Module - 3

STATISTICAL INFERENCE - 1

Z-test

→ Sample size (n):

If $n \leq 30$ the given small sample t-test
if $n > 30$ → large sample Z-test

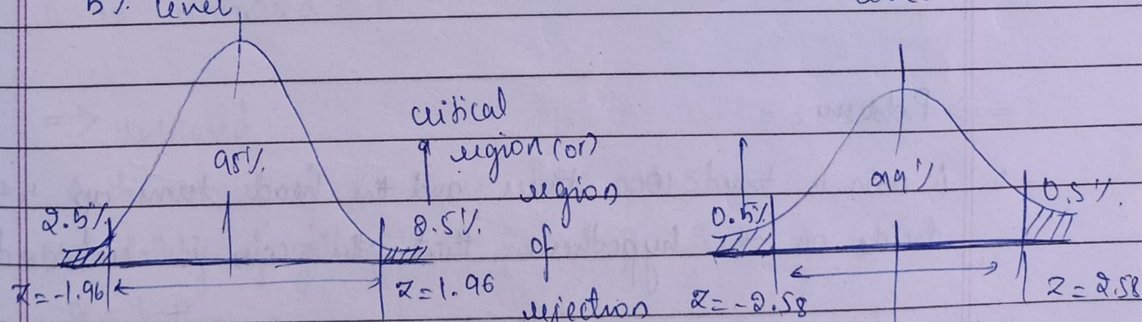
→ Critical values of Z-Test

	5% of level (0.05)	1% level (0.01)
two tailed test	-1.96 and 1.96	-2.58 and 2.58
one tailed test	-1.645 (or) 1.645	-2.33 (or) 2.33

→ Significance levels

* There are two types of significance levels they are 5% and 1% level of significance, 5% level of significance is also denoted by 0.05 and 1% level 0.01

5% level — 1% level



* At 5% level of significance, 95% of confidence that we can accept the null Hypothesis (Ho).
* 1% level of significance, 99% of confidence that we can accept the null Hypotesis (Ho).
* If the point will fall on accepted region, hence

we accept Hypothesis (H_0)

* If the point will fall on critical region hence, we reject H_0.

* At 5% level of significance, it is denoted by $z_{0.05}$ in z-test.

* At 1% level of significance, it is denoted by $z_{0.01}$ in z-test.

* If the calculated value (CV < TV) we can accept H_0.

* If calculated value more than the table value, we reject the null hypothesis.

→ Test of significance of proportion

Standard Normal Variate (SNV) $z = \frac{x - np}{\sqrt{npq}}$

Here p = probability of success

q = probability of failure

n = ~~no~~ sample size

x = observed number of successes

z = SNV

⇒ Problems:

1. A coin is tossed 1000 times and the head turns up 540 times, decide on the hypothesis that the coin is unbiased

⇒ Unbiased = fair coin   Biased = damaged coin

H_0 = null hypothesis

H_0 = hypothesis that the coin is unbiased.

$n = 1000$

$x = 540$

probability of getting head $= \frac{1}{2}$

$p + q = 1$

$q = 1 - p$

$\boxed{q = \frac{1}{2}}$

$Z = \frac{x - np}{\sqrt{npq}}$ $\quad Z = \frac{540 - 1000(0.5)}{\sqrt{1000 \times 0.5 \times 0.5}}$ $\quad \boxed{Z = 2.53}$

$Z = 2.53$ $\begin{cases} > Z_{0.05} = 1.96 \text{ (in two tailed test)} & H_0 \text{ rejected} \\ < Z_{0.05} = 2.58 \text{ (in two tailed test)} & H_0 \text{ accepted} \end{cases}$

(or)

$Z = 2.53$ $\begin{cases} Z_{0.01} = 1.645 \text{ (in one tailed test)} & H_0 \text{ rejected} \\ Z_{0.01} = 2.33 \text{ (in one tailed test)} & H_0 \text{ ~~accepted~~ rejected} \end{cases}$

2. In 324 throws of a six face die and on odd numbers turned up 181 times, if is reasonable to think that the die is an unbiased one.

