Constrained adaptation for time delay estimation with multipath propagation

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Abstract: The problem of estimating the time difference between two sensor outputs in the presence of a multipath propagation is addressed. Two adaptive configurations are proposed. One configuration assumes that the multipath delay is an integral multiple of the sampling interval. The other configuration allows a non-integral delay. In either case, filter coefficients are restricted to take on samples of a *sinc* function only. The role of the adaptive filter is to provide appropriate time shifts to the input signals. Incorporating this constraint on the coefficients, which are adapted by the LMS algorithm, reduces the computations and considerably speeds up the convergence rate. Theoretical derivations and computer simulations are presented to demonstrate the effectiveness of the scheme for time delay measurement and to show its ability to accurately track time-varying parameters.

1 Introduction

There are many applications where it is necessary to determine the location of a radiating source by passive means, a well known example is passive sonar. A standard method uses three or more receivers at separate but known locations. By measuring the differences in arrival times of the signals at these receivers, the bearing and range of the source can be determined by triangulation [1]. Other problems, such as the determination of the centre of earthquakes or underground explosions, navigation (position fixing) and speed measurement [2], also require time delay estimation.

Let the outputs of two sensors be

$$x(t) = s(t) + \sum_{i=1}^{N} \alpha_i s(t - M_i) + \phi(t) \tag{1a}$$

and

$$y(t) = s(t - D) + \sum_{j=1}^{L} \alpha_j s(t - M_j) + \psi(t) \tag{1b}$$

where $s(t)$, $\phi(t)$ and $\psi(t)$ are jointly stationary random processes, uncorrelated with each other and $s(t - D)$ is a delayed version of $s(t)$. The multipath transmissions are of attenuation α_i and α_j and delays M_i and M_j, respectively. The parameters N and L are the number of multipaths contained in $x(t)$ and $y(t)$. In the case of sonar, multipath signals come from bottom bounces or reflections from the ocean surface, and reflections from buildings or mountains in the case of radio transmissions. The delay D can be time dependent because of relative motion between the source and the receivers.

In the absence of multipath signals, $\alpha_i = \alpha_j = 0$, the two sensor outputs can be cross-correlated and the lag at which the cross correlation peaks is the delay estimate [1]. Finding delay D in the presence of multipaths in which the parameters, α_i, α_j, M_i, and M_j are all unknown is by no means a trivial problem. Ignoring the presence of multipaths and using standard methods would obviously cause erroneous time delay estimates. Tractable solutions do exist when some simplifying assumptions can be made. If D is constant, $|M_i|, |M_j| \gg D$, and $|M_i - M_j| \gg D$, then the location of the first peak of the cross correlation between $x(t)$ and $y(t)$ would yield an estimate of D. Another common approach [3] takes the cepstrum of $x(t)$ separately to determine the existence of multipath receptions and then removes them through cepstrum filtering techniques. This method only works for small M_i and M_j.

When the delay is time-varying, it is necessary to make the estimator adaptive to track the non-stationary signals that include those of the parameters of the multipaths. Only the non-stationary of time delay has been previously considered, where no multipath transmission is assumed [4–6]. Two new constrained adpative system configurations for time delay estimation in the presence of a single multipath are proposed. One configuration assumes that the multipath delay is an integral multiple of the sampling interval. This assumption is removed in the other configuration. The least-mean-square (LMS) algorithm [7] is used to adjust the filter parameters. Constrained adaptive time delay estimation (CATDE) which restricts filter coefficients to samples of a *sinc* function has been proposed [8] to speed up the convergence rate of measuring the time difference between two sensor outputs. The same constraint is incorporated in this study and, as will be shown later, many coefficients needed to be adapted and without the judicious use of constraints, convergence, if indeed achievable, usually required impractically long duration. Preliminaries of adaptive time delay estimation will be elaborated. The proposal for two adaptive configurations for CATDE with a multipath is described and the constrained algorithms and simulation results are included for performance evaluation.

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2 Preliminaries

The adaptive configuration for TDE in the absence of multipaths is shown in Fig. 1. The time shift, is modelled as a finite impulse response (FIR) filter of one process.

