5. Iterative Methods

5.1Gauss Jacobi Iterative Method

Question - 1

```
In[ ]:= ClearAll

In[ ]:= ClearAll
Out[ ]= ClearAll

In[ ]:= A = {{4, 1, 1}, {1, 5, 2}, {1, 2, 3}};
        X = {x1, x2, x3};
        x = {0.5, -0.5, -0.5};
        b = {2, -6, -4};
        Print[
         "**************************************************************************"]
        Print[" The general form of the equation is : AX = b"]
        Print["The System of equations is : ",
         MatrixForm[A].MatrixForm[X], "=", MatrixForm[b]]
        l = LowerTriangularize[A, -1];
        u = UpperTriangularize[A, 1];
        d = DiagonalMatrix[{4, 5, 3}]
        Print[
         "**************************************************************************"]
        For[k = 1, k ≤ 15, k++, y = Inverse[d].(b - l.x - u.x);
         x = N[y];
         Print[k, "th approximation solution is : ", MatrixForm[x]]]

        **************************************************************************

         The general form of the equation is : AX = b

        The System of equations is :  ⎛4 1 1⎞ . ⎛x1⎞ = ⎛ 2⎞
                                      ⎜1 5 2⎟   ⎜x2⎟   ⎜-6⎟
                                      ⎝1 2 3⎠   ⎝x3⎠   ⎝-4⎠

Out[ ]= {{4, 0, 0}, {0, 5, 0}, {0, 0, 3}}
```

****

1th approximation solution is : $\begin{pmatrix} 0.75 \\ -1.1 \\ -1.16667 \end{pmatrix}$

2th approximation solution is : $\begin{pmatrix} 1.06667 \\ -0.883333 \\ -0.85 \end{pmatrix}$

3th approximation solution is : $\begin{pmatrix} 0.933333 \\ -1.07333 \\ -1.1 \end{pmatrix}$

4th approximation solution is : $\begin{pmatrix} 1.04333 \\ -0.946667 \\ -0.928889 \end{pmatrix}$

5th approximation solution is : $\begin{pmatrix} 0.968889 \\ -1.03711 \\ -1.05 \end{pmatrix}$

6th approximation solution is : $\begin{pmatrix} 1.02178 \\ -0.973778 \\ -0.964889 \end{pmatrix}$

7th approximation solution is : $\begin{pmatrix} 0.984667 \\ -1.0184 \\ -1.02474 \end{pmatrix}$

8th approximation solution is : $\begin{pmatrix} 1.01079 \\ -0.987037 \\ -0.982622 \end{pmatrix}$

9th approximation solution is : $\begin{pmatrix} 0.992415 \\ -1.00911 \\ -1.01224 \end{pmatrix}$

10th approximation solution is : $\begin{pmatrix} 1.00534 \\ -0.993588 \\ -0.9914 \end{pmatrix}$

11th approximation solution is : $\begin{pmatrix} 0.996247 \\ -1.00451 \\ -1.00605 \end{pmatrix}$

12th approximation solution is : $\begin{pmatrix} 1.00264 \\ -0.996828 \\ -0.995744 \end{pmatrix}$

13th approximation solution is : $\begin{pmatrix} 0.998143 \\ -1.00223 \\ -1.00299 \end{pmatrix}$

14th approximation solution is : $\begin{pmatrix} 1.00131 \\ -0.998431 \\ -0.997894 \end{pmatrix}$

15th approximation solution is : $\begin{pmatrix} 0.999081 \\ -1.0011 \\ -1.00148 \end{pmatrix}$

Question - 2

In[•]:=
```
A = {{26, 2, 2}, {3, 27, 1}, {2, 3, 17}};
X = {x1, x2, x3};
x = {0, 0, 0};
b = {12.6, -14.3, 6};
Print[
 "**************************************************************************"]
Print[" The general form of the equation is : AX = b"]
Print["The System of equations is : ",
 MatrixForm[A].MatrixForm[X], "=", MatrixForm[b]]
l  = LowerTriangularize[A, -1];
u = UpperTriangularize[A, 1];
d = DiagonalMatrix[{26, 27, 17}]
Print[
 "**************************************************************************"]
For[k = 1, k ≤ 15, k++, y = Inverse[d].(b - l.x - u.x);
 x = N[y];
 Print[k, "th approximation solution is : ", MatrixForm[x]]]
```

```
**************************************************************************
 The general form of the equation is : AX = b
```

