Q1. Train a **single perceptron and SVM** to learn an AND gate with two inputs x1 and x2. Assume that all the weights of the perceptron are initialized as 0. Show the calulation for each step and also draw the decision boundary for each updation.

Importing the necessary libraries

In [ ]:
```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
import seaborn as sns
from cvxopt import matrix, solvers
```

Single Perceptron

Defining the necessary functions used

In [ ]:
```
def decision_plot(w, x_min, x_max, y_min, y_max):
    inp_0 = []
    inp_1 = []

    x_min = x_min - np.abs(((x_max - x_min) / 10))
    y_min = y_min - np.abs(((y_max - y_min) / 10))

    x_max = x_max + np.abs(((x_max - x_min) / 10))
    y_max = y_max + np.abs(((y_max - y_min) / 10))

    for i in np.arange(x_min, x_max, np.abs(x_max - x_min) / 100):
        for j in np.arange(y_min, y_max, np.abs(x_max - x_min) / 100):
            y_pred = np.dot(np.array([[1, i, j]]), w)

            if y_pred < 0:
                inp_0.append(np.array((i, j)))

            else:
                inp_1.append(np.array((i, j)))

    return inp_0, inp_1
```

In [ ]:
```
def perceptron_training_alg(X, Y, learning_rate):
  # Adding Bias to the input and Randomly initializing weights
  num_samples, num_features = X.shape

  X = np.hstack((np.ones((num_samples, 1)), X))
  W = np.zeros(num_features + 1).reshape((-1, 1))

  weight_vector = []
  weight_vector.append(W)

  print(W, "\n")

  # Running the algorithm until all the data points are correctly classified
  samples_misclassified = True
  cnt=0
  while samples_misclassified and cnt < 100000:
    # Initially we'll assume that there are no misclassified samples
    samples_misclassified = False

    for index, x in enumerate(X):
      if cnt >= 100000:
        return W, np.array(weight_vector)
```

```
      x = Y[index] * x

      if np.dot(x.reshape((1, -1)), W) <= 0: # if samples misclassified
        samples_misclassified = True
        W = W + (learning_rate * x.reshape((-1,1)))     # Gradient Descent Step
        weight_vector.append(W)

        print(W, "\n")
        cnt += 1

  return W, np.array(weight_vector)
```

In [ ]:

```
def sorter(X, Y):
  X_new0 = []
  Y_new0 = []

  X_new1 = []
  Y_new1 = []

  for x, y in zip(X, Y):
    if y == 0:
      Y_new0.append(y)
      X_new0.append(x)

    else:
      Y_new1.append(y)
      X_new1.append(x)

  X0 = []
  Y0 = []

  X1 = []
  Y1 = []

  if len(X_new0) > 0:
    X0 = np.stack(X_new0)
    Y0 = np.stack(Y_new0)

  elif len(X_new1) > 0:
    X1 = np.stack(X_new1)
    Y1 = np.stack(Y_new1)

  if len(X0) > 0 and len(X1) > 0:
    X = np.concatenate((X0, X1))
    Y = np.concatenate((Y0, Y1))

  elif len(X0) > 0:
    X = X0
    Y = Y0

  else:
    X = X1
    Y = Y1

  return X, Y
```

In [ ]:

```
def decision_boundary(W, X, Y):
  X, Y = sorter(X, Y)
  num_samples, num_features = X.shape
  np2 = np.hstack((np.ones((num_samples, 1)), X))

  X=np2

  x1, y1 = decision_plot(W, min(X[:,1]), max(X[:,1]), min(X[:,2]), max(X[:,2]))

  plt.scatter([i[0] for i in x1], [i[1] for i in x1])
```

```
    plt.scatter([i[0] for i in y1], [i[1] for i in y1])
    plt.scatter(X[:np.argmax(Y), 1], X[:np.argmax(Y), 2])
    plt.scatter(X[np.argmax(Y):, 1], X[np.argmax(Y):, 2])
    plt.show()
```

Code

In [ ]:

```
X = np.array([[1, 1], [0, 1], [1, 0], [0, 0]])
Y = np.array([[1], [-1], [-1], [-1]])
ans = np.array([[1], [0], [0], [0]])

len(X)
```