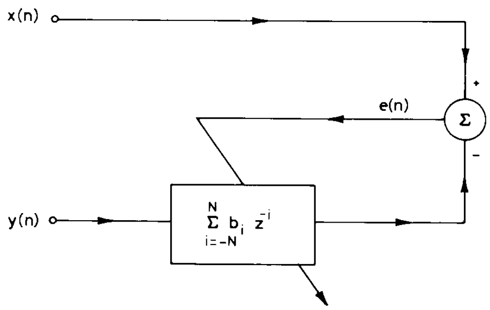

Fig. 1 *Adaptive TDE configuration without multipath*

Filter coefficients are adapted using Widrow's LMS algorithm to minimise the output square error $e(n)^2$. The system error and the coefficient updating equation can be explicitly expressed as

$$e(n) = x(n) - \sum_{i=-N}^{N} b_{i,n} y(n-i) \tag{2a}$$

$$b_{i,n+1} = b_{i,n} + 2\mu e(n) y(n-i) \quad i = -N, \ldots, N \tag{2b}$$

where $b_{i,n}$ is the ith filter coefficients at time n and μ is a control parameter that determines stability and rate of convergence of the adaptive algorithm. When the sampling rate is high enough, D can be assumed to be an integral multiple of the sampling interval T. The filter tap position that corresponds to the largest weight is then a delay estimate of $-D$. If this restriction is removed, a *sinc* function interpolation of the adaptive weights would be performed to provide a resolution smaller than the sampling interval [6].

It has been shown that [4] using Fourier series representation of a time delay, under the assumption that the discrete signal to be delayed is band limited between $-\pi$ to π

$$e^{-j\omega D} = \sum_{i=-\infty}^{\infty} \xi_i e^{j\omega i} \tag{3}$$

where

$$\xi_i = \frac{\sin \pi(i-D)}{\pi(i-D)} = \text{sinc}\,(i-D) \tag{4}$$

It follows from the convolution theorem that the time shifted version of a given sequence $s(n)$, $s(n-D)$, can be generated from

$$s(n-D) = \sum_{i=-\infty}^{\infty} \text{sinc}\,(i-D) s(n-i) \tag{5}$$

This represents the delayed signal is the output of a FIR filter whose coefficients are sinc $(i-D)$ and input is $s(n)$. For a filter to function as a pure time shifter, its coefficients must take on the values of the samples of a sinc function. The filter order has to be truncated for physical realisation and a detail analysis of the filter truncation error can be found in Reference 4. The requisite filter order for most circumstances is rather modest. For example, summing i from -8 to 8 in eqn. 5 will have a worst case error of less than 5%.

Ching and Chan [8] forced the FIR filter shown in Fig. 1 to behave as a time shifter by imposing the sinc function constraint on the weights. In this case, only the filter coefficient with the largest amplitude, say b_m, is required to be updated and it is used as an index to obtain other coefficients from a lookup table. The table is a two-dimensional matrix called $\boldsymbol{M}$ of size $k \times (2N+1)$ that contains samples of sinc function with delay ranging from 0 to 0.5. The elements of the table are

$$h_{ij} = \frac{\sin \pi(j - D_i)}{\pi(j - D_i)} \tag{6}$$

where $D_i = i/\{2(k-1)\}$, $i = 0, 1, \ldots, (k-1)$ and $j = -N, \ldots, -1, 0, 1, \ldots, N$.

The elements of the ith row of matrix $\boldsymbol{M}$ are identical to the samples of a sinc function with delay equals to D_i. The maximum of the sinc vector occurs at $h_{i,0}$ for a given D_i within the range $0 < D_i < 0.5$. Even though the value of D_i in the table only extends from 0 to 0.5, other values in the range 0.5–1.0 are also covered, because sinc $(D_i + j)$ equals sinc $[(1 - D_i) - (j + 1)]$, for $0.5 < D_i < 1$. If the delay is outside the range [0, 1], then it is expressed as $Q + D_i$, where Q is an integer and $0 < D_i < 1$, and the corresponding sinc function is given by $h_{i,j-Q}$. For negative D_i, which is a time advance, the since vector is simply $h_{i,-j}$. The maximum of b_i, $-N < i < N$, denoted by b_m, is used to locate the closest $h_{i,0}$ in the table during adaptation. Let this be $h_{q,0}$ and all the other b_i are set according to $h_{q,i}$, $b_{m+j} = h_{q,j}$ and are not adapted further.

