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Measuring soap bubble thickness with color matching

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This paper describes a laboratory experiment designed to measure thickness variations across a soap bubble. The experiment uses the phenomenon of thin film interference and the principles of color perception to measure the thickness of the soap film at various points across the surface of the bubble. The students review the classical theory of interference and use a digital camera to make the measurements. The apparatus required for the experiment is inexpensive and easy to construct. This experiment is suitable for a senior undergraduate course and can potentially be used to study hydrodynamic effects in soap films.

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I. INTRODUCTION

Bubbles are the subject of active research that focuses on several interesting dynamical phenomena such as the thinning of bubbles due to drainage of fluid[1–3] and convection in bubbles due to thermal fluctuations as well as fluctuations in the concentration of the solution.[4,5] Convection can be observed in the form of rising globules,[6] commonly referred to as "thermals," which often take on mushroom-like shapes, such as those indicated by the arrows in Fig. 1. Thermals are areas of film which differ in thickness, and therefore in areal density, from that of the ambient film. The thermals are driven by the buoyancy force. The sequence of images in Fig. 1 shows the typical evolution of a bubble. A newly created bubble, shown in Fig. 1(a), contains areas of varying thickness due to stirring by air blown from a tube. These areas are convectively unstable and break into thermals, as shown in Figs. 1(b) and 1(c). The bubble begins to thin as fluid drains away from the top surface. When the bubble is sufficiently thin, no light will be reflected due to destructive interference between rays reflected from the front surface, which undergo a 180° phase change, and rays reflected from the back surfaces of the film. This is the so-called "black film" which can be observed at the top of the bubble in Fig. 1(d). The area of very thin black film increases until the bubble bursts.

In this paper, a simple, low-cost experiment for the senior undergraduate laboratory is described which utilizes the iridescent colors often seen in soap bubbles. The colors observed can be used to obtain accurate measurements of the thickness of the film at various points on the surface and, hence, a map of the thickness for the entire bubble can be generated. While relatively straightforward, the experiment exposes students to methods that are often encountered in advanced research.

II. EXPERIMENT

The apparatus is shown schematically in Fig. 2. A shallow dish is filled with a solution consisting of 100 cm^3 water with $\sim$1% liquid soap by volume. The bottom of the dish is covered with black paper to reduce unwanted reflections. A bubble is blown using a straw held just below the surface of the liquid. Glycerin can be added to increase the viscosity of the solution and prolong the life of the bubble. A bubble can exist for several seconds to minutes and even hours if the environmental conditions in the laboratory are favorable (a moist atmosphere without drafts and temperature fluctuations).

The bubble height h, shown in Fig. 3, is measured by looking at the bubble from the side. The bubble is then covered by a cone made of white card stock, which serves a dual purpose. It shields the bubble from drafts and provides uniform illumination from the light source. The light source is a single high-power light emitting diode (LED) with triple red, green and blue (RGB) emitters (LUXdrive Endor Star™ 7007), placed inside the cone such that the light scatters from its inner surface. The emitters are controlled individually using DC power supplies and are adjusted such that the intensities of the red, green, and blue lights are approximately equal. This is achieved by visual inspection of the light color inside the cone: the light must be approximately

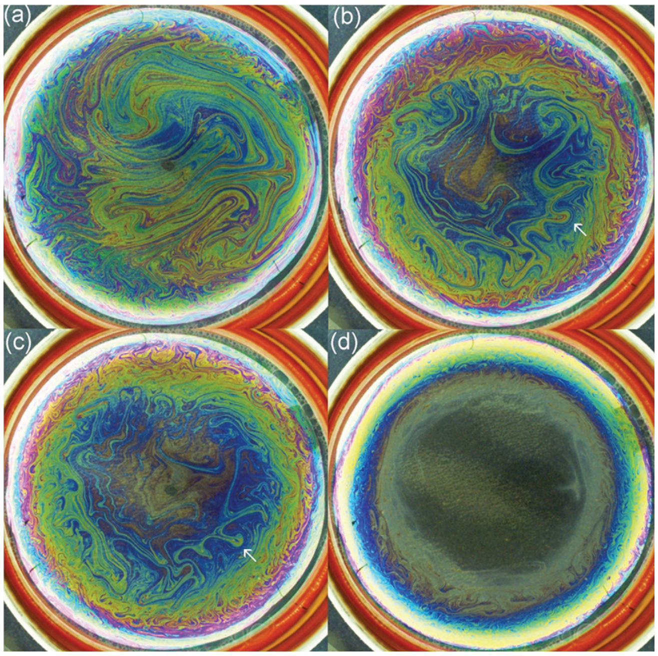

Fig. 1. Images of a soap bubble with a diameter $D = 18$ cm, taken at time intervals of approximately 4 s. Many rising and descending thermals can be seen. One of the descending mushroom-shaped thermals is indicated by the arrows in (b) and (c). The "black film" can be observed in the center of the bubble (d). The small dark circle at the center of images (a)–(c) is the reflection of the cone opening.

white, without a blue or purple tint. A more accurate adjustment can be performed if a spectrometer is available, though this is not required for this experiment. Since this method relies on color rather than light intensity, the illumination across the surface of the bubble may be non-uniform and the position of the light source is not particularly important.