$\Rightarrow$ Unbiased $\quad n = 324 \quad x = 181$

A die consists of six faces {1, 2, 3, 4, 5, 6}

probability of getting odd faces $p = \frac{3}{6} = \frac{1}{2}$

$q = 1 - p$

$\boxed{q = \frac{1}{2}}$

$Z = \frac{x - np}{\sqrt{npq}} = \frac{181 - 324(0.5)}{\sqrt{324 \times 0.5 \times 0.5}}$ $\quad Z = 2.11$

H_0 = Die is an unbiased one

$z = 2.11$ $\begin{cases} z_{0.05} = 1.96 & \text{Ho rejected (two tailed test)} \\ z_{0.01} = 2.58 & \text{Ho accepted (one tailed test)} \end{cases}$

(or)

$z = 2.11$ $\begin{cases} z_{0.05} = 1.645 & \text{Ho rejected (two tailed test)} \\ z_{0.01} = 2.33 & \text{Ho } \cancel{\text{rejected}} \text{ accepted (one tailed test)} \end{cases}$

3. A die is thrown 9000 times and a throw of a 3 (or) 4 was observed 3240 times. Show that the die ~~test~~ cannot be observed as recorded as unbiased one.

$n = 9000$

$x = 3240$ times

Here, Ho = die is biased

probability of getting a die of faces 3 (or) 4 is {1, 2, 3, 4, 5, 6}

$p(3 \text{ or } 4) = \frac{2}{6} = \frac{2}{6} = \frac{1}{3}$

$q = p + 1$

$\Rightarrow q = 1 + \frac{1}{3} = \frac{2}{3}$

$z = \frac{x - np}{\sqrt{npq}} = \frac{3240 - 9000(1/3)}{\sqrt{\frac{2}{3}}}$

$\boxed{z = 5.37}$

$z = 5.37$ $\begin{cases} > z_{0.05} = 1.96 \text{ (is two tailed) Ho rejected} \\ > z_{0.01} = 2.58 \text{ (is one tailed) Ho rejected.} \end{cases}$

⇒ Probability Limits

①. $p \pm 2.58 \sqrt{\frac{pq}{n}}$ are the probable limits at 1% of significance.

NOTE :- ②. $p \pm 1.96 \sqrt{\frac{pq}{n}}$ are probable limits at 5% of significance.

p = probability of success

q = probability of failure

n = sample size.

$$\boxed{\sqrt{\frac{pq}{n}} = \text{S.D or S.E}}$$

4. A sample of 900 days was taken in a town or city and it was found that one hundred days, the whether was very hot. Calculate the probable limits % of very hot wheater.

$n = 900$

probability of getting very hard whether $p = \frac{100}{900}$

$p = \frac{100}{900} = \frac{1}{9}$ $\quad q = 1 - p$

$q = \frac{8}{9}$

$\cancel{z = \frac{x - np}{\sqrt{npq}}}$ $= p \pm 2.58 \left(\sqrt{\frac{pq}{n}}\right)$

$\frac{1}{9} \pm 2.58 \left(\frac{1}{9} \times \frac{8}{9}\right)$

$= 0.1111 \pm 0.027$

$= 0.1111 + 0.027$ and $0.1111 - 0.027$

0.1381 and 0.084

0.1381×1000 and 0.084×1000

$= \cancel{1.3}\ 13.81\%$ and 8.4%

The probability of hot whether $\Rightarrow$ 13.81% and 8.4%

5. In a sample of 500 men it was found that 60% of them had overweight. What can be infere about the proportion of people having overweight in the population?

$\Rightarrow$ Probability of people having overweight $p = 60\% = \frac{60}{100}$

$= 0.6$

wkt, $p + q = 1$

$q = 1 - p$

$q = 1 - 0.6$

$\boxed{q = 0.4}$

$n = 500$

Probable limits $p \pm 2.58 \sqrt{\frac{pq}{n}}$

at 1% level of significance.

