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Make a suitable title: my paper on the cosmic microwave background and formation of structures in our Universe

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ABSTRACT

An abstract for the paper. Describe the paper. What is the paper about, what are the main results, etc.

Key words. cosmic microwave background – large-scale structure of Universe

1. Introduction

Write an introduction here. Give context to the paper. Citations to relevant papers. You only need to do this in the end for the last milestone.

2. Milestone I

In this section we want to study the evolution of the uniform background in the Universe. Our main goal of this section will be to implement methods that computes the Hubble parameter, as well as related time- and distance measures Rewrite. To do this, we will solve simple ordinary differential equations (ODEs) numerically. For the fiducial cosmology we will use results from the Planck Collaboration [1]. We will then test our implementation with known theoretical solutions, to assess the validity of our methods.

The main goal of this section is mainly concerned with using known parameter values to predict/solve the background cosmology. Another interesting aspect is to use data to constrain such cosmological parameters. To do this, we will use data from supernova observations [2], containing luminosity distance associated with different values of redshift. Implementing our solver, we will implement a simple Markov chain Monte Carlo (MCMC) algorithm, in order to estimate the best fits of h, Ω_m and Ω_Λ, which are the Hubble parameter, and the density parameter of matter and dark energy, respectively.

2.1. Theory

The theory behind this milestone. Define ALL parameters

Neutrinos are on the loose. Catch them before it's too late.

Define most important stuff here

2.1.1. Density parameters

The Friedmann equation can be written in terms of density parameters, $\Omega_i \equiv \rho_i/\rho_c$, where $\rho_c \equiv 3H^2/8\pi G$ is the critical density. We will assume that a given species, i, of the Universe can be described by a constant equation of state, $w_i \equiv P_i/\rho_i$, where P_i denotes the pressure. The density of a species evolves as [3, Eq. (2.72)]

$$\rho_i(t) \propto a(t)^{-3(1+w_i)}. \tag{1}$$

For baryons and cold dark matter (CDM), we have $w = 0$, for photons and massless neutrinos (Write massless assumption) we have $w = 1/3$ and for a cosmological constant we have $w = -1$. Including a curvature parameter Ω_k, with $w = -1/3$, the Friedmann Equation can be written as [3, Eq. (3.14)]

$$H = H_0\sqrt{\Omega_{m0}a^{-3} + \Omega_{r0}a^{-4} + \Omega_{k0}a^{-2} + \Omega_{\Lambda0}}, \tag{2}$$

where $H \equiv \dot{a}/a$ is the Hubble parameter, with the dot denoting a derivative with respect to cosmic time, t. For brevity, we have expressed the density parameters of matter and radiation as $\Omega_{m0} = \Omega_{b0} + \Omega_{\mathrm{CDM0}}$ and $\Omega_{r0} = \Omega_{\gamma0} + \Omega_{\nu0}$, respectively. A subscript 0 indicates the value of a given parameter today, at $a = 1$. The density parameters of radiation follow from the CMB temperature, and are given by

$$\Omega_{\gamma0} = 2\cdot\frac{\pi^2}{30}\frac{(k_b T_{\mathrm{CMB0}})^4}{\hbar^3 c^5}\cdot\frac{8\pi G}{3H_0^2}, \tag{3}$$

$$\Omega_{\nu0} = N_{\mathrm{eff}}\cdot\frac{7}{8}\cdot\left(\frac{4}{11}\right)^{4/3}\Omega_{\gamma0}. \tag{4}$$

The value of $\Omega_{\Lambda0}$ is fixed by the requirement that $H(a=1) = H_0$, yielding

$$\Omega_{\Lambda0} = 1 - (\Omega_{m0} + \Omega_{r0} + \Omega_{k0}). \tag{5}$$

We also introduce the scaled Hubble factor, $\mathcal{H} \equiv aH$. Rather than working with the scale factor, $a(t)$, we will mainly be working with the logarithm of the scale factor

$$x \equiv \ln a, \quad ' \equiv \frac{\mathrm{d}}{\mathrm{d}x}. \tag{6}$$

The resulting expression for $\mathcal{H}(x)$ is thus

$$\mathcal{H}(x) = H_0\sqrt{\Omega_{m0}e^{-x} + \Omega_{r0}e^{-2x} + \Omega_{k0} + \Omega_{\Lambda0}e^{2x}}. \tag{7}$$