The delay estimate is taken to be either $m - D_i$ or $m + D_i$, depending on the signs of the two adjustment elements of b_m. The interpolation step for obtaining the time delay is no longer required. This method is referred to as the constrained adaptive time delay estimation (CATDE). Incorporating the constraint has the merits of not only simplifying the adaptive process and reducing computational load, but also of achieving faster convergence.

3 Two adaptive configurations for CATDE with a multipath

Consider the discrete version of eqn. 1 with only a single multipath

$$x(n) = s(n) + \alpha s(n - \Delta) + \phi(n) \tag{7a}$$

$$y(n) = s(n - D) + \psi(n) \tag{7b}$$

where $nT = t$ with T, the sampling period, assumed to be unity. In eqn. 7 D can be any real number, Δ is a positive variable and $0 \leqslant \alpha \leqslant 1$. These conditions on the multipath parameters arise from the knowledge that multipath transmission is always a delayed signal and that it is always attenuated.

Having a single multipath in only one sensor is not a very restrictive assumption. It is quite common, in target localisation by passive sonar, for the two sensors to be separated by a distance of one or two miles. The water conditions can be such that one sensor will receive the direct signal plus the bottom bounce (a multipath) but the same bottom bounce could be refracted away from the other sensor, which is then multipath free. The problem of deciding which of the two sensors contain the multipath is more important. This is solved by checking the auto-correlation of the sensor output. If it contains a peak comparable in magnitude with the peak at zero shift, it is a good indication that a multipath is present.

It is better to recover $s(n)$ from $x(n)$ to make use of the existing configuration shown in Fig. 1. Substituting eqn. 5 into eqn. 7*a*

$$x(n) = s(n) + \alpha \sum_{i=-\infty}^{\infty} \text{sinc}\,(i - \Delta)s(n - i) + \phi(n) \tag{8}$$

When Δ is an integer, sinc $(i - \Delta)$ is essentially a δ function with non-zero magnitude at time Δ. In this case, taking the z-transform of eqn. 8 gives

$$X(z) = [1 + \alpha z^{-\Delta}]S(z) + \Phi(z) \tag{9}$$

where $X(z)$, $S(z)$ and $\Phi(z)$ are the z-domain representation of $x(n)$, $s(n)$ and $\phi(n)$, respectively. When noise is absent, rearranging eqn. 9 yields

$$S(z) = \frac{1}{1 + \alpha z^{-\Delta}} X(z) = H^*(z)X(z) \tag{10}$$

That means that $s(n)$ can be recovered by passing $x(n)$ through an infinite impulse response (IIR) filter $H^*(z)$. Once $s(n)$ is available, the adaptive configuration shown in Fig. 1 can be utilised for delay estimation. The multipath parameters α and Δ are unknown time varying quantities and the IIR filter cannot be determined before adaptation. It is necessary to make both parameters adaptive along with other filter coefficients on a sample-by-sample basis. The adaptive system can be configured as illustrated in Fig. 2. Since Δ is uncertain, it is necessary

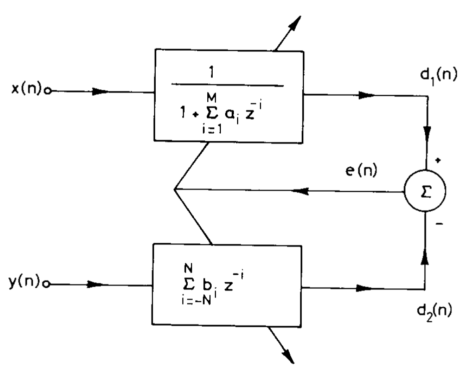

Fig. 2 *Adaptive configuration 1 for TDE with a multipath*

to model the IIR filter as

$$H(z) = \frac{1}{1 + \sum_{i=1}^{M} a_i z^{-i}} \tag{11}$$

where $\Delta \leqslant M$. From Fig. 2, the output error is given by

$$\begin{aligned} e(n) &= d_1(n) - d_2(n) \\ &= x(n) - \sum_{i=1}^{M} a_i d_1(k - i) - \sum_{i=-N}^{N} b_i y(k - i) \end{aligned} \tag{12}$$

and the filter parameters a_i and b_i are iteratively adapted to minimise the output square error. At equilibrium, the tap position corresponding to the largest tap weight will be the multipath delay Δ. This weight value will be an estimate of the attenuation factor α. The time delay estimate for D can be extracted from b_i by using a look up table.

