AI1103-Assignment-2

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Download all python codes from

```
https://github.com/Vikhyath-vec/AI1103/tree/main/
    Assignment-2/codes
```

and latex-tikz codes from

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https://github.com/Vikhyath-vec/AI1103/blob/main
    /Assignment-2/Assignment-2.tex
```

Question

Two cards are drawn successively with replacement from a well shuffled deck of 52 cards. Find the probability distribution of the number of aces.

Solution

Let $X \in \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$ represent the random variable, where 0 represents an ace card, 1 represents a card numbered '2', 2 represents a card numbered '3'... 9 represents a card numbered '10', 10 represents the J card, 11 represents the Q card, and 12 represents the K card. These are independent of the suit. From the given information, we know that there exists 4 different card of each number in a well-shuffled deck of 52 cards. Thus

$$n(X = i) = 4, i \in \{0, 1, 2 \ldots 10, 11, 12\} \quad (0.0.1)$$

Since the deck is well shuffled and complete. the probability of finding a card of a given number is:

$$\Pr(X = i) = \begin{cases} \frac{4}{52} = \frac{1}{13} & i \in \{0, 1, 2 \ldots 10, 11, 12\} \\ 0 & \text{otherwise} \end{cases} \quad (0.0.2)$$

Since we are picking only 2 cards, the number of aces which can be obtained is either 0 or 1 or 2. Let Y $\in \{0, 1, 2\}$ represent the random variable, where 0 represents the case where no aces are selected, 1 represents the case where one ace is selected, 2 represents the case where 2 aces are selected.
Let Z $\in \{0, 1\}$ represent the random variable, where 0 represents an ace card is picked while 1 represents a non-ace card is picked. From equation (0.0.2), probability of selecting an ace card is:

$$\Pr(Z = 0) = \frac{1}{13} \quad (0.0.3)$$

Similarly, probability of selecting a non-ace card is:

$$\Pr(Z = 1) = \Sigma_{i=1}^{12} Pr(X = i) \quad (0.0.4)$$
$$= \Sigma_{i=1}^{12} \frac{1}{13} \quad (0.0.5)$$
$$= \frac{12}{13} \quad (0.0.6)$$

Now, for finding the probability distribution of the number of aces, $\Pr(Y = 0)$ would mean 0 aces are selected or 2 non-ace cards are selected. Thus,

$$\Pr(Y = 0) = \frac{12}{13} \times \frac{12}{13} = \frac{144}{169} \quad (0.0.7)$$
$$\Pr(Y = 0) = 0.852071 \quad (0.0.8)$$

$\Pr(Y = 1)$ would mean 1 ace and 1 non-ace card are selected. This can be done in 2 ways namely:

1) first selecting an ace card and then selecting a non-ace card
2) first selecting a non-ace card and then selecting an ace card

Thus,

$$\Pr(Y = 1) = \frac{1}{13} \times \frac{12}{13} + \frac{12}{13} \times \frac{1}{13} = \frac{24}{169} \quad (0.0.9)$$
$$\Pr(Y = 1) = 0.142012 \quad (0.0.10)$$

$\Pr(Y = 2)$ would mean 2 aces are selected. Thus,

$$\Pr(Y = 2) = \frac{1}{13} \times \frac{1}{13} = \frac{1}{169} \quad (0.0.11)$$
$$\Pr(Y = 2) = 0.005917 \quad (0.0.12)$$

Serial number	Case	Probability of the case
1	$\Pr(Y = 0)$	0.852071
2	$\Pr(Y = 1)$	0.142012
3	$\Pr(Y = 2)$	0.005917

Above is the probability distribution table of the

number of aces obtained when two cards are drawn successively with replacement from a well shuffled deck of 52 cards.
Below is the graph with theoretical and simulated result of the probability distribution.

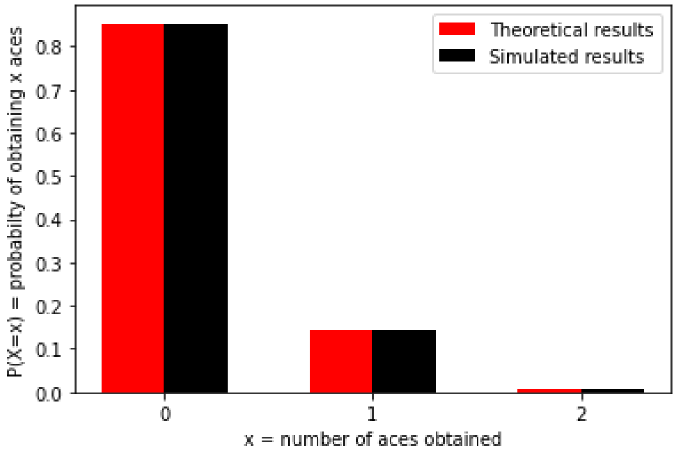

Fig. 2: theoretical and simulated probability results