MATHEMATICAL INTELLIGENCE SYSTEMS

GROUP ASSIGNMENT

TEAM MEMBERS

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QUESTION 1

Perform linear convolution of the below two sequences and verify using DFT in MATLAB (Using FFT command) x=[1 2 2 1];y=[1 3 2 1];

MATLAB CODE

```
x1 = 1221
y1 = 1321
z1 = x1*y1
x = num2str(x1) - '0'
y = num2str(y1) - '0'


%Linear Convolution
z = conv(x,y)
x0 = zeros(1,7);%
y0 = zeros(1,7);
x0(:,1:4) = x
y0(:,1:4) = y
z0_fft = round(ifft(fft(x0).*fft(y0)),0)
zc_1 = cconv(x,y,7)
```

```
x1 = 1221
y1 = 1321
z1 = 1612941
x = 1x4
     1     2     2     1

y = 1x4
     1     3     2     1

z = 1x7
     1     5    10    12     9     4     1

x0 = 1x7
     1     2     2     1     0     0     0
```

```
x0 = 1x7
     1     2     2     1     0     0     0

y0 = 1x7
     1     3     2     1     0     0     0

z0_fft = 1x7
     1     5    10    12     9     4     1

zc_1 = 1x7
  1.0000   5.0000  10.0000  12.0000   9.0000   4.0000   1.0000
```

QUESTION 12

Perform linear convolution of sequence {1,1} 50 times using FFT in MATLAB.

MATLAB CODE

MIS_Q2.m

```
1  seq = [1,1];
2  temp = seq;
3  for i=1:50
4      convo_length = length(temp)+length(seq)-1;
5      x = [seq zeros(1,convo_length-length(seq))];
6      y = [temp zeros(1,convo_length-length(temp))];
7      temp = nonzeros(ifft(fft(x).*fft(y)))'
8  end
```

First page OUTPUT

```
>> MIS_Q2

temp =

     1     2     1


temp =

     1     3     3     1


temp =

    1.0000    4.0000    6.0000    4.0000    1.0000


temp =

  Columns 1 through 5

    1.0000    5.0000   10.0000   10.0000    5.0000
```

.....

Last Page OUTPUT

```
temp =

   1.0e+14 *

  Columns 1 through 5

    0.0000    0.0000    0.0000    0.0000    0.0000

  Columns 6 through 10

    0.0000    0.0000    0.0000    0.0000    0.0000

  Columns 11 through 15

    0.0001    0.0005    0.0016    0.0048    0.0129

  Columns 16 through 20

    0.0319    0.0717    0.1477    0.2790    0.4846
```

Last Page OUTPUT

```
Columns 21 through 25

  0.7754    1.1446    1.5608    1.9679    2.2959

Columns 26 through 30

  2.4796    2.4796    2.2959    1.9679    1.5608

Columns 31 through 35

  1.1446    0.7754    0.4846    0.2790    0.1477

Columns 36 through 40

  0.0717    0.0319    0.0129    0.0048    0.0016

Columns 41 through 45

  0.0005    0.0001    0.0000    0.0000    0.0000

Columns 46 through 50

  0.0000    0.0000    0.0000    0.0000    0.0000
Columns 51 through 52

  0.0000    0.0000
```

QUESTION 3

A coin is tossed 3 times. Find the probability of getting 0 Head, 1 Head, 2 Head and 3 Head using convolution. Also verify using FFT in MATLAB.

CALCULATING NUMERICALLY

We already know how to calculate the probability of getting 0 Head, 1 Head and 2 Head and 3 Head numerically using the simple formulae.
All the outcomes = {HHH,HHT,HTH, HTT ,THH,TTT,TTH,THT}

P(0) = Number of zero heads / Total number of outcomes
= 1/8 = 0.125
P(1) = Number of one heads / Total number of outcomes
= 3/8 = 0.325
P(2) = Number of two heads / Total number of outcomes
= 3/8 = 0.325
P(3) = Number of three heads/ Total number of outcomes
= 1/8 = 0.125

Using Convolution

The probability mass function $p_Z(z)$ of the sum $Z = X + Y$ can be derived by using one of the following two formulae:

$$p_Z(z) = \sum_{y \in R_Y} p_X(z-y) p_Y(y)$$

$$p_Z(z) = \sum_{x \in R_X} p_Y(z-x) p_X(x)$$

Probability of getting 0 Head, 1 Head , 2 Heads and 3 Heads using MATLAB

MIS_Q3.m

```
% Coin probability

coin = [1/2 , 1/2];

% For 0 H, sum will be 0
% For 1,2,3 H sum will be 1,2,3 respectively

coin2 = conv(coin,coin);
coin3 = conv(coin2,coin)
```