$0.6 \pm 2.58 \sqrt{\frac{0.6(0.4)}{500}}$

$= 0.6 \pm 2.58 \sqrt{\frac{0.6(0.4)}{500}}$

$= 0.6 \pm 0.06$

$0.6 + 0.06$ $\qquad$ $0.6 - 0.06$

$= 0.66$ $\qquad$ $\Rightarrow 0.54 \times 100$

$= 0.66 \times 100$ $\qquad$ $= \cancel{0}\ 54\%$

$= 66\%$

Thus, the probable limits of people having overweight is 66% and 54%

6. A survey was conducted in a slum locality of 200 families by selecting a sample of 800. It was revealed that 180 families were illiterate. Find the probable limits of illiterate families in the population of 2000.

$\Rightarrow n = 800$ Probability of illiterate families $p = \frac{180}{800} = 0.225$

$p + q = 1$

$q = 1 - p$ $\boxed{q = 0.775}$

Probable limits $p \pm 2.58 \sqrt{\frac{pq}{n}}$

$= 0.225 \pm 2.58 \sqrt{\frac{0.225(0.775)}{800}}$

$= 0.225 \pm 0.038$

~~= 0.263~~ $0.225 + 0.038$ $\quad$ $0.225 - 0.038$

$= 0.263 \times 2000$ $\quad$ $= 0.187 \times 2000$

$= 526$ and $= 374$

thus, 374 to 526 probable illiterate families out of 2000 families.

7. A sample of 100 days in taken from records of a certain district and 10 of them are found to be foggy. What are the probable limits of the percentage of foggy days in the district? ~~what~~

$\Rightarrow n = 100$ probability of foggy day $p = \frac{10}{100} = 0.1$

$p + q = 1$

$0.1 + q = 1$

$q = 1 - 0.1$

$\boxed{q = 0.9}$

Probable limits 1%. $p \pm 2.58 \sqrt{\frac{pq}{n}}$

$$0.1 \pm 2.58 \sqrt{\frac{0.1(0.9)}{100}}$$

$= 0.1 \pm 0.0774$

$= 0.1 + 0.0774$ $\qquad$ $0.1 - 0.0774$

$= 0.1774 \times 100$ and $= 0.0226 \times 100$

17.74% $\qquad$ $= 2.26\%$

Probable limits at 5% level signifance.

$$p \pm 1.96 \sqrt{\frac{pq}{n}}$$

$$0.1 \pm 1.96 \sqrt{\frac{0.1(0.9)}{100}}$$

0.1 ± 0.0588

$0.1 + 0.0588$ $\qquad$ $0.1 - 0.0588$

$= 0.1588 \times 100$ and $= 0.0412 \times 100$

$= 15.88\%$ $\qquad$ $= 4.12\%$

8. A random sample of 500 apples was taken from a large consignment and 65 were found to be bad. Estimate the proportion of bad apples in the consignment as well as the standard error of the estimate. Also find the perentage of bad apples in the consignment.

$\Rightarrow n = 500$

probability of bad ~~$p \pm 2.58$~~ $p \Rightarrow \frac{65}{500} = 0.13$

$p + q = 1$

$q = 1 - p$

$\boxed{q = 0.87}$

Probable limits 1% $p \pm 2.58 \sqrt{\frac{pq}{n}}$

$$\Rightarrow 0.13 \pm 2.58 \sqrt{\frac{0.13(0.87)}{500}}$$

$$= 0.13 \pm 0.038$$

$= 0.1 + 0.038$ $\quad$ and $\quad$ $= 0.13 - 0.038$

$= 0.1688 \times 100$ $\quad$ $= 0.092 \times 100$

$= 16.88\%$ $\quad$ $= 9.2\%$

Standard error $= \sqrt{\frac{pq}{n}} = \sqrt{\frac{0.13(0.87)}{500}} = \boxed{0.015}$

Probable limits 5% $p \pm 1.96 \sqrt{\frac{pq}{n}}$

$$= 0.13 \pm 1.96 \sqrt{\frac{0.13(0.87)}{500}}$$

$0.13 + 1.96 \sqrt{\frac{0.13(0.87)}{500}}$ $\quad$ and $\quad$ $0.13 - 1.96 \sqrt{\frac{0.13(0.87)}{500}}$

$= \cancel{0.1594 \times 100}$ $\quad$ $= \cancel{\times 0}\ 0.1006$

$= \cancel{15.94\%}$ $\quad$ $= 11.006\%$

15.94%

$=$

9. In a locality of 18000 families, a sample of 840 families were selected at random. Of these 840 families, 206 families were found to have montly income of ~~nice~~ Rs Rs 2500 or less. It was desired to estimate how many of 18000 families have montly income of Rs 2500/- or less within what limits would you place your estimate.

