1. Perform linear convolution of the below two sequences and verify using DFT in MATLAB (Using FFT command)

   x=[1 2 2 1];

   y=[1 3 2 1];
2. Perform linear convolution of sequence {1,1} 50 times using FFT in MATLAB
3. A coin is tossed 3 times. Find the probability of getting 0 Head, 1 Head, 2 Head and 3 Head using convolution. Also verify using FFT in MATLAB
4. A die is thrown 20 times. Compute probabilities of getting all possible sums (20 to 120) using FFT in MATLAB.
5. A die is thrown 10 times. Compute probability of getting the sum on the die as more than 40 using FFT in MATLAB
6. Explain how you will convert product of two 3 digit numbers into a convolution followed by a sum of sequences

Convolution appears in probability theory

- Convolution is also a frequently applied tool in probability theory.
- It is used to find the probability mass function of a variable representing sum of two independent variables.
- Let X and Y be two independent random variable and let Z = X + Y .
- Let pX(x), pY(y) represent probability mass function of X and Y respectively.
- Then the probability mass function of Z is given

$$p_Z(z) = P(Z = z) = \sum_x p_X(x) p_Y(z - x).$$

Let $\left\{\begin{array}{cccc} X=x & 1 & 2 & 3 \\ p_X(x) & 0.5 & 0.4 & 0.1 \end{array}\right\}$ and $\left\{\begin{array}{cccc} Y=y & 2 & 3 & 4 \\ p_Y(y) & 0.3 & 0.4 & 0.3 \end{array}\right\}$ be the mass function

It can be easily seen that the required probability is given by a general formula,

$\mathrm{P}(\mathrm{Z}=\mathrm{z})=\sum_{\mathrm{x}} \mathrm{P}(\mathrm{X}=\mathrm{x}) \mathrm{P}(\mathrm{Y}=\mathrm{z}-\mathrm{x})$, which is a convolution sum.

- Step1 : Write down the two probability sequences.

$$\begin{array}{cccccccc}
\mathrm{X} = & & 1 & 2 & 3 & & \\
p_x & . & 0.5 & 0.4 & 0.1 & . & . \\
p_y & . & 0.3 & 0.4 & 0.3 & . & . \\
\mathrm{Y} = & & 2 & 3 & 4 & &
\end{array}$$

- Step2 : Fold the second sequence of probabilities, multiply to get $\mathrm{P}\left(\mathrm{Z}=3\right)$

$$\begin{array}{cccccc}
\mathrm{X} = & & & 1 & 2 & 3 \\
 & & & 0.5 & 0.4 & 0.1 \\
 & 0.3 & 0.4 & 0.3 & & \\
\mathrm{Y} = & 4 & 3 & 2 & &
\end{array}$$

$$\mathrm{P}\left(\mathrm{Z}=3\right) \ = \ \left(0.5\right) \left(0.3\right) \ = 0.15$$

- Step3 : Shift the folded sequence, multiply and add to get P (Z = 4)

$$
\begin{array}{lccccc}
X = & & 1 & 2 & 3 \\
 & & 0.5 & 0.4 & 0.1 \\
 & 0.3 & 0.4 & 0.3 & \\
Y = & 4 & 3 & 2 &
\end{array}
$$

$$P\,(Z = 4) = (0.5)\,(0.4) + (0.4)\,(0.3) = 0.32$$

- Step4 : Shift the folded sequence again, multiply and add to get P (Z = 5)

$$
\begin{array}{lccc}
X = & 1 & 2 & 3 \\
 & 0.5 & 0.4 & 0.1 \\
 & 0.3 & 0.4 & 0.3 \\
Y = & 4 & 3 & 2
\end{array}
$$

$$P\,(Z = 5) = (0.5)\,(0.3) + (0.4)\,(0.4) + (0.1)\,(0.3) = 0.34$$

- Step5 : Shift the folded sequence again, multiply and add to get $P(Z=6)$

$$\begin{array}{lcccccc}
X = & 1 & 2 & 3 & & \\
 & 0.5 & 0.4 & 0.1 & & \\
 & & 0.3 & 0.4 & 0.3 & \\
Y = & & 4 & 3 & 2 &
\end{array}$$

$$P(Z=6) = (0.4)(0.3) + (0.1)(0.4) = 0.16$$

- Step6 : Shift the folded sequence again, multiply and add to get $P(Z=7)$

$$\begin{array}{lcccccc}
X = & 1 & 2 & 3 & & \\
 & 0.5 & 0.4 & 0.1 & & \\
 & & & 0.3 & 0.4 & 0.3 \\
Y = & & & 4 & 3 & 2
\end{array}$$

$$P(Z=7) = (0.1)(0.3) = 0.03$$

Thus, we obtain the probability mass function of Z as

$Z=z$	3	4	5	6	7
$p_Z(z)$	0.15	0.32	0.34	0.16	0.03