Once $\mathcal{H}(x)$ is known, we can compute the value of the density parameters at any given x, with

$$\Omega_k(x) = \frac{\Omega_{k0}}{\mathcal{H}(x)^2/H_0^2}, \tag{8}$$

$$\Omega_m(x) = \frac{\Omega_{m0}}{e^x\mathcal{H}(x)^2/H_0^2}, \tag{9}$$

$$\Omega_r(x) = \frac{\Omega_{r0}}{e^{2x}\mathcal{H}(x)^2/H_0^2}, \tag{10}$$

$$\Omega_\Lambda(x) = \frac{\Omega_{\Lambda 0}}{e^{-2x}\mathcal{H}(x)^2/H_0^2}, \tag{11}$$

2.1.2. Conformal time

One of the main time variables we will be working with is the conformal time, η, which is a measure of the distance light have been able to travel since $t = 0$. Using its definition in terms of t [3, Eq. (2.90)], we can express it in terms of x as

$$\eta = \int_0^t \frac{c\,\mathrm{d}t'}{a(t')} = \int_{-\infty}^{x'} \frac{c\,\mathrm{d}x'}{\mathcal{H}(x')}. \tag{12}$$

This leads us to the following differential equation that we will solve numerically

$$\frac{\mathrm{d}\eta}{\mathrm{d}x} = \frac{c}{\mathcal{H}}. \tag{13}$$

The initial condition we have is $\eta(-\infty) = 0$. Noting from Eq. (7) that $\mathcal{H}(x) \to H_0\sqrt{\Omega_{r0}}e^{-x}$ as $x \to -\infty$, we get an analytical approximation for the initial condition of η at early times

$$\eta(x_{\text{start}}) \approx \int_{-\infty}^{x_{\text{start}}} \frac{c\,\mathrm{d}x'}{H_0\sqrt{\Omega_{r0}}}e^{x'} = \frac{c}{\mathcal{H}(x_{\text{start}})}. \tag{14}$$

Move sentence below to check section. From this, we also get a way of checking that our solution is reasonable, by checking if $\eta(c)\mathcal{H}(x)/c = 1$ at low x.

In addition to η, we also want to know the value of t at different points in the Universe's evolution. By definition, $H = \dot{a}/a$, and from the chain rule we obtain a differential equation for $t(x)$ which we can solve numerically,

$$\frac{\mathrm{d}t}{\mathrm{d}x} = \frac{1}{H(x)}. \tag{15}$$

To get an initial condition for t, we consider the radiation dominating era, with the following integral expression

$$t(x) = \int_{-\infty}^{x} \frac{\mathrm{d}x'}{H(x')}. \tag{16}$$

Comparing with Eq. (14), we see that the two integrands only differ by a factor e^x. The initial condition for t is therefore easily seen to be

$$t(x_{\text{start}}) = \frac{1}{2H(x_{\text{start}})}. \tag{17}$$

2.1.3. Distance measures

Rewrite this, express with x.

For the supernova fitting, we will need a measure of the luminosity distance, d_L, defined as

$$d_L(a) = \frac{d_A}{a^2} = \frac{r}{a}, \tag{18}$$

where $d_A = ar$ is the angular distance of the source at a distance r away from us. Photons move on 0-geodesics, and from the line-element in spherical coordinates,

$$ds^2 = -c^2dt^2 + a^2\left(\frac{dr^2}{1-kr^2} + r^2d\theta^2 + r^2\sin^2\theta d\phi^2\right), \tag{19}$$

we get the co-moving distance by integrating the line element of radially moving photons. For a photon emitted at, (t, r), reaching an observer at $(t_0, 0)$, we get

$$\int_0^r \frac{\mathrm{d}r'}{\sqrt{1-kr'}} = \int_t^{t_0} \frac{c\,\mathrm{d}t}{a}. \tag{20}$$

(Define $\Omega_{k0} = -kc^2/H_0^2$)

The RHS is the co-moving distance, χ, which in terms of conformal time is given as

$$\chi = \int_t^{t_0} \frac{c\,\mathrm{d}t}{a} = \int_x^0 \frac{c\,\mathrm{d}x'}{\mathcal{H}(x')} = \eta(0) - \eta(x). \tag{21}$$