If Δ is not an integral multiple of the sampling interval, configuration 1 will not work. This is because according to eqn. 8, a non-causal filter is required in the upper channel. A second configuration, shown in Fig. 3, is proposed. The sensor output $x(n)$ is considered a desired response and $y(n)$ is passed through two time shifters to obtain a replica of $x(n)$ by minimising the

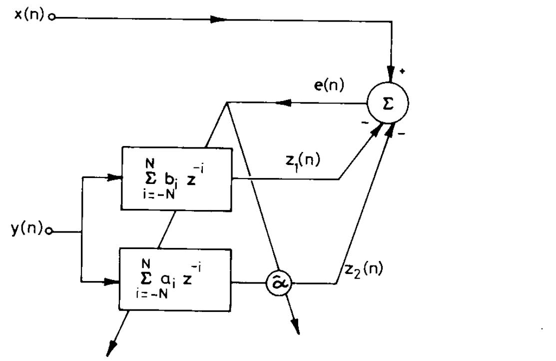

Fig. 3 *Adaptive configuration 2 for TDE with a multipath*

output square error. From Fig. 3, the output error can be written as

$$e(n) = x(n) - \sum_{i=-N}^{N} b_i y(n - i) - \hat{\alpha} \sum_{i=-N}^{N} a_i y(n - i) \tag{13}$$

After convergence, assuming no noise and no truncation errors, the two filter outputs will be an estimate of $s(n)$ and $\alpha s(n - \Delta)$. Let D_a and D_b be the time shift values obtained from a_i and b_i, respectively. The estimates of the time delay and multipath delay, $\hat{D}$ and $\hat{\Delta}$, are then given by

$$D = -D_b \tag{14a}$$

$$\hat{\Delta} = D_a - D_b \tag{14b}$$

The multipath attenuation parameter can easily be obtained from the gain $\hat{\alpha}$.

4 Constrained algorithm for two configurations

In both systems, the filter coefficients are updated on a sample-by-sample basis, according to Widrow's LMS algorithm, as follows:

$$a_{i,n+1} = a_{i,n} - \mu \frac{\partial e(n)^2}{\partial a_i} \tag{15a}$$

$$b_{i,n+1} = b_{i,n} - \mu \frac{\partial e(n)^2}{\partial b_i} \tag{15b}$$

In implementing the algorithm, it is beneficial and indeed necessary, as the number of adaptive coefficients is typically in the order of 40 or more, to utilise the a priori constraints to obtain convergence in a reasonable time. The constraints appear in the form of forcing the a_i and b_i coefficients to correspond to the values given by the sinc function and limiting $\hat{\alpha}$ to $0 \leqslant \hat{\alpha} \leqslant 1$.

For configuration 1, the constrained adaptive algorithm is as follows:

(i) The coefficients are adapted according to eqn. 15 for the first 200 iterations. The initial conditions are $a_i = 0$, $b_i = 0$ for all i.

(ii) At the 201st iteration

(a) Find the maximum of a_i. Let this be a_h. If $a_h < 0.5$, set $a_h = 0.5$. If $a_h > 0.5$, leave it unchanged. Set all other $a_i = 0$.

(*b*) Find the maximum of b_i. Let this be b_m. If $b_m > 0.9$, leave it unchanged. If $b_m < 0.9$, set $b_m = 0.9$. This corresponds to a fractional delay D_i of 0.25 or -0.25.

The proper value is determined from the signs of the two adjacent coefficients of b_m. Once the value of D_i is determined, the values for the other b_i are retrieved directly from the look-up table. The delay estimate in this case is given by $m + D_i$.

