

Practical - 3

To solve 3rd order and higher order linear differential equations and plotting its particular solutions

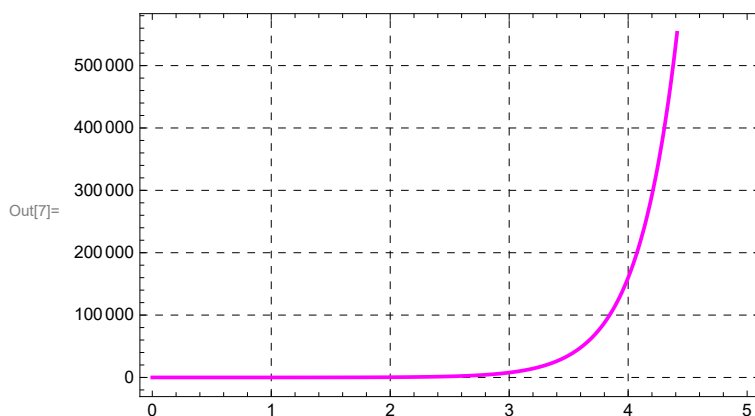
Question - 1 : $y''' - 6y'' + 11y' - 6y = 0$

```
In[4]:= eq1 = y'''[x] - 6 * y''[x] + 11 * y'[x] - 6 * y[x] == 0
DSolve[{y'''[x] - 6 * y''[x] + 11 * y'[x] - 6 * y[x] == 0}, y[x], x]
sol1 = DSolve[
  {y'''[x] - 6 * y''[x] + 11 * y'[x] - 6 * y[x] == 0, y''[0] == 9, y'[0] == 5, y[0] == 4}, y[x], x]
Plot[y[x] /. sol1, {x, 0, 5}, PlotLegends -> {eq1},
  PlotStyle -> {{Magenta, Thickness[0.006]}, {Red, Thickness[0.01]}},
  Frame -> True, GridLines -> Automatic, GridLinesStyle -> Directive[Black, Dashed]]
```

Out[4]= $-6 y[x] + 11 y'[x] - 6 y''[x] + y^{(3)}[x] == 0$

Out[5]= $\left\{ \left\{ y[x] \rightarrow e^x c_1 + e^{2x} c_2 + e^{3x} c_3 \right\} \right\}$

Out[6]= $\left\{ \left\{ y[x] \rightarrow e^x (4 - e^x + e^{2x}) \right\} \right\}$