Out[ ]: 4

In [ ]:

```
W, weight_vector = perceptron_training_alg(X, Y, 0.5)
```

```
[[0.]
 [0.]
 [0.]]

[[0.5]
 [0.5]
 [0.5]]

[[0. ]
 [0.5]
 [0. ]]

[[-0.5]
 [ 0. ]
 [ 0. ]]

[[0. ]
 [0.5]
 [0.5]]

[[-0.5]
 [ 0.5]
 [ 0. ]]

[[-1.]
 [ 0.]
 [ 0.]]

[[-0.5]
 [ 0.5]
 [ 0.5]]

[[-1. ]
 [ 0.5]
 [ 0. ]]

[[-0.5]
 [ 1. ]
 [ 0.5]]

[[-1.]
 [ 1.]
 [ 0.]]

[[-1.5]
 [ 0.5]
 [ 0. ]]

[[-1. ]
 [ 1. ]
 [ 0.5]]

[[-1.5]
```

```
 [ 0.5]
 [ 0.5]]

[[-1.]
 [ 1.]
 [ 1.]]

[[-1.5]
 [ 1. ]
 [ 0.5]]

[[-1. ]
 [ 1.5]
 [ 1. ]]

[[-1.5]
 [ 1.5]
 [ 0.5]]

[[-2. ]
 [ 1. ]
 [ 0.5]]

[[-1.5]
 [ 1.5]
 [ 1. ]]

[[-2.]
 [ 1.]
 [ 1.]]

[[-1.5]
 [ 1.5]
 [ 1.5]]

[[-2. ]
 [ 1.5]
 [ 1. ]]
```

In [ ]:

```
W
```

Out[ ]:

```
array([[-2. ],
       [ 1.5],
       [ 1. ]])
```

In [ ]:

```
weight_vector
```

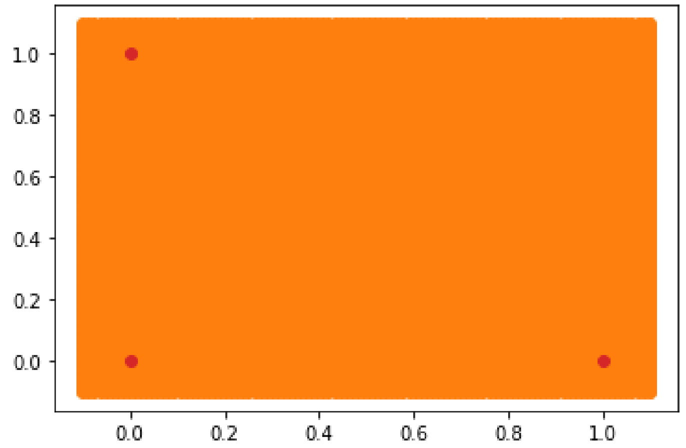
Out[ ]:

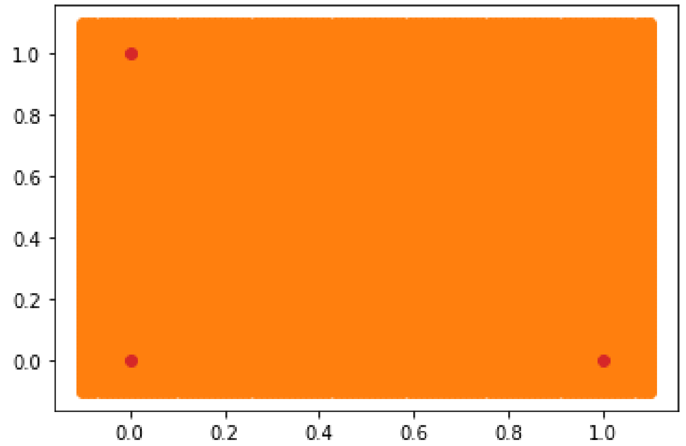
```
array([[[ 0. ],
        [ 0. ],
        [ 0. ]],

       [[ 0.5],
        [ 0.5],
        [ 0.5]],

       [[ 0. ],
        [ 0.5],
        [ 0. ]],

       [[-0.5],
        [ 0. ],
        [ 0. ]],

       [[ 0. ],
        [ 0.5],
        [ 0.5]],

       [[-0.5],
        [ 0.5],
        [ 0. ]],
```

