(i) for a function to be probability density function, following properties must follow :-

(I) $f(x) \geq 0$

given continuous random variable is

$$f(x) = \begin{cases} ce^{-x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

Now $ce^{-x} \geq 0$, for all values of x e^{-x} will be +ve and for all positive values of constant c

$$ce^{-x} \geq 0$$

(i) Sum of all probabilities = 1

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\int_{0}^{\infty} ce^{-x}\,dx + \int_{-\infty}^{0} 0\,dx = 1$$

$$\left[-ce^{-x}\right]_{0}^{\infty} = 1$$

$$ce^{-0} - \lim_{b \to \infty} ce^{-b} = 1$$

$$c(1) - 0 = 1$$

$$c = 1$$

(ii) ~~[illegible]~~

$$F(x) = \begin{cases} \int e^{-x} dx & x \geq 0 \\ \int 0 \, dx & x < 0 \end{cases}$$

$$F(x) = \begin{cases} -e^{-x} + c & x \geq 0 \\ 0 & \end{cases}$$

to solve for c

W.R.T $f(\infty) = 1$

$$-e^{-\infty} + c = 1$$

$$c = 1$$

$$f(x) = \begin{cases} (-e^{-x} + 1) \; ; & x \geq 0 \\ 0 \; ; & x < 0 \end{cases}$$

(iii) Probability of Random Variable not in (3, 5)

$= P(x < 3 \ \& \ x > 5)$

$= \int_{-\infty}^{3} f(x)\,dx + \int_{5}^{\infty} f(x)\,dx$

$= \int_{-\infty}^{0} \overset{0}{\cancel{f(x)\,dx}} + \int_{0}^{3} f(x)\,dx + \int_{5}^{\infty} f(x)\,dx$

$= 0 + \left[-e^{-x}\right]_0^3 + \left[-e^{-x}\right]_5^{\infty}$

$= -e^{-3} - (-1) + \left(-0 - \left(-\frac{1}{e^5}\right)\right)$

$= 1 + e^{-5} - e^{-3}$

$= 1 + 0.0067 - 0.0497$

$= 0.9569$