10/08/23 Lec 3

Claim:- The adversary strategy forces any dec. tree algo. to make $\binom{n}{2}$ queries.

Observation 1:- G_{YUM} is connected throughout.

Proof:- by inspecting the adversary strategy.

Observation 2 :- G_Y is disconnected until

$$G_Y = G_{Y \cup M}$$

Proof of the claim assuming Obs. 2 :

Suppose the algo. didn't make $\binom{n}{2}$ queries.

This implies $\exists$ an edge, say e_i, that has not been queried.

$\Rightarrow Y \neq Y \cup M$

$\Rightarrow G_Y \neq G_{Y \cup M}$

$\Rightarrow$ Obs 2, G_Y is disconnected.

whereas $G_{Y \cup M}$ is connected.

So the algorithm cannot distinguish between G_Y and $G_{Y \cup M}$. Hence, we can make the algorithm give a wrong answer.

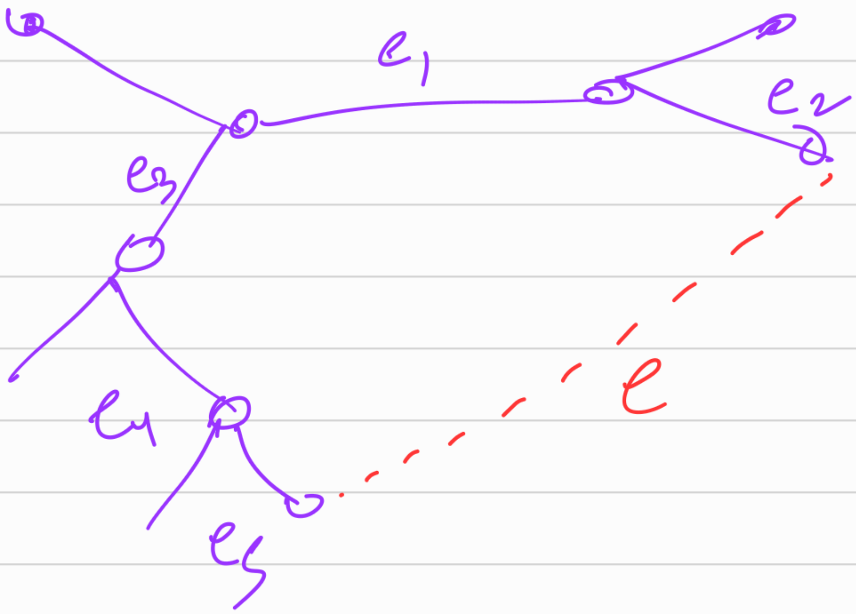
$\boxtimes$

Proof of Obs 2:

Suppose not.

$Y \neq Y \cup M$ and G_Y is connected

$\exists$ a spanning tree in G_Y.



Let e be the edge in M.

Let C be the cycle made by e in G_Y.

