1 Instructions:

- There are some questions which have functions with discrete valued domains (such as day, month, year etc). For simplicity, we treat them as continuous functions.

- **Notations:**

    - $\mathbb{R}$= Set of real numbers
    - $\mathbb{N}$= Set of natural numbers

- The set of natural numbers includes 0.

- If a question has more than one answer, any one answer will be acceptable.

1. Let the curves represented by the three degree polynomials $f(x), g(x)$, and $h(x)$ meet at points A and B. Given that

$$g(x) = x^3 - 9x^2 + 20x$$

$$h(x) = x^3 - 7x^2 + 10x$$
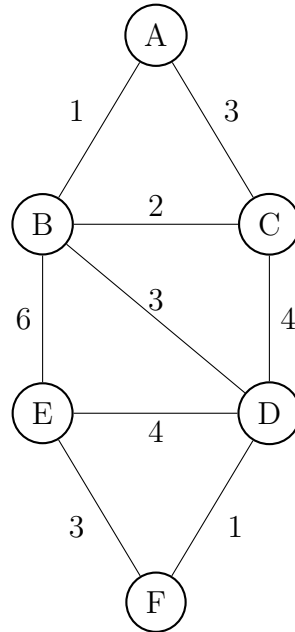
$$f(2) = 2g(2)$$

$$3f(7) = g(7)$$

Find the value of $f(10)$.

(MCQ)   [Marks: 3]

Answer: 200

2. Suppose Dijkstra's algorithm is run on the graph below, starting at node A. In what order do the shortest distances to the other vertices get finalized?
(NAT)   [Marks: 2]

Options:

1. A,C,B,D,E,F

2. A,C,B,D,F,E

3. A,B,C,D,E,F

4. A,B,C,D,F,E

Answer: Option 4.

3. A company manufactures 10 chemicals $c_1, c_2, c_3, ...c_{10}$. A relation R is defined as

$$R = \{(X_i, A) \mid A \subset \{c_1, c_2, c_3, ...c_{10}\}\}$$

Elements of A belong to the X_i ($i \in \{1, 2, 3, 4, 5\}$) type of hazardous chemicals and any pair of elements from A is incompatible and would cause explosions if brought into contact.

Given that,
$(X_1, \{c_1, c_4, c_5, c_{10}\}), (X_2, \{c_2, c_3, c_5\}), (X_3, \{c_3, c_2, c_9\}), (X_4, \{c_4, c_2, c_7\}),$ and $(X_5, \{c_5, c_8, c_{10}\})$
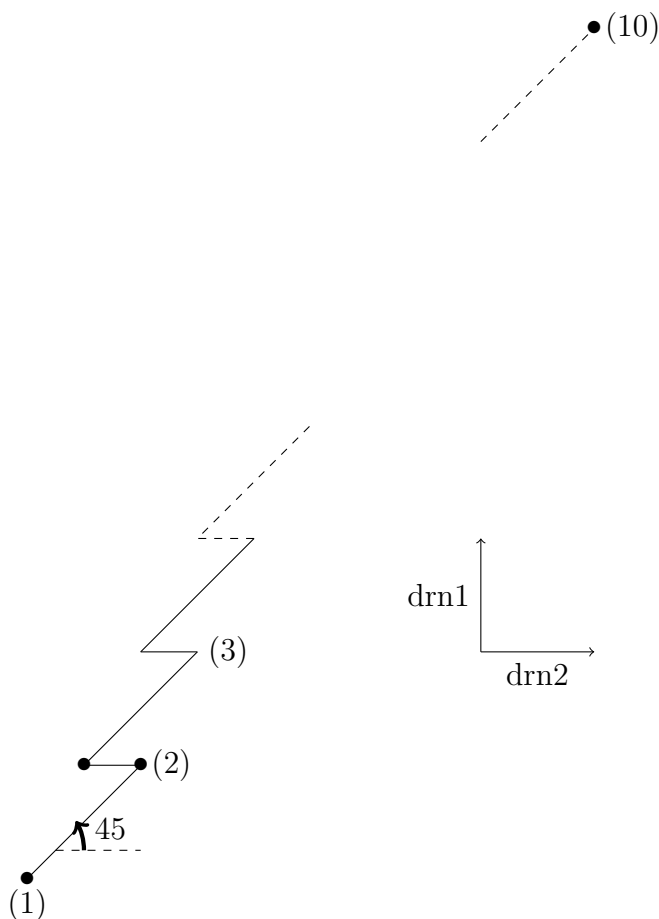are the elements of R.

As a precautionary measure the company wishes to partition its warehouse into compartments, and store incompatible chemicals in different compartments. What is the least number of compartments into which the warehouse should be partitioned?
(NAT)   [Marks: 4]

Answer: 4

4. Seema wants to draw a rangoli on the floor. She has an idea of drawing some zig-zig line. If she starts from a point named as (1), she draws a line segment of $2\sqrt{2}ft$ at an angle of 45 with respect to drn2 and reach till point (2). From point (2) she draws a line segment of $1ft$ towards the opposite direction of drn2. From there she again repeats the above procedure to reach till point (3). If her final point is point (10) then what should be the minimum length of floor in the direction of drn1? (Note that figure is not for scale purpose and $tan45 = 1$.)
(NAT)   [Marks: 3]

Answer: 18

5. Let for some fixed $x > 0$, $p(x)q(x) = 1$, where $m > 1$. Given that

$$p(x) = m^{\log_5 x^2}$$

$$q(x) = x^{\log_5 m}$$

Find the value of x.

(NAT) [Marks: 2]

Answer: 1

6. Let functions $f(x) = x^2 - 8$, $g(x) = log_{10}x$, and $h(x) = g(f(x))$ respectively. Choose the set of correct options regarding $h(x)$.