Solving Eq. (20) with respect to r, we get (check punctuation after cases)

$$r = \begin{cases} \chi \cdot \frac{\sin(\sqrt{|\Omega_{k0}|}H_0\chi/c)}{(\sqrt{|\Omega_{k0}|}H_0\chi/c)}, & \Omega_{k0} < 0 \\ \chi, & \Omega_{k0} = 0 \\ \chi \cdot \frac{\sinh(\sqrt{|\Omega_{k0}|}H_0\chi/c)}{(\sqrt{|\Omega_{k0}|}H_0\chi/c)}, & \Omega_{k0} > 0 \end{cases} \tag{22}$$

2.1.4. Analytical solutions (Working title)

We will need the first and second derivative of $\mathcal{H}(x)$. To simplify the resulting expressions, we define the function, $g(x)$, as the derivative of the term inside the square root in Eq. (7), namely

$$g(x) \equiv -\Omega_{m0}e^{-x} - 2\Omega_{r0}e^{-2x} + 2\Omega_{\Lambda 0}e^{2x}. \tag{23}$$

The first two derivatives of $\mathcal{H}(x)$ are

$$\frac{\mathrm{d}\mathcal{H}(x)}{\mathrm{d}x} = \frac{H_0^2}{2\mathcal{H}(x)}g(x), \tag{24}$$

$$\frac{\mathrm{d}^2\mathcal{H}(x)}{\mathrm{d}x^2} = \frac{H_0^2}{2\mathcal{H}(x)}\left[g'(x) - \frac{1}{2}\left(\frac{H_0 g(x)}{\mathcal{H}(x)}\right)^2\right], \tag{25}$$

If we assume that the Universe is dominated by a fluid with a constant equation of state parameter, w, the density of a given fluid evolves as $\rho \propto a^{-3(1+w)}$.

$$H(t)^2 \propto a^{-3(1+w)} \implies \mathcal{H}(x) = c_1 e^{-\frac{3}{2}(1+w)x}, \tag{26}$$

where c_1 is a constant. This gives us an analytical expression for $\mathcal{H}'(x)/\mathcal{H}(x)$, in terms of w, as c_1 and the exponent vanishes

$$\frac{1}{\mathcal{H}(x)}\frac{\mathrm{d}\mathcal{H}}{\mathrm{d}x} = -\frac{1+3w}{2} = \begin{cases} -1, & w = 1/3 \\ -1/2, & w = 0 \\ 1, & w = -1 \end{cases} \tag{27}$$

Similarly, the expression for $\mathcal{H}''(x)/\mathcal{H}(x)$ becomes

$$\frac{1}{\mathcal{H}(x)^2}\frac{\mathrm{d}^2\mathcal{H}}{\mathrm{d}x^2} = \frac{(1+3w)^2}{2} = \begin{cases} 1, & w = 1/3 \\ 1/4, & w = 0 \\ 1, & w = -1 \end{cases} \tag{28}$$

2.2. Implementation details

Something about the numerical work.

2.2.1. Splining and data analysis (Working title)

2.2.2. MCMC (Working title)

The supernova data we will use contains data of luminosity distance, $d_L^{\text{obs}}(z_i)$, with associated measurement errors, σ_i, at different redshifts, z_i. Using these measurements, we want to constrain the three-dimensional parameter space

$$C = \left\{\hat{h},\ \hat{\Omega}_{m0},\ \hat{\Omega}_{k0}\right\}, \tag{29}$$

where the hat is used to distinguish them from the fiducial values. Fiducial?

We will assume that the measurements at different redshifts are normal distributed and uncorrelated. The likelihood function is then given by $L \propto e^{-\chi^2/2}$, where

$$\chi^2(C) = \sum_{i=1}^{N} \frac{[d_L(z_i, C) - d_L^{\text{obs}}(z_i)]^2}{\sigma_i^2}, \tag{30}$$

is the function we want to minimize. Little h needs to be defined. To do this, we will sample parameter values randomly by a MCMC process. The parameter space we will explore is limited to

$$\begin{aligned} 0.5 &< \hat{h} < 1.5, \\ 0 &< \hat{\Omega}_{m0} < 1, \\ -1 &< \hat{\Omega}_{k0} < 1. \end{aligned} \tag{31}$$

To generate a new sample, we update each parameter by generating a random number $P \sim \mathcal{N}(0,1)$, and multiplying it by a step size. We will use step sizes of $\Delta\hat{h} = 0.007$, $\Delta\hat{\Omega}_{m0} = 0.05$, $\Delta\hat{\Omega}_{k0} = 0.05$. To determine whether a new configuration should be included in the sample we use the Metropolis algorithm, where we always accept a state if it yields a lower value of χ^2 compared to the previous state that was accepted. If the new value of χ^2 is greater than the old one, we accept it if the ratio of the likelihood functions $L(\chi^2_{\text{new}})/L(\chi^2_{\text{old}}) > p$, where $p \sim \mathcal{U}(0,1)$. We continue drawing samples until we get a total of $\hat{n} = 10^4$ samples. For the samples generated, we omit the first 1000 samples of the chain from our analysis.

