

A close-up, slightly blurred photograph of a desk. In the background, a silver laptop is partially visible. In the foreground, a spiral-bound notebook with lined pages is open. A black pencil lies horizontally across the notebook. A green paperclip and a green pushpin are also on the notebook. To the right of the notebook, a white ruler with black markings is visible. The text 'SHATTERPROOF' is printed on the ruler. The overall lighting is soft and natural.

# MATHEMATICAL INTELLIGENCE SYSTEM

In-Class Group Assignment

# TEAM PRESENTATION

BATCH-A TEAM-7		
GAJULA SRI VATSANKA	CB.EN.U4AIE.21010	Q1
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Given that 42% of high school students would admit to lying at least once to a teacher during the past year and that 25% of students are male and would admit to lying at least once to a teacher during the past year. Assume that 50% of the students are male.

- i. What is the probability that a randomly selected student is either male or would admit to lying to a teacher, during the past year?
- ii. A student is selected from the subpopulation of those who would admit to lying to a teacher during the past year. What is the probability that the student is female?

## SOLUTION

*Event  $A$  = student admit lying to teacher at least once during past year*

*Event  $M$  = student who is selected is a male*

*Event  $(A \cap M)$  = student who admit lying to teacher during past year is a male*

*Event  $F$  = student who is selected is a female*

**So as given in the question students who admit lying is 42%, student selected is a male is 50% and student who admit lying is a boy is 25%**

**Then the probability of occurring event A will be 42%**

**Then the probability of occurring event M will be 50%**

**Then the probability of occurring event  $(A \cap M)$  will be 25%**

$$P(A) = 0.42$$

$$P(M) = 0.50$$

$$P(F) = 0.50$$

$$P(A \cap M) = 0.25$$

**i)** The probability that a randomly selected student is either male or would admit lying to a teacher during the past year is given by  $P(A \cup M)$  .

$$P(A \cup M) = P(A) + P(M) - P(A \cap M)$$

$$= 0.42 + 0.5 - 0.25$$

$$= 0.67$$

$$P(A \cup M) = 0.67$$

**ii)** Now the probability that the selected student is a female given that student would admit to lying to a teacher during the past exam is given by  $P(F/A)$  and we can write

$$P(F/A) = 1 - P(M/A)$$

$$P(M/A) = P(A \cap M) / P(A) \quad (\text{Conditional probability formula})$$

$$= 0.25 / 0.42$$

$$P(M/A) = 0.5952$$

Hence,

$$\begin{aligned} P(F/A) &= 1 - P(M/A) \\ &= 1 - 0.5952 \end{aligned}$$

$$P(F/A) = 0.4048$$

i)  **$P(A \cup M) = 0.67$**

ii)  **$P(F/A) = 0.4048$**

A household is categorized as 'Prosperous' if its income exceeds \$80,000. Similarly, a household is categorized as 'Educated' if at least one of the members has completed college. The current Population Survey says that of all the households 15.2% are prosperous, 34.1% are educated, and 9% are both prosperous and educated. From this information, estimate.

- i. The probability that a house hold selected is either prosperous or educated?
- li. The probability that at least one person in a household is educated, given it is not prosperous.





i) Probability of a family being prosperous is  $P(\text{pros}) = 0.152$

Probability of a family being educated is  $P(\text{edu}) = 0.341$

Probability of a family being prosperous and educated  $P(\text{pros} \cap \text{edu}) = 0.09$

Let the probability of the event, a family being either prosperous or educated be  $P(A)$

$$\begin{aligned} P(A) &= P(\text{pros} \cup \text{edu}) = P(\text{pros}) + P(\text{edu}) - P(\text{pros} \cap \text{edu}) \\ &= 0.152 + 0.341 - 0.09 \\ &= 0.403 \end{aligned}$$

Probability of a family being NOT prosperous is  $P(\text{not pros}) = 1 - 0.152 = 0.848$

Probability of a family being prosperous and educated

$$P(\text{not pros} \cap \text{edu}) = P(\text{edu}) - P(\text{pros} \cap \text{edu})$$

$$= 0.341 - 0.09 = 0.251$$

Let the probability of the event, atleast one person in a household is educated given it is not prosperous be  $P(B)$

$$P(B) = P(\text{pros} \cup \text{edu}) = P(\text{not pros} \cap \text{edu}) / P(\text{not pros})$$

$$= 0.251 / 0.848$$

$$= 0.296$$

People with albinism have little pigment in their skin, hair, and eyes. The gene that governs albinism has two forms (called alleles), which we denote by  $a$  and  $A$ . Each person has a pair of these genes, one inherited from each parent. A child inherits one of each parent's two alleles independently with probability 0.5. Albinism is a recessive trait, so a person is albino only if the inherited pair is  $aa$ .