    (MSQ) [Marks: 3]

    1. Domain of $h(x)$ will be $(-\infty, -2\sqrt{2}) \cup (2\sqrt{2}, \infty)$.

    2. Domain of $h(x)$ will be $(-\infty, -3) \cup (3, \infty)$.

    3. If $h(x)$ is positive then, $x \in (-\infty, -2\sqrt{2}] \cup [2\sqrt{2}, \infty)$.

    4. If $h(x)$ is positive then, $x \in (-\infty, -3) \cup (3, \infty)$.

    5. $h(x)$ has only one asymptote.

    6. $h(x)$ has at least three asymptotes.

    Answer: Options 1 and 4.

7. Polynomial fit for the data given in the table recorded by a student is

$$y = f(x) = log_{101}11^{log_{1001}101^{x^5-5x^3+4x}} + K5^{x^5-5x^3+4x}$$

Find the value of K, so that SSE (sum squared error) will be minimum?
(NAT) [Marks: 3]

x	-2	-1	1	1	2
y	1	1	-1	1	1

Answer: 0.6

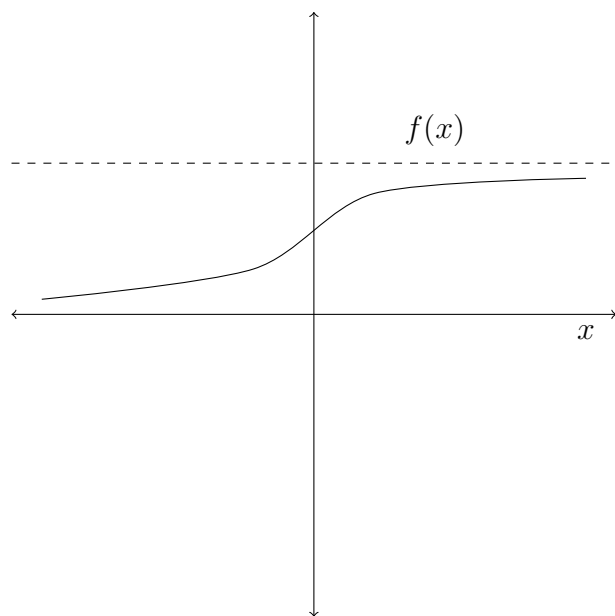8. Choose the option which has the most possible representation of function

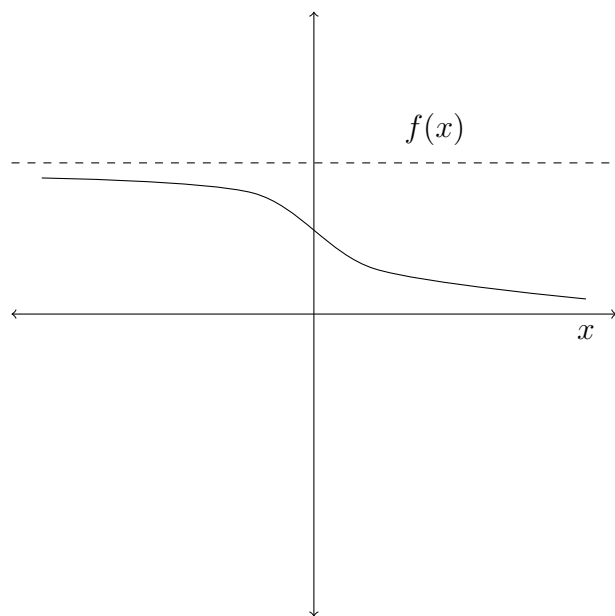$$f(x) = \frac{1}{1 + e^x}$$

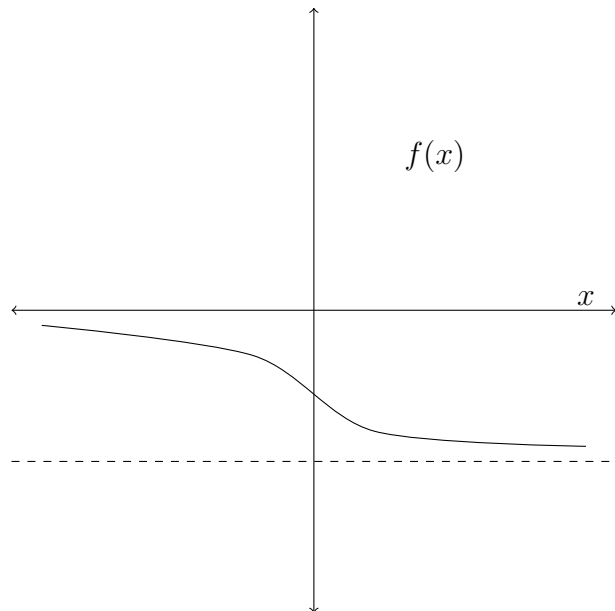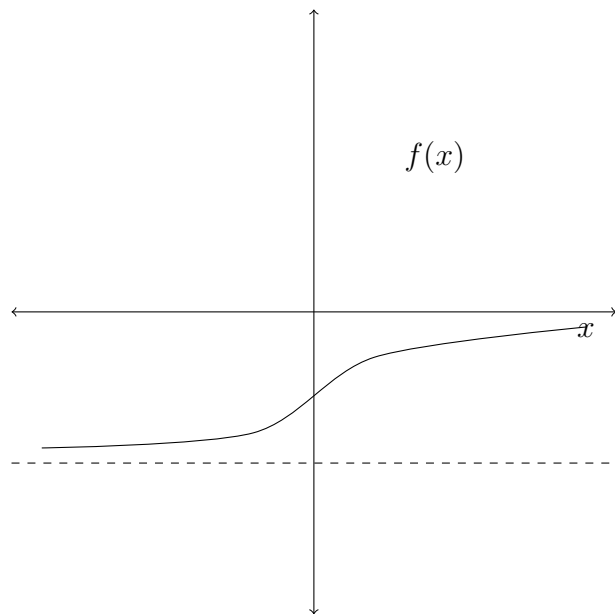(MCQ)   [Marks: 2]

1.