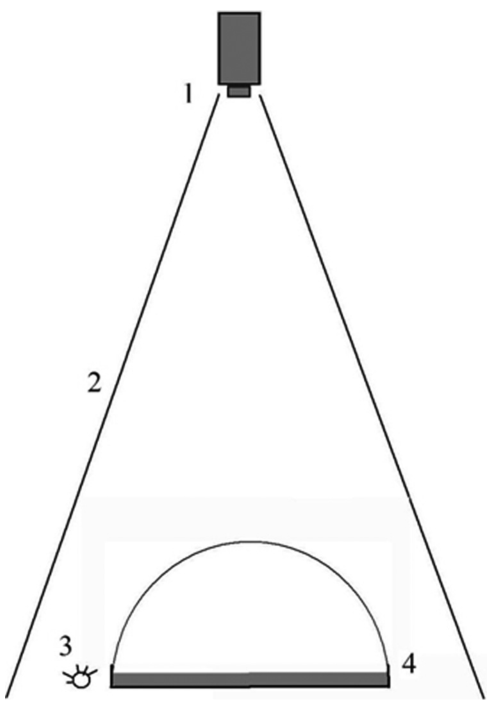

Fig. 2. Schematic of the experimental set-up showing the photo/video camera (1), the cardboard cone (2), the RGB LED light source (3), and the dish with the soap solution (4).

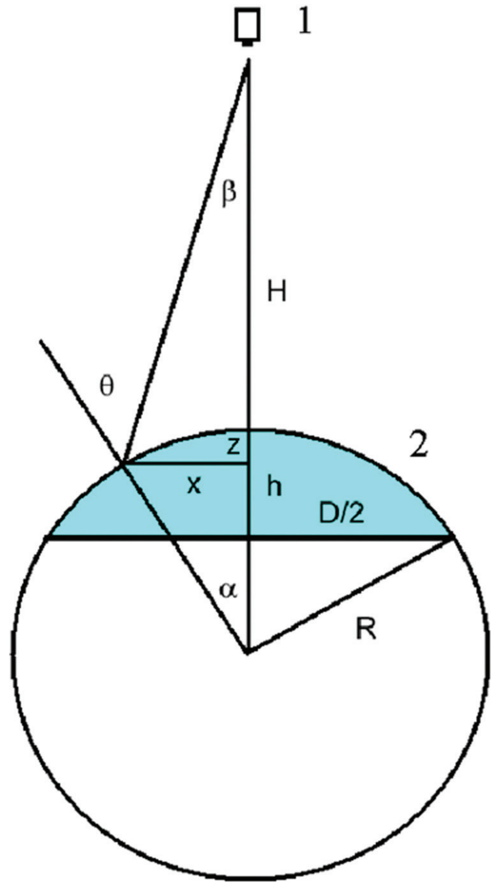

Fig. 3. Schematic of the geometry of the soap bubble (2) and the photo/video camera (1).

A camera is placed directly over the opening of the cone and the top surface of the bubble is photographed. The camera must be set to "manual" so that its settings, such as exposure and aperture, remain fixed during the experiment. In the present work, pictures were taken using a Canon Powershot G11™ digital camera with an exposure time of 1/40 s, an aperture of f/4, and an ISO number of 1600. A series of pictures are taken during the life of the bubble to document its evolution. A picture of the background, without the bubble, is also taken so that it can be subtracted from each bubble image.

The geometry of the system is described in terms of the height h of the bubble, its diameter D, and the height H of the camera above the base of the bubble, as shown in Fig. 3. In our experiments, $H = 60$ cm. Since the bubble is a section of a sphere, the height h of the bubble is in general not equal to half of its diameter.