(iii) At each iteration after the 201st iteration

(*a*) If $a_h < TH$, then all a_i are adjusted according to eqn. 15*a*. If $a_h > TH$, only a_h is adjusted according to eqn. 15*a*. Set all other $a_i = 0$. The threshold TH is chosen as the minimum possible multipath gain.

(*b*) Compute

$$\begin{aligned}\delta_{m-1} &= \text{sgn}\,\{b_{m-1,n+1} - b_{m-1,n}\}\\ &= \text{sgn}\,\{e(n)y(n-m+1)\}\end{aligned} \tag{16a}$$

$$\begin{aligned}\delta_{m} &= \text{sgn}\,\{b_{m,n+1} - b_{m,n}\}\\ &= \text{sgn}\,\{e(n)y(n-m)\}\end{aligned} \tag{16b}$$

$$\begin{aligned}\delta_{m+1} &= \text{sgn}\,\{b_{m+1,n+1} - b_{m+1,n}\}\\ &= \text{sgn}\,\{e(n)y(n-m-1)\}\end{aligned} \tag{16c}$$

where sgn is the signum operation. If $D_i > 0$ and the sign conditions

$$\{\delta_{m-1} > 0, \delta_m > 0, \delta_{m+1} < 0\} \tag{17a}$$

or

$$\{\delta_{m-1} < 0, \delta_m < 0, \delta_{m+1} > 0\} \tag{17b}$$

are satisfied, then update b_m according to eqn. 15*b*. From this new b_m, get an updated D_i from the table together with updated values for all other b_i. If condition eqn. 17 is violated, leave b_m unchanged. Similar procedures are followed if $D_i < 0$.

A detailed explanation of the particular steps taken in the algorithm is given. The constraints are not applied for the initial 200 iterations because the coefficients may be peaking at wrong positions during the initial transients, especially when noise is present. The minimum value for the $h_{i,0}$ column in the matrix $\boldsymbol{M}$ is 0.6366, corresponding to $D_i = 0.5$. Using the bounds that $0.6366 \leqslant h_{i,0} \leqslant 1$, the adaptation can be made faster by setting $b_m = 0.9$, if it is below 0.9, on the first application of the constraint at the 201st iteration. The choice of 0.9 is ad hoc, any value between 0.6366 and unity will suffice. The check on sign conditions in step iii(*b*) is to ensure that any alternations of the three most significant coefficients b_{m-1}, b_m and b_{m+1} will move in the correct directions.

For configuration 2, the constrained adaptive algorithm can be applied with very little modifications, $\hat{\alpha}$ is initially set at 0.5 to speed up the adaptation and is subsequently updated using the LMS algorithm. The filter weights b_i remain unchanged in the first 100 iterations to avoid the condition of lock-up of the two filters. Subsequently, in each iteration, the parameter estimates of $\hat{\text{D}}$ and $\hat{\Delta}$ are computed according to eqn. 14. If $\hat{\Delta} < 0$ then set $\hat{\Delta} = 0.5$ and reassign the filter vector b_j. All other procedures are similar to those described previously.

5 Simulation results

Simulations were conducted to study the validity and performance of the two constrained adaptive schemes for time delay estimation with a single multipath. The signals and noises were all generated using a random number generator with Gaussian distribution and an approximately white spectrum. The size of the lookup table for time difference and filter weight conversion was fixed to 512 × 21 which gives a delay estimates of approximately $0.001T$ resolution. The delayed signal were obtained according to eqn. 5 with the order of the FIR filter fixed to 40. In both configurations, the order of the filters are chosen such that $M = 10$ and $N = 10$. When the acceptable modelling error is chosen to be 9%, then maximum delays of $\pm 5T$ are adequately covered. The results provided are the average of 10 independent runs with the signal to noise ratio set to 20 dB. The sampling interval is unity.

Fig. 4 shows the ability of adaptive configuration 1 to estimate the system parameters under a non-stationary environment in which all three parameters were given step offsets after each 5000 iterations. Control parameters for the IIR and time shift filter were both fixed to 0.002 and TH was set to 0.1. It can be seen that the adaptive system responded to these changes and was able to give accurate estimates after transients although the multipath delay transient was much shorter. To illustrate the novelty of applying constraint, the case without constraint is also provided in Fig. 4 for comparison. It is observed that although comparable multipath delay trajectory is obtained, the system cannot converge to the desired multipath gain and time delay values.