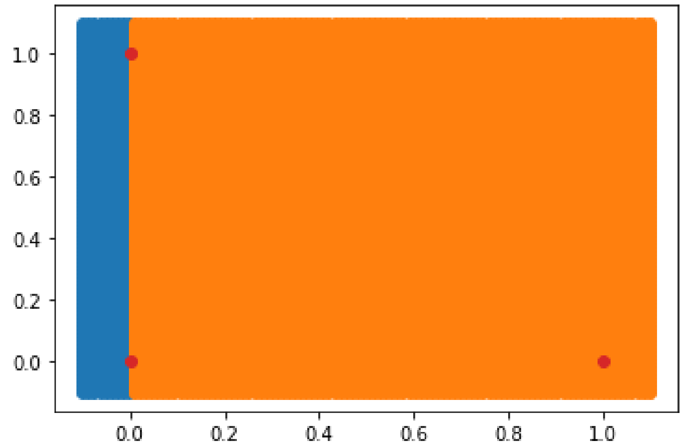
```
        [[-1. ],
         [ 0. ],
         [ 0. ]],

        [[-0.5],
         [ 0.5],
         [ 0.5]],

        [[-1. ],
         [ 0.5],
         [ 0. ]],

        [[-0.5],
         [ 1. ],
         [ 0.5]],

        [[-1. ],
         [ 1. ],
         [ 0. ]],

        [[-1.5],
         [ 0.5],
         [ 0. ]],

        [[-1. ],
         [ 1. ],
         [ 0.5]],

        [[-1.5],
         [ 0.5],
         [ 0.5]],

        [[-1. ],
         [ 1. ],
         [ 1. ]],

        [[-1.5],
         [ 1. ],
         [ 0.5]],

        [[-1. ],
         [ 1.5],
         [ 1. ]],

        [[-1.5],
         [ 1.5],
         [ 0.5]],

        [[-2. ],
         [ 1. ],
         [ 0.5]],

        [[-1.5],
         [ 1.5],
         [ 1. ]],

        [[-2. ],
         [ 1. ],
         [ 1. ]],

        [[-1.5],
         [ 1.5],
         [ 1.5]],

        [[-2. ],
         [ 1.5],
         [ 1. ]]]])
```

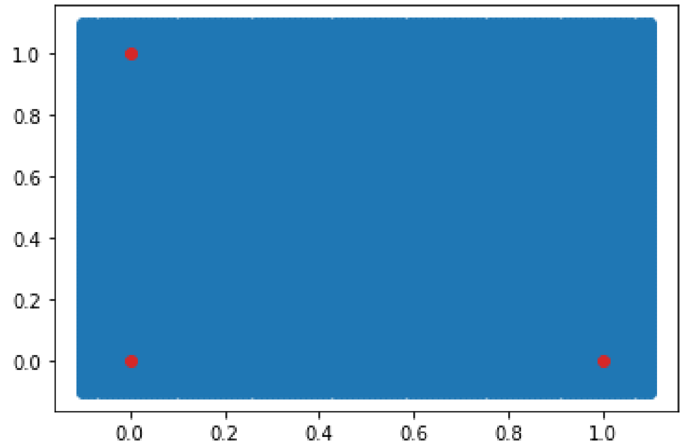
In [ ]:

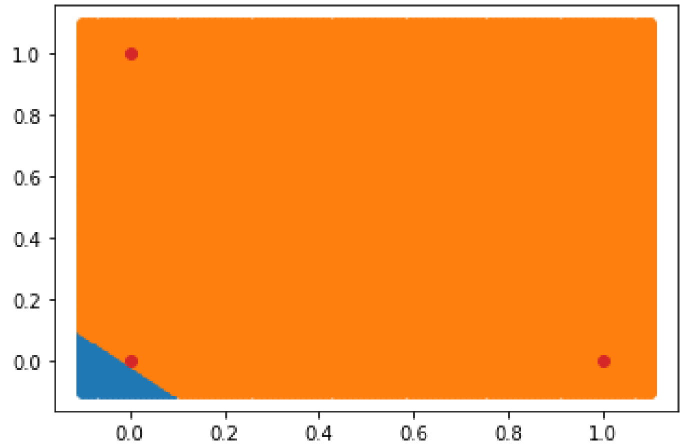
```
for w in weight_vector:
  decision_boundary(w, X, ans)
```

