Project 3 - FYS4150

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We provide an overview of how to structure a scientific report. For concreteness, we consider the example of writing a report about an implementation of the midpoint rule of integration. For each section of the report we briefly discuss what the purpose of the given section is. We also provide examples of how to properly include equations, tables, algorithms, figures and references.

I. INTRODUCTION

Write scientific introduction here

Equations (temporary title)

We will consider an ideal Penning trap, where the electric field is defined by the electric potential

$$V(x, y, z) = \frac{V_0}{2d^2}(2z^2 - x^2 - y^2), \tag{1}$$

where V_0 is the potential applied to the electrodes, d is the characteristic dimension. And ... Define equation more appropriately.

For a single particle, the electric field is given by the negative gradient of the potential in equation (1)

$$\mathbf{E} = -\nabla V = \frac{V_0}{d^2}(x, \, y, \, -2z) \tag{2}$$

A homogeneous magnetic field is imposed in the z-direction

$$\mathbf{B} = B_0 \hat{e}_z = (0, 0, B_0). \tag{3}$$

The force $\mathbf{F}$ on a single particle with charge q, is given by the Lorentz Force

$$\mathbf{F} = q\mathbf{E} + q\mathbf{v} \times \mathbf{B}. \tag{4}$$

Inserting the expressions for $\mathbf{E}$ and computing the cross product $\mathbf{v} \times \mathbf{B}$ the resulting Lorentz force on the particle is

$$\begin{aligned}
\mathbf{F} &= \frac{qV_0}{d^2}(x, \, y, \, -2z) + qB_0(\dot{y}, \, -\dot{x}, \, 0) \\
&= \frac{m}{2}\omega_z^2(x, \, y, \, -2z) + m\omega_0(\dot{y}, \, -\dot{x}, 0),
\end{aligned} \tag{5}$$

where we defined $\omega_0 \equiv \frac{qB_0}{m}$ and $\omega_z^2 \equiv \frac{2qV_0}{md^2}$. From Newton's second law, we have $\ddot{\mathbf{r}} = \mathbf{F}/m$. Dividing equation (5) by m, we get three equations of motion, one for each spatial component. These differential equations are

$$\ddot{x} - \omega_0\dot{y} - \frac{1}{2}\omega_z^2 x = 0, \tag{6}$$

$$\ddot{y} + \omega_0\dot{x} - \frac{1}{2}\omega_z^2 y = 0, \tag{7}$$

$$\ddot{z} + \omega_z^2 z = 0. \tag{8}$$

Equation (8) has the general solution

$$z(t) = c_1\cos(\omega_z t) + c_2\sin(\omega_z t). \tag{9}$$

For this project, we will assume $q > 0$. (Move this?)

Equations (6) and (7) are coupled. We therefore introduce a complex function $f(t) = x(t) + iy(t)$. Explaining maths.

The single differential equation is therefore

$$\ddot{f} + i\omega_0\dot{f} - \frac{1}{2}\omega_z^2 f = 0. \tag{10}$$

The general solution to equation (10) is

$$f(t) = A_+ e^{-i(\omega_+ t + \phi_+)} + A_- e^{-i(\omega_- t + \phi_-)}, \tag{11}$$

where ϕ_+ and ϕ_- are constant phases, A_+ and A_- are positive amplitudes and

$$\omega_\pm = \frac{\omega_0 \pm \sqrt{\omega_0^2 - 2\omega_z^2}}{2}. \tag{12}$$

$$\ddot{x}_i - \omega_{0,i}\dot{y}_i - \frac{1}{2}\omega_{z,i}^2 x_i - k_e\frac{q_i}{m_i}\sum_{j \neq i} q_j \frac{x_i - x_j}{|\mathbf{r}_i - \mathbf{r}_j|^3} = 0, \tag{13}$$

$$\ddot{y}_i + \omega_{0,i}\dot{x}_i - \frac{1}{2}\omega_{z,i}^2 y_i - k_e\frac{q_i}{m_i}\sum_{j \neq i} q_j \frac{y_i - y_j}{|\mathbf{r}_i - \mathbf{r}_j|^3} = 0, \tag{14}$$

$$\ddot{z}_i + \omega_{z,i}^2 z_i - k_e\frac{q_i}{m_i}\sum_{j \neq i} q_j \frac{z_i - z_j}{|\mathbf{r}_i - \mathbf{r}_j|^3} = 0, \tag{15}$$

II. METHODS

The algorithm

use equation [1]

III. RESULTS

IV. DISCUSSION

V. CONCLUSION

In this section we state three things in a concise manner: what we have done, what we have found, and what should or could be done in the future.

Algorithm 1 Midpoint rule for integration

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procedure MIDPOINT RULE(f, a, b, n)

    $I \leftarrow 0$ $\triangleright$ Initialize the integral variable

    $h \leftarrow (b-a)/n$ $\triangleright$ Compute the interval length

    for $i = 1, 2, \ldots, n$ **do**

        $x \leftarrow a + (i - 1/2)h$ $\triangleright$ Assign x to the midpoint

        $I \leftarrow I + f(x)$ $\triangleright$ Add contribution to integral

    $I \leftarrow Ih$ $\triangleright$ Finalize the computation

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[1] A. Faul, "A concise introduction to numerical analysis," (CRC Press, 2016) Chap. 5, p. 131.