Explain: $\Omega_{b0} = 0.05$, splining, etc...

2.3. Results

2.3.1. Sanity checks

Show and discuss the results.

3. Milestone II

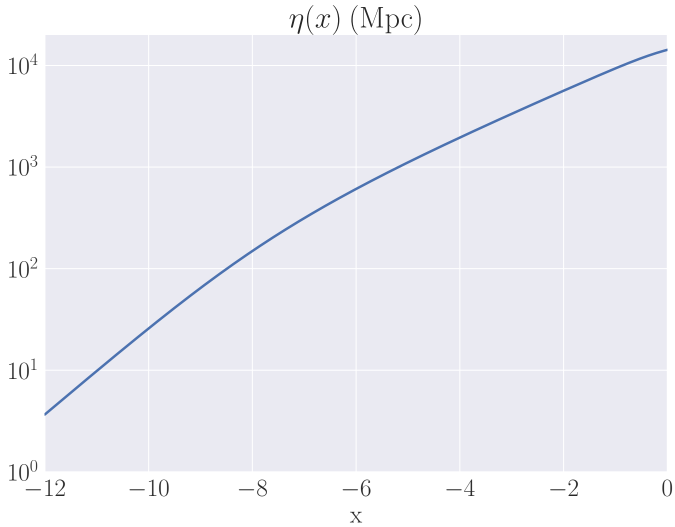
Some introduction about what it is all about. And more so that

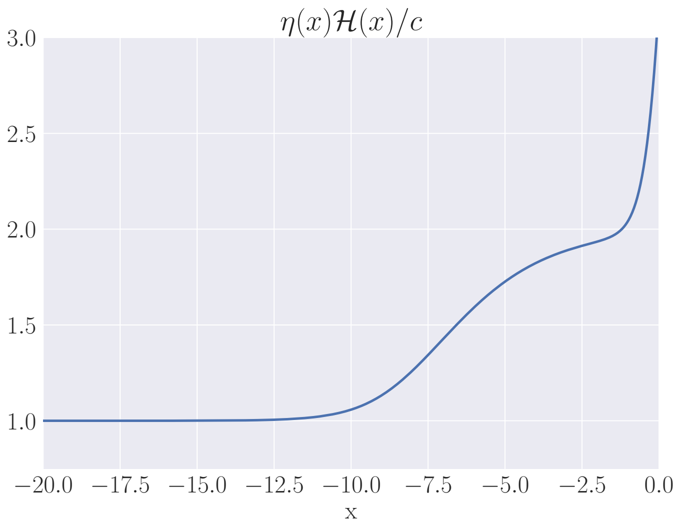
3.1. Theory

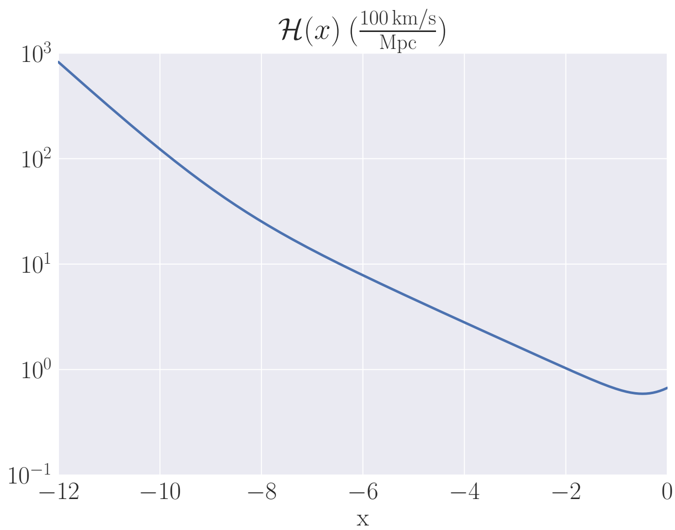
The theory behind this milestone.

3.2. Implementation details

Something about the numerical work.