The System of equations is : $\begin{pmatrix} 26 & 2 & 2 \\ 3 & 27 & 1 \\ 2 & 3 & 17 \end{pmatrix} . \begin{pmatrix} x1 \\ x2 \\ x3 \end{pmatrix} = \begin{pmatrix} 12.6 \\ -14.3 \\ 6 \end{pmatrix}$

Out[•]= {{26, 0, 0}, {0, 27, 0}, {0, 0, 17}}

****

1th approximation solution is : $\begin{pmatrix} 0.484615 \\ -0.52963 \\ 0.352941 \end{pmatrix}$

2th approximation solution is : $\begin{pmatrix} 0.498207 \\ -0.596548 \\ 0.389392 \end{pmatrix}$

3th approximation solution is : $\begin{pmatrix} 0.50055 \\ -0.599408 \\ 0.399602 \end{pmatrix}$

4th approximation solution is : $\begin{pmatrix} 0.499985 \\ -0.600046 \\ 0.399831 \end{pmatrix}$

5th approximation solution is : $\begin{pmatrix} 0.500017 \\ -0.599992 \\ 0.40001 \end{pmatrix}$

6th approximation solution is : $\begin{pmatrix} 0.499999 \\ -0.600002 \\ 0.399997 \end{pmatrix}$

7th approximation solution is : $\begin{pmatrix} 0.5 \\ -0.6 \\ 0.400001 \end{pmatrix}$

8th approximation solution is : $\begin{pmatrix} 0.5 \\ -0.6 \\ 0.4 \end{pmatrix}$

9th approximation solution is : $\begin{pmatrix} 0.5 \\ -0.6 \\ 0.4 \end{pmatrix}$

10th approximation solution is : $\begin{pmatrix} 0.5 \\ -0.6 \\ 0.4 \end{pmatrix}$

11th approximation solution is : $\begin{pmatrix} 0.5 \\ -0.6 \\ 0.4 \end{pmatrix}$

12th approximation solution is : $\begin{pmatrix} 0.5 \\ -0.6 \\ 0.4 \end{pmatrix}$

13th approximation solution is : $\begin{pmatrix} 0.5 \\ -0.6 \\ 0.4 \end{pmatrix}$

14th approximation solution is : $\begin{pmatrix} 0.5 \\ -0.6 \\ 0.4 \end{pmatrix}$

15th approximation solution is : $\begin{pmatrix} 0.5 \\ -0.6 \\ 0.4 \end{pmatrix}$

Question - 3

In[1]:=

```
A = {{1, 1, 1}, {1, 2, 2}, {1, 3, 1}};
X = {x1, x2, x3};
x = {0, 0, 0};
b = {7, 13, 13};
Print[
 "**************************************************************************"]
Print[" The general form of the equation is : AX = b"]
Print["The System of equations is : ",
 MatrixForm[A].MatrixForm[X], "=", MatrixForm[b]]
l  = LowerTriangularize[A, -1];
u = UpperTriangularize[A, 1];
d = DiagonalMatrix[{26, 27, 17}]
Print[
 "**************************************************************************"]
For[k = 1, k ≤ 15, k++, y = Inverse[d].(b - l.x - u.x);
 x = N[y];
 Print[k, "th approximation solution is : ", MatrixForm[x]]]
```

```
**************************************************************************
 The general form of the equation is : AX = b
The System of equations is :  ⎛1 1 1⎞ . ⎛x1⎞ = ⎛ 7⎞
                              ⎜1 2 2⎟   ⎜x2⎟   ⎜13⎟
                              ⎝1 3 1⎠   ⎝x3⎠   ⎝13⎠
```