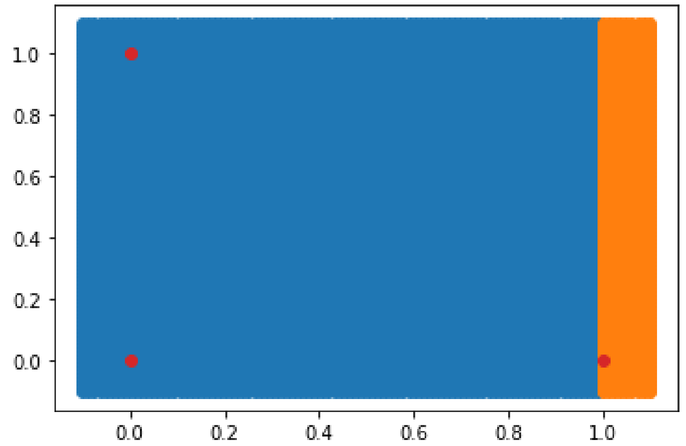

1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0


1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0


1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0


1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0


1.0
0.8
0.6
0.4
0.2
0.0
0.0 0.2 0.4 0.6 0.8 1.0


1.0
0.8
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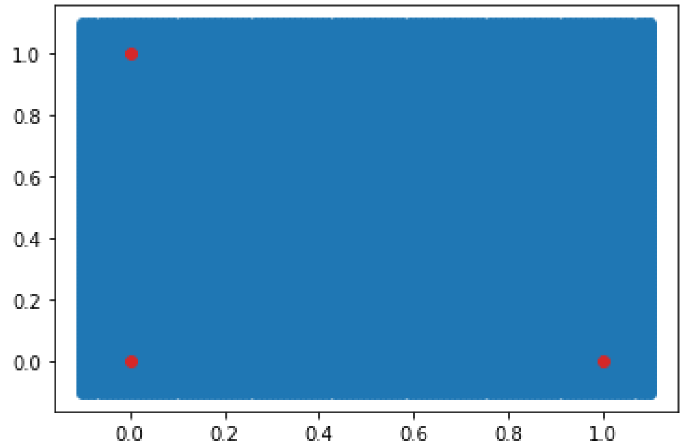
1.0
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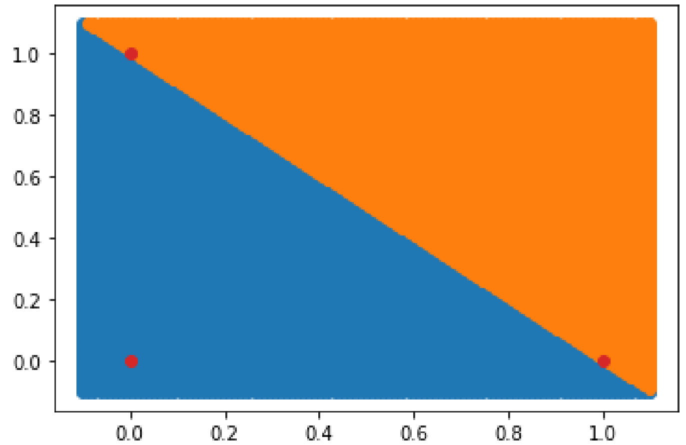
1.0
