

If $b^2 = 4mk$ term under square root is 0
and characteristic polynomial has repeated
roots $\frac{-b}{2m}, \frac{-b}{2m}$

We use these repeated roots to

Solve eqⁿ $ms^2 + bs + k = 0$

We get only 1 exponential

Solⁿ, so to get second
Solⁿ, we multiply it (i.e. first solⁿ)
by t .

our Basic Solⁿ comes out to be
 $e^{-bt/2m}, te^{-bt/2m}$

General Solⁿ

$$x(t) = e^{-bt/2m} (c_1 + c_2 t)$$

for critical damping,
Repeated roots of (1) we are getting
values = -1

where $b = 40$

Solⁿ for this case

$$m = 20$$

$$k = 20$$

$$e^{-\frac{40t}{2(20)}} \text{ , } te^{-\frac{40t}{2(20)}}$$

$$x(t) = e^{-\frac{40t}{40}} (c_1 + c_2 t)$$

$$x(0) = 1$$

$$\dot{x}(0) = 0$$

$$x(0) = e^{-1(0)} ((c_1) + 0)$$

$$\boxed{1 = c_1}$$

Egⁿ $20\ddot{x} + 40\dot{x} + 20x = 0$

$$\ddot{x} + 2\dot{x} + x = 0$$

$$s^2 + 2s + 1 = 0$$

$$(s+1)^2 = 0$$

$$\dot{x}(0) = -e^{-1(0)} (c_1) + c_2(e^{-1(0)})$$

$$c_2(1)(e^{-1(0)})$$

$$0 = 1(c_1) + c_2$$

$$\boxed{-1 = c_2}$$

final Egⁿ $(x(t) = e^{-bt/2m} (1 - t)) \star$

we know that here $b^2 = 4mk$

term under square root = 0

characteristic polynomial has repeated solⁿ

$$ms^2 + bs + k = 0$$

$$m\ddot{x} + b\dot{x} + kx = 0 \text{ --- (1)}$$