2.

3.

4.

Answer: Option 2

9. Let $f(x)$ and $g(x)$ be two functions defined from $\mathbb{R}$ to $\mathbb{R}$ such that

$$f(x) = \begin{cases} x^4 - 20x^2 + 64 & \text{if } x \in (-\infty, -4) \cup (-2, 2) \cup (4, \infty) \\ -(x^4 - 20x^2 + 64) & \text{if } x \in (-4, -2) \cup (2, 4) \\ 1 & \text{if } x \in \{-4, -2, 2, 4\} \end{cases}$$

,

$$g(x) = log_2 x$$

Find the value of $f(g(4)) + g(f(0)) + g(\frac{f(3)}{7} - 1)$.
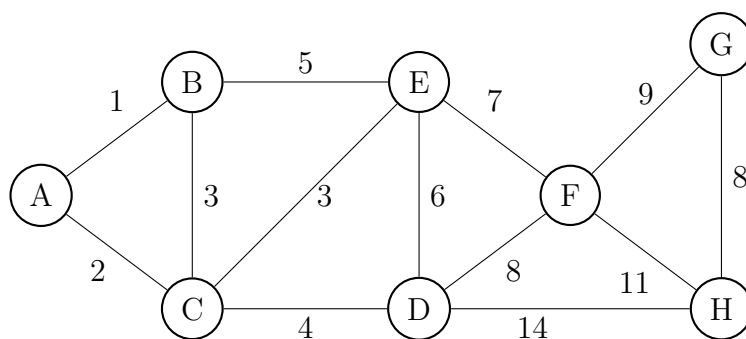(NAT)   [Marks: 4]

Answer: 9

10. Consider two sets A, B defined as $A = \{(a, b) \in \mathbb{R} \times \mathbb{R} \mid |a - 3| < 1, |b - 3| < 1\}$, $B = \{(a, b) \in \mathbb{R} \times \mathbb{R} \mid b - 1 \leq (a - 3)^2\}$. Choose the correct option. (MCQ) [Marks: 5]

1. $A \cup B = B$
2. $A \cap B = \phi$
3. $A \cup B = A$
4. $A \cap B = B$

Answer: Option 2

11. What is the weight of a minimum cost spanning tree of the graph given below?
    (NAT)   [Marks: 2]

Answer: 34

12. In an undirected graph G, the number of vertices having odd and even degrees are M and N respectively. Which of these values are possible for M and N? (MCQ) [Marks: 2]

    1. $M = 96$, $N = 11$

    2. $M = 101$, $N = 10$

    3. $M = 97$, $N = 31$

    4. $M = 103$, $N = 12$

Answer: Option 1

13. We use matrix multiplication to find A^2 from A^1. What is the value of $2\alpha + \beta$?

A^1	i	j	k	l	m
i	0	0	0	0	1
j	0	0	1	0	1
k	0	0	0	1	0
l	0	0	0	0	1
m	0	0	0	0	0

A^2	i	j	k	l	m
i	0	α	0	0	0
j	0	0	0	1	0
k	0	0	0	0	β
l	0	0	0	0	0
m	0	0	0	0	0

(NAT)   [Marks: 2]

Answer: 1

14. Liala and Vinay both have to travel to various locations for advertising their company's products. The company reimburses their expenses such as accommodation, food etc. The company also blacklists an employee whenever the employee's expenditure in a given month exceeds ₹ 12000. The accounts department fits the data of monthly expenditure to the polynomial $E_l(x)$ and $E_v(x)$ (in ₹) for Liala and Vinay respectively, where x is the number of months since they joined the company (i.e., $x = 1$ represents the completion of one month). The polynomial fit is known to be applicable for a period of 30 months (i.e., $x \leq 30$). If $E_l(x) - 12,000 = a(x - 5.1)(x - 11)(x - 20.5)$, $a > 0$ and $E_v(x) - 12,000 = a(x - 2)^2(x - 8)(x - 25.5)$, $a > 0$. Let N_v, N_l, and N_{vl} are the sets defined as

$$N_v = \{x \mid x \text{ is the month number when Vinay got blacklisted}\}$$

$$N_l = \{x \mid x \text{ is the month number when Liala got blacklisted}\}$$

$$N_{vl} = \{x \mid x \text{ is the month number when Liala and Vinay both got blacklisted}\}$$

Find the cardinality of $N_l \cup N_v$.
(NAT)   [Marks: 3]

Answer: 19

15. Let A be the set of positive integers divisible by both 2 and 3 but not by 12, B be the set of positive integers divisible by both 3 and 5 but not by 15, and C be the set of positive integers strictly less than 100 and divisible by both 5 and 2 but not by 20 or 30. Find the cardinality of set G where

$$G = \{x \mid x \in A \cap B \cap C\}$$

(NAT) [Marks: 2]

Answer : 0

16. Consider the path of a ball thrown upwards is represented in the form:

$$S(t) = -\tfrac{g}{2}t^2 + v_0 t + h_0$$

where S is the vertical distance of the ball from the ground (ground is $S = 0$), t is time elapsed since the ball was thrown ($t = 0$ when the ball is thrown), g is the acceleration due to gravity of the Earth, v_0 is the initial velocity of the ball and h_0 is the initial height. $h_0 = 0$ means the ball is thrown from the ground, $h_0 = 1$ means the ball is thrown from a height of 1 unit from the ground. Note that all the variables are consistent in their units. Take $g = 10$ for all the calculations. Which of the following options are correct? (MSQ) [Marks: 6]

Options:

1. Two balls thrown with the same initial velocity must reach the same maximum height

2. Two balls thrown with the same initial velocity and initial height must reach the same maximum height

3. Two balls thrown with the same initial velocity may not reach the same maximum height but must reach their respective maximum heights after the same time t

4. Time taken by a ball thrown from the ground to reach back to the ground is given by $2v_0/g$

5. Time taken by a ball thrown from the ground to reach back to the ground is given by v_0/g

6. A ball is thrown with an initial velocity of 10 units and from a height of 2 units. Another ball is thrown with an initial velocity of 10 units and from the ground. Both the balls reach the ground at the same time.

