

AI1103-Assignment-6

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Download all python codes from

<https://github.com/Vikhyath-vec/AI1103/tree/main/Assignment-6/codes>

and latex-tikz codes from

<https://github.com/Vikhyath-vec/AI1103/blob/main/Assignment-6/Assignment-6.tex>

Then X is a binomial distributions with parameter:

$$p = \frac{\text{number of red coloured faces}}{\text{total number of faces}} \quad (0.0.1)$$

$$= \frac{2}{6} \quad (0.0.2)$$

$$= \frac{1}{3} \quad (0.0.3)$$

Then,

$$\Pr(X = i) = \begin{cases} {}^3C_i(p)^i(1-p)^{3-i} & i \in \{0, 1, 2, 3\} \\ 0 & \text{otherwise} \end{cases} \quad (0.0.4)$$

$$\Pr(X = i) = \begin{cases} {}^3C_i(\frac{1}{3})^i(1-\frac{1}{3})^{3-i} & i \in \{0, 1, 2, 3\} \\ 0 & \text{otherwise} \end{cases} \quad (0.0.5)$$

Serial number	Case	Probability of the case
1	$\Pr(X = 0)$	${}^3C_0\left(\frac{1}{3}\right)^0\left(\frac{2}{3}\right)^3 = \frac{8}{27}$
2	$\Pr(X = 1)$	${}^3C_1\left(\frac{1}{3}\right)^1\left(\frac{2}{3}\right)^2 = \frac{12}{27}$
3	$\Pr(X = 2)$	${}^3C_2\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^1 = \frac{6}{27}$
4	$\Pr(X = 3)$	${}^3C_3\left(\frac{1}{3}\right)^3\left(\frac{2}{3}\right)^0 = \frac{1}{27}$

TABLE 0: PMF of X

QUESTION

Consider an unbiased cubic dice with opposite faces coloured identically and each face coloured red, blue or green such that each colour appears only two times on the dice. If the dice is thrown thrice, the probability of obtaining red colour on top face of the dice at least twice is _____ .

SOLUTION

Let $X \in \{0, 1, 2, 3\}$ be the random variable representing the number of times a red face is obtained.

$$F_X(i) = \begin{cases} \sum_{k=0}^i {}^3C_k(p)^k(1-p)^{3-k} & i \in \{0, 1, 2, 3\} \\ 0 & \text{otherwise} \end{cases} \quad (0.0.6)$$

Serial number	Case	Cumulative Probability of the case
1	$F_X(0)$	$\frac{8}{27}$
2	$F_X(1)$	$\frac{8}{27} + \frac{12}{27} = \frac{20}{27}$
3	$F_X(2)$	$\frac{8}{27} + \frac{12}{27} + \frac{6}{27} = \frac{26}{27}$
4	$F_X(3)$	$\frac{8}{27} + \frac{12}{27} + \frac{6}{27} + \frac{1}{27} = \frac{27}{27}$

TABLE 0: CDF of X

$$\Pr(X \geq 2) = 1 - \Pr(X < 2) \quad (0.0.7)$$

$$= 1 - \Pr(X \leq 1) \quad (0.0.8)$$

$$= 1 - F_X(1) \quad (0.0.9)$$

$$= 1 - \frac{20}{27} \quad (0.0.10)$$

$$= \frac{7}{27} \quad (0.0.11)$$

Thus, the probability of obtaining red colour on top face of the dice at least twice is $\frac{7}{27} = 0.25926$.

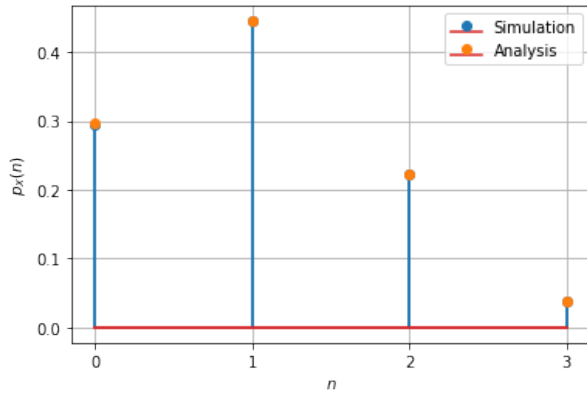


Fig. 0: The PMF distribution of X

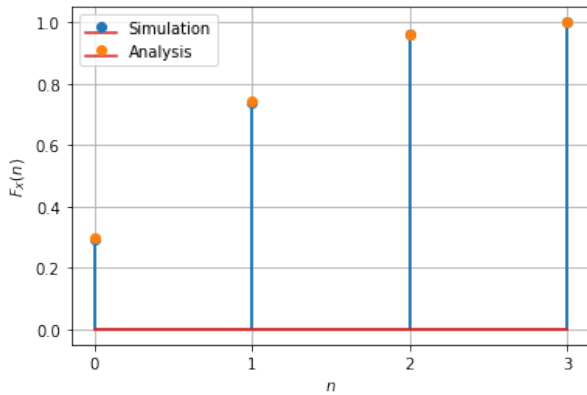


Fig. 0: The CDF distribution of X