$n = 840$

Probability of the familyies having the income less than 2500/.

is $\frac{206}{840}$ $p = 0.2452$

$q = 1 - p$

$\boxed{q = 0.7548}$

$n = 840$

Standard error $= \sqrt{\frac{pq}{n}} = \sqrt{\frac{0.2452(0.7548)}{840}}$

$= 0.0148$

Probable limits of 1% $= p \pm 2.58 \sqrt{\frac{pq}{n}}$

$\cancel{p \pm 2.58}$ $0.2452 \pm 2.58(0.0148)$

$= 0.2452 + 2.58(0.0148)$ and $0.2452 - 2.58(0.0148)$

$= 0.2489 \times 18000$ and 0.2415

$= 4480.2$ 4347

Probable limit of 5% $= p \pm 1.96 \sqrt{\frac{pq}{n}}$

$= 0.2452 \pm 1.96(0.0148)$

$= 0.2452 + 1.96(0.0148)$ and $0.2452 - 1.96(0.0148)$

$= 0.2742 \times 18000$ $= 0.2161 \times 18000$

$= 4935.7$ $= 3891.4$

→ Test of significance for difference of means

$$* \; z = \frac{(\bar{x}_1 - \bar{x}_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

$\bar{x}_1, \bar{x}_2 \rightarrow$ means

$\sigma_1, \sigma_2 \rightarrow$ S.D

$n_1, n_2 \rightarrow$ sample size

$$* \; (\bar{x}_1 - \bar{x}_2) \pm 1.96 \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \quad 5\% \text{ l.s.}$$

$$* \; (\bar{x}_1 - \bar{x}_2) \pm 2.58 \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \quad 1\% \text{ l.s}$$

If only one SD is given ie,

$$z = \frac{(\bar{x}_1 - \bar{x}_2)}{\sigma\sqrt{1/n_1 + 1/n_2}}$$

1. In an elementary school examination the mean grade of 32 boys was 72 with S.D 8 while the mean grade of 36 girls was 75 with S.D 6. Test the hypothesis that the performance of girls are better than boys.

Ho = Hypothesis that the performance of girls are better than boys.

By the data, Boys ~~$\bar{x}_1 = \sigma_1 =$~~ $\bar{x}_1 = 72$ $\sigma_1 = 8$ $n_1 = 32$

Girls $\bar{x}_2 = 75$ $\sigma_2 = 6$ $n_2 = 36$

$$Z = \frac{(\bar{x}_1 - \bar{x}_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{(72-75)}{\sqrt{\frac{8^2}{32} + \frac{6^2}{36}}}$$

$$= \frac{\cancel{3}}{\cancel{\sqrt{\ }}} \quad Z = -1.73$$

$$|Z| = 1.73$$

$Z = 1.73 \left\{ \begin{array}{ll} < Z_{0.05} \text{ (two tailed test)} = 1.96 & \text{Ho accepted} \\ < Z_{0.01} \text{ (} \cancel{\text{two}} \text{ tailed test)} = 2.58 & \text{Ho accepted} \end{array} \right.$

$Z = 1.73 \left\{ \begin{array}{ll} Z_{0.05} = 1.645 \text{ (One tailed test)} & = \text{Ho rejected} \\ Z_{0.01} = 2.33 \text{ one tailed Test} & = \text{Ho accepted} \end{array} \right.$

2. A sample of 100 bulbs produced by a company A, showed a mean life of 1190 hours and SD of 90 hours. Also, a sample of 75 bulbs produced by company B a mean life of 1230 hours and SD of 120 hours, is there difference between the mean lifetime of the bulbs produced by 2 companies at 5 % L.S and 1% L.S