Command Window

```
>> MIS_Q3

coin3 =

    0.1250    0.3750    0.3750    0.1250
```

Using FFT in MATLAB

Editor - C:\Users\HP\Desktop\MIS_Q3.m

MIS_Q3.m

```
1   coin = [1/2,1/2];
2   coin_padded = [coin, zeros(1,length(coin))];
3   coin2 = ifft(fft(coin_padded).* fft(coin_padded));
4   coin2_padded = [coin2, zeros(1,length(coin2))];
5   coun_padded = [coin, zeros(1,length(coin2_padded) - length(coin))];
6   coin_3 = ifft(fft(coin2_padded).*fft(coun_padded))
```

Command Window

```
>> MIS_Q3

coin_3 =

  Columns 1 through 5

    0.1250    0.3750    0.3750    0.1250    0.0000

  Columns 6 through 8

         0         0         0
```

fx

QUESTION 4

A die is thrown 20 times. Compute probabilities of getting all possible sums (20 to 120) using FFT in MATLAB.

Convolution using FFT in MATLAB

```
prob = [1/6 1/6 1/6 1/6 1/6 1/6];
temp = prob;

for i = 1:20
    prob_len = length(prob);
    temp_len = length(temp);

    temp_padding = [temp zeros(1, temp_len)];
    prob_padding = [prob zeros(1, (length(temp_padding)-prob_len))];

    % convolution

    temp = ifft(fft(prob_padding).*fft(temp_padding));


end

result = temp([1:101]);

values = [20:120];

output = table(values', result')

```

output =

101×2 table

Var1	Var2
20	4.5722e-17
21	9.5737e-16
22	1.053e-14
23	8.0731e-14
24	4.8439e-13
25	2.4219e-12
26	1.0494e-11
27	4.0461e-11
28	1.4146e-10
29	4.5484e-10
30	1.3594e-09
31	3.8089e-09
32	1.0072e-08
33	2.5278e-08
34	6.048e-08
35	1.3849e-07
36	3.0447e-07
37	6.445e-07
38	1.3167e-06
39	2.6017e-06
40	4.981e-06
41	9.2548e-06
42	1.6713e-05
43	2.9369e-05
44	5.0281e-05
45	8.3953e-05
46	0.00013683
47	0.00021788
48	0.00033919
49	0.00051664
50	0.00077038
51	0.0011253
52	0.0016108
53	0.002261
54	0.0031132
55	0.0042066
56	0.0055799
57	0.0072685
58	0.0093006
59	0.011694
60	0.01445
61	0.017553
62	0.020966
63	0.024627
64	0.028453
65	0.032339
66	0.036162
67	0.039789
68	0.043082
69	0.045908
70	0.048145
71	0.049696
72	0.050489
73	0.050489
74	0.049696
75	0.048145
76	0.045908
77	0.043082
78	0.039789
79	0.036162
80	0.032339
81	0.028453
82	0.024627
83	0.020966
84	0.017553
85	0.01445
86	0.011694
87	0.0093006
88	0.0072685
89	0.0055799
90	0.0042066
91	0.0031132
92	0.002261
93	0.0016108
94	0.0011253
95	0.00077038

95	0.00077038
96	0.00051664
97	0.00033919
98	0.00021788
99	0.00013683
100	8.3953e-05
101	5.0281e-05
102	2.9369e-05
103	1.6713e-05
104	9.2548e-06
105	4.981e-06
106	2.6017e-06
107	1.3167e-06
108	6.445e-07
109	3.0447e-07
110	1.3849e-07

111	6.048e-08
112	2.5278e-08
113	1.0072e-08
114	3.8089e-09
115	1.3594e-09
116	4.5484e-10
117	1.4146e-10
118	4.0461e-11
119	1.0494e-11
120	2.4219e-12