Out[10]= {{26, 0, 0}, {0, 27, 0}, {0, 0, 17}}

****

1th approximation solution is : $\begin{pmatrix} 0.269231 \\ 0.481481 \\ 0.764706 \end{pmatrix}$

2th approximation solution is : $\begin{pmatrix} 0.2213 \\ 0.414865 \\ 0.663901 \end{pmatrix}$

3th approximation solution is : $\begin{pmatrix} 0.22774 \\ 0.424107 \\ 0.678477 \end{pmatrix}$

4th approximation solution is : $\begin{pmatrix} 0.226824 \\ 0.422789 \\ 0.676467 \end{pmatrix}$

5th approximation solution is : $\begin{pmatrix} 0.226952 \\ 0.422972 \\ 0.676753 \end{pmatrix}$

6th approximation solution is : $\begin{pmatrix} 0.226934 \\ 0.422946 \\ 0.676714 \end{pmatrix}$

7th approximation solution is : $\begin{pmatrix} 0.226936 \\ 0.42295 \\ 0.676719 \end{pmatrix}$

8th approximation solution is : $\begin{pmatrix} 0.226936 \\ 0.422949 \\ 0.676719 \end{pmatrix}$

9th approximation solution is : $\begin{pmatrix} 0.226936 \\ 0.422949 \\ 0.676719 \end{pmatrix}$

10th approximation solution is : $\begin{pmatrix} 0.226936 \\ 0.422949 \\ 0.676719 \end{pmatrix}$

11th approximation solution is : $\begin{pmatrix} 0.226936 \\ 0.422949 \\ 0.676719 \end{pmatrix}$

12th approximation solution is : $\begin{pmatrix} 0.226936 \\ 0.422949 \\ 0.676719 \end{pmatrix}$

13th approximation solution is : $\begin{pmatrix} 0.226936 \\ 0.422949 \\ 0.676719 \end{pmatrix}$

14th approximation solution is : $\begin{pmatrix} 0.226936 \\ 0.422949 \\ 0.676719 \end{pmatrix}$

15th approximation solution is : $\begin{pmatrix} 0.226936 \\ 0.422949 \\ 0.676719 \end{pmatrix}$

5.2 Gauss Seidel Iterative Method

Question - 1

In[•]:=
```
A = {{2, -1, 0}, {-1, 2, -1}, {0, -1, 2}};
X = {x1, x2, x3};
x = {0, 0, 0};
b = {7, 1, 1};
Print[
 "************************************************************************"]
Print[" The general form of the equation is : AX = b"]
Print["The System of equations is : ",
 MatrixForm[A].MatrixForm[X], "=", MatrixForm[b]]
l  = LowerTriangularize[A, -1];
u = UpperTriangularize[A, 1];
d = DiagonalMatrix[{2, 2, 2}]
Print[
 "************************************************************************"]
For[k = 1, k ≤ 15, k++, y = Inverse[d + l].(b - u.x);
 x = N[y];
 Print[k, "th approximation solution is : ", MatrixForm[x]]]
```

```
************************************************************************
 The general form of the equation is : AX = b
The System of equations is :  ⎛ 2  -1  0 ⎞ . ⎛x1⎞ = ⎛7⎞
                              ⎜-1   2 -1 ⎟   ⎜x2⎟   ⎜1⎟
                              ⎝ 0  -1  2 ⎠   ⎝x3⎠   ⎝1⎠
```

Out[•]=
```
{{2, 0, 0}, {0, 2, 0}, {0, 0, 2}}
```

****

1th approximation solution is : $\begin{pmatrix} 3.5 \\ 2.25 \\ 1.625 \end{pmatrix}$

2th approximation solution is : $\begin{pmatrix} 4.625 \\ 3.625 \\ 2.3125 \end{pmatrix}$

3th approximation solution is : $\begin{pmatrix} 5.3125 \\ 4.3125 \\ 2.65625 \end{pmatrix}$

4th approximation solution is : $\begin{pmatrix} 5.65625 \\ 4.65625 \\ 2.82813 \end{pmatrix}$