When the multipath is not an integral multiple of the sampling interval, configuration 2 is used and a typical result is shown in Fig. 5. All the three unknown parameters were given step offsets after each 10 000 iterations to demonstrate the tracking ability of the system. Control

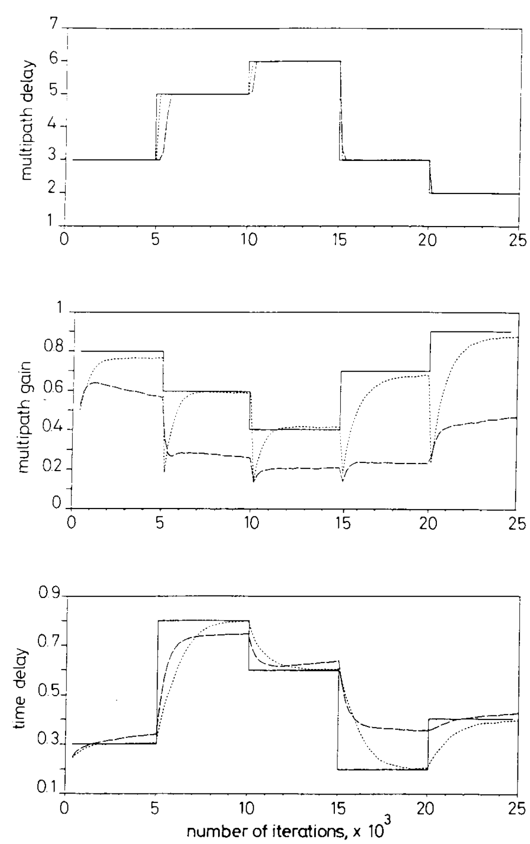

Fig. 4 *Adaptation characteristics of configuration 1*

------- with constraint
- - - - without constraint
——— actual value

parameter μ in the LMS update equation was fixed at 0.008. Examination of Fig. 5 shows that the parameters can be precisely estimated after each step offset. They converged at approximately the same speed as configuration 1 but the multipath delay had a larger transient in this case. When there is no constraint, it was found that convergence to the optimal solution is impossible. This is because there is no guarantee that a_i and b_i will become sinc function samples as desired.

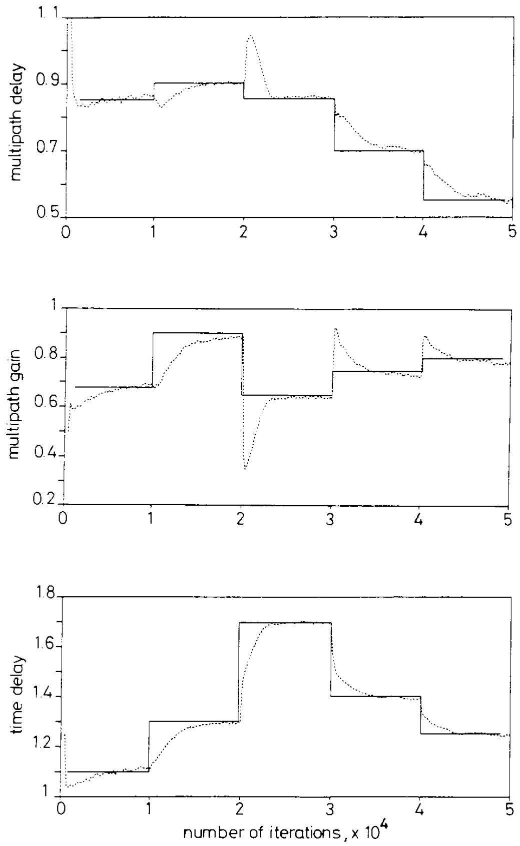

Fig. 5 *Adaptation characteristics of configuration 2*
------- estimated value
——— actual value

CATDE has to be performed in real time in many practical situations. It is therefore necessary to further reduce the computational requirement. This can be achieved by restricting the alternation of the parameters to fixed amounts with polarity equal to that of stochastic gradients. For delay estimates, the sign of the noisy gradient of the largest coefficient is used as an indicator to decide whether it is to move one step up or down in the lookup table and hence only a change in indexing is involved. The result is that the multiplication operation associated in the adaptive algorithm is eliminated. Figs. 6 and 7 showed the trajectories of the adapting parameters for configurations 1 and 2, respectively. The operating environment was the same as before. The amount of adjustment for the IIR filter in configuration 1 was 0.004. The adjustment for $\hat{\alpha}$ in configuration 2 was 0.001. The convergence performance is only marginally degraded. The simplified version thus serves as a good alternative for reducing complexity in CATDE with a multipath.