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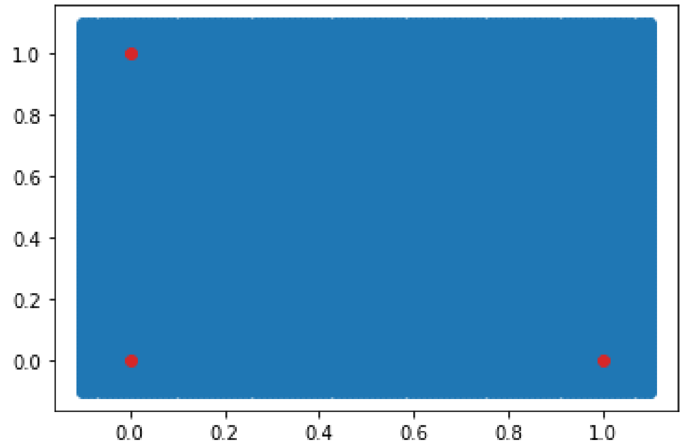
1.0
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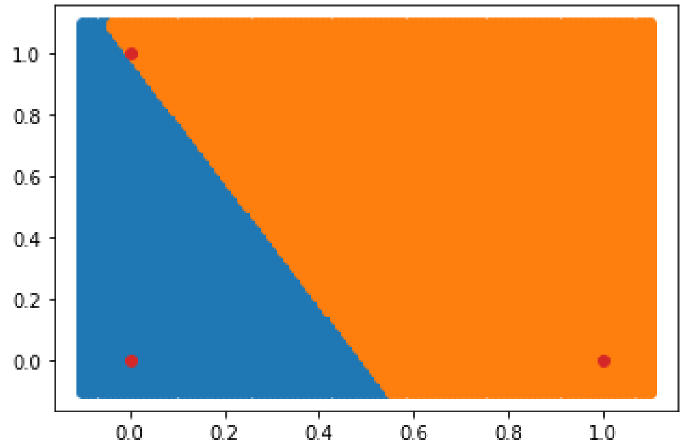
0.0
0.0 0.2 0.4 0.6 0.8 1.0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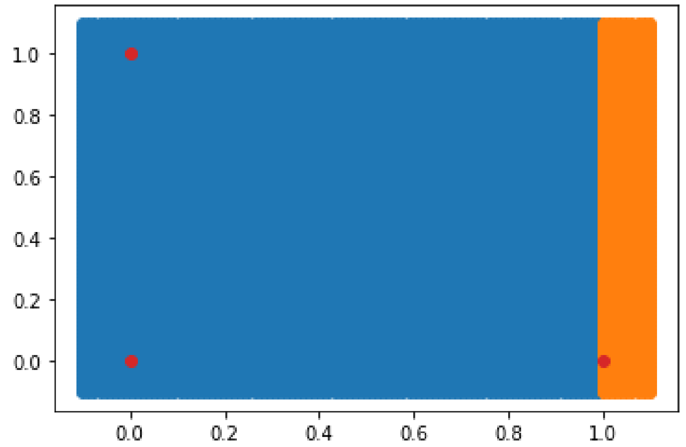
1.0
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0.0
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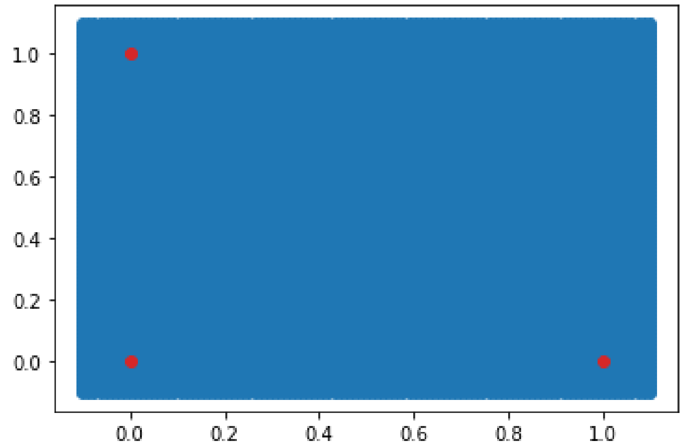
1.0
0.8
0.6
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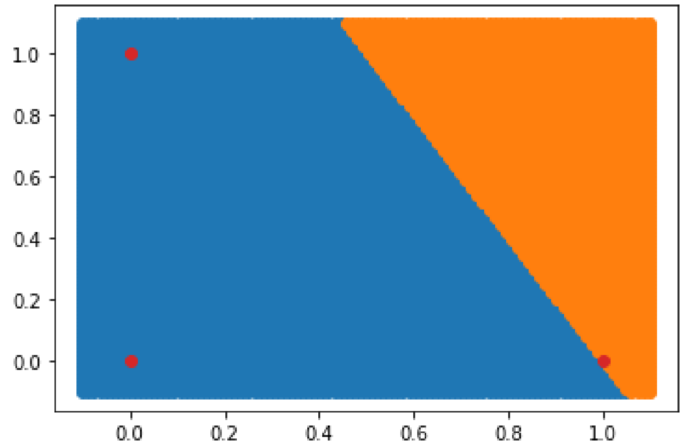
0.0
0.0 0.2 0.4 0.6 0.8 1.0


