Practical - 4

To solve differential equation using variation of parameter

Question -1: y''(x)+ y = sec(x) (Solving without any program)

In[120]:=
```
sol = DSolve[y''[x] + y[x] == 0, y[x], x]
y1 = y[x] /. sol[[1]] /. {C[1] → 1, C[2] → 0}
y2 = y[x] /. sol[[1]] /. {C[1] → 0, C[2] → 1}
w = Wronskian[{y1, y2}, x]
v1 = -Integrate[(y2 * Sec[x])/w, x]
v2 = Integrate[(y1 * Sec[x])/w, x]
pi = Simplify[v1 * y1 + v2 * y2]

Plot[{y1, y2}, {x, -10, 10}, PlotRange → {-3, 3}, PlotLegends → {y1, y2},
 PlotStyle → {{Red, Thickness[0.006]}, {Blue, Thickness[0.006]}}, Frame → True,
 GridLines → Automatic, GridLinesStyle → Directive[Black, Dashed]]
```

Out[120]= $\{\{y[x] \to c_1 \text{Cos}[x] + c_2 \text{Sin}[x]\}\}$

Out[121]= $\text{Cos}[x]$

Out[122]= $\text{Sin}[x]$

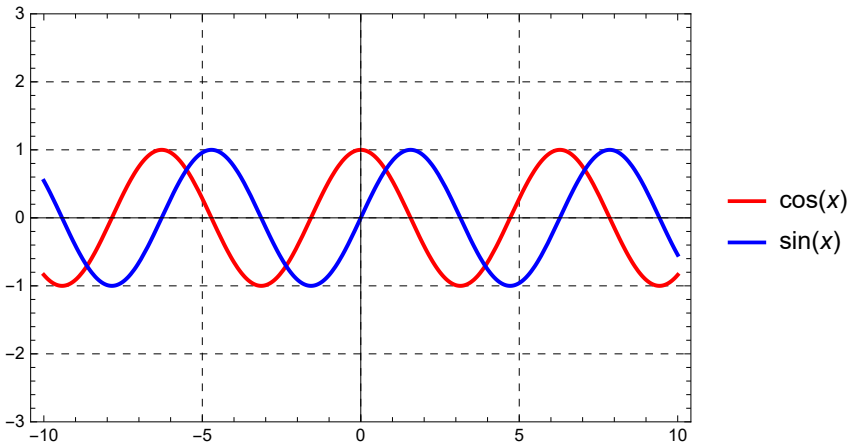
Out[123]= 1

Out[124]= $\text{Log}[\text{Cos}[x]]$

Out[125]= x

Out[126]= $\text{Cos}[x]\ \text{Log}[\text{Cos}[x]] + x\ \text{Sin}[x]$

Out[127]=



In[144]:=

Creating function for 2nd order differential equation

In[166]:=

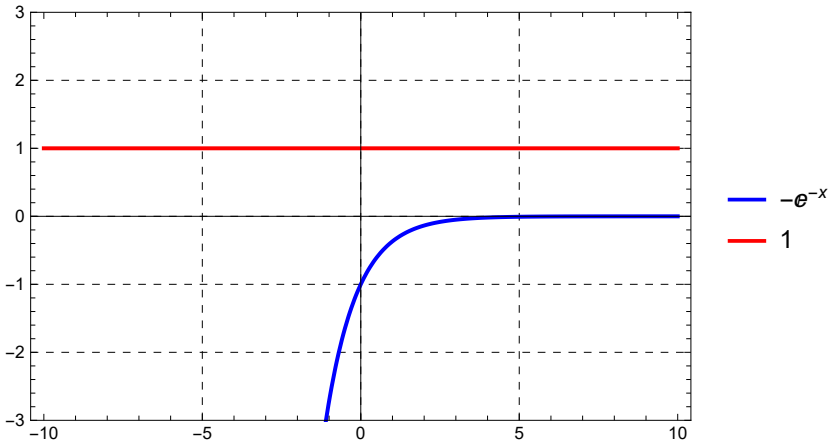
```
VariationPara[homo_, rhs_, y_, x_] := Block[{sol, y1, y2, w, v1, v2},
  sol = DSolve[homo, y[x], x];
  y1 = y[x] /. sol[[1]] /. {C[1] → 1, C[2] → 0};
  y2 = y[x] /. sol[[1]] /. {C[1] → 0, C[2] → 1};
  w = Wronskian[{y1, y2}, x];
  v1 = -Integrate[((y2 * rhs)/w), x];
  v2 = Integrate[((y1 * rhs)/w), x];
  q1 = Simplify[v1 * y1 + v2 * y2];
  Print[q1];
  Plot[{y1, y2}, {x, -10, 10}, PlotRange → {-3, 3}, PlotLegends → {y1, y2},
   PlotStyle → {{Blue, Thickness[0.006]}, {Red, Thickness[0.006]}}, Frame → True,
   GridLines → Automatic, GridLinesStyle → Directive[Black, Dashed]]]
```

