Project 1 FYS4150

Vetle Vikenes, Johan Mylius Kroken and Nanna Bryne
(Dated: August 26, 2022)

List a link to your github repository here!

PROBLEM 1

We have the equation

$$-\frac{\mathrm{d}^2u}{\mathrm{d}x^2} = 100e^{-10x}, \quad \text{where} \quad x \in [0,1], \quad u(0) = u(1) = 0. \tag{1}$$

We want to check that

$$u(x) = 1 - (1 - e^{-10})x - e^{-10x} \tag{2}$$

is the solution of eq. 1. We first control that eq. 2 satisfies the boundary conditions.

$$\begin{aligned} u(0) &= 1 - 0 - e^{0} = 0 \\ u(1) &= 1 - (1 - e^{-10}) - e^{-10} = 0 \end{aligned}$$

We find the double derivative of $u(x)$,

$$\frac{\mathrm{d}u}{\mathrm{d}x} = -(1 - e^{-10}) - (-10)e^{-10x} \quad \Rightarrow \quad \frac{\mathrm{d}^2u}{\mathrm{d}x^2} = -100e^{-10x},$$

and see that this satisfies the differential eq. 1.

PROBLEM 2

PROBLEM 3

We have the discretisation of the second order derivative as follows:

$$\left.\frac{\mathrm{d}^2u}{\mathrm{d}x^2}\right|_{x_i} = u_i'' = \frac{u_{i+1} - 2u_i + u_{i-1}}{h^2} + \mathcal{O}(h^2) \approx \frac{v_{i+1} - 2v_i + v_{i-1}}{h^2} = v_i''$$

where we have approximated u_i with v_i by neglecting the error term, and h is the step length. We further insert this into the Poisson equation and obtain the following:

$$\begin{aligned} -\frac{\mathrm{d}^2u}{\mathrm{d}x^2} \approx -\left.\frac{\mathrm{d}^2v}{\mathrm{d}x^2}\right|_{x_i} &= -v_i'' = f(x_i) = f_i \\ \implies -v_i'' &= f_i \\ \implies \frac{-v_{i+1} + 2v_i - v_{i-1}}{h^2} &= f_i \\ \implies -v_{i+1} + 2v_i - v_{i-1} &= h^2 f_i \equiv g_i \end{aligned}$$

Algorithm 1 Some algorithm

Some maths, e.g $f(x) = x^2$. ▷ Here's a comment
for $i = 0, 1, ..., n - 1$ **do**
    Do something here
while Some condition **do**
    Do something more here
Maybe even some more math here, e.g $\int_0^1 f(x)\mathrm{d}x$