6. Gauss Jordan

Question 1

Solve the following system of equations(without partial pivoting):

x1 + 1x2 + 3x3 = 1

2x1 + 6x2 + 10x3 = 0

3x1 + 14x2 + 28x3 = -8

In[ ]:=

```
A = {{1, 2, 3}, {2, 6, 10}, {3, 14, 28}};
A // MatrixForm
x = {x1, x2, x3};
x // MatrixForm
b = {{1}, {0}, {-8}};
b // MatrixForm
aug = ArrayFlatten[{{A, b}}];
aug // MatrixForm
aug[[2]] = aug[[2]] - 2 aug[[1]];
aug[[3]] = aug[[3]] - 3 aug[[1]];
aug // MatrixForm
aug[[2]] = aug[[2]] * (1 / 2);
aug // MatrixForm
aug[[1]] = aug[[1]] - 2 aug[[2]];
aug[[3]] = aug[[3]] - 8 aug[[2]];
aug // MatrixForm
aug[[3]] = aug[[3]] * (1 / 3);
aug // MatrixForm
aug[[1]] = aug[[1]] + aug[[3]];
aug[[2]] = aug[[2]] - 2 aug[[3]];
aug // MatrixForm
IdentityMatrix[3] = Take[aug, 3, 3];
IdentityMatrix[3] // MatrixForm
c = Take[aug, 3, -1];
c // MatrixForm
IdentityMatrix[3].x == c
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 6 & 10 \\ 3 & 14 & 28 \end{pmatrix}$$

Out[ ]//MatrixForm=

$$\begin{pmatrix} x1 \\ x2 \\ x3 \end{pmatrix}$$

Out[ ]//MatrixForm=

$$\begin{pmatrix} 1 \\ 0 \\ -8 \end{pmatrix}$$

Out[ ]//MatrixForm=

$$\begin{pmatrix} 1 & 2 & 3 & 1 \\ 2 & 6 & 10 & 0 \\ 3 & 14 & 28 & -8 \end{pmatrix}$$

Out[ ]//MatrixForm=

$$\begin{pmatrix} 1 & 2 & 3 & 1 \\ 0 & 2 & 4 & -2 \\ 0 & 8 & 19 & -11 \end{pmatrix}$$

Out[ ]//MatrixForm=

$$\begin{pmatrix} 1 & 2 & 3 & 1 \\ 0 & 1 & 2 & -1 \\ 0 & 8 & 19 & -11 \end{pmatrix}$$

Out[ ]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & -1 & 3 \\ 0 & 1 & 2 & -1 \\ 0 & 0 & 3 & -3 \end{pmatrix}$$

Out[ ]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & -1 & 3 \\ 0 & 1 & 2 & -1 \\ 0 & 0 & 1 & -1 \end{pmatrix}$$

Out[ ]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -1 \end{pmatrix}$$

... Set: Tag IdentityMatrix in IdentityMatrix[3] is Protected.

Out[ ]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Out[ ]//MatrixForm=

$$\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$$

Out[ ]= {x1, x2, x3} == {{2}, {1}, {-1}}

Question 2

Solve the following system of equations(without partial pivoting):

x1 + x2 + x3 = 1

4x1 + 3x2 - 1x3 = 6

3x1 + 5x2 + 3x3 = 4

In[ ]:=

```
A = {{1, 1, 1}, {4, 3, -1}, {3, 5, 3}};
A // MatrixForm
x = {x1, x2, x3};
x // MatrixForm
b = {{1}, {6}, {4}};
b // MatrixForm
aug = ArrayFlatten[{{A, b}}];
aug // MatrixForm
aug[[2]] = aug[[2]] - 4 * aug[[1]];
aug[[3]] = aug[[3]] - 3 * aug[[1]];
aug // MatrixForm
aug[[1]] = aug[[1]] + aug[[2]];
aug[[3]] = aug[[3]] + 2 * aug[[2]];
aug // MatrixForm
aug[[1]] = aug[[1]] - (4 / 10) * aug[[3]];
aug[[2]] = aug[[2]] - (5 / 10) * aug[[3]];
aug // MatrixForm
aug[[2]] = aug[[2]] * (-1);
aug[[3]] = aug[[3]] * (1 / 10) * (-1);
aug // MatrixForm
IdentityMatrix[3] = Take[aug, 3, 3];
IdentityMatrix[3] // MatrixForm
c = Take[aug, 3, -1];
c // MatrixForm
IdentityMatrix[3].x == c
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} 1 & 1 & 1 \\ 4 & 3 & -1 \\ 3 & 5 & 3 \end{pmatrix}$$

Out[ ]//MatrixForm=

$$\begin{pmatrix} x1 \\ x2 \\ x3 \end{pmatrix}$$

Out[ ]//MatrixForm=

$$\begin{pmatrix} 1 \\ 6 \\ 4 \end{pmatrix}$$

Out[ ]//MatrixForm=

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 4 & 3 & -1 & 6 \\ 3 & 5 & 3 & 4 \end{pmatrix}$$

Out[ ]//MatrixForm=

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & -1 & -5 & 2 \\ 0 & 2 & 0 & 1 \end{pmatrix}$$

Out[ ]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & -4 & 3 \\ 0 & -1 & -5 & 2 \\ 0 & 0 & -10 & 5 \end{pmatrix}$$

Out[ ]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & -1 & 0 & -\frac{1}{2} \\ 0 & 0 & -10 & 5 \end{pmatrix}$$

Out[•]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & -\frac{1}{2} \end{pmatrix}$$

Set: Tag IdentityMatrix in IdentityMatrix[3] is Protected.

