Lec-10

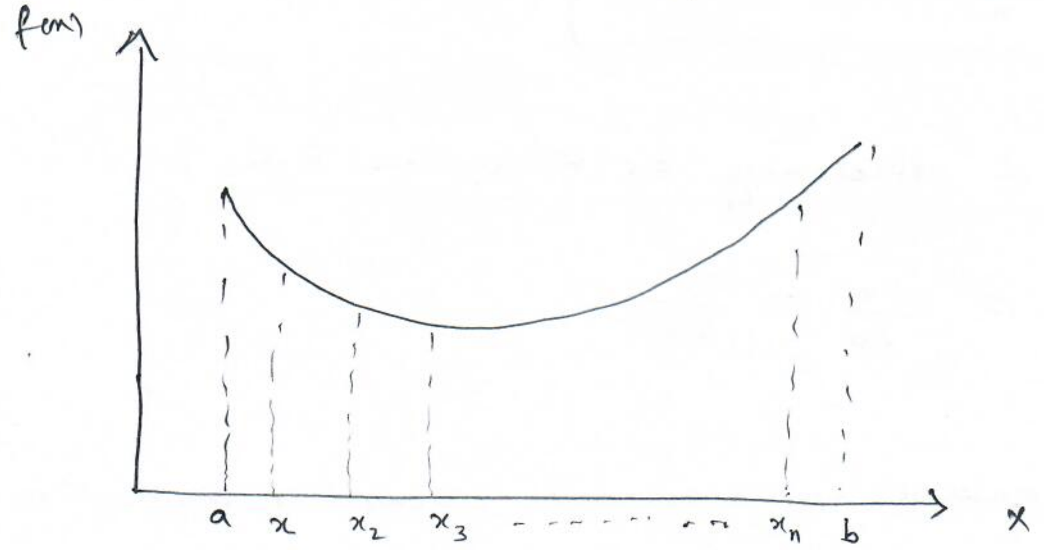
Exhaustive Search Technique.

In this lecture we will study some technique for region elimination Method. Here, we assume our objective function is unimodal.

Step-I :- Given an interval (initial interval) we derive $(n+1)$ numbers of intervals with equal length.

Step-II :- ~~If~~ ~~to~~ Compute the functional value at each end points of the intervals.

Step-III :- If the functional value minimum at x_K then final interval of uncertainity $[x_{K-1}, x_{K+1}]$.



- Each interval length $L_k = 2 \cdot \dfrac{(b-a)}{n+1}$.
- L_0 : initial interval ~~of~~ length of uncertainity $[a, b]$

  $L_0 = b - a$.

- Find n. (Based on how much error we allowed)
- Efficiency / Reduction Ratio $= \dfrac{L_n}{L_0}$.
- $n \rightarrow$ number of experiments
- After nth experiment interval of uncertainty is $[x_{k+1}, x_{k-1}]$
- L_n : Length of interval of uncertainty after n experiments

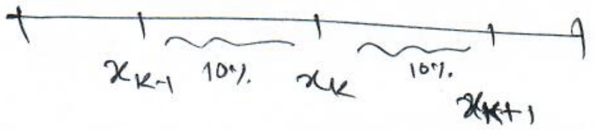
$$\boxed{L_n = 2 \cdot \frac{L_0}{n+1} = \frac{2(b-a)}{n+1}}$$

- <u>Measure of efficiency or Reduction Ratio</u> :-

$$= \frac{L_n}{L_0} = \frac{2}{n+1}.$$

<u>Note</u>:- The method is discussed about minimization problem. The same logic can be extended for maximization problem also.

<u>Example</u> :- solve for minimum value of $f(x) = x(x-1)$ in $[0,1]$. obtain minimum value within 10% of exact value.



i.e. $\frac{L_n}{2} < \frac{L_0}{10} \Rightarrow \frac{1}{n+1} < \frac{1}{10}$.