⇒ Company A $\bar{x}_1 = 1190$ hours $\sigma_1 = 90$ hours $n_1 = 100$

Company B $\bar{x}_2 = 1230$ $\sigma_2 = 120$ $n_2 = 75$

$$Z = \frac{(\bar{x}_1 - \bar{x}_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

$$= \frac{(1190 - 1230)}{\sqrt{\frac{90^2}{100} \quad \frac{120^2}{75}}} = -2.4209$$

$$= \Rightarrow 2.4209$$

$z = 2.42$ $\begin{cases} Z_{0.05} = 1.96 \text{ (two tailed test)} & \text{Ho rej} \\ Z_{0.01} = 2.58 \text{ (two tailed test)} & \text{Ho accepted} \end{cases}$

$Z = \cancel{1.645}$ 2.42 $\begin{cases} > Z_{0.05} = 1.645 \text{ (One tailed test)} & \text{Ho reject} \\ < Z_{0.01} = 2.33 \text{ (one tailed test)} & \text{Ho accep} \end{cases}$

3. A random sample for 1000 workers in a company has mean wage of 50 per/day and S.D = 15, another sample of ~~ttO~~ 1500 workers from another company has mean wage of 45/per day and SD of 20. Does the mean rate of wages varies between the two companies. Find 95% of condifence limit for the difference of ~~mkeage~~ wages of the population of 2 Companies.

$\Rightarrow$ ~~Co~~ Company $A = \bar{x}_1 = 50$ $\sigma_1 = 15$ $n_1 = 1000$

$B = \bar{x_2} = 45$ $\sigma_2 = 20$ $n_2 = 1500$

$\cancel{Z} = (50 -$

$$Z = (x_1 - x_2) \pm 1.96 \left(\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}\right)$$

$$Z = (50 - 45) \pm 1.96 \sqrt{\frac{15^2}{1000} + \frac{20^2}{1500}}$$

$Z = 5 \pm 1.374$

$Z = 5 \pm 1.374$ and $Z = 5 - 1.374$

$Z = 6.374$ and 3.625

$z = 7.13$

4. The mean of two large samples of 1000 and 2000 members are 168.75 cm and 170 cm respectively. Can the samples be recorded as drawn same population of S.D 6.25 cm at 5% and 1% L.S

$\Rightarrow$ $n_1 = 1000$ $\quad \bar{x}_1 = 168.75$ cm $\quad \sigma = 6.25$

$n_2 = 2000$ $\quad \bar{x}_2 = 170$ cm

$$z = \frac{(\bar{x}_1 - \bar{x}_2)}{\sigma\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \frac{(168.75 - 170)}{6.25\left(\sqrt{\frac{1}{1000} + \frac{1}{2000}}\right)}$$

$= 5.16$

$z = 5.16$ $\begin{cases} z_{0.05} = 1.96 \text{ (two tailed test)} & H_0 \text{ rejected} \\ z_{0.01} = 2.58 \text{ (two tailed test)} & H_0 \text{ rejected} \end{cases}$

$z = 5.16$ $\begin{cases} z_{0.05} = 1.645 \text{ (one tailed test)} & H_0 \text{ rejected} \\ z_{0.01} = 2.33 \quad " & H_0 \text{ rejected} \end{cases}$

Test of significance for difference of properties for two samples.

$$z = \frac{P_1 - P_2}{\sqrt{pq\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

Here $P = \frac{n_1 P_1 + n_2 P_2}{n_1 + n_2}$

$q = 1 - p$

P_1 and P_2 be the sample proportions
n_1, n_2 be the sample sizes
$p \to$ probability of success
$q \to$ probability of failure

1. In an exit poll enquiry it was revealed that 600 voters in one locality and 400 voters from another locality favoured 55% and 48% respectively, a particular party comes to power, test the hypothesis that there is a difference of in the locality in respect of the opinion.