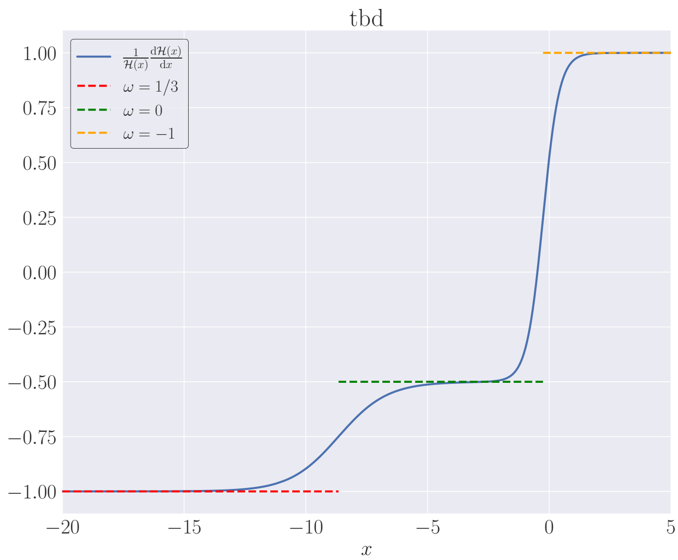


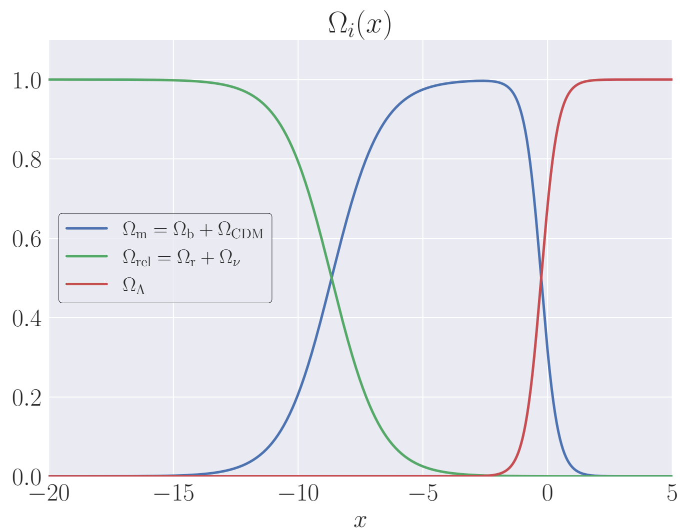
3.3. Results

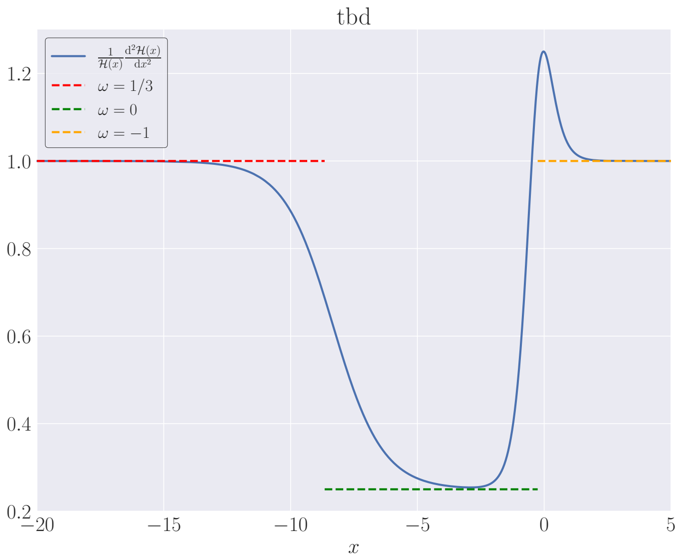
Show and discuss the results.

