

AI1103-Assignment-2

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Download all python codes from

<https://github.com/Vikhyath-vec/AI1103/tree/main/Assignment-2/codes>

and latex-tikz codes from

<https://github.com/Vikhyath-vec/AI1103/blob/main/Assignment-2/Assignment-2.tex>

QUESTION

Two cards are drawn successively with replacement from a well shuffled deck of 52 cards. Find the probability distribution of the number of aces.

SOLUTION

Let $X \in \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$ represent the random variable, where 0 represents an ace card, 1 represents a card numbered '2', 2 represents a card numbered '3'... 9 represents a card numbered '10', 10 represents the J card, 11 represents the Q card, and 12 represents the K card. These are independent of the suit.

$$n(X = i) = 4, i \in \{0, 1, 2 \dots 10, 11, 12\} \quad (0.0.1)$$

$$\Pr(X = i) = \begin{cases} \frac{4}{52} = \frac{1}{13} & i \in \{0, 1, 2 \dots 10, 11, 12\} \\ 0 & \text{otherwise} \end{cases} \quad (0.0.2)$$

Let $Y \in \{0, 1, 2\}$ represent the random variable, where 0 represents the case where no aces are selected, 1 represents the case where one ace is selected, 2 represents the case where 2 aces are selected.

Let $Z \in \{0, 1\}$ represent the random variable, where 0 represents an ace card is picked while 1 represents a non-ace card is picked.

$$\Pr(Z = 0) = \frac{1}{13} \quad (0.0.3)$$

$$\Pr(Z = 1) = \sum_{i=1}^{12} \Pr(X = i) \quad (0.0.4)$$

$$= \sum_{i=1}^{12} \frac{1}{13} \quad (0.0.5)$$

$$= \frac{12}{13} \quad (0.0.6)$$

Now, for finding the probability distribution of the number of aces,

$$\Pr(Y = 0) = \Pr(Z = 1) \times \Pr(Z = 1) \quad (0.0.7)$$

$$\Pr(Y = 0) = \frac{12}{13} \times \frac{12}{13} = \frac{144}{169} \quad (0.0.8)$$

$$\Pr(Y = 0) = 0.852071 \quad (0.0.9)$$

$$\Pr(Y = 1) = \Pr(Z = 0) \times \Pr(Z = 1) + \Pr(Z = 1) \times \Pr(Z = 0) \quad (0.0.10)$$

$$\Pr(Y = 1) = \frac{1}{13} \times \frac{12}{13} + \frac{12}{13} \times \frac{1}{13} \quad (0.0.11)$$

$$\Pr(Y = 1) = \frac{24}{169} = 0.142012 \quad (0.0.12)$$

$$\Pr(Y = 2) = \Pr(Z = 0) \times \Pr(Z = 0) \quad (0.0.13)$$

$$\Pr(Y = 2) = \frac{1}{13} \times \frac{1}{13} = \frac{1}{169} \quad (0.0.14)$$

$$\Pr(Y = 2) = 0.005917 \quad (0.0.15)$$

Serial number	Case	Probability of the case
1	$\Pr(Y = 0)$	$\frac{144}{169}$
2	$\Pr(Y = 1)$	$\frac{24}{169}$
3	$\Pr(Y = 2)$	$\frac{1}{169}$

TABLE 0: Probability distribution table

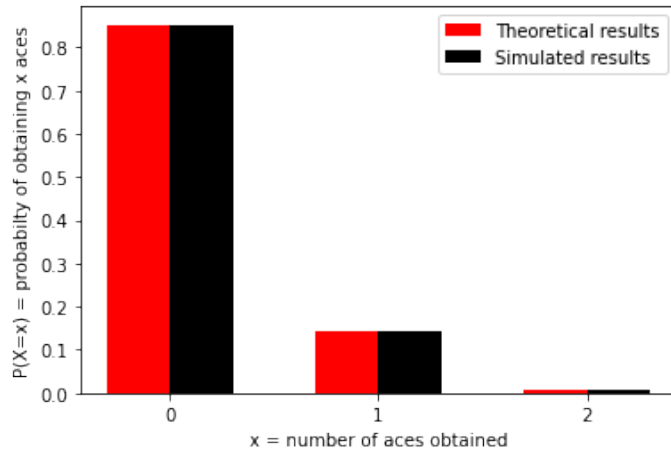


Fig. 0: theoretical and simulated probability results