7. It is never possible that one ball thrown from the ground and another thrown from a finite height h reach the ground at the same time

8. Maximum height reached by a ball with an initial velocity v and thrown from a height h is always higher than that reached by the ball thrown with an initial velocity v and from the ground

Answer: Options 2, 3, 4, 8

18

17. If A and B are two sets such that the cardinality of $A \times B$ is 6. If three elements of $A \times B$ are $(4, 5)$ $(1, 2)$ $(7, 2)$, then $A \times B$ is
    (MCQ)   [Marks: 2]

    1. $\{(5, 4), (5, 1), (5, 7), (2, 4), (2, 1), (2, 7)\}$
    2. $\{(4, 5), (2, 4), (5, 1), (1, 2), (5, 7), (7, 2)\}$
    3. $\{(4, 5), (2, 4), (5, 1), (1, 2), (5, 7), (7, 2)\}$
    4. $\{(4, 5), (4, 2), (1, 5), (1, 2), (7, 5), (7, 2)\}$

Answer: Option 4

18. Consider the relation $R = \{(x, y) \in \mathbb{R} \times \mathbb{R} \mid 0 < |x - y| < 2\}$ on the set of real numbers. Which of the option(s) is(are) correct?
(MSQ) [Marks: 3]

    1. R is a reflexive relation.

    2. R is a symmetric relation.

    3. R is a transitive relation.

    4. R is an equivalence relation.

Answer: Option 2

19. A mobile company offers two plans. Plan A costs ₹300 and offers 1000 free minutes per month with a charge of 15 paise per minute for every additional minute. Plan B costs ₹400 and offers 1500 free minutes per month with a charge of 10 paise per minute for every additional minute. Let $C_A(t)$ and $C_B(t)$ represent the total cost per month for the plan A and the plan B respectively, where t represents the number of minutes used. If Riya uses 1800 minutes per month, then choose the set of correct options with respect to Riya.
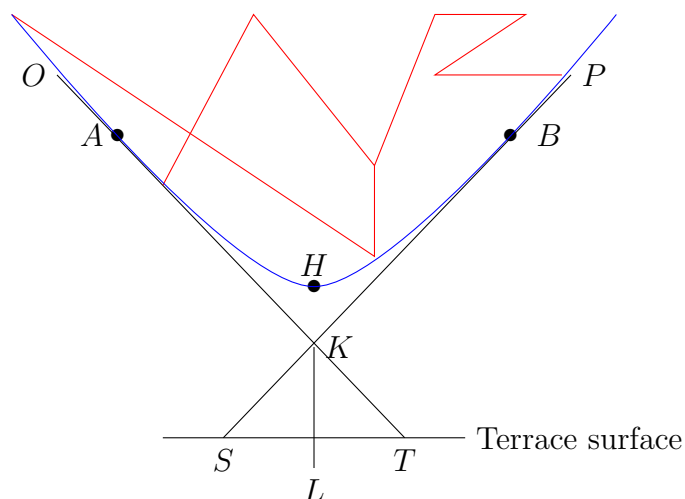   (MSQ) [Marks: 3]

    1. Plan A costs less than plan B.
    2. Plan B costs less than plan A.
    3. $C_A(t) = 300$, for all $t \in [0, 1000]$.
    4. $C_B(t) = 400$, for all $t \in [0, 1500]$.
    5. $C_A(t) = 0.1t + 400$, for all $t \in [1500, 1800]$.
    6. $C_A(t) = 0.15t + 150$, for all $t \in [1000, 1500]$.

Answer: Options 1, 3,4

20. A parabolic banner of a company XYZ is supported by two rods OAT and PBS on the terrace of the company. The lowest point H of the banner is $5ft$ above from the terrace. Both the rods OAT and PBS touch the banner at points A and B only. Both the points are horizontally $2ft$ away from H. Both the points are at the height of $9ft$ from the terrace. To increase the strength of the support, a small rod KL is fixed between the terrace and the point where both the rods meet. If the rod KL is $0.5ft$ inside the terrace, then find the total length of rod KL in ft. (Note that the figure is not for scale purpose and for simplicity assume all the parts of banner and the all the rods are in the same plane.)

(NAT) [Marks: 4]

Answer: 1.5

Mock Test paper

Mathematics for Data Science - 1

Max Marks: 100

1 Instructions:

- There are some questions which have functions with discrete valued domains (such as day, month, year etc). For simplicity, we treat them as continuous functions.

- **Notations:**

  - $\mathbb{R}$= Set of real numbers

  - $\mathbb{Q}$= Set of rational numbers

  - $\mathbb{Z}$= Set of integers

  - $\mathbb{N}$= Set of natural numbers

- The set of natural numbers includes 0.

- **Standard acronyms:**

  - m - metres

  - cm - centimetres

  - kg - kilograms

  - kcal - kilocalories

  - ft - feet

2 MCQ

1. If $p(x) = 2^{\log_{\sqrt{2}}(x-1)} + 1, q(x) = e^{\ln x} - x$, and for some fixed $x > 1$, $p(x) + q(x) = 2$, then choose the correct option.