output =

Var1	Var2
20	4.5722e-17
21	9.5737e-16
22	1.053e-14
23	8.0731e-14
24	4.8439e-13
25	2.4219e-12
26	1.0494e-11
27	4.0461e-11
28	1.4146e-10
29	4.5484e-10
30	1.3594e-09
31	3.8089e-09
32	1.0072e-08
33	2.5278e-08
34	6.048e-08
35	1.3849e-07
36	3.0447e-07
37	6.445e-07
38	1.3167e-06
39	2.6017e-06
40	4.981e-06
41	9.2548e-06
42	1.6713e-05
43	2.9369e-05
44	5.0281e-05
45	8.3953e-05
46	0.00013683
47	0.00021788
48	0.00033919
49	0.00051664
50	0.00077038
51	0.0011253
52	0.0016108
53	0.002261
54	0.0031132
55	0.0042066
56	0.0055799
57	0.0072685
58	0.0093006
59	0.011694
60	0.01445
61	0.017553
62	0.020966
63	0.024627
64	0.028453
65	0.032339
65	0.032339
66	0.036162
67	0.039789
68	0.043082
69	0.045908
70	0.048145
71	0.049696
72	0.050489
73	0.050489
74	0.049696
75	0.048145
76	0.045908
77	0.043082
78	0.039789
79	0.036162
80	0.032339
81	0.028453
82	0.024627
83	0.020966
84	0.017553
85	0.01445
86	0.011694
87	0.0093006
88	0.0072685
89	0.0055799
90	0.0042066
91	0.0031132
92	0.002261
93	0.0016108
94	0.0011253
95	0.00077038
96	0.00051664
97	0.00033919
98	0.00021788
99	0.00013683
100	8.3953e-05
101	5.0281e-05
102	2.9369e-05
103	1.6713e-05
104	9.2548e-06
105	4.981e-06
106	2.6017e-06
107	1.3167e-06
108	6.445e-07
109	3.0447e-07
110	1.3849e-07
111	6.048e-08
112	2.5278e-08
113	1.0072e-08
114	3.8089e-09
115	1.3594e-09
116	4.5484e-10
117	1.4146e-10
118	4.0461e-11
119	1.0494e-11
120	2.4219e-12

QUESTION 5

A die is thrown 10 times. Compute probability of getting the sum on the die as more than 40 using FFT in MATLAB

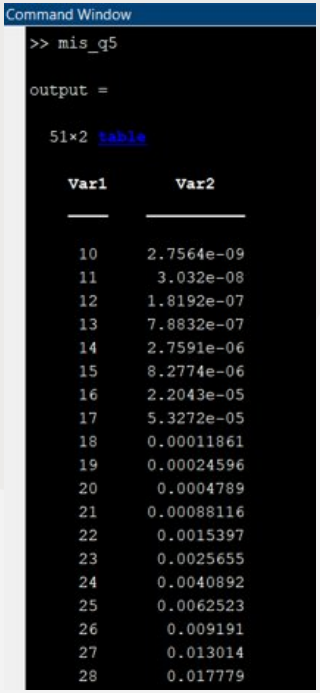
MATLAB Code:

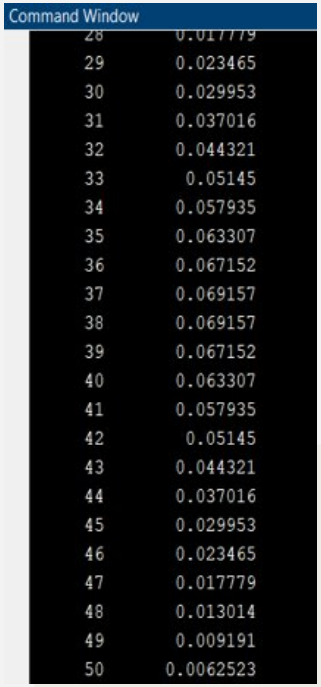
Editor - D:\matlab\mis_q5.m

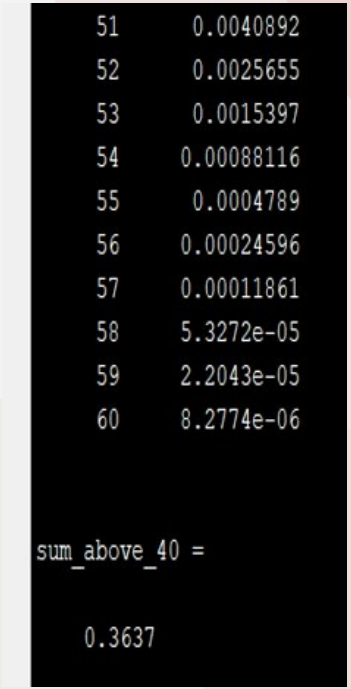
mis_q5.m | mis_q4.m | mis_q3.m | mis_q2.m

```
seq = [1/6 1/6 1/6 1/6 1/6 1/6];
temp = seq;
% convolution 20 times on the sequence

for i = 1:10

    seq_length = length(seq);
    temp_length = length(temp);

    temp_padded = [temp zeros(1, temp_length)];

    % array size
    zero_for_seq = length(temp_padded) - seq_length;
    seq_padded = [seq zeros(1, zero_for_seq)];

    % result
    temp = ifft(fft(seq_padded).*fft(temp_padded));

end
% we know there are 51 possibilities
final = temp([1:51]);
sum_values = [10:60];
output = table(sum_values' ,final')

% sum 40 lies in the 31 element
sum_above_40 = sum(final([31:51]))
```

Output:







Probability of getting the sum on the die as more than 40 is **0.3637**

QUESTION 6

Explain how you will convert product of two 3-digit numbers into a convolution followed by a sum of sequences.

