Assignment - 1

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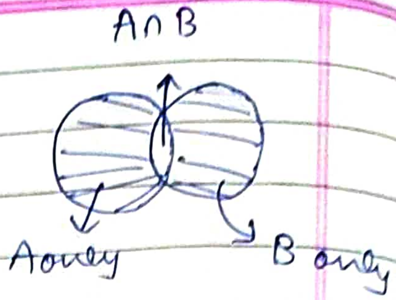

Sol. 1 $P(A) = 0.3$, $P(B) = 0.4$,
$P(A \cap B) = 0.2$

(a) $P(A \text{ only}) + P(B \text{ only})$

$= \{P(A) - P(A \cap B)\} + \{P(B) - P(A \cap B)\}$

$= 0.3 - 0.2 + 0.4 - 0.2$

$= \underline{0.3}$

(b) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.3 + 0.4 - 0.2$

$= \underline{0.5}$

(c) $P((A \cup B)') = 1 - P(A \cup B) = 1 - 0.5 = \underline{0.5}$

Sol. 2 let E_1, E_2, E_3 & A be the events such that

$E_1 \rightarrow$ Car is behind door 1

$E_2 \rightarrow$ Car is behind door 2

$E_3 \rightarrow$ Car is behind door 3

$A \rightarrow$ Host opens door 3

Probability that the car is behind door 1 given that Host opens door 3,

$$P(E_1|A) = \frac{P(A|E_1) \cdot P(E_1)}{P(A|E_1) \cdot P(E_1) + P(A|E_2) \cdot P(E_2) + P(A|E_3) \cdot P(E_3)}$$

$P(E_1) = P(E_2) = P(E_3) = 1/3$

$P(A|E_1) = 1/2$,

$P(A|E_2) = 1$,

$P(A|E_3) = 0$

$P(E_1|A) = 1/3$

$P(E_2|A) = 1 - P(E_1|A)$

$= 1 - 1/3 = 2/3$

Hence, If contestant does not switch door, then he has 1/3 prob. of winning the car, but If contestant doesn't switch, he has 2/3 prob. of winning the car.

Sol. ③ let E_1, E_2, E_3, E_4 & A be events such that,

$E_1 \rightarrow$ No. ball is red out of remaining 3 balls.

$E_2 \rightarrow$ 1 ball is red out of rem. 3 balls.

$E_3 \rightarrow$ 2 balls are red out of rem. 3 balls.

$E_4 \rightarrow$ all 3 balls are red out of rem. 3 balls.

$A \rightarrow$ Drawing 3 red balls.

$$P(A|E_1) = \frac{{}^3C_3}{{}^6C_3} = \frac{1}{20},$$

$$P(A|E_2) = \frac{{}^4C_3}{{}^6C_3} = \frac{4}{20}, \quad P(A|E_3) = \frac{{}^5C_3}{{}^6C_3} = \frac{10}{20}$$

$$P(A|E_4) = 1$$

$$P(E_1) = P(E_2) = P(E_3) = P(E_4) = 1/4$$

$$P(A) = P(A|E_1)\cdot P(E_1) + P(A|E_2)\cdot P(E_2) + P(A|E_3)\cdot P(E_3) + P(A|E_4)\cdot P(E_4)$$

$$= \frac{1}{20} \times \frac{1}{4} + \frac{4}{20} \times \frac{1}{4} + \frac{10}{20} \times \frac{1}{4} + 1 \times \frac{1}{4}$$

$$P(A) = \frac{35}{80}$$

Prob. that all balls in bag are red,

$$P(E_4|A) = \frac{P(A|E_4)\cdot P(E_4)}{P(A)} = \frac{1\times 1/4}{35/80} = \frac{20}{35} = \frac{4}{7}$$

Sol. 4

$$P_X(x) = \begin{cases} 0.1 & , x = 0.2 \\ 0.2 & , x = 0.4 \\ 0.2 & , x = 0.5 \\ 0.3 & , x = 0.8 \\ 0.2 & , x = 1 \\ 0 & , \text{otherwise} \end{cases}$$

$P(X<0.5) = 0.1 + 0.2 = \underline{0.3}$

$P(0.25 < X < 0.75) = 0.2 + 0.2 = \underline{0.4}$

$$P(X=0.2 \mid X<0.6) = \frac{0.1}{0.1+0.2+0.2} = \underline{0.2}$$

Sol. 6 X : R.V. uniformly distributed over [0, 1]

$$f_X(x) = \begin{cases} 0 & , x<0 \\ 1 & , 0 \le x \le 1 \\ 0 & , x>1 \end{cases}$$

→ $E[X] = \int_0^1 x f(x)\, dx = \int_0^1 x\, dx = 1/2$

$E[X^2] = \int_0^1 x^2 f(x)\, dx = \int_0^1 x^2\, dx = 1/3$

→ $Var[X] = E[X^2] - (E[X])^2 = 1/3 - 1/4 = 1/12$

$E[X^2 + Y^2] = 1 \Rightarrow E[X^2] + E[Y^2] = 1$ (Given)

$E[Y^2] = 1 - 1/3 = 2/3$

$Var[Y] = 5/9 \Rightarrow E[Y^2] - (E[Y])^2 = 5/9$

→ $\Rightarrow E[Y] = 1/3$

→ $E[X+Y] = E[X] + E[Y] = 1/2 + 1/3 = 5/6$

Sol. 5 X : R.V. with cdf,

$$F(x) = \begin{cases} 0, & x < 0 \\ 2/3, & 0 \le x < 1 \\ \frac{7-6c}{6}, & 1 \le x < 2 \\ \frac{4c^2-9c+6}{4}, & 2 \le x \le 3 \\ 1, & x > 3 \end{cases}$$

(i) for finding c, $F(1^+) = F(1^-)$

$$\Rightarrow \frac{7-6c}{6} = \frac{2}{3} \Rightarrow \boxed{c = \frac{1}{2}}$$ but $c \in \left(\frac{1}{4}, \frac{1}{2}\right)$

Take $c = 1/3$

$$F(x) = \begin{cases} 0, & x < 0 \\ 2/3, & 0 \le x < 1 \\ 5/6, & 1 \le x < 2 \\ 31/36, & 2 \le x \le 3 \\ 1, & x > 3 \end{cases}$$

(ii) $P(1 < X < 2) = F(1) - F(1) = 0$

$P(2 \le X < 3) = F(2) - F(1) = 31/36 - 5/6 = 1/36$

$P(0 < X \le 1) = F(1) - F(0) = 5/6 - 2/3 = 1/6$

$P(1 \le X \le 2) = F(2) - F(0) = 31/36 - 2/3 = 7/36$

$P(X \ge 3) = 1 - P(X < 3) = 1 - P(X \le 2)$

$= 1 - F(2) = 1 - 31/36 = 5/36$