickness[0.01]}},
        Frame → True, GridLines → Automatic, GridLinesStyle → Directive[Black, Dashed]]
```

$Out[ ]=$ $-6 y[x] + y'[x] + y''[x] == 0$

$Out[ ]=$ $\left\{\left\{y[x] \to e^{-3x} c_1 + e^{2x} c_2\right\}\right\}$

$Out[ ]=$ $\left\{\left\{y[x] \to \dfrac{e^{-3x}\left(-9 e^6 + 2 e^{10} + 3 e^{5x} + 9 e^{6+5x}\right)}{3 + 2 e^{10}}\right\}\right\}$


$Out[ ]=$ — $y''(x) + y'(x) - 6 y(x) = 0$

Question 2 : y'' - 9y' + 20y = sin x

In[◦]:= `eq1 = y''[x] + 9 * y'[x] + 20 * y[x] == Sin[x]`
`DSolve[{y''[x] + 9 * y'[x] + 20 * y[x] == Sin[x]}, y[x], x]`
`sol2 = DSolve[{y''[x] + 9 * y'[x] + 20 * y[x] == Sin[x], y[0] == 2, y'[2] == 7}, y[x], x]`
`Plot[y[x] /. sol2, {x, -2, 2}, PlotLegends → {eq1},`
`  PlotStyle → {{Red, Thickness[0.006]}, {Red, Thickness[0.01]}}, Frame → True,`
`  GridLines → Automatic, GridLinesStyle → Directive[Black, Dashed]]`

Out[◦]= $20\, y[x] + 9\, y'[x] + y''[x] == \text{Sin}[x]$

Out[◦]= $\left\{\left\{y[x] \to e^{-5x}\, c_1 + e^{-4x}\, c_2 + \dfrac{1}{442}\left(-9\,\text{Cos}[x] + 19\,\text{Sin}[x]\right)\right\}\right\}$

Out[◦]= $\Big\{\Big\{y[x] \to -\dfrac{1}{442\left(-5 + 4\,e^2\right)}$
$e^{-5x}\left(-3572\,e^2 - 3094\,e^{10} + 4465\,e^x + 3094\,e^{10+x} + 19\,e^{10}\,\text{Cos}[2] - 19\,e^{10+x}\,\text{Cos}[2] - 45\,e^{5x}\,\text{Cos}[x] + \right.$