Question - 2 : y''(x)+x^2*y(x)=Cos[x] & y''[x] +4y'[x]==Sin[2x]*Sex [2][2x]

In[167]:=
```
VariationPara[{y''[x] + y'[x] == 0}, ln[x], y, x]
```

$$\int \ln[x]\, dx + e^{-x} \int \frac{\ln[x]}{-\cosh[x] + \sinh[x]}\, dx$$

Out[167]=

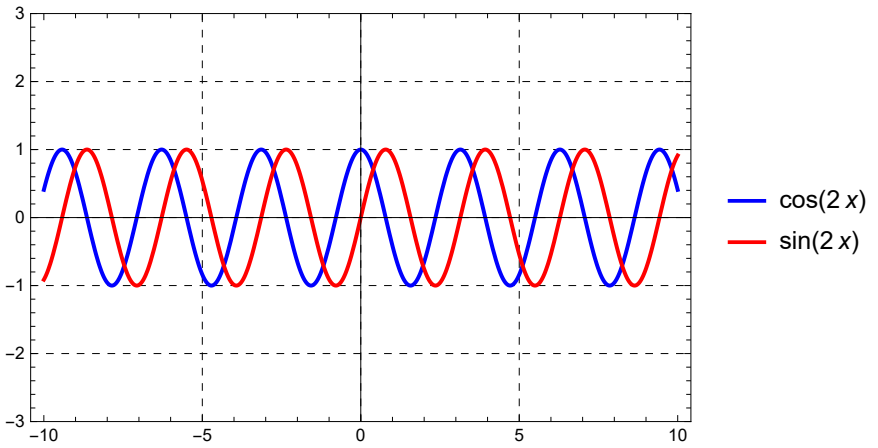


In[143]:=

In[168]:=
```
VariationPara[{y''[x] + 4 * y[x] == 0}, Sin[2 * x] * Sec²[2 * x], y, x]
```

$$\frac{1}{4}\left(-2\cos[2x]\int \sec^2[2x]\sin[2x]^2\, dx + \left(\int \sec^2[2x]\sin[4x]\, dx\right)\sin[2x]\right)$$

Out[168]=

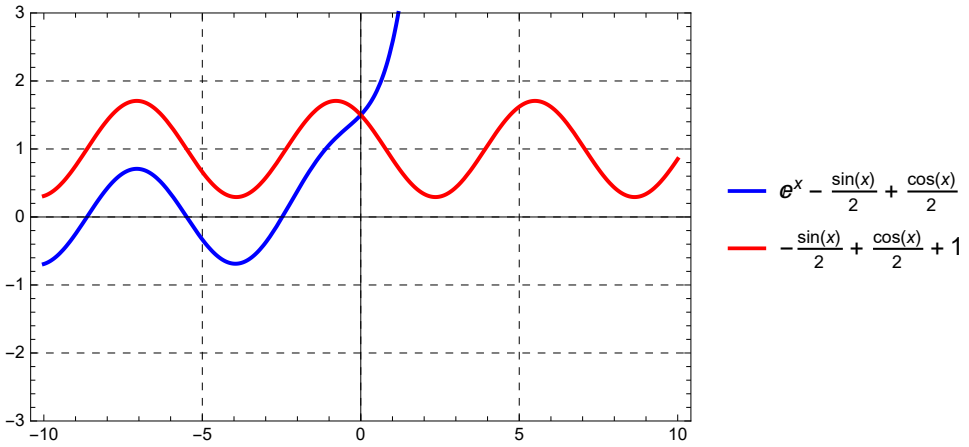


Question -3: y''[x] -y'[x]-Sin[x]=Sec[x]

In[169]:=
```
VariationPara[{y''[x] - y'[x] - Sin[x] == 0}, Sec[x], y, x]
```

$$\frac{1}{2}\left(\left(\int \frac{\text{Sec}[x]\,\left(2\,e^{x}+\text{Cos}[x]-\text{Sin}[x]\right)}{\text{Cos}[x]-2\,e^{x}\,(1+\text{Cos}[x])+\text{Sin}[x]}\,dx\right)\left(2+\text{Cos}[x]-\text{Sin}[x]\right)-\left(\int \frac{\text{Sec}[x]\,\left(2+\text{Cos}[x]-\text{Sin}[x]\right)}{\text{Cos}[x]-2\,e^{x}\,(1+\text{Cos}[x])+\text{Sin}[x]}\,dx\right)\left(2\,e^{x}+\text{Cos}[x]-\text{Sin}[x]\right)\right)$$