5th approximation solution is : $\begin{pmatrix} 5.82813 \\ 4.82813 \\ 2.91406 \end{pmatrix}$

6th approximation solution is : $\begin{pmatrix} 5.91406 \\ 4.91406 \\ 2.95703 \end{pmatrix}$

7th approximation solution is : $\begin{pmatrix} 5.95703 \\ 4.95703 \\ 2.97852 \end{pmatrix}$

8th approximation solution is : $\begin{pmatrix} 5.97852 \\ 4.97852 \\ 2.98926 \end{pmatrix}$

9th approximation solution is : $\begin{pmatrix} 5.98926 \\ 4.98926 \\ 2.99463 \end{pmatrix}$

10th approximation solution is : $\begin{pmatrix} 5.99463 \\ 4.99463 \\ 2.99731 \end{pmatrix}$

11th approximation solution is : $\begin{pmatrix} 5.99731 \\ 4.99731 \\ 2.99866 \end{pmatrix}$

12th approximation solution is : $\begin{pmatrix} 5.99866 \\ 4.99866 \\ 2.99933 \end{pmatrix}$

13th approximation solution is : $\begin{pmatrix} 5.99933 \\ 4.99933 \\ 2.99966 \end{pmatrix}$

14th approximation solution is : $\begin{pmatrix} 5.99966 \\ 4.99966 \\ 2.99983 \end{pmatrix}$

15th approximation solution is : $\begin{pmatrix} 5.99983 \\ 4.99983 \\ 2.99992 \end{pmatrix}$

Question 2

In[ ]:=
```
A = {{45, 2, 3}, {-3, 22, 2}, {5, 1, 20}};
X = {x1, x2, x3};
x = {0, 0, 0};
b = {58, 47, 67};
Print[
 "**********************************************************************"]
Print[" The general form of the equation is : AX = b"]
Print["The System of equations is : ",
 MatrixForm[A].MatrixForm[X], "=", MatrixForm[b]]
l  = LowerTriangularize[A, -1];
u = UpperTriangularize[A, 1];
d = DiagonalMatrix[{45, 22, 20}]
Print[
 "**********************************************************************"]
For[k = 1, k ≤ 15, k++, y = Inverse[d + l].(b - u.x);
 x = N[y];
 Print[k, "th approximation solution is : ", MatrixForm[x]]]
```

```
**********************************************************************
 The general form of the equation is : AX = b
The System of equations is :  ⎛45  2  3⎞ . ⎛x1⎞ = ⎛58⎞
                              ⎜-3 22  2⎟   ⎜x2⎟   ⎜47⎟
                              ⎝ 5  1 20⎠   ⎝x3⎠   ⎝67⎠
```

Out[ ]=
```
{{45, 0, 0}, {0, 22, 0}, {0, 0, 20}}
```

```
**************************************************************************
```

1th approximation solution is : $\begin{pmatrix} 1.28889 \\ 2.31212 \\ 2.91217 \end{pmatrix}$

2th approximation solution is : $\begin{pmatrix} 0.991983 \\ 2.00689 \\ 3.00166 \end{pmatrix}$

3th approximation solution is : $\begin{pmatrix} 0.999583 \\ 1.99979 \\ 3.00011 \end{pmatrix}$

4th approximation solution is : $\begin{pmatrix} 1. \\ 1.99999 \\ 3. \end{pmatrix}$