$\left. 36\,e^{2+5x}\,\text{Cos}[x] + 9\,e^{10}\,\text{Sin}[2] - 9\,e^{10+x}\,\text{Sin}[2] + 95\,e^{5x}\,\text{Sin}[x] - 76\,e^{2+5x}\,\text{Sin}[x]\right)\Big\}\Big\}$

Out[◦]=

$$y''(x) + 9\,y'(x) + 20\,y(x) = \sin(x)$$
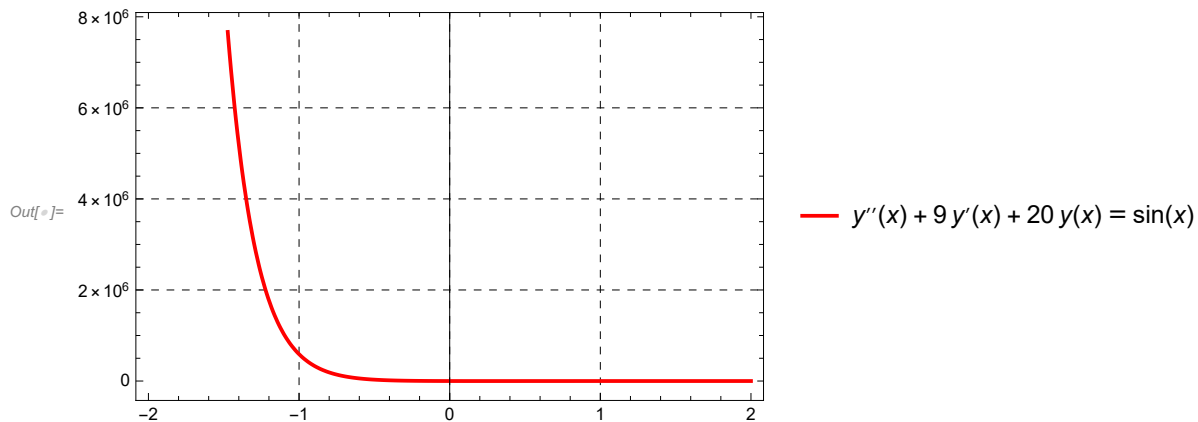
# Question 3: y'' - 6y' + 5y = e-3x

In[◦]:= `eq1 = y''[x] - 6 * y'[x] + 5 * y[x] == e^{-3*x}`
`DSolve[{y''[x] - 6 * y'[x] + 5 * y[x] == e^{-3*x}}, y[x], x]`
`sol2 = DSolve[{y''[x] - 6 * y'[x] + 5 * y[x] == e^{-3*x}, y[0] == 4, y'[2] == 7}, y[x], x]`
`Plot[y[x] /. sol2, {x, -2, 2}]`
`Plot[y[x] /. sol2, {x, -10, 10}, PlotLegends → {eq1},`
`  PlotStyle → {{Blue, Thickness[0.006]}, {Red, Thickness[0.01]}}, Frame → True,`
`  GridLines → Automatic, GridLinesStyle → Directive[Black, Dashed]]`

Out[◦]= $5\, y[x] - 6\, y'[x] + y''[x] == e^{-3x}$

Out[◦]= $\left\{\left\{y[x] \to \dfrac{e^{-3x}}{32} + e^x\, c_1 + e^{5x}\, c_2\right\}\right\}$

Out[◦]= $\left\{\left\{y[x] \to -\dfrac{e^{-8-3x}\left(e^8 - 5\,e^{16} + 3\,e^{4x} - 3\,e^{8x} + 224\,e^{6+4x} - 635\,e^{16+4x} - 224\,e^{6+8x} + 127\,e^{8+8x}\right)}{32\left(-1 + 5\,e^8\right)}\right\}\right\}$

Out[●]=
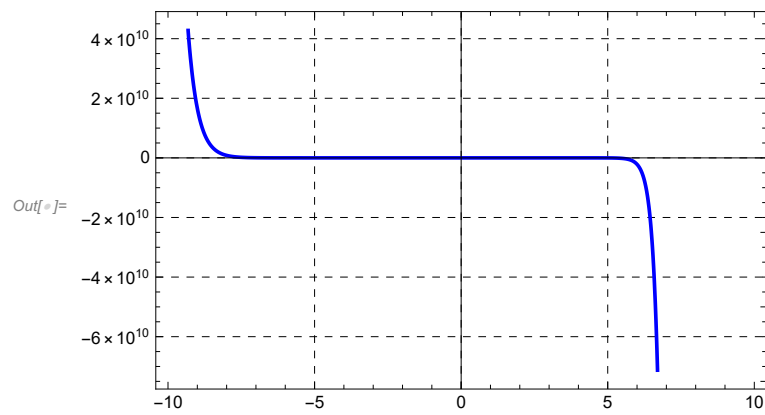
$$y''(x) - 6\,y'(x) + 5\,y(x) = e^{-3\,x}$$

Question 4: $y''[x] - 4*y'[x] + 13*y[x] == (e^{2*x}) *Cos[3*x]$
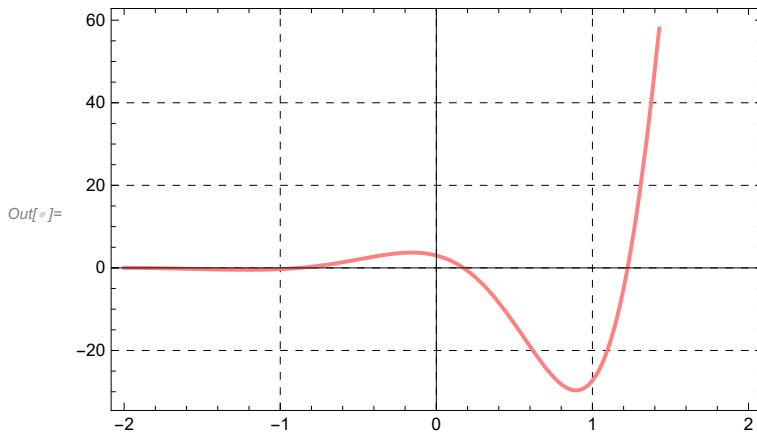