Out[•]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Out[•]//MatrixForm=

$$\begin{pmatrix} 1 \\ \frac{1}{2} \\ -\frac{1}{2} \end{pmatrix}$$

Out[•]= $\{x1, x2, x3\} == \left\{\{1\}, \left\{\frac{1}{2}\right\}, \left\{-\frac{1}{2}\right\}\right\}$

Question 3:

x+3y+2z=2

2x+7y+7z=−1

2x+5y+2z=7

In[117]:=
```
A = {{1, 3, 2}, {2, 7, 7}, {2, 5, 2}};
A // MatrixForm
x = {x1, x2, x3};
x // MatrixForm
b = {{2}, {-1}, {7}};
b // MatrixForm
aug = ArrayFlatten[{{A, b}}];
aug // MatrixForm
aug[[2]] = aug[[2]] - 2 aug[[1]];
aug[[3]] = aug[[3]] - 2 aug[[1]];
aug[[3]] = aug[[3]] - (-1) aug[[2]];
aug[[2]] = aug[[2]] - (3) aug[[3]];
aug[[1]] = aug[[1]] - (3) aug[[2]];
aug[[1]] = aug[[1]] - (2) aug[[3]];
aug // MatrixForm
IdentityMatrix[3] = Take[aug, 3, 3];
IdentityMatrix[3] // MatrixForm
c = Take[aug, 3, -1];
c // MatrixForm
IdentityMatrix[3].x == c
```

Out[118]//MatrixForm=

$$\begin{pmatrix} 1 & 3 & 2 \\ 2 & 7 & 7 \\ 2 & 5 & 2 \end{pmatrix}$$

Out[120]//MatrixForm=

$$\begin{pmatrix} x1 \\ x2 \\ x3 \end{pmatrix}$$

Out[122]//MatrixForm=

$$\begin{pmatrix} 2 \\ -1 \\ 7 \end{pmatrix}$$

Out[124]//MatrixForm=

$$\begin{pmatrix} 1 & 3 & 2 & 2 \\ 2 & 7 & 7 & -1 \\ 2 & 5 & 2 & 7 \end{pmatrix}$$

Out[131]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -2 \end{pmatrix}$$

... Set: Tag IdentityMatrix in IdentityMatrix[3] is Protected.

Out[133]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Out[135]//MatrixForm=

$$\begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix}$$

Out[136]= {x1, x2, x3} == {{3}, {1}, {-2}}

Question 4:

x + 2y + 6z = 15

3x + 4y + z = 16

$$6x - y - z = 20$$

In[533]:=
```
A = {{1, 2, 6}, {3, 4, 1}, {6, -1, -1}};
A // MatrixForm
x = {x1, x2, x3};
x // MatrixForm
b = {{15}, {16}, {20}};
b // MatrixForm
aug = ArrayFlatten[{{A, b}}];
aug // MatrixForm
aug[[2]] = aug[[2]] - 3 aug[[1]];
aug[[3]] = aug[[3]] - 6 aug[[1]];
aug[[3]] = aug[[3]] - (13 / 2) aug[[2]];
aug[[3]] = aug[[3]] * (2 / 147);
aug[[2]] = aug[[2]] - (-17) * aug[[3]];
aug[[2]] = aug[[2]] * (-1 / 2);
aug[[1]] = aug[[1]] - 2 * aug[[2]];
aug[[1]] = aug[[1]] - 6 aug[[3]];
aug // MatrixForm
IdentityMatrix[3] = Take[aug, 3, 3];
IdentityMatrix[3] // MatrixForm
c = Take[aug, 3, -1];
c // MatrixForm
IdentityMatrix[3].x == c
```

Out[534]//MatrixForm=
$$\begin{pmatrix} 1 & 2 & 6 \\ 3 & 4 & 1 \\ 6 & -1 & -1 \end{pmatrix}$$

Out[536]//MatrixForm=
$$\begin{pmatrix} x1 \\ x2 \\ x3 \end{pmatrix}$$

Out[538]//MatrixForm=
$$\begin{pmatrix} 15 \\ 16 \\ 20 \end{pmatrix}$$

Out[540]//MatrixForm=
$$\begin{pmatrix} 1 & 2 & 6 & 15 \\ 3 & 4 & 1 & 16 \\ 6 & -1 & -1 & 20 \end{pmatrix}$$

Out[549]//MatrixForm=
$$\begin{pmatrix} 1 & 0 & 0 & \frac{183}{49} \\ 0 & 1 & 0 & \frac{39}{49} \\ 0 & 0 & 1 & \frac{79}{49} \end{pmatrix}$$

... Set: Tag IdentityMatrix in IdentityMatrix[3] is Protected.

Out[551]//MatrixForm=
$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Out[553]//MatrixForm=

$$\begin{pmatrix} \frac{183}{49} \\ \frac{39}{49} \\ \frac{79}{49} \end{pmatrix}$$

Out[554]= $\{x1, x2, x3\} == \left\{\left\{\frac{183}{49}\right\}, \left\{\frac{39}{49}\right\}, \left\{\frac{79}{49}\right\}\right\}$

In[ ]:=

In[ ]:=

In[ ]:=

In[ ]:=

In[ ]:=

In[ ]:=