H_0 = ~~hypothesis~~ difference in two localities

By the data $n_1 = 600$ $n_2 = 400$

$P_1 = 55\%$ i.e., $\frac{55}{100} = 0.55$

$P_2 = 48\%$ i.e., $\frac{48}{100} = 0.48$

$$p = \frac{n_1 P_1 + n_2 P_2}{n_1 + n_2} = \frac{600(0.55) + 400(0.48)}{600 + 400} = \boxed{0.522}$$

$q = 1 - p$
$= 1 - 0.522$
$\boxed{q = 0.478}$

$$z = \frac{P_1 - P_2}{\sqrt{pq\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{0.55 - 0.48}{\sqrt{(0.522)(0.478)\left[\frac{1}{600} + \frac{1}{400}\right]}}$$

$= z = 2.1709$

$\boxed{z = 2.171}$

$$z = 2.171 \begin{cases} > Z_{0.05} = 1.96 \text{ (two tailed test)} & = H_0 \text{ rejected} \\ Z_{0.01} = 2.58 & = H_0 \text{ accepted} \end{cases}$$

$$z = 2.171 \begin{cases} > Z_{0.05} = 1.645 \text{ (one tailed test)} & = H_0 \text{ rejected} \\ Z_{0.01} = 2.33 & = H_0 \text{ accepted} \end{cases}$$

2. A random sample of 1000 engineering students from city A and 800 from City B were taken. It was found that 400 in each of the sample were from payment quota. Does the data reveal difference b/w the two cities in respect of payment quota student

$n_1 = 1000 \quad n_2 = 800$

By the data, $P_1 = \frac{400}{1000} = 0.4$

$P_2 = \frac{400}{800} = 0.5$

$p = \frac{n_1 P_1 + n_2 P_2}{n_1 + n_2} = \frac{1000(0.4) + 800(0.5)}{1000 + 800} = 0.44$

$q = 1 - p$

$q = 0.55$

$$z = \frac{P_1 - P_2}{\sqrt{pq\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{0.4 - 0.5}{\sqrt{0.44(0.55)\left(\frac{1}{1000} + \frac{1}{800}\right)}}$$

$z = 4.24$

$z = 4.24 \begin{cases} Z_{0.05} = 1.96 \text{ (two tailed test)} & H_0 \text{ rej} \\ Z_{0.01} = 2.58 & H_0 \text{ rej} \end{cases}$

$Z = 4.24 \begin{cases} Z_{0.05} = 1.645 \text{ (one tailed test)} & H_0 \text{ rej} \\ Z_{0.01} = 2.58 & H_0 \text{ rej} \end{cases}$

3. In a city A, 20% of random sample of 900 schools boys had a certain ~~had~~ slight physical defect. In another city B, 18.5% of a random sample of 1600 school boys had the same defect. ~~It~~ Is the difference b/w the proportion significant at 5% level of signifance.

By the data, $n_1 = 900$ $n_2 = 1600$

$P_1 = \frac{20}{100} = \cancel{0.022}\ 0.2$

$P_2 = \frac{18.5}{100} = 0.185$

$P = \frac{n_1 P_1 + n_2 P_2}{n_1 + n_2} = \frac{900(0.2) + 1600(0.185)}{900 + 1600} = 0.1904$

$q = 1 - P$

$\boxed{q = 0.8096}$

$z = \frac{P_1 - P_2}{\sqrt{pq\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{0.2 - 0.185}{\sqrt{0.1904(0.8096)\left(\frac{1}{900} + \frac{1}{1600}\right)}} = 0.9169$

$Z = 0.9169$ $\begin{cases} Z_{0.05} = 1.96 \text{ (two tailed test)} & = H_0 \text{ accept} \\ Z_{0.01} = 2.58 & = H_0 \text{ accept} \end{cases}$

$Z = 0.9169$ $\begin{cases} Z_{0.05} = 1.645 \text{ (one tailed test)} & = H_0 \text{ accept} \\ Z_{0.01} = 2.33 & H_0 \text{ accept.} \end{cases}$

4. A sample of 600 men ~~from~~ from a city. 450 are found ~~so~~ smokers. In another sample of 900 men from another city, 450 are smoker. Do they indicate that the significantly different with respect to the habit of smoking among the men. Test at 5% of significance

By the data, $n_1 = 600$ $n_2 = 900$

$P_1 = \frac{450}{600} = 0.75$

$P_2 = \frac{450}{900} = 0.5$

$p = \frac{n_1 P_1 + n_2 P_2}{n_1 + n_2} = \frac{600(0.75) + 900(0.5)}{600 + 900} = 0.6$

$\cancel{p} = q = 1 - p$

$q = 0.4$

$Z = \frac{P_1 - P_2}{\sqrt{pq\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{0.75 - 0.5}{\sqrt{0.6(0.4)\left[\frac{1}{600} + \frac{1}{900}\right]}}$

$Z = 9.68$

Z test :

5. One type of aircraft is found develop engine trouble in 5 flights out of a total of 100 and another type in 7 flights out of a total of 200 flights, is there a significance difference in the two types of aircrafts so far as engine defect ~~are conserned~~ concerned! Test at 5% L.S.

