Transportation Problem

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Introduction

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The model can be extended in a direct manner to cover practical situations in the areas of inventory control, employment scheduling, personnel assignment, cash flow and many others. The model also can be modified to account for multiple commodities. The transportation model is basically an LPP. However, its special structure allows development of a solution procedure, called the transportation technique, that is computationally more efficient.

Formulation of Transportation Problem

Let m be the number of origins and n be the number of destinations. The cost of transporting one unit of the commodity from origin i to destination j is c_{ij}. Let $a_i \geq 0$ for all $i = 1, 2, \ldots, m$ be the quantity of the commodity available at origin i and $b_j \geq 0$ for all $j = 1, 2, \ldots, n$ be the quantity required at destination j. If $x_{ij} \geq 0$ is the quantity transported from origin i to destination j, then the general formulation of the transportation problem is:

$$\text{Minimise } Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} \tag{7.1a}$$

$$\text{subject to } \sum_{j=1}^{n} x_{ij} = a_i, \ i = 1, 2, \ldots, m \tag{7.1b}$$

$$\sum_{i=1}^{m} x_{ij} = b_j, \ j = 1, 2, \ldots, n \tag{7.1c}$$

$$x_{ij} \geq 0. \tag{7.1d}$$

Formulation of Transportation Problem

The above represents an LP with (mn) variables and $(m+n)$ constraints.
A transportation problem is said to be *balanced*, if

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j.$$

Otherwise, it is said to be *unbalanced*.

Suppose,

$$\sum_{i=1}^{m} a_i > \text{or} < \sum_{j=1}^{n} b_j,$$

then a fictitious destination or fictitious origin with requirement

$$\sum_{i=1}^{m} a_i - \sum_{j=1}^{n} b_j$$

or with available quantity

$$\sum_{j=1}^{n} b_j - \sum_{i=1}^{m} a_i$$

can be taken so that unit cost of transportation to this destination from all the m origins or unit cost of transportation from origin to all n-destinations will be taken as zero. This way an unbalanced problem can be converted into balanced problem.

Formulation of Transportation Problem

A transportation problem can be represented by a tableau of the following type:

Destinations

		D_1	D_2		D_j		D_{n-1}	D_n	
	O_1	c_{11}	c_{12}	$\cdots$	c_{1j}	$\cdots$	$\cdots$	c_{1n}	a_1
	O_2	c_{21}	c_{22}	$\cdots$	c_{2j}	$\cdots$	$\cdots$	c_{2n}	a_2
		$\vdots$	$\vdots$		$\vdots$			$\vdots$	
Origins	O_i	c_{i1}	c_{i2}	$\cdots$	c_{ij}	$\cdots$	$\cdots$	c_{in}	a_i
		$\vdots$	$\vdots$		$\vdots$			$\vdots$	
	O_m	c_{m1}	c_{m2}	$\cdots$	c_{mj}	$\cdots$	$\cdots$	c_{mn}	a_m
		b_1	b_2		b_j			b_n	

The above table has $(m\,n)$ cells arranged in m rows and n columns. The cell in the ith row and jth column of the table is called (i, j) cell. In the transportation table a sequence of cells is said to form a *loop*, if

Formulation of Transportation Problem

(i) each adjacent pair of cells either lies in the same column or in the same row;

(ii) not more than two consecutive cells in the sequence lie in the same row or in the same column;

(iii) the first and the last cells in the sequence lie either in the same row or in the same column;

(iv) the sequence must involve at least two rows or two columns of the table.

Illustration of Loops

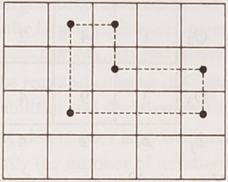

Fig. 1 (1,2) (1,3) (2,3) (2,5) (3,5) (3,2)

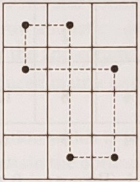

Fig. 2 (1,1) (1,2) (4,2) (4,3) (2,3) (2,1)

Note that every loop contains an *even* number of cells; every row or column either contains no cells or an even number of cells in the loop. Therefore, a set of cells S_c of a transportation table is said to contain a loop if the set S_c or a subset of S_c can be sequenced to form a loop.

Some Important Results

Theorem: A necessary and sufficient condition for the existence of a feasible solution to the transportation problem is that

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j.$$

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Theorem: The number of basic variables in a transportation problem is at most $(m + n - 1)$.

Remark: All basic variables may not be positive, some of them may be zero. When all basic variables are positive, the solution is called a non-degenerate B.F.S. When at least one basis variable is zero, the solution is called a degenerate B.F.S.

The number of basic cells will be exactly $(m + n - 1)$ all of which contain $(m + n - 1)$ variables which are either all positive basic variables or some variables may be 0 (we will consider this case later).