   (MCQ) [Marks: 6]

   Options:

   1. $x = 2$
   2. $x = 2.5$
   3. $x = 3$
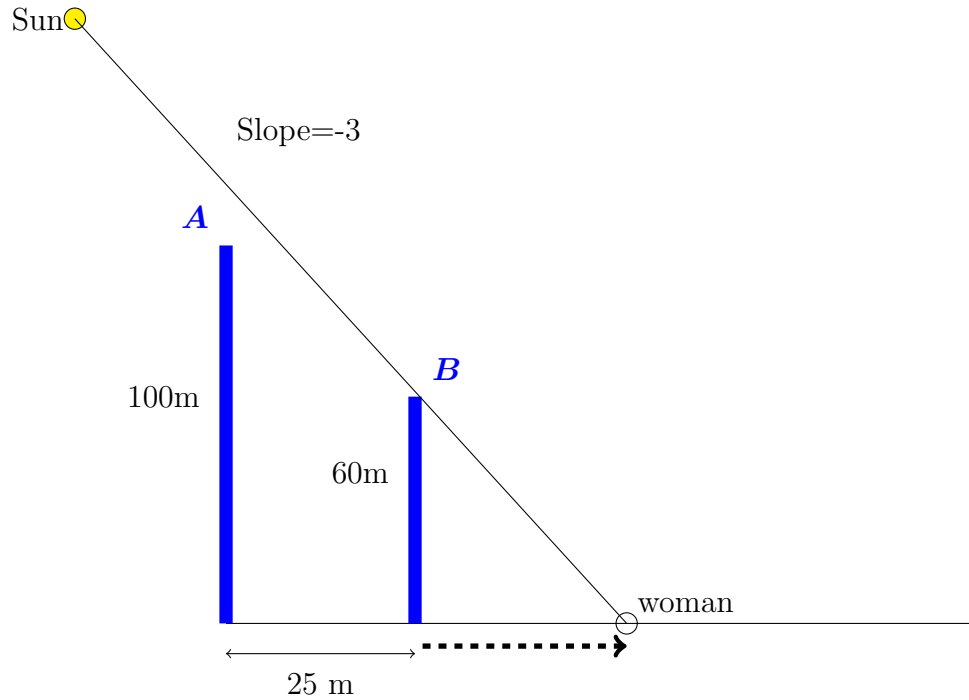   4. $x = 3.5$

   Answer: Option 1.

2. Let $p(x)$ and $q(x)$ be the polynomial of degree four. Suppose $p(x) = q(x)$ for 4 different real values of x (say α, β, γ and δ) and c is any arbitrary constant $(c \neq 0)$. Which of the following may represent the polynomial $p(x) - q(x)$?

   Options:

   1. $c(x^4 + \alpha)$
   2. $c(x^2 - (\alpha + \beta)x + \alpha\beta)(x - \gamma)(x - \delta)$
   3. $c(x^2 + (\alpha - \beta)x - \alpha\beta)(x - \gamma)(x + \delta)$
   4. $c(x^2 + (\alpha - \beta)x + \alpha\beta)(x + \gamma)(x - \delta)$

   Answer: Option 2.

3. Two buildings A and B are as shown in the below diagram. A woman walks out of building B to get some sunshine. How much minimum distance in metres will she have to walk to the right of building B, according to the situations given below? Assume the right direction of buildings as positive and ground as $X-axis$ and the first sun ray not blocked by B has a slope of -3.(Consider both the building and the sun are in the same plan)

Sun

Slope=-3

A

100m

B

60m

woman

25 m

Options:

1. 20
2. 25
3. 35
4. 45

Answer: Option 1.

4. A student was calculating SSE (sum squared error) and she found that SSE is a function of b as follows: SSE $= f(b) = b^2 - 6b + 9$. What will be the best fit value obtained by minimizing SSE using $f(b)$.

   Options:

   1. 4
   2. 1
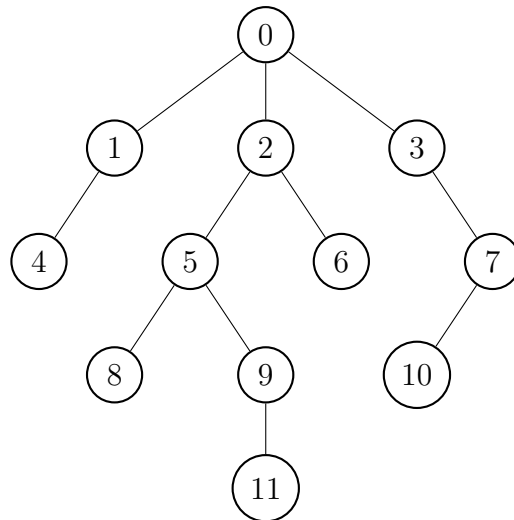   3. 0
   4. -2.25

   Answer: Option 3.

5. External dimensions of a metallic cuboid are $l = (x - 10)$ unit, $b = (x - 15)$ unit, $h = (x - 20)$ unit and the thickness of the metal is 2.5 units from all sides. If the volume of the metal contained in the cuboid formed is $V = 300(x - 15)$ cubic unit. What is the possible value of x. Note: l, b, h and V are the functions of x.
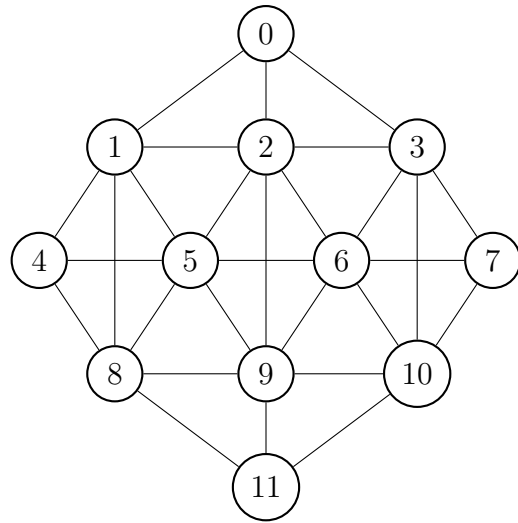(MCQ)   [Marks: 6]

   Options:

   1. 20
   2. 25
   3. 40
   4. 50

   Answer: Option 3.
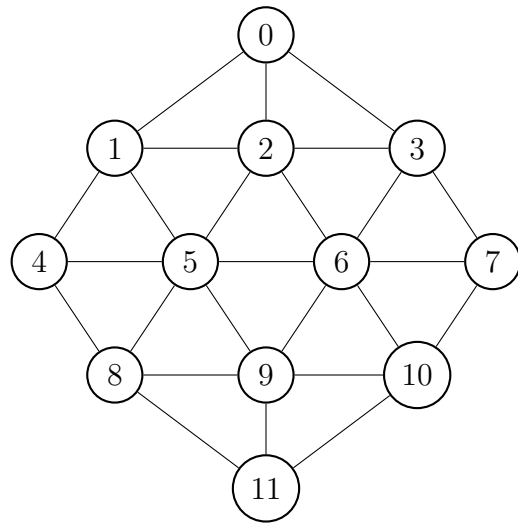
6. The BFS (Breadth First Search) tree of a graph G is shown below. Choose the option which may represent the original graph G.
(MCQ)   [Marks: 6]
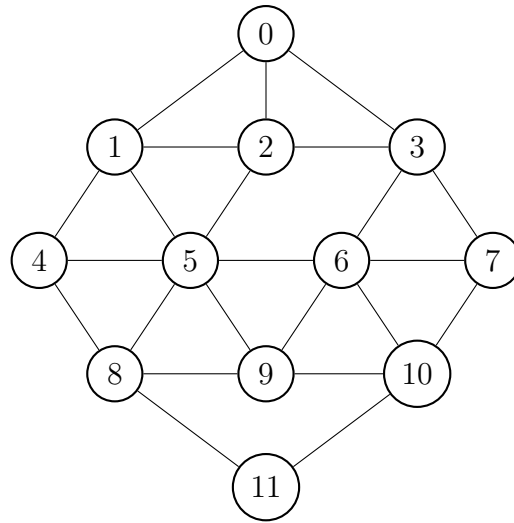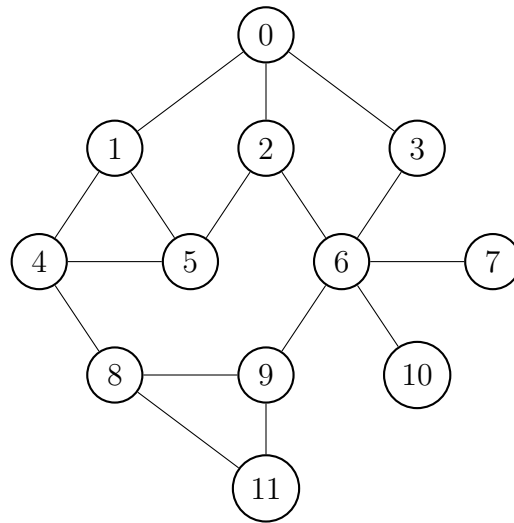

   Options:

1.

2.

3.

4.

Answer: Option 2.

7. In a shop if the number of items bought at a time is n, then the shop provides discount of $M(n)\%$ on the total payable amount. Let $M(n) = -n^2 + 18n - 72$, for $n \in \{6, 7, 8, 9\}$ and the cost of each item be ₹1000. If Aliya buys 7 items, then how much she has to pay?
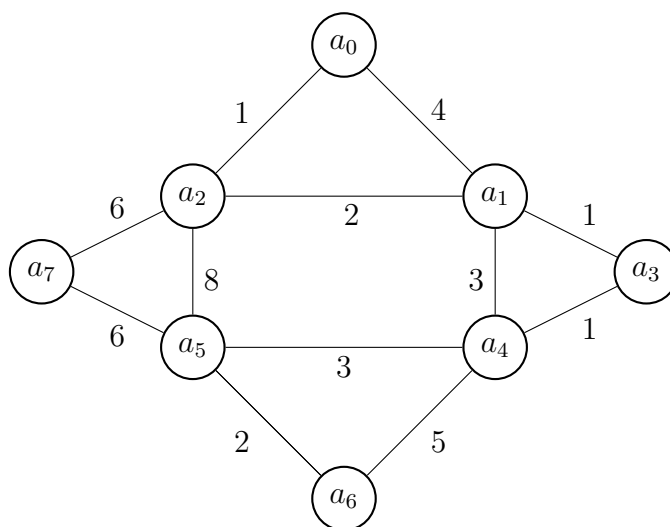(MCQ) [Marks: 6]

1. ₹ 7000

2. ₹ 6650

3. ₹ 6500

4. ₹ 6370

Answer: Option 2.

8. Suppose in the graph below each vertex denotes an oil deposit, edges are pipelines between 2 oil deposits and weights on edges indicates the time taken for the fire to travel through the pipeline. Considering fire travels at uniform speed along each pipeline and fire is set at oil deposit which is at vertex a_0, then the order in which the oil deposits are burnt will be?
(MCQ)   [Marks: 6]

Options:

1. a_0, a_2, a_3, a_1, a_4, a_5, a_7, a_6

2. a_0, a_2, a_1, a_3, a_5, a_4, a_6, a_7

3. a_0, a_2, a_1, a_3, a_4, a_5, a_6, a_7

4. a_0, a_2, a_1, a_3, a_4, a_7, a_5, a_6

Answer: Option 4.

9. Choose the interval in which $f(x) = x^3 + 6x^2 + 11x + 6$ is one to one function.
(MCQ)   [Marks: 6]

   1. $x \in (-3, \infty)$

   2. $x \in (-3, -1)$

   3. $x \in (-\infty, -1)$

   4. $x \in (-1, \infty)$

Answer: Option 4.