$\Rightarrow n+1 > 10$

$\Rightarrow n \geq 9$.

x_i	0.1	0.2	0.3	0.4	$\boxed{0.5}$	0.6	0.7	0.8	0.9
$f(x_i)$	-0.09	-0.16	-0.21	-0.24	$\boxed{-0.25}$	-0.24	-0.21	-0.16	-0.09.

$x^* = 0.5$. , $f(x^*) = -0.25$.

Interval $= [0.4, 0.6]$.

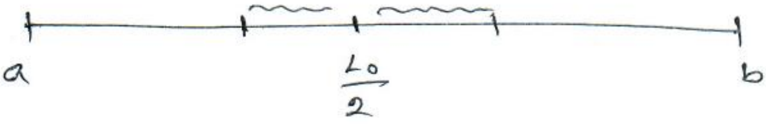
<u>Note</u> :- If minimum occurs at two adjecent points Consider the middle value.

Dichotomous Search Method :-

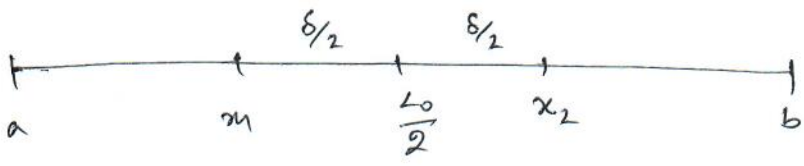
Given information is similar like the previous method. The given function is unimodal (i.e. only one minimum or maximum in the given domain). Also, number of experiments or how much accuracy we want that will be given before hand.

<u>Note</u> :- All the searching techniques we are discussing which are applicable not only continuous function (non-linear), these are also applicable for discontinuous non-linear functions as well.

<u>Step-I</u> :- Given initial interval of uncertainty $[a, b]$ and choose δ, which is very small positive value.



• <u>Step-II</u> :- Choose two points x_1, x_2 which is ~~from sa m~~ same length apart from the middle value.



The distance will be $\delta/2$ (δ is given)

Step III: Evaluate function values at x_1 and x_2

i.e. $f(x_1)$ and $f(x_2)$.

Step-IV: For minimization problem

- If $f(x_1) < f(x_2) \Rightarrow$ Eliminate $[x_2, b]$

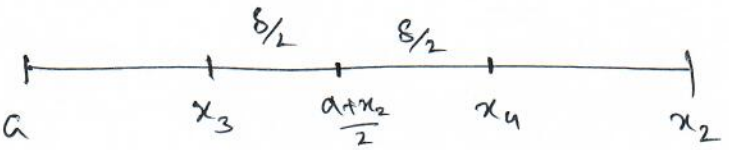
  New interval of uncertainity $[a, x_2]$

- For maximization problem eliminate $[a, x_2)$ and new interval of uncertainity will be $[x_2, b]$.

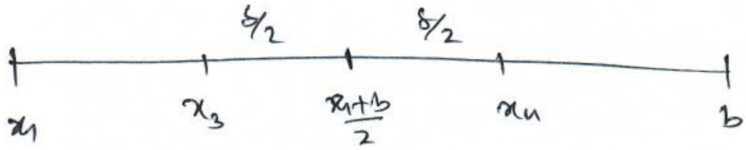
- $f(x_1) > f(x_2) \Rightarrow$ Eliminate $[a, x_1)$

  New interval of uncertainty $[x_1, b]$.

Step-V:



for $f(x_1) < f(x_2)$



for $f(x_1) > f(x_2)$,

Step-VI: According to the value of n or according to our desired accuracy we will stop our iteration.

No of Exp.	Interval	Length of uncertainity.
1	L_0	$b - a$
2	$[a, x_2] / [x_1, b]$	$L_0/2 + \delta/2$
4	$[x_1, x_4] / [x_3, b]$	$\frac{1}{2}\left(\frac{L_0}{2} + \frac{\delta}{2}\right) + \frac{\delta}{2} = \frac{L_0}{2^2} + \delta\left(1 - \frac{1}{2^2}\right)$.
⋮		
n	nth interval	$\frac{L_0}{2^{n/2}} + \delta\left(1 - \frac{1}{2^{n/2}}\right)$