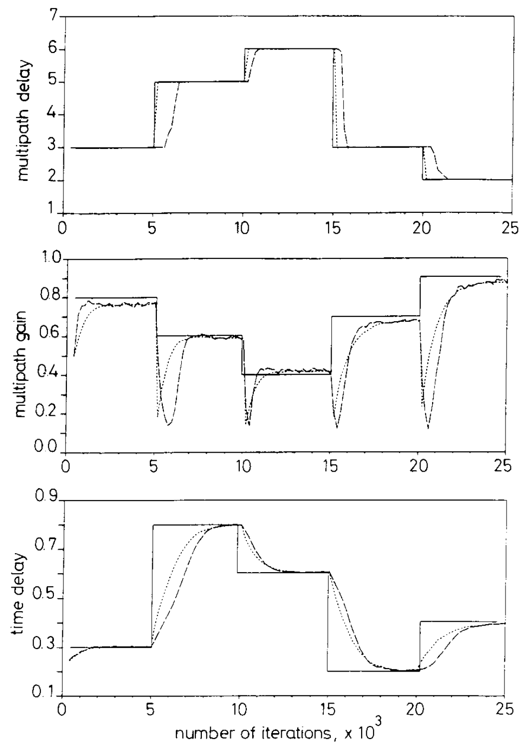

Fig. 6 *Adaptation characteristics of configuration 1 with simplified adaptation rule*
------- original ——— actual
- - - - simplified

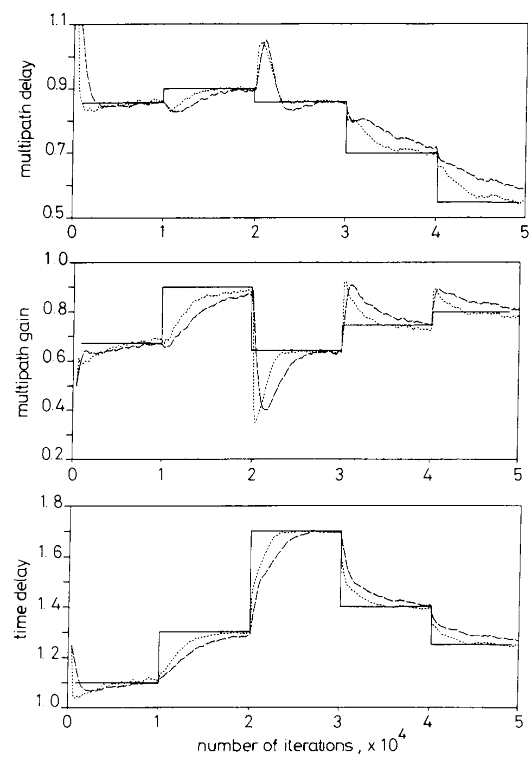

Fig. 7 *Adaptation characteristics of configuration 2 with simplified adaptation rule*
------- original ——— actual
- - - - simplified

6 Conclusions

A constrained adaptive algorithm for time delay estimation in the presence of a multipath reception has been presented. The scheme is based on the LMS algorithm but through constraints, the number of parameters to be adaptive is reduced and the convergence time is greatly decreased, enabling the real time applicability of the proposed adaptive scheme. Convergence would not have been possible in many cases without constraints. Simulation results are included which demonstrate the effectiveness of this method and its ability to track time-varying parameters. Although the proposed system would not work if multipaths are present in both sensors, the assumption of multipath being observed only in one sensor is not too restrictive and would not limit the practical application of CATDE approach. A modified CATDE algorithm in conjunction with a different filter configuration is currently under investigation to tackle the problem of nonstationary time delay estimation with more than one multipath.

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