1.0
0.8
0.6
0.4
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0.0
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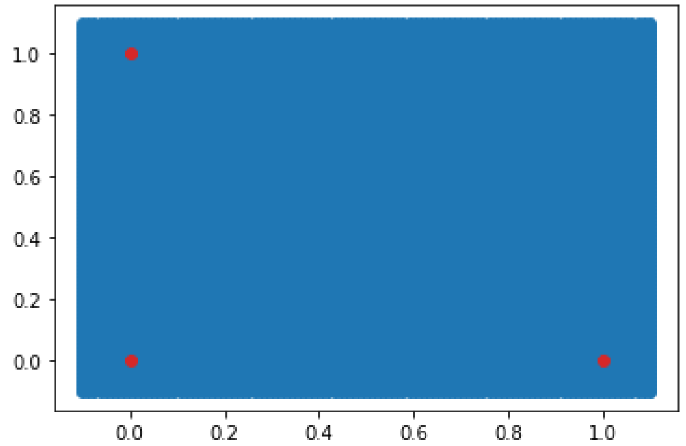
0.4
0.6
0.8
1.0


1.0
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0.2
0.4
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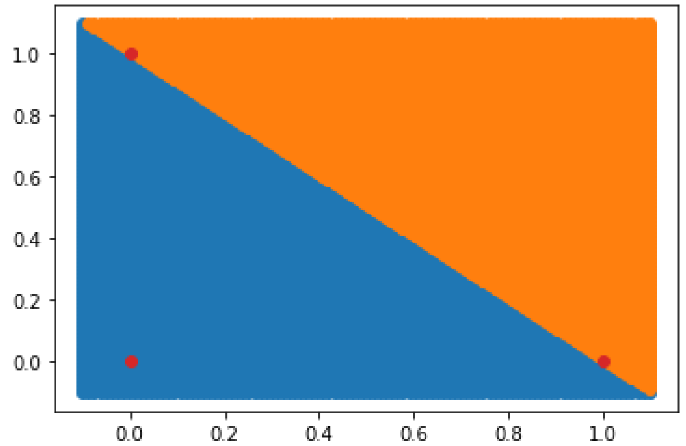
1.0


1.0
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0.6
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0.0
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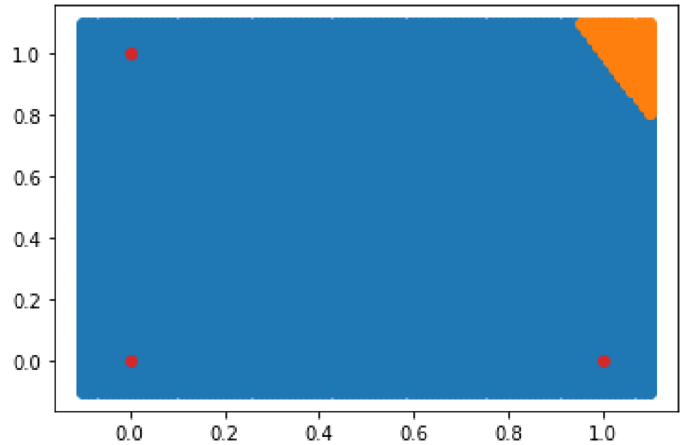
0.2
0.4
0.6
0.8
1.0


1.0
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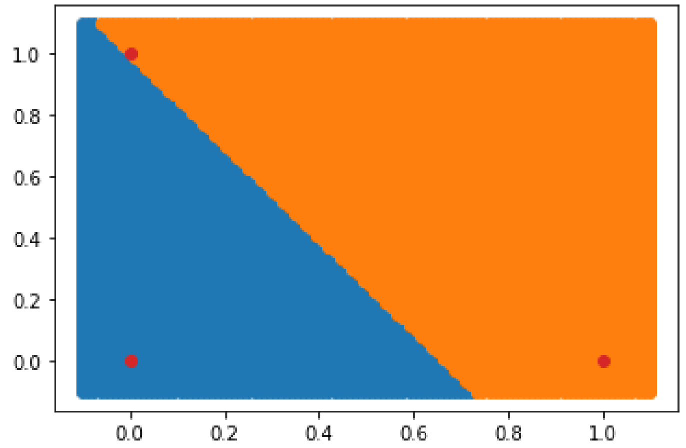
0.0
0.0
0.2
0.4
0.6
0.8
1.0


1.0
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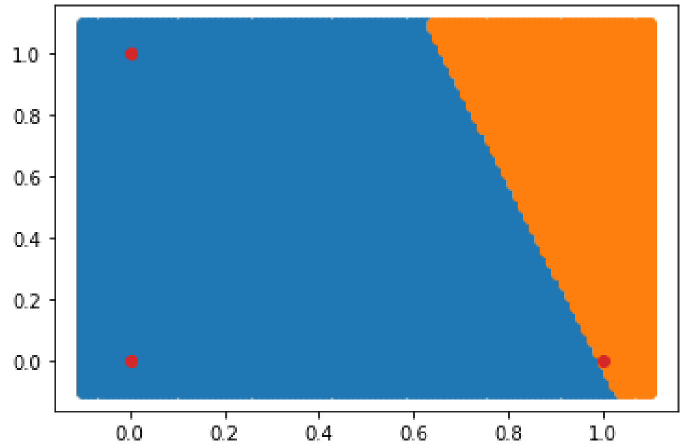
0.0 0.2 0.4 0.6 0.8 1.0