3 MSQ

1. Let functions $f(x) = x^2 - 6x + 11, g(x) = log_{10}x$, and $h(x) = g(f(x))$ respectively. Choose the set of correct options regarding $h(x)$.
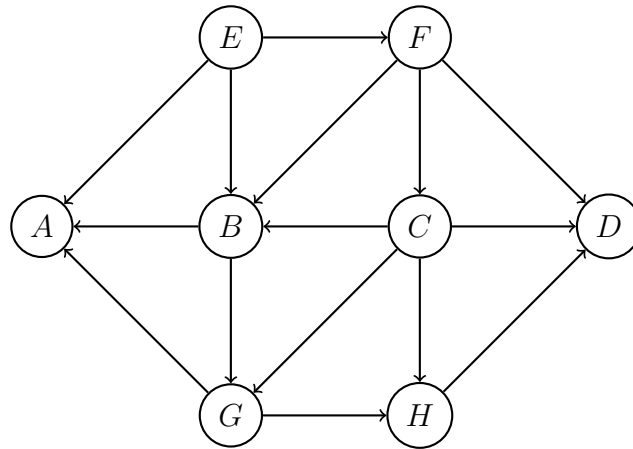   (MSQ) [Marks: 8]
   Option:

   1. Domain of $h(x)$ will be $\mathbb{R}$.

   2. $h(x)$ is an even function.

   3. $h(x)$ is positive for all $\mathbb{R}$.

   4. $h(x)$ has at least one asymptote.

   Answer: Options 1 and 3.

2. The topological sequence of the given DAG is.
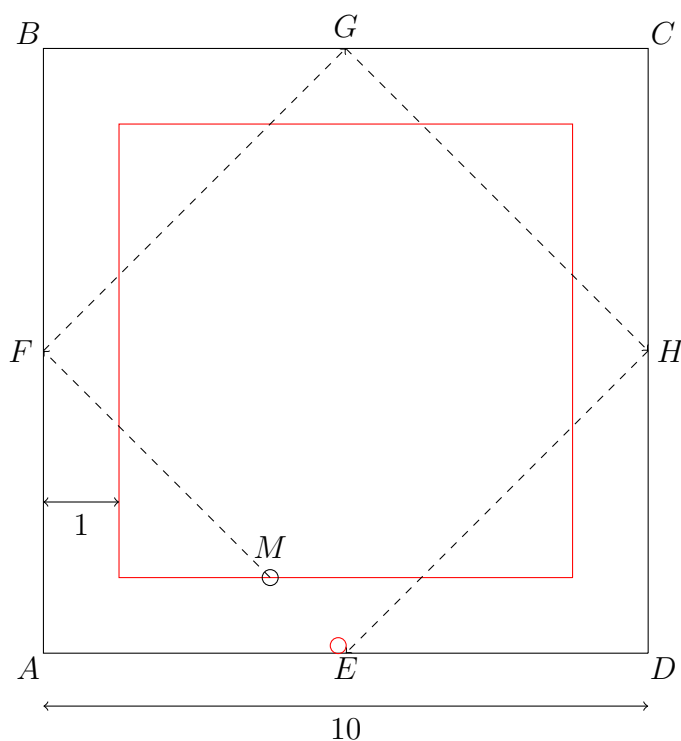   (MSQ)   [Marks: 8]

   1. *E, F, C, B, A, G, H, D*
   2. *E, F, B, C, G, A, H, D*
   3. *E, F, C, B, G, A, H, D*
   4. *E, F, C, B, G, H, D, A*

Answer: Option (3), Option (4)

3. A carrom board is a square board with a symmetrical design as shown in the diagram below. E, F, G, and H are the midpoints of AD, BA, CB, and DC respectively. Rohit wants to pocket his last coin which is at E. Taking the laws of reflection to be applicable, he strikes from the point M, so that the striker reflects at F, then at G, then at H and finally hits the coin at E. Choosing A to be the origin (right direction is $+ve\,X-axis$ and upward direction is +ve Y-axis), choose the correct options.
(MSQ)   [Marks: 8]

Options:

1. FH and EG are perpendicular line segments.

2. Segment GH intersects the inner square at $(6, 9)$ and $(9, 6)$.

3. Slope of line EF is -1.

4. Distance between M and E is $\sqrt{2}$.

Answer: Option(1), (2), (3) and (4)

4. Let $f(x) = e^{a(x^2+4x+5)}$, $a \in \mathbb{R}$ then choose the set of correct options.
(MSQ) [Marks: 8]

    1. $f(x)$ will be symmetric around y axis irrespective of value of a.

    2. $f(x)$ will be positive for $x \in (-5, 1)$ if $a > 0$.

    3. $f(x)$ will be positive for $x \in (-5, 1)$ if $a < 0$.

    4. If $a = 1$ then we will get two $X-$ intercepts of $f(x)$.

Answer: Options 2 and 3.

4 NAT

1. Let us consider the following sets,

   - $A = \{x \in \mathbb{N} \mid x \bmod 3 = 0 \text{ and } 1 \le x \le 25\}$
   - $B = \{x \in \mathbb{N} \mid x \bmod 4 = 0 \text{ and } 1 \le x \le 25\}$
   - $C = \{x \in \mathbb{N} \mid x \bmod 6 = 0 \text{ and } 1 \le x \le 25\}$

   What is the cardinality of $((A \setminus (B \cup C)) \cup (B \setminus (A \cup C)) \cup (C \setminus (B \cup A)))$?
   (NAT) [Marks: 7
   Answer: 8

2. Find the number of solutions in the interval $(2, \infty)$ of the equation $\log(x^4 + 7x^3 - x^2 - 67x - 59) = 0$.
   Note: No need to check the domain of $\log(x^4 + 7x^3 - x^2 - 67x - 60)$.
   Hint: $x^4 + 7x^3 - x^2 - 67x - 60$ is divisible by $x^2 + 6x + 5$.
   (NAT) [Marks: 7]
   Answer: 1.