5th approximation solution is : $\begin{pmatrix} 1. \\ 2. \\ 3. \end{pmatrix}$

6th approximation solution is : $\begin{pmatrix} 1. \\ 2. \\ 3. \end{pmatrix}$

7th approximation solution is : $\begin{pmatrix} 1. \\ 2. \\ 3. \end{pmatrix}$

8th approximation solution is : $\begin{pmatrix} 1. \\ 2. \\ 3. \end{pmatrix}$

9th approximation solution is : $\begin{pmatrix} 1. \\ 2. \\ 3. \end{pmatrix}$

10th approximation solution is : $\begin{pmatrix} 1. \\ 2. \\ 3. \end{pmatrix}$

11th approximation solution is : $\begin{pmatrix} 1. \\ 2. \\ 3. \end{pmatrix}$

12th approximation solution is : $\begin{pmatrix} 1. \\ 2. \\ 3. \end{pmatrix}$

13th approximation solution is : $\begin{pmatrix} 1. \\ 2. \\ 3. \end{pmatrix}$

14th approximation solution is : $\begin{pmatrix} 1. \\ 2. \\ 3. \end{pmatrix}$

15th approximation solution is : $\begin{pmatrix} 1. \\ 2. \\ 3. \end{pmatrix}$

Question - 3

In[13]:=

```
A = {{2, 1, 1}, {3, 5, 2}, {2, 1, 4}};
X = {x1, x2, x3};
x = {0, 0, 0};
b = {5, 15, 8};
Print[
 "************************************************************************"]
Print[" The general form of the equation is : AX = b"]
Print["The System of equations is : ",
 MatrixForm[A].MatrixForm[X], "=", MatrixForm[b]]
l  = LowerTriangularize[A, -1];
u = UpperTriangularize[A, 1];
d = DiagonalMatrix[{2, 2, 2}]
Print[
 "************************************************************************"]
For[k = 1, k ≤ 15, k++, y = Inverse[d + l].(b - u.x);
 x = N[y];
 Print[k, "th approximation solution is : ", MatrixForm[x]]]
```

```
************************************************************************
 The general form of the equation is : AX = b
The System of equations is :  ⎛2 1 1⎞ . ⎛x1⎞ = ⎛ 5 ⎞
                              ⎜3 5 2⎟   ⎜x2⎟   ⎜15 ⎟
                              ⎝2 1 4⎠   ⎝x3⎠   ⎝ 8 ⎠
```

Out[22]= {{2, 0, 0}, {0, 2, 0}, {0, 0, 2}}

****

1th approximation solution is : $\begin{pmatrix} 2.5 \\ 3.75 \\ -0.375 \end{pmatrix}$

2th approximation solution is : $\begin{pmatrix} 0.8125 \\ 6.65625 \\ -0.140625 \end{pmatrix}$

3th approximation solution is : $\begin{pmatrix} -0.757813 \\ 8.77734 \\ 0.369141 \end{pmatrix}$

4th approximation solution is : $\begin{pmatrix} -2.07324 \\ 10.2407 \\ 0.952881 \end{pmatrix}$

5th approximation solution is : $\begin{pmatrix} -3.0968 \\ 11.1923 \\ 1.50064 \end{pmatrix}$

6th approximation solution is : $\begin{pmatrix} -3.84648 \\ 11.7691 \\ 1.96194 \end{pmatrix}$

7th approximation solution is : $\begin{pmatrix} -4.36551 \\ 12.0863 \\ 2.32235 \end{pmatrix}$

8th approximation solution is : $\begin{pmatrix} -4.70434 \\ 12.2342 \\ 2.58726 \end{pmatrix}$

9th approximation solution is : $\begin{pmatrix} -4.91071 \\ 12.2788 \\ 2.77131 \end{pmatrix}$

10th approximation solution is : $\begin{pmatrix} -5.02505 \\ 12.2663 \\ 2.89192 \end{pmatrix}$

11th approximation solution is : $\begin{pmatrix} -5.0791 \\ 12.2267 \\ 2.96573 \end{pmatrix}$

12th approximation solution is : $\begin{pmatrix} -5.09623 \\ 12.1786 \\ 3.00692 \end{pmatrix}$

13th approximation solution is : $\begin{pmatrix} -5.09277 \\ 12.1322 \\ 3.02665 \end{pmatrix}$

14th approximation solution is : $\begin{pmatrix} -5.07944 \\ 12.0925 \\ 3.03319 \end{pmatrix}$

15th approximation solution is : $\begin{pmatrix} -5.06285 \\ 12.0611 \\ 3.03231 \end{pmatrix}$

In[ ]:=

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