1

(0.0.4)

AI1103-Assignment-5

Name: Vikhyath Sai Kothamasu Roll Number: CS20BTECH11056



Download all python codes from

https://github.com/Vikhyath-vec/AI1103/tree/main/ Assignment-5/codes

and latex-tikz codes from

https://github.com/Vikhyath-vec/AI1103/blob/main/Assignment-5/Assignment-5.tex

QUESTION

Let X be the number of heads in 4 tosses of a fair coin by Person 1 and let Y be the number of heads in 4 tosses of a fair coin by Person 2. Assume that all the tosses are independent. Then the value of $\mathbb{P}(X = Y)$ correct up to three decimal places is ______.

SOLUTION

Let $X \in \{0, 1, 2, 3, 4\}$ be the random variable representing the number of heads obtained by Person 1 in 4 tosses. Similarly, Let $Y \in \{0, 1, 2, 3, 4\}$ be the random variable representing the number of heads obtained by Person 2 in 4 tosses. Then X and Y are binomial distributions with parameter:

$$p = \frac{1}{2} \tag{0.0.1}$$

Then,

$$\Pr(X = i) = \begin{cases} {}^{4}C_{k}(p)^{k}(1-p)^{4-k} & i \in \{0, 1, 2, 3, 4\} \\ 0 & \text{otherwise} \end{cases}$$

$$(0.0.2)$$

$$\Pr(X = i) = \begin{cases} {}^{4}C_{k}(\frac{1}{2})^{k}(1-\frac{1}{2})^{4-k} & i \in \{0, 1, 2, 3, 4\} \\ 0 & \text{otherwise} \end{cases}$$

$$(0.0.3)$$

$$\Pr(X = i) = \begin{cases} {}^{4}C_{k} \times (\frac{1}{2})^{4} & i \in \{0, 1, 2, 3, 4\} \\ 0 & \text{otherwise} \end{cases}$$

| Serial number | Case | Probability of the case |
|---------------|-----------|-----------------------------------------|
| 1 | Pr(X=0) | $\frac{^4C_0}{16} = \frac{1}{16}$ |
| 2 | Pr(X = 1) | $\frac{{}^{4}C_{1}}{16} = \frac{4}{16}$ |
| 3 | Pr(X = 2) | $\frac{^4C_2}{16} = \frac{6}{16}$ |
| 4 | Pr(X = 3) | $\frac{^4C_3}{16} = \frac{4}{16}$ |
| 5 | Pr(X = 4) | $\frac{^4C_4}{16} = \frac{1}{16}$ |

TABLE 0: Probability distribution table of X

Similar is the distribution of Y.

$$\Pr(X = Y) = \sum_{i=0}^{4} \Pr(X = i) \times \Pr(Y = i)$$
 (0.0.5)

$$\Pr(X = Y) = \left(\frac{1}{16} \times \frac{1}{16}\right) + \left(\frac{4}{16} \times \frac{4}{16}\right) + \left(\frac{6}{16} \times \frac{6}{16}\right) + \left(\frac{4}{16} \times \frac{4}{16}\right) + \left(\frac{1}{16} \times \frac{1}{16}\right) \quad (0.0.6)$$

$$Pr(X = Y) = \frac{1}{256} + \frac{16}{256} + \frac{36}{256} + \frac{16}{256} + \frac{1}{256}$$

$$= \frac{70}{256}$$

$$= \frac{35}{128}$$

$$= 0.2734375$$

$$(0.0.10)$$

The the value of $\mathbb{P}(X = Y)$ correct up to three decimal places is 0.273.

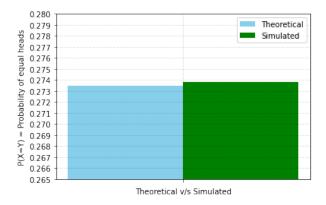


Fig. 0: Theoretical and simulated results