By the data, $n_1 = 100$ $\quad n_2 = 200$

$$P_1 = \frac{5}{100} = 0.05$$

$$P_2 = \frac{7}{200} = 0.035$$

$$p = \frac{n_1 P_1 + n_2 P_2}{n_1 + n_2} = \frac{100(0.05) + 200(0.035)}{100 + 200}$$

$$p = 0.04$$

$$p = 1 - q$$
$$q = 1 - p$$
$$q = 0.96$$

$$Z = \frac{P_1 - P_2}{\sqrt{pq\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{0.05 - 0.035}{\sqrt{0.04(0.96)\left(\frac{1}{100} + \frac{1}{200}\right)}}$$

$$Z = 0.625 = 2 \text{ lei}^{h}$$

6. In an examination, given too students at a large number of different schools, the mean grade was 74.5 and S.D grade was 8. At one particular school where 200 students took the examination the mean grade was 75.9. Discuss the significance of this

at both 5% and 1% L.S.

$n_1 = 200$ $n_2 = 200$

$\sigma = 8$

$\bar{x}_1 = 74.8$

$\bar{x}_2 = 75.9$

→ Confidence limits

⇒ 95% of confidence limits $\bar{x} \pm 1.96 \left(\frac{\sigma}{\sqrt{n}}\right)$

⇒ 99% of confidence limits $\bar{x} \pm 2.58 \left(\frac{\sigma}{\sqrt{n}}\right)$

1. The mean and SD of the maximum loads supported by 60 cabels are 11.09 tonnes and 0.73 tonnes respectively. Find 95% and 99% confidence limits for mean of the maximum loads of all cabels produced by the company.

⇒ mean $= 11.09$

SD $= 0.73$

$n = 60$

⇒ 95% of confidence limits $\bar{x} \pm 1.96 \left(\frac{\sigma}{\sqrt{n}}\right)$

$= 11.09 \pm 1.96 \left(\frac{0.73}{\sqrt{60}}\right)$

$= 11.09 + 1.96 \left(\frac{0.73}{\sqrt{60}}\right)$ and $11.09 - 1.96 \left(\frac{0.73}{\sqrt{60}}\right)$

$= 11.2747$ $= 10.9052$

$= 99\%$ of confidence limits $\bar{x} \pm 2.58\left(\frac{\sigma}{\sqrt{n}}\right)$

$11.09 \pm 2.58\left(\frac{0.73}{\sqrt{60}}\right)$

$11.09 + 2.58\left(\frac{0.73}{\sqrt{60}}\right)$ $11.09 - 2.58\left(\frac{0.73}{\sqrt{60}}\right)$

$= 11.3331$ 10.8468

2. The weights of 1500 balls bearings are normally distributed with mean of 635 gms and SD of 1.36 gms. If 300 randomly samples of size 36 are drawn from this population determine the expected mean and S.D. If the sampling distribution of means if sampling is done.

⇒ ~~$\bar{x} = 635$ gms~~ i). With replacement

~~$\sigma = 1.36$ gms~~ ii). without replacement.

⇒ Here $N = 1500$

$\mu = 635$

$\sigma = 1.36$

$n = 36$

i). With replacement:

Expected mean $\mu = 635$

Expected S.D $\sigma_x = \frac{\sigma}{\sqrt{n}} = \frac{1.36}{\sqrt{36}} = 0.227$

ii). Without replacement:

~~Expect~~ $\mu = 635$

Expected variance $\sigma_x^2 = \frac{\sigma^2}{n}\left[\frac{N-n}{N-1}\right] = \frac{1.36^2}{36}\left[\frac{1500-36}{1500-1}\right]$

$\sigma_x^2 = 0.05$

$\sigma_x = \sqrt{\sigma_x^2} = \sqrt{0.05}$ $= 0.2236$