1. Consider a sequence $\{a_n\}$ defined as

$$a_n = \begin{cases} \frac{\lfloor \frac{n}{2} \rfloor + 2n}{3n+2} & \text{when } n \text{ is odd} \\ \frac{5n+3}{6n+2} & \text{when } n \text{ is even} \end{cases}$$

Find the limit of the sequence $\{b_n\}$ defined as $b_n = 36a_n^2 + 18a_n$. [Ans: 40]

2. Consider a function $f(x)$ defined as

$$f(x) = \begin{cases} \frac{2(\cos 2x - \cos x)}{x^2} & \text{if } x \neq 0 \\ b & \text{if } x = 0 \end{cases}$$

If $f(x)$ is continuous at $x = 0$, then find the value of b. [Ans: -3]

3. Consider a function $f(x)$ defined as $f(x) = \ln(2\sqrt{x} - \frac{2}{\sqrt{x}}) + x \tan^{-1} 2x$. If derivative of the function $f'(x) = A\frac{x+B}{x(x+C)} + \frac{Dx}{1+Ex^2} + F \tan^{-1} 2x$, then find the value of $2A + B + C + D + E + F$. [Ans: 8]

The quantity (in a unit) of production of a product A is given by the function

$$p(x) = \begin{cases} 180(x + \frac{4}{x}) + 280 & \text{if } 0 < x < 10 \\ xe^x + 1100 & \text{if } x \geq 10 \end{cases}$$

where x is the quantity (in a unit) of the raw materials. The quality of the product A is measured by a scale, based on the value of the linear approximation of the function $p(x)$ with respect to the 1 unit raw material (i.e if $L_p(x)$ is the linear approximation of the function $p(x)$ at $x = 1$, then quality of the product A, when the used raw material is m units, is $L_p(m)$).
Use the information above to answer questions 4, 5 and 6.

4. If a is the rate of increment in the quantity of the production of the product A when the used quantity of the raw materials is 15 units, then find the value of $\frac{a}{e^{15}}$. [Ans: 16]

5. Find the minimum quantity of the production of the product A. [Ans: 1000]

6. Find the quality of the product A, when the production is minimum. [Ans: 640]

The curve represented by the functions $f(x) = |x^2 - 8x + 7|$ and $g(x) = \lfloor x + 1 \rfloor$ in the interval $[0, 9]$ represents the number of products sold in two online grocery stores: Store A and Store B, respectively, from 12PM to 9PM, where 12PM is taken as $x = 0$ and 9 PM is taken as $x = 9$. The area under the curve bounded by positive X-axis in the given interval is also measured to determine which one is more active in that time period (the more area under the curve bounded by the positive X-axis means the more activity in the online grocery store). Figure T2M2Q1.1 shows the graph of function $f(x) = |x^2 - 8x + 7|$, whereas, Figure T2M2Q1.2 shows the graph of function $g(x) = \lfloor x + 1 \rfloor$, in the interval $[0, 9]$.
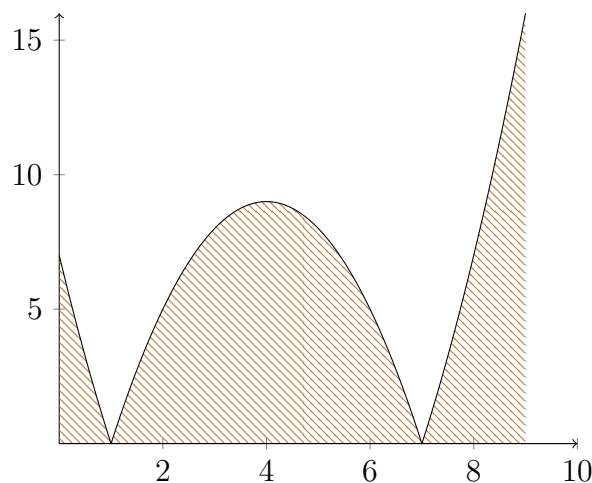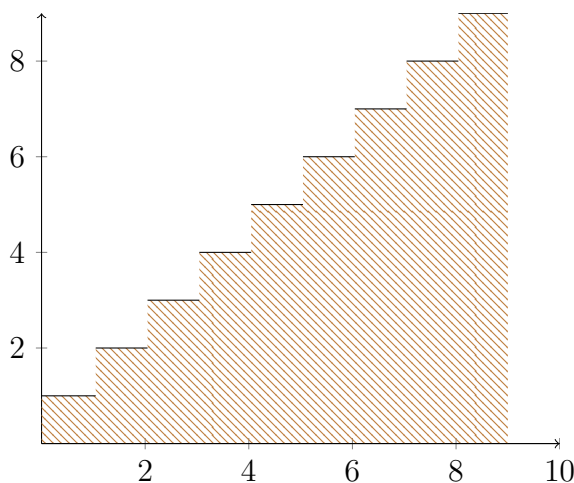

Figure: T2M2Q1.1

Figure: T2M2Q1.2

Use the information above answer questions 7, 8 and 9.

7. If the critical points of the functions $f(x)$ and $g(x)$ correspond to the grocery shops A and B are not active at that time, then how many times the both grocery shops A and B are not active between 2.30PM to 8.30PM. [Ans: 8]

8. Choose the set of correct statements. (MSQ)

○ **Option 1:** f is continuous everywhere in the interval $(0, 9)$, but g is not.

○ Option 2: g is continuous everywhere in the interval $(0, 9)$, but f is not.

○ Option 3: Both f and g are continuous everywhere in the interval $(0, 9)$.

○ Option 4: Store A was less active than Store B in the given time interval. interval $(0, 9)$.

○ **Option 5:** Store B was less active than Store A in the given time interval. interval $(0, 9)$.

9. Find the estimated area of the region bounded by the graph of the function $g(x)$, above the interval [0,9] using the trapezoidal Riemann sums by taking 9 subintervals of equal length. [Answer: 49.5]

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