Project 4 - FYS4150

Nanna Bryne, Johan Mylius Kroken, Vetle A. Vikenes
(Dated: November 14, 2022)

Abstract. The code is found in the following GitHub repository: https://github.com/Vikenes/FYS4150/tree/main/project3.

I. INTRODUCTION

[1]

II. METHODS

III. RESULTS

IV. DISCUSSION

V. CONCLUSION

Appendix A: Analytical results

$N_\uparrow$	$E(s)$	$M(s)$	Degeneracy
4	$-8\,\mathrm{J}$	4	1
3	0	2	4
2	$8\,\mathrm{J}$	0	2
2	0	0	4
1	0	-2	4
0	$-8\,\mathrm{J}$	-4	1

TABLE I.

$$Z = 12 + 2e^{-8\beta J} + 2e^{8\beta J} = 12 + 4\cosh(8\beta J) \tag{A1}$$

$$\langle \epsilon \rangle = -\frac{\partial \ln Z}{\partial \beta} = -\frac{8J}{Z}\sinh(8\beta J), \tag{A2}$$

$$\langle \epsilon^2 \rangle = \sum_s \epsilon(s)^2 \frac{1}{Z} e^{-\beta E(s)} = \frac{16J^2}{Z}\cosh(8\beta J), \tag{A3}$$

$$\langle |m| \rangle = \sum_s |m(s)| \frac{1}{Z} e^{-\beta E(s)} = \frac{2}{Z}\left(2 + e^{8\beta J}\right), \tag{A4}$$

$$\langle m^2 \rangle = \sum_s m(s)^2 \frac{1}{Z} e^{-\beta E(s)} = \frac{2}{Z}\left(1 + e^{8\beta J}\right). \tag{A5}$$

$$\begin{aligned} C_V &= \frac{\beta}{NT}\left(\langle E^2 \rangle - \langle E \rangle^2\right) \\ &= \frac{64\beta J^2}{ZT}\left[\cosh(8\beta J) - \frac{4}{Z}\sinh^2(8\beta J)\right], \end{aligned} \tag{A6}$$

$$\begin{aligned} \chi &= \frac{\beta}{N}\left(\langle M^2 \rangle - \langle M \rangle^2\right) \\ &= \frac{8\beta}{Z}\left[\left(1 + e^{8\beta J}\right) - \frac{2}{Z}\left(2 + e^{8\beta J}\right)^2\right]. \end{aligned} \tag{A7}$$

[1] Onsager, L. (1944). Crystal statistics. i. a two-dimensional model with an order-disorder transition. *Phys. Rev.*, 65:117–149.