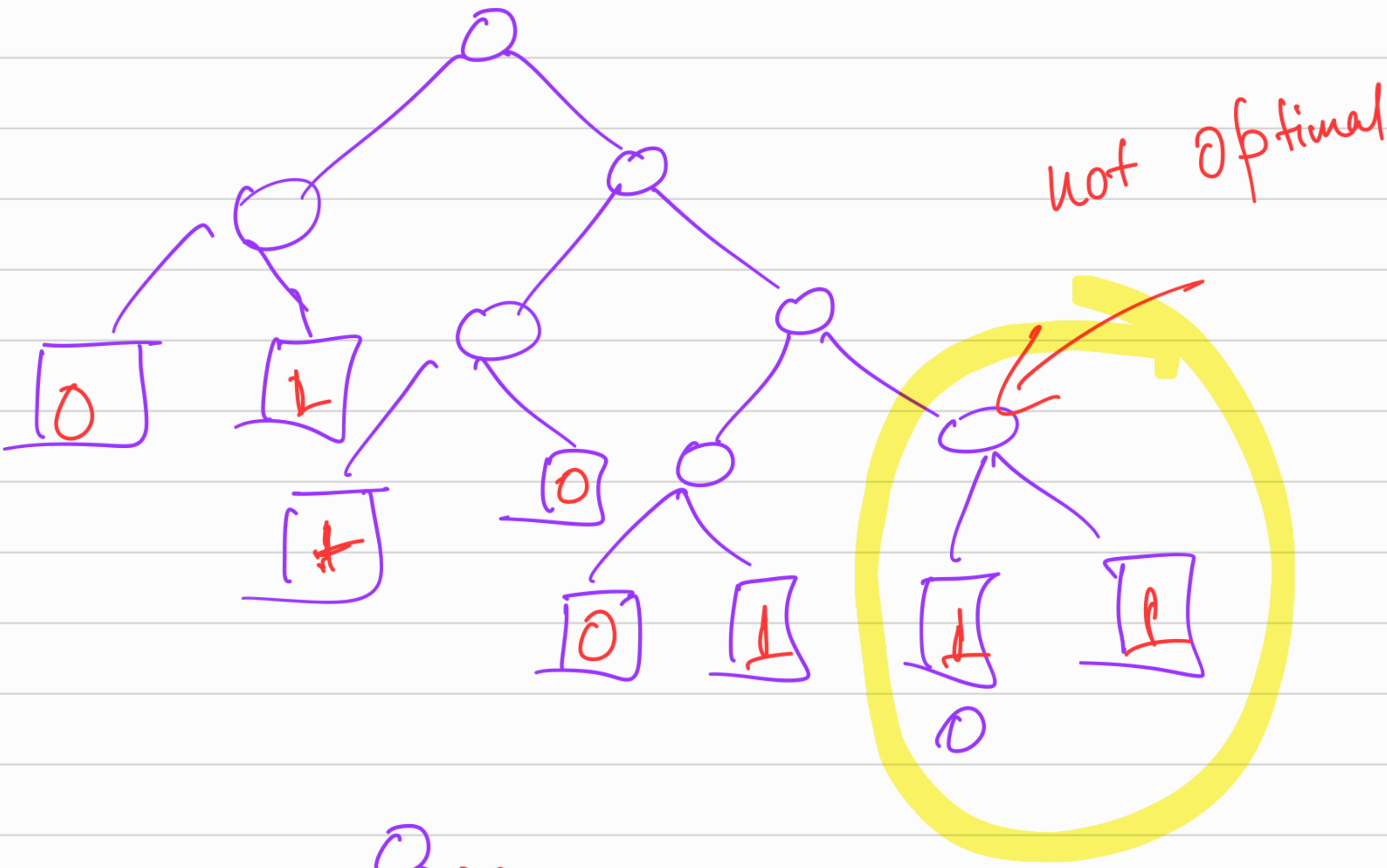
Let e_1 be the first edge among edges in C to be added to y.

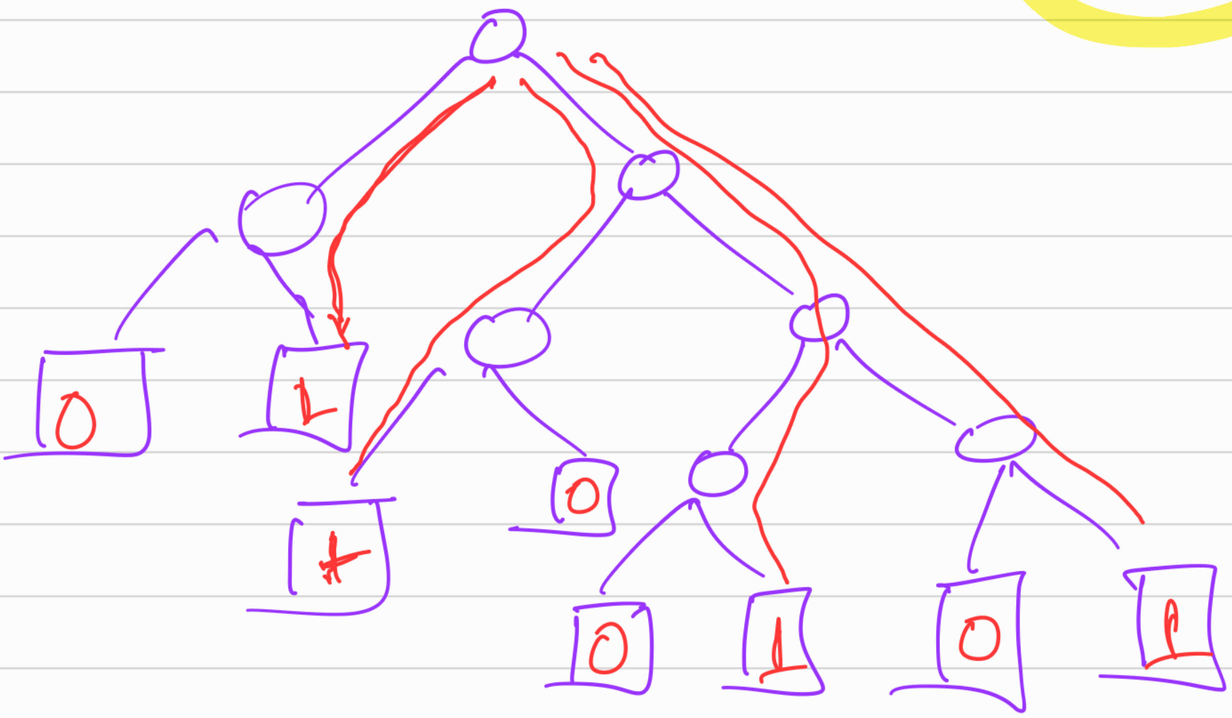
We set $x_{e_1} = 1$ iff removing e_1 from G_{rem} disconnects the graph.