1.0
0.8
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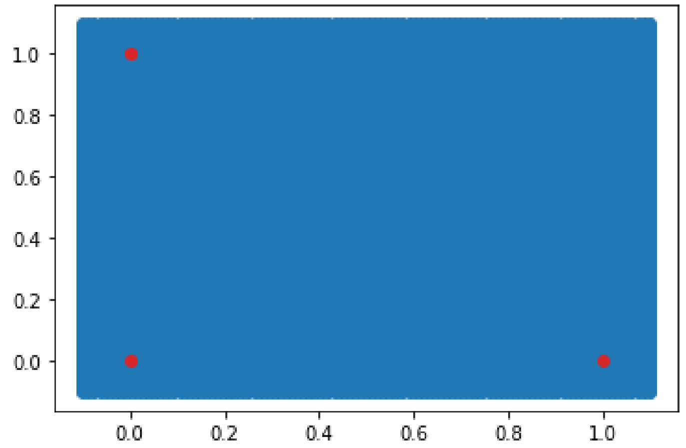
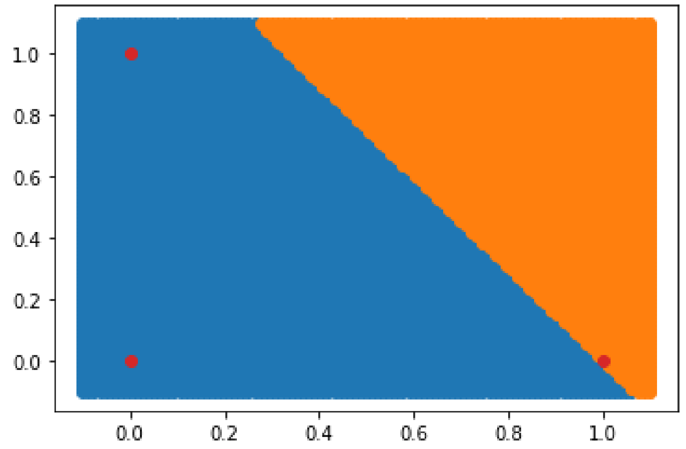
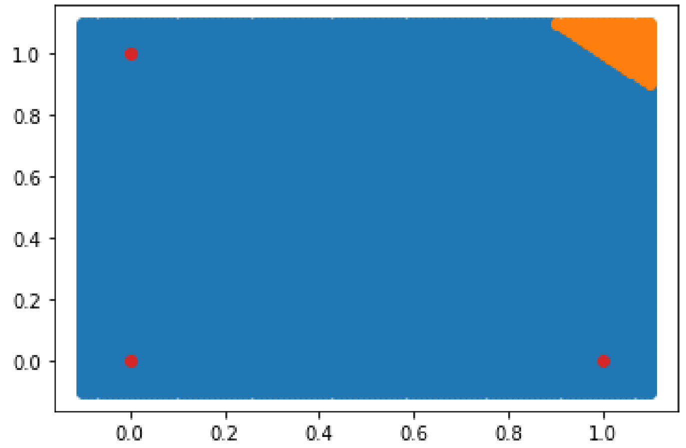
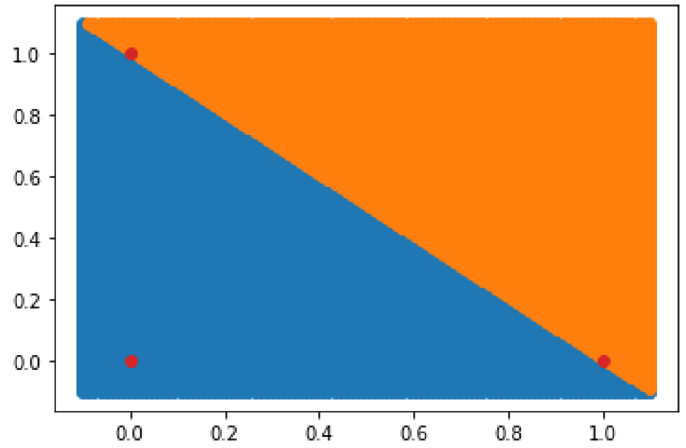
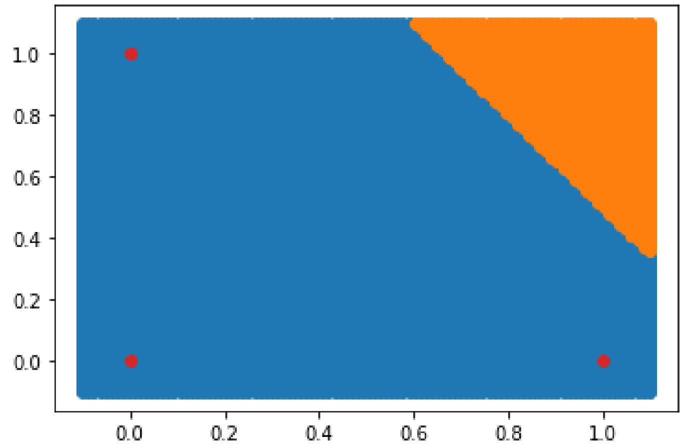
0.4
0.2
0.0
0.0 0.2 0.4 0.6 0.8 1.0


