

# 12. Euler's Method

Q1. To solve Differential equation using Euler's Method:

$$\frac{dy}{dx} = yx^3 - 1.5y; \quad y(0) = 1 \quad \text{in the interval } [0,2] \quad h = 0.1$$

```
In[ ]:=
```

```
ClearAll
```

```
Out[ ]:= ClearAll
```

```
In[ ]:=
```

```
Euler[a0_, b0_, h0_, f_, alpha_] := Module[
  {a = N[a0], b = N[b0], h = N[h0], n, x},
  n =  $\frac{b - a}{h}$ ;
  y[0] = alpha;
  For[i = 0, i ≤ n, i++, x[i] = a + h * i;
    y[i + 1] = y[i] + h * f[x[i], y[i]];
    Print["Value of x[" , i, "] = ", x[i], " is : ", y[i]]];];
```

```
In[ ]:=
```

```
f[x_, y_] := y * x3 - 1.5 * y;
Euler[0, 2, 0.1, f, 1]

Value of x[0] = 0. is : 1
Value of x[1] = 0.1 is : 0.85
Value of x[2] = 0.2 is : 0.722585
Value of x[3] = 0.3 is : 0.614775
Value of x[4] = 0.4 is : 0.524219
Value of x[5] = 0.5 is : 0.448941
Value of x[6] = 0.6 is : 0.387212
Value of x[7] = 0.7 is : 0.337494
Value of x[8] = 0.8 is : 0.298446
Value of x[9] = 0.9 is : 0.268959
Value of x[10] = 1. is : 0.248222
Value of x[11] = 1.1 is : 0.235811
Value of x[12] = 1.2 is : 0.231826
Value of x[13] = 1.3 is : 0.237112
Value of x[14] = 1.4 is : 0.253638
Value of x[15] = 1.5 is : 0.285191
Value of x[16] = 1.6 is : 0.338664
Value of x[17] = 1.7 is : 0.426582
Value of x[18] = 1.8 is : 0.572174
Value of x[19] = 1.9 is : 0.82004
Value of x[20] = 2. is : 1.2595
```

Q2. To solve Differential equation using Euler's Method:

$$\frac{dy}{dx} = x^2 + y^2; \quad y(0) = 0 \quad \text{in the interval } [0,8] \quad h = 0.5$$

In[ ]:=

```
f[x_, y_] := x^2 + y^2;
```

```
Euler[0, 8, 0.5, f, 0]
```

Value of x[0] = 0. is : 0

Value of x[1] = 0.5 is : 0.

Value of x[2] = 1. is : 0.125

Value of x[3] = 1.5 is : 0.632813

Value of x[4] = 2. is : 1.95804

Value of x[5] = 2.5 is : 5.875

Value of x[6] = 3. is : 26.2578

Value of x[7] = 3.5 is : 375.493

Value of x[8] = 4. is : 70879.2

Value of x[9] = 4.5 is :  $2.512 \times 10^9$

Value of x[10] = 5. is :  $3.15508 \times 10^{18}$

Value of x[11] = 5.5 is :  $4.97727 \times 10^{36}$

Value of x[12] = 6. is :  $1.23866 \times 10^{73}$

Value of x[13] = 6.5 is :  $7.67139 \times 10^{145}$

Value of x[14] = 7. is :  $2.94251 \times 10^{291}$

Value of x[15] = 7.5 is :  $4.329177860188018 \times 10^{582}$

Value of x[16] = 8. is :  $9.37089047257105 \times 10^{1164}$

Q3. To solve Differential equation using Euler's Method:

$$\frac{dy}{dx} = y+x; \quad y(0) = 1 \quad \text{in the interval } [0,3] \quad h = 0.1$$

In[ ]:=

```
f[x_, y_] := x + y;
```

```
Euler[0, 3, 0.1, f, 1]
```

```

Value of x[0] = 0. is : 1
Value of x[1] = 0.1 is : 1.1
Value of x[2] = 0.2 is : 1.22
Value of x[3] = 0.3 is : 1.362
Value of x[4] = 0.4 is : 1.5282
Value of x[5] = 0.5 is : 1.72102
Value of x[6] = 0.6 is : 1.94312
Value of x[7] = 0.7 is : 2.19743
Value of x[8] = 0.8 is : 2.48718
Value of x[9] = 0.9 is : 2.8159
Value of x[10] = 1. is : 3.18748
Value of x[11] = 1.1 is : 3.60623
Value of x[12] = 1.2 is : 4.07686
Value of x[13] = 1.3 is : 4.60454
Value of x[14] = 1.4 is : 5.195
Value of x[15] = 1.5 is : 5.8545
Value of x[16] = 1.6 is : 6.58995
Value of x[17] = 1.7 is : 7.40894
Value of x[18] = 1.8 is : 8.31983
Value of x[19] = 1.9 is : 9.33182
Value of x[20] = 2. is : 10.455
Value of x[21] = 2.1 is : 11.7005
Value of x[22] = 2.2 is : 13.0805
Value of x[23] = 2.3 is : 14.6086
Value of x[24] = 2.4 is : 16.2995
Value of x[25] = 2.5 is : 18.1694
Value of x[26] = 2.6 is : 20.2364
Value of x[27] = 2.7 is : 22.52
Value of x[28] = 2.8 is : 25.042
Value of x[29] = 2.9 is : 27.8262
Value of x[30] = 3. is : 30.8988

```

Q3. To solve Differential equation using Euler's Method:

$$\frac{dy}{dx} = -2x(y^2); \quad y(0) = 1 \quad \text{in the interval } [0,4] \quad h = 0.2$$

```

In[ ]:= f[x_, y_] := x + y;
Euler[0, 4, 0.2, f, 1]

```

```
Value of x[0] = 0. is : 1
Value of x[1] = 0.2 is : 1.2
Value of x[2] = 0.4 is : 1.48
Value of x[3] = 0.6 is : 1.856
Value of x[4] = 0.8 is : 2.3472
Value of x[5] = 1. is : 2.97664
Value of x[6] = 1.2 is : 3.77197
Value of x[7] = 1.4 is : 4.76636
Value of x[8] = 1.6 is : 5.99963
Value of x[9] = 1.8 is : 7.51956
Value of x[10] = 2. is : 9.38347
Value of x[11] = 2.2 is : 11.6602
Value of x[12] = 2.4 is : 14.4322
Value of x[13] = 2.6 is : 17.7986
Value of x[14] = 2.8 is : 21.8784
Value of x[15] = 3. is : 26.814
Value of x[16] = 3.2 is : 32.7769
Value of x[17] = 3.4 is : 39.9722
Value of x[18] = 3.6 is : 48.6467
Value of x[19] = 3.8 is : 59.096
Value of x[20] = 4. is : 71.6752
```