# 7. Gauss Elimination Method

Solving the following system of equations (without partial pivoting):

$x1 + 2x2 + 3x3 = 1$

$2x1 + 6x2 + 10x3 = 0$

$3x1 + 14x2 + 28x3 = -8$

In[ ]:=
```
A = {{1, 2, 3}, {2, 6, 10}, {3, 14, 28}};
x = {x1, x2, x3};
A // MatrixForm
x // MatrixForm
b = {{1}, {0}, {-8}};
b // MatrixForm
Print["The System of equations is : ",
 MatrixForm[A].MatrixForm[x], "=", MatrixForm[b]]
aug = ArrayFlatten[{{A, b}}];
aug // MatrixForm
aug[[2]] = aug[[2]] - 2 aug[[1]];
aug[[3]] = aug[[3]] - 3 aug[[1]];
aug // MatrixForm
aug[[3]] = aug[[3]] - 4 aug[[2]];
aug // MatrixForm
upper = Take[aug, 3, 3];
upper // MatrixForm
c = Take[aug, 3, -1];
c // MatrixForm
Solve[upper.x == c, x]
```

Out[ ]//MatrixForm=
$$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 6 & 10 \\ 3 & 14 & 28 \end{pmatrix}$$

Out[ ]//MatrixForm=
$$\begin{pmatrix} x1 \\ x2 \\ x3 \end{pmatrix}$$

Out[ ]//MatrixForm=
$$\begin{pmatrix} 1 \\ 0 \\ -8 \end{pmatrix}$$

The System of equations is : $\begin{pmatrix} 1 & 2 & 3 \\ 2 & 6 & 10 \\ 3 & 14 & 28 \end{pmatrix} . \begin{pmatrix} x1 \\ x2 \\ x3 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ -8 \end{pmatrix}$

Out[ ]//MatrixForm=
$$\begin{pmatrix} 1 & 2 & 3 & 1 \\ 2 & 6 & 10 & 0 \\ 3 & 14 & 28 & -8 \end{pmatrix}$$

Out[ ]//MatrixForm=
$$\begin{pmatrix} 1 & 2 & 3 & 1 \\ 0 & 2 & 4 & -2 \\ 0 & 8 & 19 & -11 \end{pmatrix}$$

Out[ ]//MatrixForm=
$$\begin{pmatrix} 1 & 2 & 3 & 1 \\ 0 & 2 & 4 & -2 \\ 0 & 0 & 3 & -3 \end{pmatrix}$$

Out[ ]//MatrixForm=
$$\begin{pmatrix} 1 & 2 & 3 \\ 0 & 2 & 4 \\ 0 & 0 & 3 \end{pmatrix}$$

Out[ ]//MatrixForm=
$$\begin{pmatrix} 1 \\ -2 \\ -3 \end{pmatrix}$$

Out[ ]= $\{\{x1 \to 2, x2 \to 1, x3 \to -1\}\}$

Solving the following system of equations (without partial pivoting):