$y^{(3)}(x) - 6y''(x) + 11y'(x) - 6y(x) = 0$

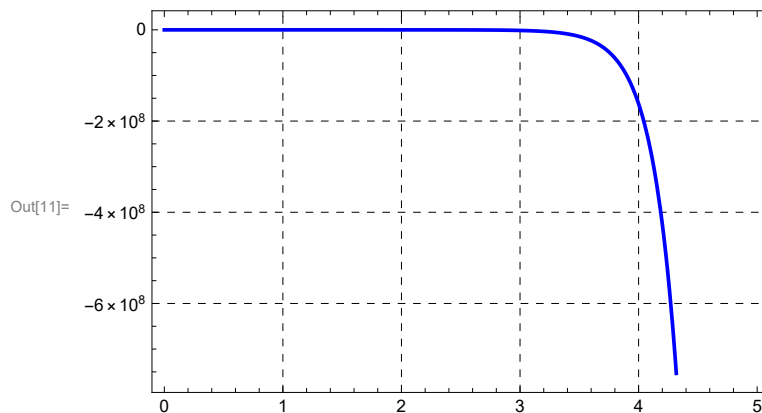
Question - 2 : $y''' - 6y'' + 4y' + 8y = e^{2x}(4 + 19x + 6x^2)$.

```
In[8]:= eq1 = y'''[x] - 6 * y''[x] + 4 * y'[x] + 8 * y[x] == e^{2x} (4 + 19 * x + 6 * x^2)
DSolve[{y'''[x] - 6 * y''[x] + 4 * y'[x] + 8 * y[x] == e^{2x} (4 + 19 * x + 6 * x^2)}, y[x], x]
sol2 = DSolve[{y'''[x] - 6 * y''[x] + 4 * y'[x] + 8 * y[x] == e^{2x} (4 + 19 * x + 6 * x^2),
  y''[0] == 3, y'[0] == 4, y[0] == 8}, y[x], x]
Plot[y[x] /. sol2, {x, 0, 5}, PlotLegends -> {eq1},
  PlotStyle -> {{Blue, Thickness[0.006]}, {Red, Thickness[0.01]}}, Frame -> True,
  GridLines -> Automatic, GridLinesStyle -> Directive[Black, Dashed]]
```

```
Out[8]= 8 y[x] + 4 y'[x] - 6 y''[x] + y^{(3)}[x] == e^{2x} (4 + 19 x + 6 x^2)
```

```
Out[9]= { {y[x] ->
  1/128 e^{-2\sqrt{2} x} \left( -19 e^{(2+2\sqrt{2}) x} - 11 \sqrt{2} e^{(2+2\sqrt{2}) x} - 19 e^{4\sqrt{2} x + (2-2\sqrt{2}) x} + 11 \sqrt{2} e^{4\sqrt{2} x + (2-2\sqrt{2}) x} -
  12 e^{(2+2\sqrt{2}) x} x - 38 \sqrt{2} e^{(2+2\sqrt{2}) x} x - 64 e^{2x+2\sqrt{2} x} x - 12 e^{4\sqrt{2} x + (2-2\sqrt{2}) x} x +
  38 \sqrt{2} e^{4\sqrt{2} x + (2-2\sqrt{2}) x} x - 12 \sqrt{2} e^{(2+2\sqrt{2}) x} x^2 - 152 e^{2x+2\sqrt{2} x} x^2 +
  12 \sqrt{2} e^{4\sqrt{2} x + (2-2\sqrt{2}) x} x^2 - 32 e^{2x+2\sqrt{2} x} x^3 \right) + e^{(2-2\sqrt{2}) x} c_1 + e^{(2+2\sqrt{2}) x} c_2 + e^{2x} c_3 } }
```

```
Out[10]= { {y[x] ->
  1/128 e^{-2\sqrt{2} x} \left( -19 e^{(2+2\sqrt{2}) x} - 11 \sqrt{2} e^{(2+2\sqrt{2}) x} + 720 e^{2x+2\sqrt{2} x} +
  171 e^{2\sqrt{2} x + (2-2\sqrt{2}) x} + 181 \sqrt{2} e^{2\sqrt{2} x + (2-2\sqrt{2}) x} - 19 e^{4\sqrt{2} x + (2-2\sqrt{2}) x} +
  11 \sqrt{2} e^{4\sqrt{2} x + (2-2\sqrt{2}) x} + 171 e^{2\sqrt{2} x + (2+2\sqrt{2}) x} - 181 \sqrt{2} e^{2\sqrt{2} x + (2+2\sqrt{2}) x} - 12 e^{(2+2\sqrt{2}) x} x -
  38 \sqrt{2} e^{(2+2\sqrt{2}) x} x - 64 e^{2x+2\sqrt{2} x} x - 12 e^{4\sqrt{2} x + (2-2\sqrt{2}) x} x + 38 \sqrt{2} e^{4\sqrt{2} x + (2-2\sqrt{2}) x} x -
  12 \sqrt{2} e^{(2+2\sqrt{2}) x} x^2 - 152 e^{2x+2\sqrt{2} x} x^2 + 12 \sqrt{2} e^{4\sqrt{2} x + (2-2\sqrt{2}) x} x^2 - 32 e^{2x+2\sqrt{2} x} x^3 \right) } }
```



— $y^{(3)}(x) - 6y''(x) + 4y'(x) + 8y(x) = e^{2x}(6x^2)$

Question -3 : $8y''' + 32y'' + 64y' + 39y = e^{(-2x)} * (4 - 15x) \cos 3x$

```

In[12]:= eq1 = 8 * y'''[x] + 32 * y''[x] + 64 * y'[x] + 39 * y[x] == e^{-2*x} * Cos[3 * x]
DSolve[{8 * y'''[x] + 32 * y''[x] + 64 * y'[x] + 39 * y[x] == e^{-2*x} * Cos[3 * x]}, y[x], x]
sol3 = DSolve[{8 * y'''[x] + 32 * y''[x] + 64 * y'[x] + 39 * y[x] == e^{-2*x} * Cos[3 * x],
  y'[0] == 1, y[0] == 2, y[0] == 3}, y[x], x]
Plot[y[x] /. sol3, {x, 0, 5}, PlotLegends -> {eq1},
  PlotStyle -> {{Magenta, Thickness[0.006]}, {Red, Thickness[0.01]}},
  Frame -> True, GridLines -> Automatic, GridLinesStyle -> Directive[Black, Dashed]]