However we argued above that deleting e_1 doesn't disconnect the graph.

That contradicts the adversary strategy.

∎





$$f : \{0,1\}^n \to \{0,1\}$$

$$|\{x \mid f(x)=1\}| =$$

$$2^{n-2} + 2^{n-3} + 2^{n-4} + 2^{n-4}$$

Q: $f : \{0,1\}^n \to \{0,1\}$

s.t. $|f^{-1}(1)|$ is odd.

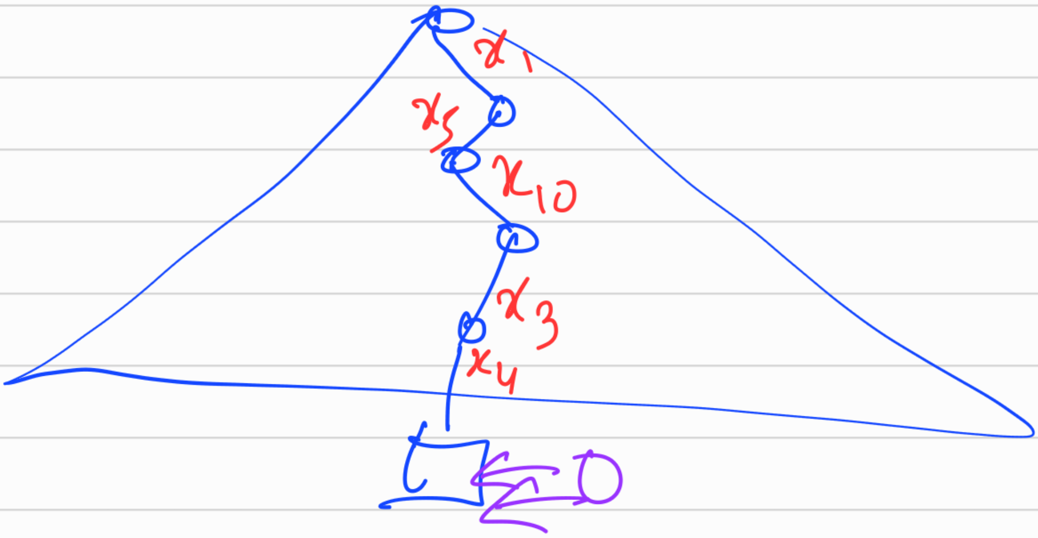
Then what is $D(f)$?

Ans:- $D(f) \geq n$.

Exercise:- complete the argument.

→ Deterministic Decision tree.

→ Non-deterministic DT.



$x_1 = 1, \; x_5 = 0, \; x_{10} = 1, \; x_3 = 0$

$x_4 = 0$

Suppose you have an input

y that reaches this

leaf and you want

to convince someone

then $f(y) = 0$.

Every root to leaf path

gives a set of input

bits s.t. every input

consistent with these

bits has the same

function value.

$(S \subseteq [n],\ \alpha : S \to \{0,1\})$

$S = \{1, 5, 10, 3, 4\}$

$$\alpha = \left\{ \begin{array}{l} x_1 \to \underline{1} \\ x_5 \to 0 \\ x_{10} \to 1 \\ x_3 \to 0 \\ x_4 \to 0 \end{array} \right\}$$

Certificate

A certificate for an input $x \in \{0,1\}^n$ w.r.t the function f.

is a tuple $(S, \alpha : S \to \{0,1\})$

s.t. $\forall y \in \{0,1\}^n$

s.t. $y_i = x_i = \alpha(i)$ $\forall i \in S$

we have $f(y) = f(x)$.

$OR_n : \{0,1\}^n \to \{0,1\}$

$C(OR_n, 0^n)$

give me a certificate for OR_n at 0^n.

In this case.

$S = [n]$

$\alpha : S \to \{0\}$

Certificates of

$0\underline{1}000 0^{n-4}$

$s = \{2\}$ $\alpha: 2 \to 1$.

$s = \{1\}$