x1 + 10x2 -1x3 = 3

2x1 + 3x2 + 20x3 = 7

10x1 - 1x2 +2x3 = 4

In[ ]:=
```
Needs["LinearAlgebra`BLAS`"]
```

In[ ]:=
```
A = {{10, -1, 2}, {2, 3, 20}, {1, 10, -1}};
x = {x1, x2, x3};
A // MatrixForm
x // MatrixForm
b = {{4}, {7}, {3}};
b // MatrixForm
Print["The System of equations is : ",
 MatrixForm[A].MatrixForm[x], "=", MatrixForm[b]]
aug = ArrayFlatten[{{A, b}}];
aug // MatrixForm
aug[[2]] = aug[[2]] - (1/5) * aug[[1]];
aug[[3]] = aug[[3]] - (1/10) * aug[[1]];
aug // MatrixForm
SWAP[aug[[2]], aug[[3]]];
aug // MatrixForm
aug[[3]] = aug[[3]] - ((16/5) * aug[[2]]) / (101/10);
aug // MatrixForm

upper = Take[aug, 3, 3];
upper // MatrixForm
c = Take[aug, 3, -1];
c // MatrixForm
ee = NumberForm[Solve[upper.x == c, x]]
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} 10 & -1 & 2 \\ 2 & 3 & 20 \\ 1 & 10 & -1 \end{pmatrix}$$

Out[ ]//MatrixForm=

$$\begin{pmatrix} x1 \\ x2 \\ x3 \end{pmatrix}$$

Out[ ]//MatrixForm=

$$\begin{pmatrix} 4 \\ 7 \\ 3 \end{pmatrix}$$

The System of equations is : $\begin{pmatrix} 10 & -1 & 2 \\ 2 & 3 & 20 \\ 1 & 10 & -1 \end{pmatrix} . \begin{pmatrix} x1 \\ x2 \\ x3 \end{pmatrix} = \begin{pmatrix} 4 \\ 7 \\ 3 \end{pmatrix}$

Out[ ]//MatrixForm=

$$\begin{pmatrix} 10 & -1 & 2 & 4 \\ 2 & 3 & 20 & 7 \\ 1 & 10 & -1 & 3 \end{pmatrix}$$

Out[ ]//MatrixForm=

$$\begin{pmatrix} 10 & -1 & 2 & 4 \\ 0 & \frac{16}{5} & \frac{98}{5} & \frac{31}{5} \\ 0 & \frac{101}{10} & -\frac{6}{5} & \frac{13}{5} \end{pmatrix}$$

Out[ ]//MatrixForm=

$$\begin{pmatrix} 10 & -1 & 2 & 4 \\ 0 & \frac{101}{10} & -\frac{6}{5} & \frac{13}{5} \\ 0 & \frac{16}{5} & \frac{98}{5} & \frac{31}{5} \end{pmatrix}$$

Out[ ]//MatrixForm=

$$\begin{pmatrix} 10 & -1 & 2 & 4 \\ 0 & \frac{101}{10} & -\frac{6}{5} & \frac{13}{5} \\ 0 & 0 & \frac{2018}{101} & \frac{543}{101} \end{pmatrix}$$

Out[ ]//MatrixForm=

$$\begin{pmatrix} 10 & -1 & 2 \\ 0 & \frac{101}{10} & -\frac{6}{5} \\ 0 & 0 & \frac{2018}{101} \end{pmatrix}$$

Out[ ]//MatrixForm=

$$\begin{pmatrix} 4 \\ \frac{13}{5} \\ \frac{543}{101} \end{pmatrix}$$

Out[ ]//NumberForm=

$$\left\{\left\{x1 \to \frac{757}{2018},\ x2 \to \frac{292}{1009},\ x3 \to \frac{543}{2018}\right\}\right\}$$

Solving the following system of equations (without partial pivoting):

3x1 + 4x2 +9x3 = 1

x1 + 9x2 + 2x3 = 4

12x1 +2x2 +2x3 =16

In[70]:=
```
A = {{3, 4, 9}, {1, 9, 2}, {12, 2, 2}};
x = {x1, x2, x3};
A // MatrixForm
x // MatrixForm
b = {{1}, {4}, {16}};
b // MatrixForm
Print["The System of equations is : ",
 MatrixForm[A].MatrixForm[x], "=", MatrixForm[b]]
aug = ArrayFlatten[{{A, b}}];
aug // MatrixForm
aug[[2]] = aug[[2]] - (1 / 3) * aug[[1]];
aug[[3]] = aug[[3]] - 4 * aug[[1]];
aug[[3]] = aug[[3]] - (3 / 23) * (-14) * aug[[2]];
aug // MatrixForm