```

Out[12]= $39 y[x] + 64 y'[x] + 32 y''[x] + 8 y^{(3)}[x] == e^{-2x} \cos[3x]$

Out[13]=

$$\begin{aligned}
 \{ \{ y[x] \rightarrow & e^{x \sqrt{-0.959...}} c_1 + e^{x \sqrt{-1.52... - 1.66... i}} c_2 + \\
 & e^{x \sqrt{... 1 ...}} c_3 + \left(e^{-x \left(2 + \sqrt{-1.52... - 1.66... i} \right)} - x \left(2 + \sqrt{-1.52... + 1.66... i} \right) \right. \\
 & \left. \left(-173056 e^{x \left(2 + \sqrt{... 1 ...} \right)} + x \sqrt{... 1 ...} \cos[3x] \sqrt{-1.92...} + \sqrt{... 958 ...} + \right. \right. \\
 & \left. \left. 12 e^{x \sqrt{... 1 ...} + \sqrt{... 1 ...}} \sqrt{... 1 ...} \sqrt{... 1 ...}^2 \sin[3x] \right) \right) / \\
 & \left(2 \left(-\sqrt{-1.92...} + \sqrt{-3.04... - 3.33... i} \right) \sqrt{... 5 ...} \left((2 - 3i) + \sqrt{-1.52... - 1.66... i} + \right. \right. \\
 & \left. \left. \sqrt{-1.52... + 1.66... i} \right) \left((2 + 3i) + \sqrt{-1.52... - 1.66... i} + \sqrt{-1.52... + 1.66... i} \right) \right) \} \}
 \end{aligned}$$

large output

show less

show more

show all

set size limit...

Out[14]=

$$\left\{ \left\{ y[x] \rightarrow \left(e^{x \sqrt{-1.52... + 1.66... i}} \left(173056 \sqrt{-1.92...} - 346112 \sqrt{-3.04... - 3.33... i} + \dots 775 \dots + 8 \sqrt{-1.92...} \sqrt{-3.04... - 3.33... i}^3 \sqrt{-3.04... + 3.33... i} \sqrt{-0.959...} \sqrt{-1.52... - 1.66... i} \sqrt{-1.52... + 1.66... i} \right) \right) \right\} / \left(2 \left(\sqrt{-1.92...} - \sqrt{-3.04... - 3.33... i} \right) \left(52 + 8 \sqrt{-3.04... - 3.33... i} + \sqrt{-3.04... - 3.33... i}^2 \right) \dots 3 \dots \left(52 + 8 \sqrt{-3.04... + 3.33... i} + \sqrt{-3.04... + 3.33... i}^2 \right) \left(\sqrt{-1.52... - 1.66... i} - \sqrt{-1.52... + 1.66... i} \right) \left(-\sqrt{-0.959...} + \sqrt{-1.52... + 1.66... i} \right) \right) + \dots 1 \dots + \dots 1 \dots + \frac{e^{-1...} \dots 1 \dots}{2 \dots 7 \dots (\dots 1 \dots)} \right\} \right\}$$

large output

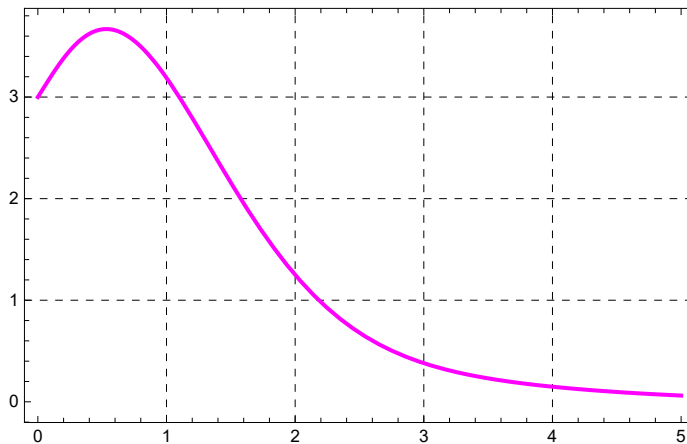
show less

show more

show all

set size limit...