III. ANALYSIS

In order to understand how to use color to analyze the photographs, we must first review the principles of color vision. Color can be characterized by three parameters. The human eye has three types of receptors that are most sensitive to short, middle, and long visible wavelengths. Photo or video cameras, which mimic human vision, contain three pixel arrays covered with red (R), green (G), and blue (B) filters. The intensities measured with the red, green, and blue pixels for light with a spectral power distribution $I(\lambda)$ are given by the integral,[7]

$$\begin{bmatrix} R \\ G \\ B \end{bmatrix} = \int_0^\infty I(\lambda) \begin{bmatrix} r(\lambda) \\ g(\lambda) \\ b(\lambda) \end{bmatrix} d\lambda \tag{1}$$

where λ is the wavelength of the light. The kernels $r(\lambda), g(\lambda),$ and $b(\lambda)$ are the spectral sensitivity functions of the human eye or the imaging array. The latter can often be obtained from the camera manufacturer. To simplify the problem, we assume that the peak in the spectrum of light emitted by an LED is very narrow and can be effectively represented by a delta function. The values of R, G, and B become simply the intensities of the spectral peaks of the LEDs.

Thin film interference theory shows that the intensity of monochromatic light reflected from a film depends on the wavelength of the incident light, the optical thickness of the film, its index of refraction, and the angle of incidence. If the film is illuminated by a light source with a spectral composition I_{in} (λ), the reflected light will have a composition $I_r(\lambda) = R_\lambda I_{in}$ (λ) where R_λ is the reflectance which is different for different components of the spectrum of the incident light. The reflectance is given by the following equation:[8,9]

$$R_\lambda = R_s^2 \frac{1-\cos\varphi}{1+R_S^4-2R_S^2\cos\varphi} + R_p^2 \frac{1-\cos\varphi}{1+R_p^4-2R_p^2\cos\varphi}, \tag{2}$$

where R_s and R_p are the amplitude coefficients of reflection for light polarized perpendicular and parallel to the plane of incidence, respectively, and φ is the phase difference between the reflected waves. The amplitude coefficients are given by the Fresnel equations,[8]

$$R_s = \frac{\cos\theta - \sqrt{n^2-\sin^2\theta}}{\cos\theta + \sqrt{n^2-\sin^2\theta}}, \quad R_p = \frac{n^2\cos\theta - \sqrt{n^2-\sin^2\theta}}{n^2\cos\theta + \sqrt{n^2-\sin^2\theta}} \tag{3}$$

where $n = 1.3$ is the index of refraction of the soapy water and θ is defined in Fig. 3. The phase difference φ is given by

$$\varphi(\lambda,\theta) = \frac{4\pi d}{\lambda}\sqrt{n^2-\sin^2\theta}, \tag{4}$$

where d is the thickness of the film. Using Eqs. (2)–(4) we can calculate the RGB values for light reflected from the bubble and determine the color perceived by the eye or the camera for a given spectral composition of the light source. Performing these calculations for all possible values of d and θ, results in a two-dimensional color map, shown as an insert in Fig. 4(c), where the horizontal axis shows the distance x from the center of the bubble and the vertical axis the film thickness d. Using the geometry shown in Fig. 3, we can determine the angle θ as a function of the position on the bubble. The angle θ is equal to

$$\theta = \alpha + \beta. \tag{5}$$

Angles α and β, shown in Fig. 3, can be found using the following relations:

$$\alpha = \sin^{-1}(x/R),$$
$$\beta = \tan^{-1}(x/(R - R\cos\alpha + H - h)). \tag{6}$$

where R is the radius of the bubble which is equal to

$$R = (D^2/4 + h^2)/(2h). \tag{7}$$

Using the look-up color map we can determine the thickness of the bubble at different locations by matching the color observed at each point to the color map, as is illustrated in Fig. 4(c). It is convenient to start at the center of the bubble where the dark film forms. This avoids confusion with determining the order of the color bands on the bubble. The black film provides a good reference point in determining the color in the adjacent color bands. To make the color matching procedure easier we recommend that a background image (Fig. 4(b)) is subtracted from each bubble image using image editing software such as "IMAGEJ."[10] The position of the LEDs can be adjusted to minimize the reflection from the bottom of the dish and to improve the quality of the image. However, the matching procedure works even when the lighting conditions are not ideal. Advanced students can attempt to write code to automate the color matching procedure by finding the minima of the difference between the observed RGB indices and those of the color map. This is tricky because of the existence of multiple minima related to color bands of different order.