upper = Take[aug, 3, 3];
upper // MatrixForm
c = Take[aug, 3, -1];
c // MatrixForm
ee = NumberForm[Solve[upper.x == c, x]]
```

Out[72]//MatrixForm=

$$\begin{pmatrix} 3 & 4 & 9 \\ 1 & 9 & 2 \\ 12 & 2 & 2 \end{pmatrix}$$

Out[73]//MatrixForm=

$$\begin{pmatrix} x1 \\ x2 \\ x3 \end{pmatrix}$$

Out[75]//MatrixForm=

$$\begin{pmatrix} 1 \\ 4 \\ 16 \end{pmatrix}$$

The System of equations is : $\begin{pmatrix} 3 & 4 & 9 \\ 1 & 9 & 2 \\ 12 & 2 & 2 \end{pmatrix} . \begin{pmatrix} x1 \\ x2 \\ x3 \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \\ 16 \end{pmatrix}$

Out[78]//MatrixForm=

$$\begin{pmatrix} 3 & 4 & 9 & 1 \\ 1 & 9 & 2 & 4 \\ 12 & 2 & 2 & 16 \end{pmatrix}$$

Out[82]//MatrixForm=

$$\begin{pmatrix} 3 & 4 & 9 & 1 \\ 0 & \frac{23}{3} & -1 & \frac{11}{3} \\ 0 & 0 & -\frac{824}{23} & \frac{430}{23} \end{pmatrix}$$

Out[84]//MatrixForm=

$$\begin{pmatrix} 3 & 4 & 9 \\ 0 & \frac{23}{3} & -1 \\ 0 & 0 & -\frac{824}{23} \end{pmatrix}$$

Out[86]//MatrixForm=

$$\begin{pmatrix} 1 \\ \frac{11}{3} \\ \frac{430}{23} \end{pmatrix}$$

Out[87]//NumberForm=

$$\left\{\left\{x1 \to \frac{557}{412},\ x2 \to \frac{169}{412},\ x3 \to -\frac{215}{412}\right\}\right\}$$

Solving the following system of equations (without partial pivoting):

7x1 + 4x2 +9x3 = 1

x1 + x2 + 8x3 = 89

0x1 +9x2 +1x3 =64

In[389]:=
```
A = {{7, 4, 9}, {1, 1, 8}, {0, 1, 1}};
x = {x1, x2, x3};
A // MatrixForm
x // MatrixForm
b = {{1}, {89}, {64}};
b // MatrixForm
Print["The System of equations is : ",
 MatrixForm[A].MatrixForm[x], "=", MatrixForm[b]]
aug = ArrayFlatten[{{A, b}}];
aug // MatrixForm
aug[[2]] = aug[[2]] - (1/7) * aug[[1]];
aug[[3]] = aug[[3]] - (7/3) * aug[[2]];
aug // MatrixForm

upper = Take[aug, 3, 3];
upper // MatrixForm
c = Take[aug, 3, -1];
c // MatrixForm
ee = NumberForm[Solve[upper.x == c, x]]
```

Out[391]//MatrixForm=

$$\begin{pmatrix} 7 & 4 & 9 \\ 1 & 1 & 8 \\ 0 & 1 & 1 \end{pmatrix}$$

Out[392]//MatrixForm=

$$\begin{pmatrix} x1 \\ x2 \\ x3 \end{pmatrix}$$

Out[394]//MatrixForm=

$$\begin{pmatrix} 1 \\ 89 \\ 64 \end{pmatrix}$$

The System of equations is : $\begin{pmatrix} 7 & 4 & 9 \\ 1 & 1 & 8 \\ 0 & 1 & 1 \end{pmatrix} . \begin{pmatrix} x1 \\ x2 \\ x3 \end{pmatrix} = \begin{pmatrix} 1 \\ 89 \\ 64 \end{pmatrix}$

Out[397]//MatrixForm=

$$\begin{pmatrix} 7 & 4 & 9 & 1 \\ 1 & 1 & 8 & 89 \\ 0 & 1 & 1 & 64 \end{pmatrix}$$

Out[400]//MatrixForm=

$$\begin{pmatrix} 7 & 4 & 9 & 1 \\ 0 & \frac{3}{7} & \frac{47}{7} & \frac{622}{7} \\ 0 & 0 & -\frac{44}{3} & -\frac{430}{3} \end{pmatrix}$$

Out[402]//MatrixForm=

$$\begin{pmatrix} 7 & 4 & 9 \\ 0 & \frac{3}{7} & \frac{47}{7} \\ 0 & 0 & -\frac{44}{3} \end{pmatrix}$$

Out[404]//MatrixForm=

$$\begin{pmatrix} 1 \\ \frac{622}{7} \\ -\frac{430}{3} \end{pmatrix}$$

Out[405]//NumberForm=

$$\left\{\left\{x1 \to -\frac{955}{22},\ x2 \to \frac{1193}{22},\ x3 \to \frac{215}{22}\right\}\right\}$$