Out[15]=



$$8 y^{(3)}(x) + 32 y''(x) + 64 y'(x) + 39 y(x) = e^{-2}$$

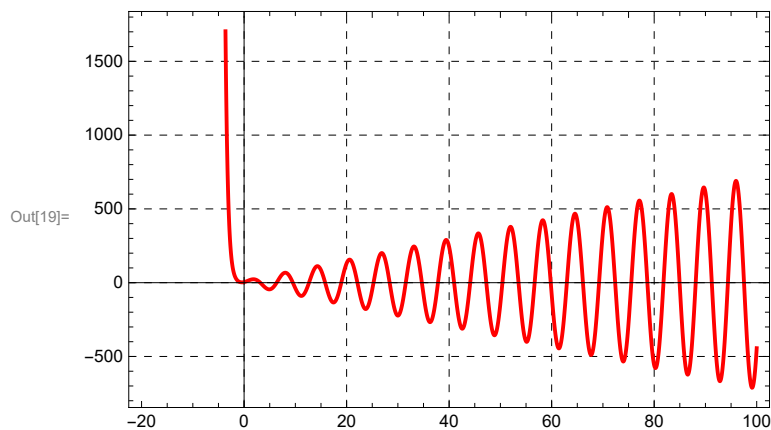
Question -4 : $y''' + 2y'' + y' + 2y = 30\cos x - 10\sin x$

```
In[16]:= eq1 = y'''[x] + 2 * y''[x] + y'[x] + 2 * y[x] == 30 * Cos[x] - 10 * Sin[x]
DSolve[{y'''[x] + 2 * y''[x] + y'[x] + 2 * y[x] == 30 * Cos[x] - 10 * Sin[x]}, y[x], x]
sol4 = DSolve[{y'''[x] + 2 * y''[x] + y'[x] + 2 * y[x] == 30 * Cos[x] - 10 * Sin[x],
  y''[0] == 16, y'[0] == 8, y[0] == 4}, y[x], x]
Plot[y[x] /. sol4, {x, -20, 100}, PlotLegends -> {eq1},
  PlotStyle -> {{Red, Thickness[0.006]}, {Red, Thickness[0.01]}}, Frame -> True,
  GridLines -> Automatic, GridLinesStyle -> Directive[Black, Dashed]]
```

```
Out[16]= 2 y[x] + y'[x] + 2 y''[x] + y(3)[x] == 30 Cos[x] - 10 Sin[x]
```

```
Out[17]= { {y[x] -> e-2 x c3 + c1 Cos[x] + c2 Sin[x] +  $\frac{1}{10} (28 \cos[x] - 10 x \cos[x] + 50 \cos[x]^3 + 4 \sin[x] + 70 x \sin[x] + 50 \cos[x]^2 \sin[x] - 25 \cos[x] \sin[2x] + 25 \sin[x] \sin[2x])$  } }
```

```
Out[18]= { {y[x] ->  $\frac{1}{10} e^{-2 x} (12 - 22 e^{2 x} \cos[x] - 10 e^{2 x} x \cos[x] + 50 e^{2 x} \cos[x]^3 + 114 e^{2 x} \sin[x] + 70 e^{2 x} x \sin[x] + 50 e^{2 x} \cos[x]^2 \sin[x] - 25 e^{2 x} \cos[x] \sin[2x] + 25 e^{2 x} \sin[x] \sin[2x])$  } }
```



— $y^{(3)}(x) + 2y''(x) + y'(x) + 2y(x) = 30 \cos(x) - 10 \sin(x)$

Question - 5: $y''''+3y''' + 5y'' - 2y' = -2e^x(\cos x - \sin x)$

```
In[24]:= eq1 = y''''[x] + 3 * y'''[x] + 5 * y''[x] - 2 * y'[x] == -2 * (Cos[x] - Sin[x])
DSolve[{y''''[x] + 3 * y'''[x] + 5 * y''[x] - 2 * y'[x] == -2 * (Cos[x] - Sin[x])}, y[x], x]
sol5 = DSolve[{y''''[x] + 3 * y'''[x] + 5 * y''[x] - 2 * y'[x] == -2 * (Cos[x] - Sin[x]),
  y'''[0] == 1, y''[0] == 2, y'[2] == 1, y[2] == 2}, y[x], x]
Plot[y[x] /. sol5, {x, -20, 100}, PlotLegends -> {eq1},
  PlotStyle -> {{Brown, Thickness[0.006]}, {Red, Thickness[0.01]}}, Frame -> True,
  GridLines -> Automatic, GridLinesStyle -> Directive[Black, Dashed]]
```

```
Out[24]= -2 y'[x] + 5 y''[x] + 3 y'''[x] + y''''[x] == -2 (Cos[x] - Sin[x])
```

```
Out[25]= {{y[x] -> C4 +  $\frac{1}{0.328...} e^{x(1.66... + 1.82... i)}$  +  $\frac{6}{0.328...} e^{x(-1.66... - 1.82... i)}$ }}
```

