AI1103-Assignment-5

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Download all python codes from

```
https://github.com/Vikhyath−vec/AI1103/tree/main/
    Assignment−5/codes
```

and latex-tikz codes from

```
https://github.com/Vikhyath−vec/AI1103/blob/main
    /Assignment−5/Assignment−5.tex
```

Question

Let X be the number of heads in 4 tosses of a fair coin by Person 1 and let Y be the number of heads in 4 tosses of a fair coin by Person 2. Assume that all the tosses are independent. Then the value of $\mathbb{P}(X = Y)$ correct up to three decimal places is ______ .

Solution

Let $X \in \{0, 1, 2, 3, 4\}$ be the random variable representing the number of heads obtained by Person 1 in 4 tosses. Similarly, Let $Y \in \{0, 1, 2, 3, 4\}$ be the random variable representing the number of heads obtained by Person 2 in 4 tosses. Then X and Y are binomial distributions with parameter:

$$p = \frac{1}{2} \tag{0.0.1}$$

Then,

$$\Pr(X = i) = \begin{cases} {}^4C_k (p)^k (1-p)^{4-k} & i \in \{0,1,2,3,4\} \\ 0 & \text{otherwise} \end{cases} \tag{0.0.2}$$

$$\Pr(X = i) = \begin{cases} {}^4C_k (\frac{1}{2})^k (1-\frac{1}{2})^{4-k} & i \in \{0,1,2,3,4\} \\ 0 & \text{otherwise} \end{cases} \tag{0.0.3}$$

$$\Pr(X = i) = \begin{cases} {}^4C_k \times (\frac{1}{2})^4 & i \in \{0,1,2,3,4\} \\ 0 & \text{otherwise} \end{cases} \tag{0.0.4}$$

Serial number	Case	Probability of the case
1	$\Pr(X = 0)$	$\frac{{}^4C_0}{16} = \frac{1}{16}$
2	$\Pr(X = 1)$	$\frac{{}^4C_1}{16} = \frac{4}{16}$
3	$\Pr(X = 2)$	$\frac{{}^4C_2}{16} = \frac{6}{16}$
4	$\Pr(X = 3)$	$\frac{{}^4C_3}{16} = \frac{4}{16}$
5	$\Pr(X = 4)$	$\frac{{}^4C_4}{16} = \frac{1}{16}$

TABLE 0: Probability distribution table of X

Similar is the distribution of Y. For finding $\Pr(X = Y)$, let $Y = y$,

$$\Pr(X = Y = y) = \frac{{}^4C_y}{16} \times \Pr(Y = y) \tag{0.0.5}$$

Generalizing this result,

$$\Pr(X = Y) = \sum_{y=0}^{4} \frac{{}^4C_y}{16} \times \Pr(Y = y) \tag{0.0.6}$$

$$= \sum_{y=0}^{4} \frac{{}^4C_y}{16} \times \frac{{}^4C_y}{16} \tag{0.0.7}$$

$$\Pr(X = Y) = \left(\frac{1}{16} \times \frac{1}{16}\right) + \left(\frac{4}{16} \times \frac{4}{16}\right) + \left(\frac{6}{16} \times \frac{6}{16}\right) + \left(\frac{4}{16} \times \frac{4}{16}\right) + \left(\frac{1}{16} \times \frac{1}{16}\right) \quad (0.0.8)$$

$$\Pr(X = Y) = \frac{1}{256} + \frac{16}{256} + \frac{36}{256} + \frac{16}{256} + \frac{1}{256} \quad (0.0.9)$$

$$= \frac{70}{256} \quad (0.0.10)$$

$$= \frac{35}{128} \quad (0.0.11)$$

$$= 0.2734375 \quad (0.0.12)$$

The the value of $\mathbb{P}(X = Y)$ correct up to three decimal places is 0.273.

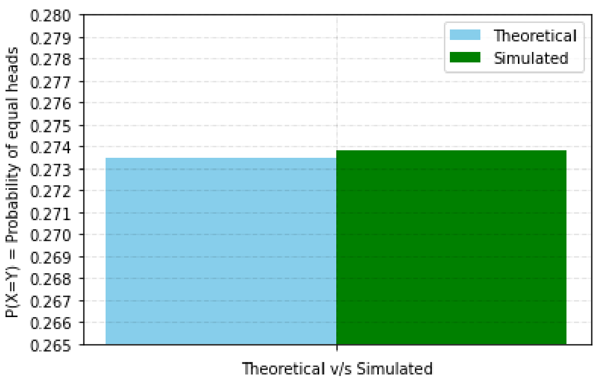

Fig. 0: Theoretical and simulated results