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Review of optimal multi-agent Pathfinding algorithms and usage in warehouse automation

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Contents

1	Introduction	1
2	Background	1
3	Single-agent pathfinding	1
4	Multi-agent pathfinding	2
5	Optimal multi-agent pathfinding algorithms	2
5.1	Conflict-based Search	2
5.2	ICTS	2
5.3	A*	2
5.4	Push and Rotate	2
6	Bounded suboptimal variants	2
7	Mixed Integer Programming	3
8	Other reduction-based techniques	3
9	Other improvements to Warehouse Automation	3
10	Conclusion	3

1 Introduction

The order picking process is the number one expense in the operating cost of warehouse systems [De Koster et al. \(2007\)](#). This project will look at warehouse automation, whereby the order-picking process is performed by automated vehicles. In particular we will be exploring Kiva systems which employs warehouse automation. More detail about Kiva systems is provided in Section 2.

When improving on the part-to-picker systems, we look at the system as a Multi-agent pathfinding (MAPF) problem.

The results of this literature review will help identify how we should position storage and picking stations in a warehouse. Additionally, we will be looking at developing a MAPF method which uses a pre-computed path oracle.

2 Background

[Sturtevant \(2012\)](#) provides a number of game maps to use for benchmarking and we will use these when comparing against other multi-agent pathfinding algorithms.

In Warehouse Automation, we look at MAPF on an orthogonal grid-map.

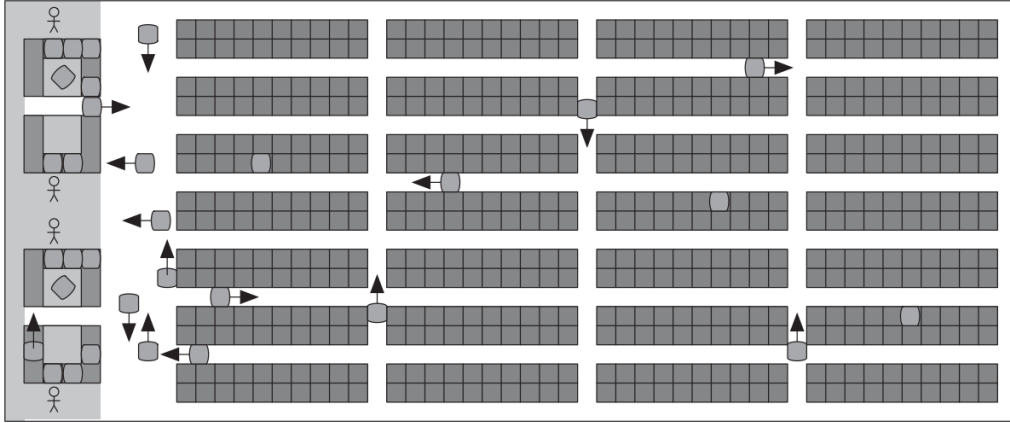


Figure 1: A Small Region of a Kiva Layout ([Wurman et al. \(2008\)](#)). Picking stations located on the left and storage pods laid out in rows.

3 Single-agent pathfinding

Single-agent pathfinding aims to find a path from start node to goal node. In this paper we won't cover the full details of single-agent pathfinding, it has been studying in detail by a number of papers.

The *grid-based path planning competition* (GPPC) is run by [Sturtevant et al. \(2015\)](#). The paper provides detailed results of the state of the art at the time in 2015. These are run on a number of game maps which are used for benchmarking [Sturtevant \(2012\)](#). While these results are a few years old, we are expecting the results from this year's GPPC.

For our simulation we will aim to assist the MAPF algorithm and will consider looking at two algorithms: *jump point search* and *compressed path databases*.

4 Multi-agent pathfinding

Multi-agent pathfinding (MAPF) involves finding a path for every agent to their goal. Warehouse Automation is commonly modeled on an orthogonal undirected grid ?.

Within MAPF, we are focusing on an optimal solution. The following sections will focus on optimal MAPF algorithms as well as suboptimal variants of these algorithms.

5 Optimal multi-agent pathfinding algorithms

Here we look at multi-agent pathfinding algorithms which aim to find the optimal solution based on some objective function. The objective function will be noted per algorithm. Our main aim is to determine the benefits and downsides of each algorithm.

Take a MAPF problem instance where we have k agents on a graph with n nodes. The possible state space is n^k and branching factor is 5^k . So the complexity of the MAPF problem is exponential in the number of agents and polynomial in the number of nodes on the graph.

- a NP-hard problem

- Coupled approach: All agents at one! A* ICTS

- Decoupled approach: Each agent individually (not optimal, not complete)

- [Krontiris et al. \(2013\)](#) looked at the feasibility.

5.1 Conflict-based Search

The conflict-based search algorithm ([Sharon et al. \(2015\)](#))

- Constraint tree. Conflict tree.

- High level: conflict tree

- Low level: find optimal paths for each agent

Although CBS is exponential in the number of conflicts, results found that compared against ICTS and A* they suggest it is best in maps with bottlenecks.

5.2 ICTS

Exponential in Δ

5.3 A*

Exponential in number of agents

5.4 Push and Rotate

[Wilde et al. \(2014\)](#)

6 Bounded suboptimal variants

A *bounded suboptimal variant* is a variant of an optimal MAPF algorithm. The defining feature of a bounded suboptimal variant is that we can guarantee that the cost of the solution, B lies within the optimal cost C and $w * C$ where w is a user-defined parameter. Hence:

$$C \leq B \leq w * C$$

Our project is aiming to make a bounded suboptimal variant of our algorithm due to the large-scale of the simulation (Section 2). By adjusting w we are able to relax the problem and find a solution faster at the loss of solution quality.

7 Mixed Integer Programming

Mixed-integer programming (MIP) is an optimization technique. It looks at an objective function subject to a list of constraints and finds the optimal solution for each variable.

[Yu and LaValle \(2013\)](#)

8 Other reduction-based techniques

Answer set programming?

9 Other improvements to Warehouse Automation

Independence detection

Operator decomposition

10 Conclusion

References

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