# Applied Estimation

EL2320 Lab 2

Authors: Weiqi Xu, weiqi@kth.se

November 29, 2020

## 1 Part I

- 1. The particles are a set of random state samples (particles) to describe the probability density.
- 2. importance weight:  $w_t = p(z_t|x_t)$  is the probability of the measurement  $z_t$  under the particle  $x_t$ . target distribution f: the distribution based on the measurements, corresponds to the target belief  $bel(x_t)$ . proposal distribution g: distribution based on prediction from previous state  $x_t$

The relationship: We can calculate importance weights for each sample x based on data fusion between f and g.

- 3. Particle deprivation can occur when the number of particles is too small in the relevant regions where there is high likelihood that the true state may lie. The danger is to generate incorrect estimation or unobtainable correct estimation
- 4. Without resampling, weights of some particles may end up very low while others are very high. Therefore, we keep good samples and desert bad samples to cover the likelihood region.
- 5. If the distribution is of the donut shape, the average of the particle set will be in the center of donut, whose likelihood is really low.
- 6. Particles need to be extended to continuous density using several different ways: 1) Gaussian approximation, 2) k-means clustering, 3) histogram bins, etc.
- 7. A high sample variance means our particle distribution is inaccurate. The remedies are using more particles or adding random particles.
- 8. A high pose uncertainty will need more particles to maintain quality of estimate.

#### 2 Part II

### 2.1 Warm up problem with the Particle Filter

- 1. Advantage: (6) will have a lower computational expense.

  Drawback: (8) can react to the disturbance if target moves off the line, while (6) won't.
- Using (9), we can model the circular motions whose linear and angular velocities are both constant. The Limitation is that we can't change these velocities, i.e. we need to fix  $v_0$  and  $\omega_0$  in advance.
- 3. To normalize the distribution.
- 4. Multinomial: M Systematic: 1
- 5.  $w = 1/M + \epsilon$ :

For Multinomial method: probability of surviving in 1 loop:  $p_s = 1/M + \epsilon$ . Thus, probability of surviving in M loop is:  $P = 1 - (1 - p_s)^M$ 

For Systematic method: 1

0 < w < 1/M:

For Multinomial method:  $p_s = w$ ;  $P = 1 - (1 - p_s)^M$ 

For Systematic method: P = Mw

Decrease in the weight means that the probability of surviving will decrease.

- 6. 'params.Sigma\_Q' is the measurement noise. 'params.Sigma\_R' is the process noise
- 7. We no longer resample, since we model the process to have no noise, i.e. there is no need to rely on the measurements. We are completely confident in the modelling. Thus, the error will then entirely depend on the initial position.
- 8. The distribution does not converge and the distribution stays uniform.

9.

When observation noise is 10000: Our particles become more diffused. As a result, the error generally grows compared to 100.

When observation noise is 0.0001: For smaller Q, the convergence is slower. If Q is too small, we are likely to not converge at all, due to lack of resampling.

10.

10000: particles become more scattered during the prediction step. Estimation can converge but the error becomes larger.

0.0001: particles become less diffused during prediction step, i.e. convergence is slower. The estimate becomes more 'concentrated'.

- 11. For non-fixed motion, we generally need a larger noise than the fixed one to compensate for the uncertainty in the control input, i.e. we need to increase process noise to compensate for a bad motion model which is a wrong representation of actual motion .
- 12. The higher dimensional motion will lead to a decreasing accuracy of estimation. We need more particles to achieve the same accuracy. We expect the estimation error to be generally larger when using linear/circular motion vs fixed.
- 13. We reject those observation far way the predicted position with high likelihoods.

14.

For fixed motion model:

The best precision is:  $e = 11.3 \pm 5.5$ ;  $Q = 5Q_0$ ,  $R = 15R_0$ 

For linear motion model:

The best precision is:  $e = 7.8 \pm 3.5$ ;  $Q = 5Q_0$ ,  $R = R_0$ 

For circular motion model:

The best precision is:  $e = 7.6 \pm 3.2; Q = 5Q_0, R = R_0$ 

Conclusion:

The filters with "wrong" motion models are generally more sensitive to parameters setting compared to those with real motion models, i.e. the filters need more appropriate parameters to get good estimation. To be specific, the fixed motion model requires the largest R to reach the largest amount of diffusion in the prediction step, followed by the linear motion model.

Q is not as important to filter's sensitive as R, but we will also need a generally increased Q for the fixed/linear case to get a wider spread of particles in the measurements. Circular is the least sensitive model. Even if the estimation temporarily diverge, the motion model may still give a better estimate compared than fixed/linear model.

#### 2.2 Main problem: Monte Carlo Localization

15. Q and  $\lambda$  will affect the outlier detection. If we set a very low Q, i.e. we have high confidence in measurements, the measurement are less likely to be detected as an outlier.

16. The weight will become uniform among the particles.

#### 2.3 Simulation

#### 2.3.1 Dataset 4

Figure 1,2,3 show that result of tracking problem is much better than global localization. It's because in the tracking model the filter knows the initial position and set the initial particles, which makes it easier to converge to the right position.



Figure 1: 1000 particles with Systematic Re-Sampling in Tracking and Global model



Figure 2: Covariance matrix of 1000 particles with Systematic Re-Sampling in Tracking and Global model



Figure 3: Error of 1000 particles with Systematic Re-Sampling in Tracking and Global model

We have two valid hypothesises since the landmarks are distributed as a rectangular. We have four hypothesises when the measurement noise is large. Using 10000 particles, the hypothesises can be kept longer. If we set only 1000 particles, the estimation will converge in a short time, which cannot cover all the regions with high likelihood and may end up in a wrong position.

The multinomial sampling is less effective to keep multiple hypothesises since random values are used to pick each particle. Figure 4 below show that hypothesises vanish in a shorter time by using multinomial sampling.

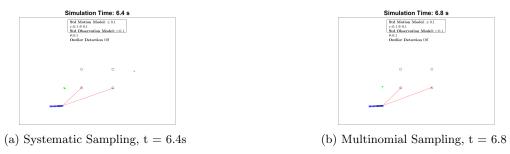


Figure 4: hypothesises of 10000 particles with Systematic and Rultinomial Re-Sampling in Global model

With a stronger measurement noise, we are less confident in the measurements and rely more on motion model. Thus, the hypothesises will be preserved for longer time.

#### 2.3.2 Dataset 5

Figure 5. and Figure 6. show that the filter converge to the right hypothesis. The symmetry is broken as the top right landmark is observed. A new valid observation can generally reduce the uncertainty.



Figure 5: Hypothesis in t = 15, 38s



Figure 6: Hypothesis in t = 157, 227s