For overdamping, $k=20$

characteristic root $r_1 = \dfrac{-b + \sqrt{b^2-4mk}}{2m}$

$m\ddot{x} + b\dot{x} + kx = 0$

$ms^2 + bs + k = 0$

$r_2 = \dfrac{-b - \sqrt{b^2-4mk}}{2m}$

$k = 20$

$b = 200$

$m = 20$

$r_1 = -0.1$

$r_2 = -9.8$

$20s^2 + 200s + 20 = 0$

$x(t) = c_1 e^{-0.1t} + c_2 e^{-9.8t}$

$s^2 + 10s + 1 = 0$

Initial condition $\dot{x}(0) = 0$

$x(0) = 1$

After Solving

$s_1 = -0.1$

$s_2 = -9.8$

$1 = \cancel{-0.1e}\ c_1 + c_2$ — (1)

$0 = -0.1\,c_1 - 9.8\,c_2$ — (2)

$-98c_2 =$ [redacted] $1c_1$

$-98c_2 = c_1$

$-1/$[redacted]$97 = c_2$

$\dfrac{98\text{[redacted]}}{\text{[redacted]}97} = c_1$

So $x(t) = \dfrac{98}{97} e^{-0.1t} + \dfrac{-1}{97} e^{-9.8t}$

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we know that $b^2 > 4mk$

Important thing to notice here is that both roots are negative here

we can find this using the formulae (quadratic) under characterstic eq^n

for critical damping, Repeated roots of 1 we are getting

Values $= -1$ where $b = 40$

Sol^n for this case $m = 20$

$k = 20$

$e^{-\frac{40t}{2(20)}}$, te

$$x(t) = e^{-\frac{40t}{40}}(c_1 + c_2 t)$$

$x(0) = 1$

$\dot{x}(0) = 0$

$$x(0) = e^{-1(0)}(c_1)$$

$$\boxed{1 = c_1}$$

Eq^n $20\ddot{x} + 40\dot{x} + 20x = 0$

$\ddot{x} + 2\dot{x} + x = 0$

$s^2 + 2s + 1 = 0$

$(s+1)^2 = 0$

$$x'(0) = -e^{-1(0)}(c_1) + c_2(e^{-1(0)})$$

$$c_2(t)(e^{-1(0)})$$

$$0 = 1(c_1) + 1(c_2)$$

$$\boxed{-1 = c_2}$$

final Eq^n $\left(x(t) = e^{-bt/2m}(1 - t)\right)$ *

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we know that here $b^2 = 4mk$

term under square root $= 0$

characterstic polynomial has repeated sol^n

$ms^2 + bs + k = 0$

$m\ddot{x} + b\dot{x} + kx = 0$

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k was formulated using all conditions

$p = \frac{b}{2m}$

$\omega = \sqrt{\frac{k}{m}}$

(Critical Damping) $p = \omega$ $\quad b^2 = 4mk$ $\quad k = 20$

(overdamping) $p > \omega$ $\quad b^2 > 4mk$ $\quad k < 500$

(underdamping) $p < \omega$ $\quad b^2 < 4mk$ $\quad k > \frac{5}{16}$

So we can see that $k = 20$ Satisfy all conditions

All other Values given in Ques.

for underdamped $k = 20$

we know $b^2 < 4mk$

term under sq root is -ve and are not real in characterstic eq^n

$$ms^2 + bs + k = 0$$

$$20s^2 + 5s + 20 = 0$$

After solving we get,

$$\left.\begin{aligned} S_1 &= -0.125 + 0.99i \\ S_2 &= -0.125 - 0.99i \end{aligned}\right\}$$ using MATLAB

we know $\omega_d = \frac{\sqrt{b^2 - 4mk}}{2m}$

characterstic root $(S_1, S_2) = -\frac{b}{2m} \pm i\omega_d$

leading to complex exponential sol^n

$$e^{(-b/2m + i\omega_d)t}, \quad e^{(-b/2m - i\omega_d)t}$$

Basic real sol^n are

$$e^{-bt/2m}\cos(\omega_d t), \quad e^{-bt/2m}\sin(\omega_d t)$$

general real sol^n is found by taking linear combination of basic sol^n

$$x(t) = c_1 e^{-bt/2m}\cos(\omega_d t) + c_2 e^{-bt/2m}\sin(\omega_d t)$$

ω_d = Damped Angular frequency

$ms^2 + bs + k = 0$

$20s^2 + 5s + 20 = 0$

Characterstic root : $-\frac{1}{8} \pm \frac{3\sqrt{7}}{8}$

General Sol^n

$$x(t) = c_1 e^{\left(-\frac{5}{40}t\right)} \cos\left(\frac{3\sqrt{7}}{8}t\right) + c_2 e^{\left(-\frac{5}{40}t\right)} \sin\left(\frac{3\sqrt{7}t}{8}\right)$$

which can ~~also be~~ written as

$$\Rightarrow Ae^{-t/2} \cos\left(\frac{3\sqrt{7}t}{8} - \phi\right)$$

$\left.\begin{array}{l} (x(0)) = 1 \\ (\dot{x}(0)) = 0 \end{array}\right\}$ using Initial Conditions

when $x(0) = 1$

$$\boxed{1 = c_1}$$

Now $\dot{x}(t) = \left(\frac{-5}{40}\right) c_1 e^{-5/40t} \cos\left(\frac{3\sqrt{7}}{8}t\right)$

$$+ c_1 e^{\frac{-5t}{40}} \left(-\sin\frac{3\sqrt{7}}{8}t\right)$$

$$+ c_2 \left(\frac{-5}{40} e^{\frac{-5t}{40}} \sin\frac{3\sqrt{7}t}{8} + e^{\frac{-5t}{40}} \cos\left(\frac{3\sqrt{7}}{8}\right.\right.$$

Now $\dot{x}(0) = 0$

$$\dot{x}(0) = \frac{-5}{40}(1) + c_2\left(e^0 \cos\left(\frac{3\sqrt{7}}{8} * 0\right)\right)$$

$$\boxed{c_2 = 0.125}$$

final Eq^n

$$x(t) = 1 e^{\left(-\frac{5}{40}t\right)} \cos\left(\frac{3\sqrt{7}t}{8}\right) + (0.125) e^{\frac{-5t}{40}} \sin\left(\frac{3\sqrt{7}t}{8}\right)$$