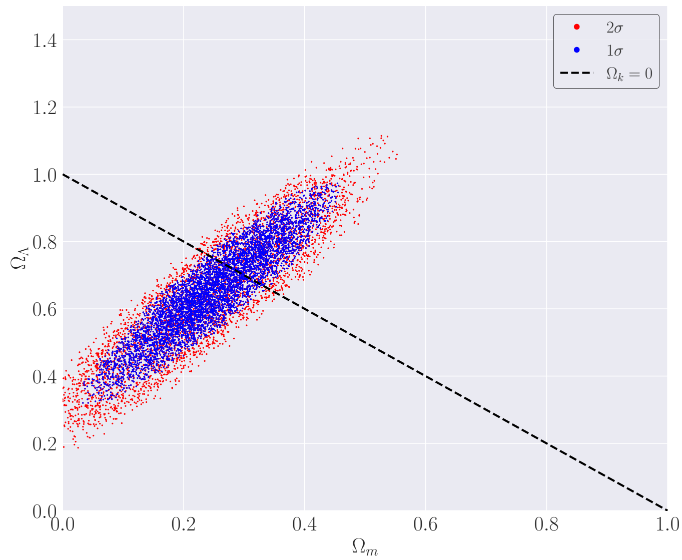
4. Milestone III

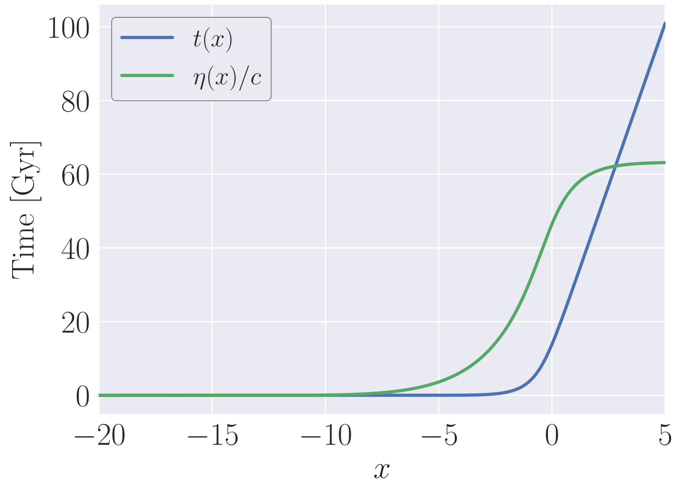
Some introduction about what it is all about.

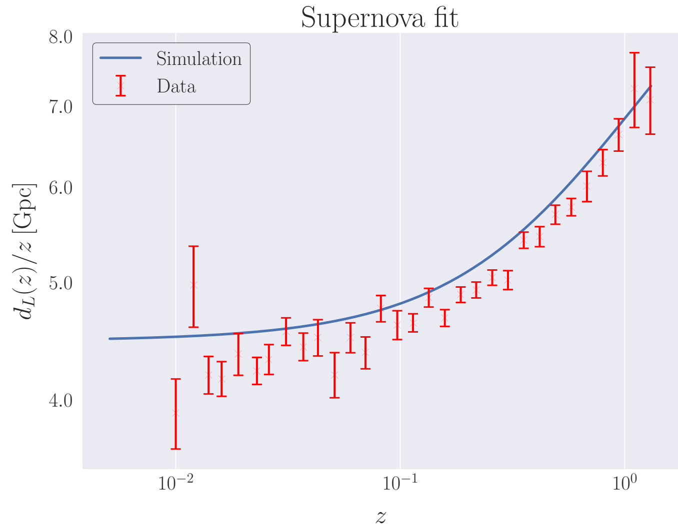










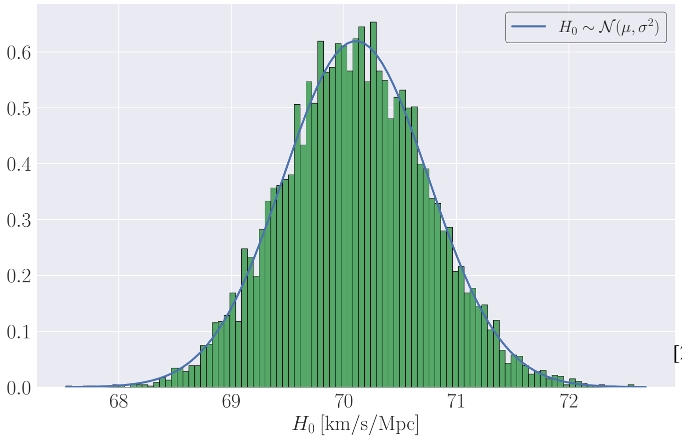


4.1. Theory

The theory behind this milestone.

4.2. Implementation details

Something about the numerical work.



4.3. Results

Show and discuss the results.

5. Milestone IV

Some introduction about what it is all about.

5.1. Theory

The theory behind this milestone.

5.2. Implementation details

Something about the numerical work.

5.3. Results

Show and discuss the results.

6. Conclusions

Write a short summary and conclusion in the end.

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References

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Appendix A: Bruh

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Appendix B: sis