Measure of Efficiency

Let us define the measure of efficiency or reduction ratio (R.R) as

- Measure of efficiency OR Reduction Ratio

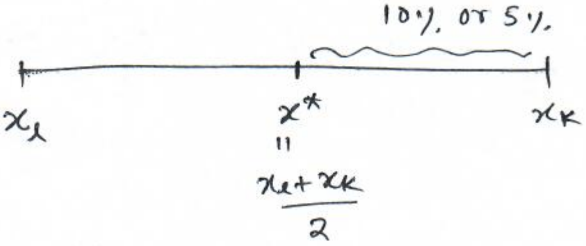
$$= \frac{\text{Length of interval of uncertainty after } n \text{ experiments}}{\text{Length of initial interval of uncertainty.}}$$

$$= \frac{L_n}{L_0}$$

- We have studied many region elemination techniques for solving unconstrained non-linear optimization problems. The measure of efficiency for a particular method is less than the other methods concludes that ~~the meth.~~ particular method is more efficient compare to the others. Let's have a look for the methods discussed before which ~~are~~ is more efficient among all the discussed method.

Elemination Technique.	Initial Interval of uncertainity	Final Interval of uncertainity	Reduction Ratio.
1. Exhaustive Search	L_0	$L_n \doteq 2 \times \frac{L_0}{n+1}$	$R.R = \frac{2}{n+1}$
2. Dichotomous Search	L_0	$L_n = \frac{L_0}{2^{n/2}} + \delta\left(1 - \frac{1}{2^{n/2}}\right)$	$RR \simeq \frac{1}{2^{n/2}}$
3. Interval Halving	L_0	$L_n = \left(\frac{1}{2}\right)^{\frac{n-1}{2}} L_0$	$RR = \left(\frac{1}{2}\right)^{\frac{n-1}{2}}$
4. Fibonacci Method	L_0	$L_n = \frac{1}{F_n} \cdot L_0$	$RR = \frac{1}{F_n}$
5. Golden Section Method	L_0	$L_n = \frac{1}{\gamma^{n-1}} \cdot L_0$	$RR = \frac{1}{\gamma^{n-1}}$ $\equiv (0.618)^{n-1}$

- As we discussed in the RET methods that after reaching the final interval of uncertainity L_n, we will take the optimal point as the middle point of that interval.



- If we allowed some error tollerency of the initial interval of uncertainity.

Therefore, $\frac{L_n}{2} \leq L_0 \times$ error allowed

- If error is 10% of exact value

- $\frac{L_n}{2} \leq \frac{L_0}{10}$

- If error is 5% of exact value

- $\frac{L_n}{2} \leq \frac{L_0}{20}$

$\Rightarrow L_n \leq \frac{L_0}{10}$

$$\boxed{\frac{L_n}{2} \le L_0 \times \text{error}}, \quad L_0 = [0,1] \text{ (assume)}$$

Elimination Technique	10% error allowed	5% error allowed
1. Exhaustive Search	$\frac{2}{n+1} \le \frac{1}{5} \Rightarrow n \geq 9$	$\frac{2}{n+1} \le \frac{1}{10} \Rightarrow n \geq 19$
2. Dichotomous Search	$\frac{1}{2^{n/2}} \le \frac{1}{5} \Rightarrow n \geq 4$	$\frac{1}{2^{n/2}} \le \frac{1}{10} \Rightarrow n \geq 6$
3. Interval Halving	$\left(\frac{1}{2}\right)^{\frac{n-1}{2}} \le \frac{1}{5} \Rightarrow n \geq 5$	$\left(\frac{1}{2}\right)^{\frac{n-1}{2}} \le \frac{1}{10} \Rightarrow n \geq 7$
4. Fibonacci Method	$\frac{1}{F_n} \le \frac{1}{5} \Rightarrow n \geq 5$	$\frac{1}{F_n} \le \frac{1}{10} \Rightarrow n \geq 7$
5. Golden section Method	~~(0.61~~ $(0.618)^{n-1} \le \frac{1}{5} \Rightarrow n \geq 5$	$(0.618)^{n-1} \le \frac{1}{10} \Rightarrow n \geq 6.$

- Fibonacci Method and Golden Section Method are very efficient method among all the methods.

Also, as $n \to \infty$ both methods are same kind of behaviour.

Exercise :-

(1) Minimize $f(x) = x^5 - 5x^3 - 20x + 5$

within the interval $[0, 5]$ by

- unrestricted search by considering step size 0.1. and starting point 0.
- Exhaustive search
- Dichotomous search, $\delta = 0.001$
- Interval halving Method
- Fibonacci Method (consider 10 % of initial interval)
- Golden section Method. (10 % of initial interval)

(2) Use Golden Section Method / Fibonacci Method to obtain the optimal solⁿ.

$\boxed{\text{error} = 0.8.}$

$$\max \ x^2 + 2x$$

$$\text{s.t.} \quad x \in [-3, 5]$$