```
In[ ]:= eq1 = y''[x] - 4 * y'[x] + 13 * y[x] == (e^{2*x}) * Cos[3 * x]
       DSolve[{y''[x] - 4 * y'[x] + 13 * y[x] == (e^{2*x}) * Cos[3 * x]}, y[x], x]
       sol4 =
        DSolve[{y''[x] - 4 * y'[x] + 13 * y[x] == (e^{2*x}) * Cos[3 * x], y[0] == 3, y'[3] == 4}, y[x], x]
       Plot[y[x] /. sol4, {x, -2, 2}, PlotLegends → {eq1},
        PlotStyle → {{Pink, Thickness[0.006]}, {Red, Thickness[0.01]}}, Frame → True,
        GridLines → Automatic, GridLinesStyle → Directive[Black, Dashed]]
```

$Out[ ]= 13\,y[x] - 4\,y'[x] + y''[x] == e^{2x}\,Cos[3\,x]$

$Out[ ]= \left\{\left\{y[x] \to e^{2x}\,c_2\,Cos[3\,x] + e^{2x}\,c_1\,Sin[3\,x] + \frac{1}{36}\,e^{2x}\,\left(Cos[3\,x]\,Cos[6\,x] + 6\,x\,Sin[3\,x] + Sin[3\,x]\,Sin[6\,x]\right)\right\}\right\}$

$Out[ ]= \left\{\left\{y[x] \to \frac{1}{36\,(3\,Cos[9] + 2\,Sin[9])}\right.\right.$
$e^{-6+2x}\left(321\,e^6\,Cos[9]\,Cos[3\,x] + 3\,e^6\,Cos[9]\,Cos[3\,x]\,Cos[6\,x] + 214\,e^6\,Cos[3\,x]\,Sin[9] + \right.$
$2\,e^6\,Cos[3\,x]\,Cos[6\,x]\,Sin[9] + 144\,Sin[3\,x] - 268\,e^6\,Cos[9]\,Sin[3\,x] + $
$18\,e^6\,x\,Cos[9]\,Sin[3\,x] - 2\,e^6\,Cos[9]\,Cos[18]\,Sin[3\,x] + 279\,e^6\,Sin[9]\,Sin[3\,x] + $
$12\,e^6\,x\,Sin[9]\,Sin[3\,x] - 3\,e^6\,Cos[18]\,Sin[9]\,Sin[3\,x] + 3\,e^6\,Cos[9]\,Sin[18]\,Sin[3\,x] - 2\,e^6\,$
