

Project 1 FYS4150

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PROBLEM 1

We have the equation

$$-\frac{d^2u}{dx^2} = 100e^{-10x}, \quad \text{where } x \in [0, 1], \quad u(0) = u(1) = 0. \quad (1)$$

We want to check that

$$u(x) = 1 - (1 - e^{-10})x - e^{-10x} \quad (2)$$

is the solution of eq. 1. We first control that eq. 2 satisfies the boundary conditions.

$$\begin{aligned} u(0) &= 1 - 0 - e^0 = 0 \\ u(1) &= 1 - (1 - e^{-10}) - e^{-10} = 0 \end{aligned}$$

We find the double derivative of $u(x)$,

$$\frac{du}{dx} = -(1 - e^{-10}) - (-10)e^{-10x} \quad \Rightarrow \quad \frac{d^2u}{dx^2} = -100e^{-10x},$$

and see that this satisfies the differential eq. 1.

PROBLEM 2

PROBLEM 3

We have the discretisation of the second order derivative as follows:

$$\left. \frac{d^2u}{dx^2} \right|_{x_i} = u''_i = \frac{u_{i+1} - 2u_i + u_{i-1}}{h^2} + \mathcal{O}(h^2) \approx \frac{v_{i+1} - 2v_i + v_{i-1}}{h^2} = v''_i$$

where we have approximated u_i with v_i by neglecting the error term, and h is the step length. We further insert this into the Poisson equation and obtain the following:

$$\begin{aligned} -\frac{d^2u}{dx^2} &\approx -\left. \frac{d^2v}{dx^2} \right|_{x_i} = -v''_i = f(x_i) = f_i \\ &\Rightarrow -v''_i = f_i \\ \Rightarrow \frac{-v_{i+1} + 2v_i - v_{i-1}}{h^2} &= f_i \\ \Rightarrow -v_{i+1} + 2v_i - v_{i-1} &= h^2 f_i \equiv g_i \end{aligned}$$

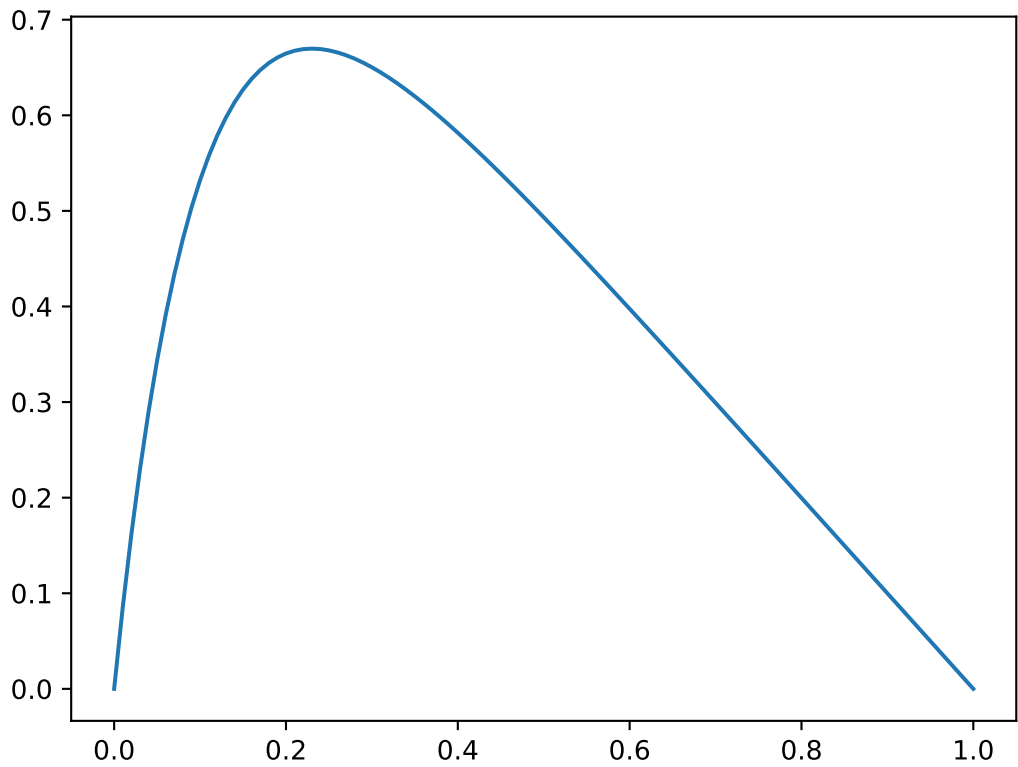


FIG. 1. CAPTION

PROBLEM 4

Algorithm 1 Some algorithm

Some maths, e.g $f(x) = x^2$.

for $i = 0, 1, \dots, n - 1$ **do**
 Do something here

while Some condition **do**
 Do something more here

Maybe even some more math here, e.g $\int_0^1 f(x)dx$

▷ Here's a comment