large output

show less

show more

show all

set size limit...

Out[26]=

```
{{y[x] ->  $\frac{e^{x(1.66... + 1.82... i)} (1.66... + 1.82... i) - (1.66... - 1.82... i) e^{x(-1.66... - 1.82... i)}}{0.328...} + \frac{(-5 + (-1.66... - 1.82... i)^2 + 2(1.66... + 1.82... i)) (1.66... + 1.82... i) + (-5 + (-1.66... - 1.82... i)^2 + 2(1.66... - 1.82... i)) (-1.66... - 1.82... i)}{0.328...} + \frac{(1+i)(-837+56i) + (1.66... + 1.82... i)^2 (1.66... + 1.82... i) (Cos[x] + i Sin[x]) + (-1.66... - 1.82... i)^2 (-1.66... - 1.82... i) (Cos[x] - i Sin[x])}{0.328...} \frac{1}{0.328...} e^{x(1.66... + 1.82... i)} + \frac{6}{0.328...} e^{x(-1.66... - 1.82... i)}$ }}
```

large output

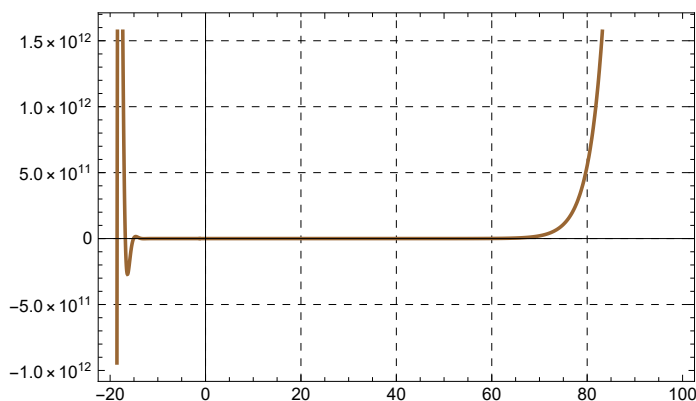
show less

show more

show all

set size limit...

Out[27]=

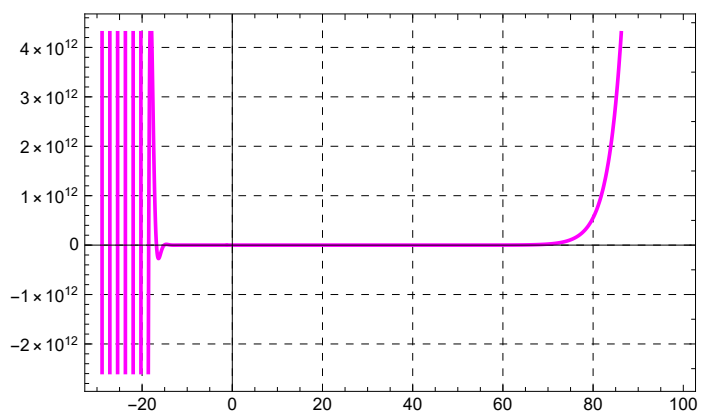


— $y^{(4)}(x) + 3y'''(x) + 5y''(x) - 2y'(x) = -2(\cos(x) - \sin(x))$

In[28]:=

```
Plot[y[x] /. sol5, {x, -30, 100}, PlotLegends → {eq1},
  PlotStyle → {{Magenta, Thickness[0.006]}, {Red, Thickness[0.01]}},
  Frame → True, GridLines → Automatic, GridLinesStyle → Directive[Black, Dashed]]
```

Out[28]=



$$y^{(4)}(x) + 3y^{(3)}(x) + 5y''(x) - 2y'(x) = -2(\cos(x))$$