$Sin[9]\,Sin[18]\,Sin[3\,x] + 3\,e^6\,Cos[9]\,Sin[3\,x]\,Sin[6\,x] + 2\,e^6\,Sin[9]\,Sin[3\,x]\,Sin[6\,x]\left.\right)\left.\right\}\right\}$

$Out[ ]=$

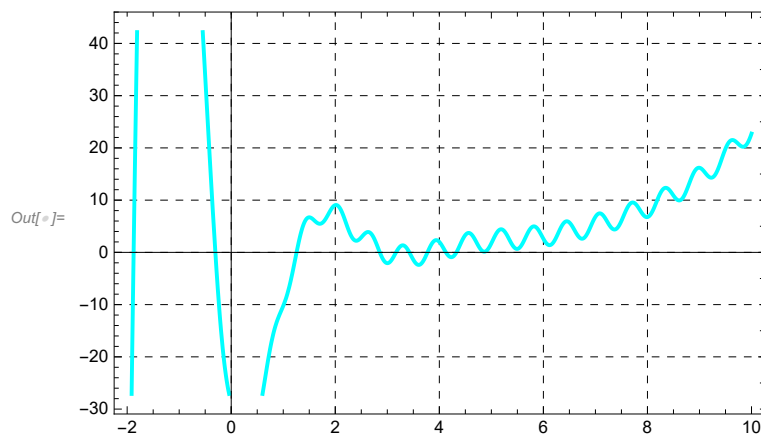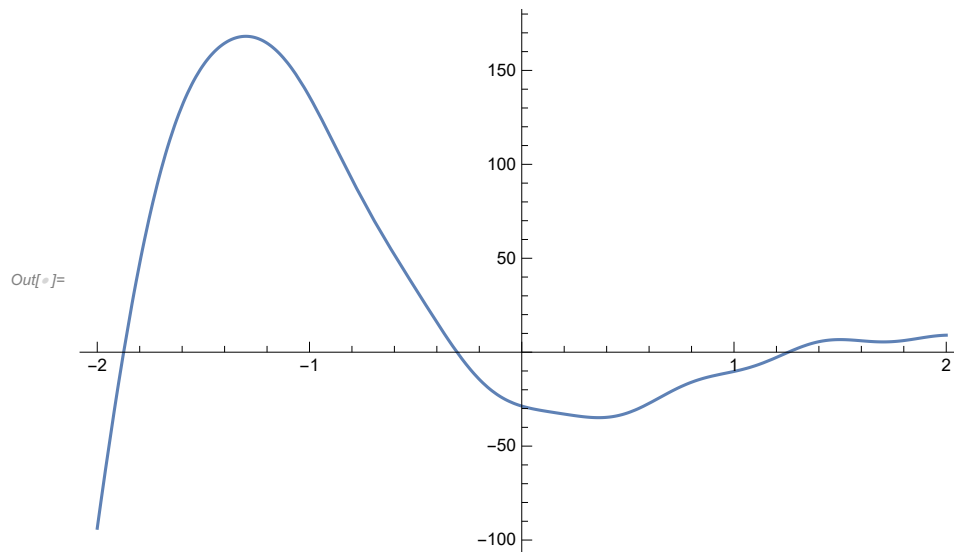$y''(x) - 4\,y'(x) + 13\,y(x) = e^{2x}\cos(3\,x)$

Question 5 :

In[ ]:=
```
DSolve[{y''[x] + 2 * y'[x] + 5 * y[x] == e^0.5*x + 40 * Cos[10 * x] - 190 * Sin[10 * x]}, y[x], x]
sol5 = DSolve[{y''[x] + 2 * y'[x] + 5 * y[x] == e^0.5*x + 40 * Cos[10 * x] - 190 * Sin[10 * x],
    y[4] == 2, y'[2] == 3}, y[x], x]
Plot[y[x] /. sol5, {x, -2, 2}]
Plot[y[x] /. sol5, {x, -2, 10}, PlotLegends → {eq1},
 PlotStyle → {{Cyan, Thickness[0.006]}, {Red, Thickness[0.01]}}, Frame → True,
 GridLines → Automatic, GridLinesStyle → Directive[Black, Dashed]]
```

Out[ ]= $\{\{y[x] \rightarrow e^{-1. x} c_2 \cos[2. x] + e^{-1. x} c_1 \sin[2. x] + 2. e^{-1. x} (0. + 0.08 e^{1.5 x} \cos[2. x]^2 + 0.08 e^{1.5 x} \sin[2. x]^2 + 1. e^{1. x} \cos[2. x]^2 \sin[10. x] + 1. e^{1. x} \sin[2. x]^2 \sin[10. x])\}\}$

Out[ ]= $\{\{y[x] \rightarrow 2. e^{-1. x} (-14.4216 \cos[2. x] + 0.08 e^{1.5 x} \cos[2. x]^2 - 20.6763 \sin[2. x] + 0.08 e^{1.5 x} \sin[2. x]^2 + 1. e^{1. x} \cos[2. x]^2 \sin[10. x] + 1. e^{1. x} \sin[2. x]^2 \sin[10. x])\}\}$

Out[ ]=

Out[ ]=

$$y''(x) - 4 y'(x) + 13 y(x) = e^{2x} \cos(3x)$$

In[ ]:=