Out[169]=



Creating function for third order differential equation

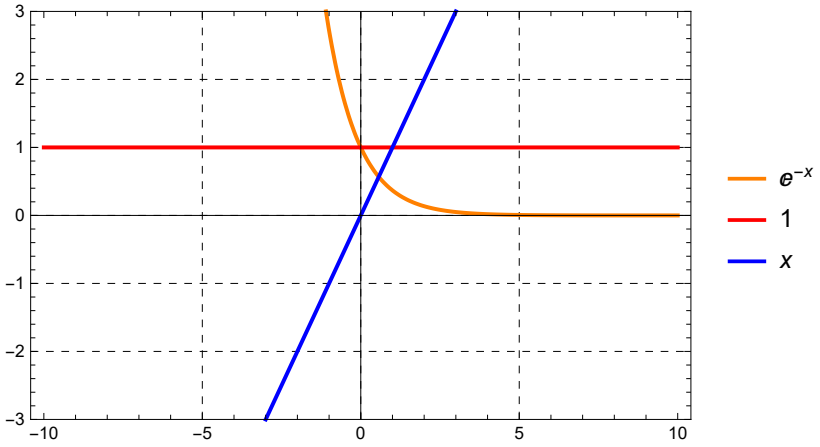
In[170]:=
```
Variation1Para[homo_, rhs_, y_, x_] :=
 Block[{sol, y1, y2, y3, w, w1, w2, w3, v1, v2, v3},
  sol = DSolve[homo, y[x], x];
  y1 = y[x] /. sol[[1]] /. {C[1] → 1, C[2] → 0, C[3] → 0};
  y2 = y[x] /. sol[[1]] /. {C[1] → 0, C[2] → 1, C[3] → 0};
  y3 = y[x] /. sol[[1]] /. {C[1] → 0, C[2] → 0, C[3] → 1};
  w = Wronskian[{y1, y2, y3}, x];
  w1 = Wronskian[{y2, y3}, x];
  w2 = Wronskian[{y1, y3}, x];
  w3 = Wronskian[{y1, y2}, x];
  v1 = Integrate[(w1 * rhs)/w, x];
  v2 = -{Integrate[(w2 * rhs)/w, x]};
  v3 = Integrate[(w3 * rhs)/w, x];
  q1 = Simplify[v1 * y1 + v2 * y2 + v3 * y3];
  Print[q1];
  Plot[{y1, y2, y3}, {x, -10, 10},
   PlotRange → {-3, 3}, PlotLegends → {y1, y2, y3}, PlotStyle →
    {{Orange, Thickness[0.006]}, {Red, Thickness[0.006]}, {Blue, Thickness[0.006]}},
   Frame → True, GridLines → Automatic, GridLinesStyle → Directive[Black, Dashed]]]
```

Question 4: y’’’[x] + y’’[x] ==Sin[x]

In[171]:=
```
Variation1Para[y'''[x] + y''[x] == 0, Sin[x], y, x]
```

$$\left\{\frac{1}{2}\left(\text{Cos}[x] - \text{Sin}[x]\right)\right\}$$

Out[171]=

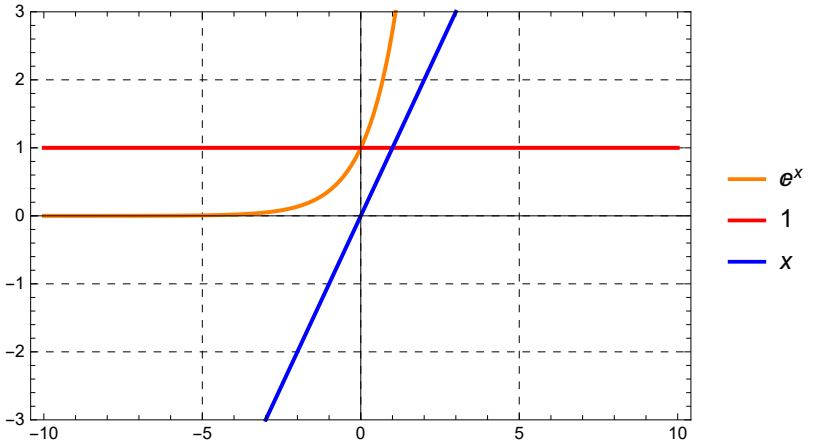


Question 5 : y’’’[x]-y’[x]==0

In[172]:=
```
Variation1Para[y'''[x] - y''[x] == 0, Sin[x], y, x]
```

$$\left\{\frac{1}{2}\left(\text{Cos}[x] + \text{Sin}[x]\right)\right\}$$

Out[172]=



In[50]:=