- i. Alan's parents and his sister Beth are not albino, but he is. What can you infer about the gene type present in Alan's parent?
- ii. Which of the types  $aa$ ,  $Aa$ ,  $AA$  could a child of Alan's parents have? What is the probability of each type?
- iii. Given Beth is not an albino. What are the probabilities for Beth's possible genetic types, given this fact?

## Part-i

According to the given question ,there are 4 different gene types: aa,aA,AA,Aa

- ❑ Now, we have been given that Alan is an albino ,so he must be of the gene type of "aa".
- ❑ As Alan parents are not albinos, there gene type probably should be "Aa" or "aA".
- ❑ Since Alan is an albino ,if anyone of his parent's gene is AA ,he won't be an Albino.
- ❑ So, it's mentioned that his are not Albino ,which means their gene type is neither "Aa" or "aA".

## Part-ii

- ❑ As we know that Alan's parent can't have a gene type of "aa" and "AA".
- ❑ So, the probability of Alan's parent's child having the gene type

1) aa,  $P(aa) = 1/4$

2) AA,  $P(AA) = 1/4$

3) Aa,  $P(Aa) = 1/4$

4) aA,  $P(aA) = 1/4$

- ❑ A child of Alan parent's can have all the 4 possible combination of gene types.
- ❑ Also if Aa and aA are same gene types  $P(Aa)=P(aA)=1/2$

## Part-iii

- ❑ But,  
Alan's parent can have gene type of "Aa" and "aA".
- ❑ As Beth is not an Albino, Probability of her having the gene type is:

1) aA,  $P(aA)=1/3$

2) Aa,  $P(Aa)=1/3$

3) AA,  $P(AA)=1/3$

if Aa and aA are same gene types

$P(aA)=2/3$  :  $P(Aa)=2/3$

- ❑  $P(Aa)=P(aA)=P(AA)=1/3$ , since she can contain only 3 gene types.
- ❑ As this selection of any one type of gene among the possible 3 gene types will be equal to  $1/3$

A continuous random variable is said to have a probability density function given below.  $f(x) = \begin{cases} ce^{-x} & x \geq 0 \\ 0 & x < 0 \end{cases}$

- I. Detail on how to verify if the given function is a PDF. Identify the values of c for which the PDF is valid.
- ii. Find the cumulative distribution function of x.
- iii. Estimate the probability of the random variable not being in the range (3,5).



(i) for a function to be probability density function, following properties must follow:-

(1)  $f(x) \geq 0$

given continuous random variable is

$$f(x) = \begin{cases} ce^{-x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

Now  $ce^{-x} \geq 0$ , for all values of  $x$   
 $e^{-x}$  will be true  
and for all positive  
values of constant  $c$   
 $ce^{-x} \geq 0$



∴ Sum of all probabilities = 1

$$\int_{-\infty}^{\infty} f(x) dx = 1$$

$$\int_0^{\infty} ce^{-x} dx + \int_{-\infty}^0 0 dx = 1$$

$$[-ce^{-x}]_0^{\infty} = 1$$

$$ce^{-0} - \lim_{b \rightarrow \infty} ce^{-b} = 1$$

$$c(1) - 0 = 1$$

$$c = 1$$

(ii) ~~Find the~~  
$$F(n) = \begin{cases} \int e^{-n} dn & n \geq 0 \\ 0 & n < 0 \end{cases}$$

$$F(n) = \begin{cases} -e^{-n} + c & n \geq 0 \\ 0 & n < 0 \end{cases}$$

to solve for  $c$

$$\text{w.r.t } f(\infty) = 1$$

$$-e^{-\infty} + c = 1$$

$$c = 1$$

$$f(n) = \begin{cases} (-e^{-n} + 1) & ; n \geq 0 \\ 0 & ; n < 0 \end{cases}$$

(iii) Probability of Random Variable not in  $(3, 5)$

$$= P(n < 3 \text{ \& } n > 5)$$

$$= \int_{-\infty}^3 f(n) dn + \int_5^{\infty} f(n) dn$$

$$= \int_{-\infty}^0 \cancel{f(n)} dn + \int_0^3 f(n) dn + \int_5^{\infty} f(n) dn$$

$$= 0 + [-e^{-n}]_0^3 + [-e^{-n}]_5^{\infty}$$

$$= -e^{-3} - (-1) + (-0 - (-\frac{1}{e^5}))$$

$$= 1 + e^{-5} - e^{-3}$$

$$= 1 + 0.0067 - 0.0497$$

$$= 0.9569$$

**THANK YOU**