Measurements based on matching color with a white-light source have certain advantages compared to those based on monochromatic light. For a monochromatic source, a region of the film will appear black whenever the film has a thickness $d = m\lambda/2n$, where $m = 0, 1, 2, 3, \ldots$ is the order of interference. Thus, for monochromatic light, it is impossible to determine the order of the interference of the black film on the top of the bubble, making it impossible to determine the orders of interference of the colored bands. For white light,

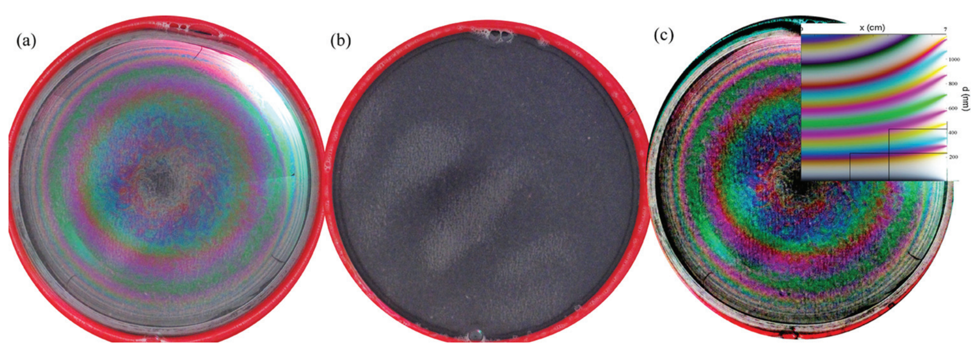

Fig. 4. (a) Image of a bubble of diameter $D = 14$ cm, (b) the background, and (c) the result of the background subtraction. The inset in (c) shows the color map; the horizontal axis shows the radial distance and the vertical axis shows the thickness d of the film. Black lines demonstrate the matching procedure when the color in a particular point of the bubble is matched to that on the color map (vertical line). The thickness is then determined by drawing a horizontal line.

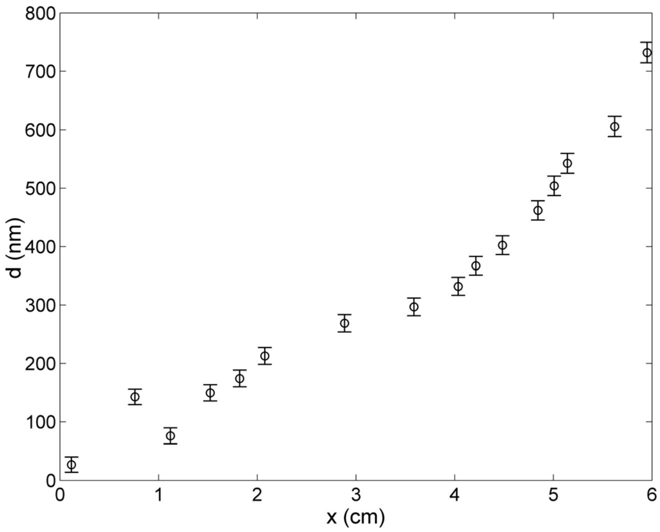

Fig. 5. Distribution of the measured film thickness d as a function of the distance x. The data were obtained from the images shown in Fig. 4. The error bars reflect the estimated uncertainties in thickness due to uncertainties in color matching.

the film appears black only when the thickness is small relative to the shortest visible wavelength, allowing an unambiguous identification of the orders of interference of the colored bands. Moreover, color matching can be performed even without reference to the black film, although it is somewhat more challenging.

Figure 5 shows the distribution of thickness measured along the radial direction as a function of horizontal distance x. Note that there is no simple theoretical result to compare these experimental data with. The distribution of thickness is also time dependent because the bubble is not in equilibrium. The draining of liquid from the bubble can be described by hydrodynamical equations[4,11] which, in most cases, have to be solved numerically.

The main source of experimental error is the uncertainty in the color matching procedure. Each student can estimate the experimental error according to his/her individual ability to match colors using the color map in Fig. 4(c), where an uncertainty in color matching across each color band, in the vertical direction, translates into an uncertainty of thickness. In the example shown in Fig. 4, the uncertainty of the thickness measurement is approximately 20 nm (see error bars in Fig. 5). The bubble shown in Fig. 4 was fairly axisymmetric with only small-scale fluctuations of thickness due to thermals. Several radial profiles of thickness can be measured and then averaged to obtain an accurate mean distribution of thickness. When large-scale thermals are observed, it is interesting to measure the difference between their thickness and the thickness of the ambient film. This allows one to calculate the areal density difference and the buoyant force on the thermal. A further useful exercise is to perform measurements of mean thickness profiles at different times to document the thinning of the bubble which results from a loss of liquid due to evaporation as well as draining at the contact line between the bubble and the dish. The rate of change of the volume of the bubble can then be determined.

IV. CONCLUSION

We have presented a laboratory experiment to measure the thickness of soap bubbles using the laws of thin-film interference. The procedure is based on matching the color observed at each point on the bubble to a reference color map. The method can be used in experiments where an accurate determination of bubble thickness is required. The MATLAB code used to create the color map is available on EPAPS.[12]

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