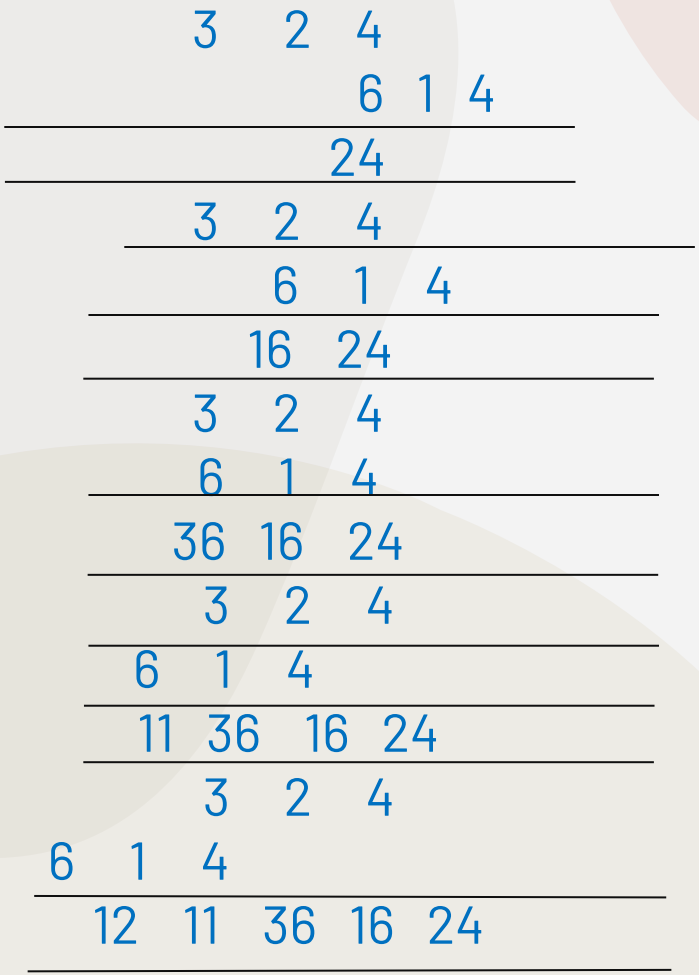
Convolution *Sum*

- A convolution Sum takes the form:
- Such types of sums have a wide range of applications in various fields.
- They are used in Linear time Invariant systems to denote the sum of scaled and shifted impulse responses with the delay k.
- It is used as a tool in Probability theory to find the probability mass function of a variable dependent in a specific way on two independent variables and the integral form of the convolution sum is used to calculate the probability density distribution.

To portray the product of two 3-digit numbers as a convolution and a sum of Sequences.

- Let us take two 3-digit numbers X and Y, X=324 and Y=416.
- We can write these as entries in X and Y as X={3,2,4} and Y={4,1,6}.
- Let z=x*y.
- $z=\Sigma x_i*10^i *\Sigma y_j*10^j = \Sigma x_i*y_j*10^{(i+j)}$
- We can write this as entries/sequences in Z.
- $z_k= \Sigma x_i*y_{(k-i)}$

- Nota bene: In the sequential representation of the numbers, the rightmost digit has index n, and each number to the left has an index value increase of 1.
- By multiplying each number by 10n (where n is the index of the number) and adding the total of sequences, the sequent ial representation of X,Y, and Z may be transformed to the c onventional decimal format.



To convert the product of two 3-digit numbers into convolution:

- Write the sequences in X and the Sequence in Y, one below the other.
- Match the indices n=0 from both X and Y, vertically below each other.
- Now, Multiply the numbers that fall along the same vertical line and add the products to get the sum.
- Note down the sum right below the position that index n=0 of Y takes, (i.e. write the sum right below 6) and shift the position of Sequence Y to the left.
- The numbers/Sequences in Z are the sums of the products that were noted down.
- The Sequence of Z can be considered as the products of the Sequences of X and Y.

- We now have the sequence in zk.
- Z = {12,11,36,16,24}
- X*Y Equals Z
- The product is not read as 12,11,36,16,24, but the indices n=0 to n=4 have a power of 10n associated with them.
- As a result, the total of the sequences is 12*104 + 11*103 + 36*102 + 16*101 + 24*100 = 1,34,784.
- After convolution, we may obtain the sum of any digit number as a sum of sequences.

/*THANK YOU*/