1.0
0.8
0.6
0.4
0.2
0.0
0.0 0.2 0.4 0.6 0.8 1.0


1.0
0.8
0.6
0.4
0.2
0.0
0.0 0.2 0.4 0.6 0.8 1.0







SVM

Defining the necessary functions used

In [ ]:

```
def SVM(C1, C2):
  x1 = np.array(C1.to_numpy())
  x2 = np.array(C2.to_numpy())

  x = np.vstack((x1, x2))
  y = np.hstack((np.ones(len(x1)), np.ones(len(x2)) * -1))

  X = np.dot(x, np.transpose(x))
  Y = np.outer(y, y)

  n = X.shape[0]

  H = matrix(Y * X)
  f = matrix(np.ones(n) * -1)
  A = matrix(np.diag(np.ones(n) * -1))
  a = matrix(np.zeros(n))
```

```
    B = matrix(y, (1, n))
    b = matrix(0.0)

    # solve QP problem
    solution = solvers.qp(H, f, A, a, B, b)
    print('')

    # Lagrange multipliers
    t1 = np.ravel(solution['x'])

    # Support vectors have non zero lagrange multipliers
    for i in range(0, len(t1)):
      if(t1[i] < 1e-4):
        t1[i] = 0.0
      else:
        t1[i] = round(t1[i], 4)

    t1 = np.array(t1)
    print(t1)

    # Weight vector
    w = np.zeros(X.shape[1])
    for i in range(X.shape[1]):
      w[i] = t1[i] * y[i]

    w = np.dot(w, x)

    # Intercept
    b = 0
    for i in range(len(t1)):
      if(t1[i] != 0.0):
        b = round(1 - np.dot(w, np.transpose(x[i])), 4)
        break

    return w, b
```

Code

In [ ]:
```
X = [[0, 0], [0, 1], [1, 0]]

C1 = pd.DataFrame(X)
C1
```

Out[ ]:

	0	1
0	0	0
1	0	1
2	1	0

In [ ]:
```
X = [[1, 1]]

C2 = pd.DataFrame(X)
C2
```

Out[ ]:

	0	1
0	1	1

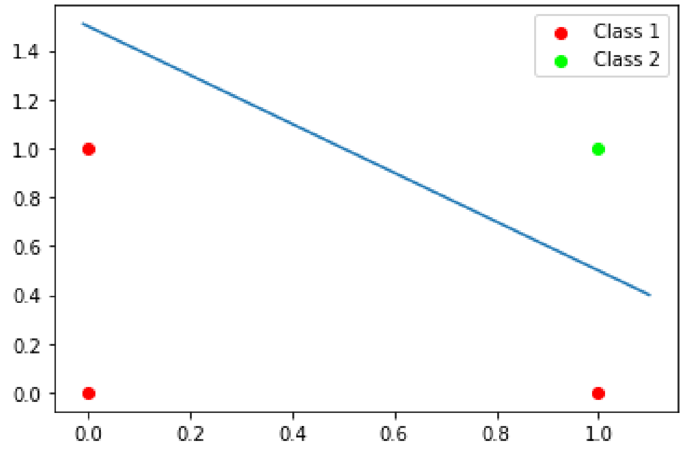
In [ ]:
```
w, b = SVM(C1, C2)


print(w)
```

```
print('')
print(b)
```

```
     pcost       dcost       gap    pres   dres
 0: -1.7500e+00 -4.2500e+00  8e+00  2e+00  2e+00
 1: -3.6653e+00 -4.3941e+00  1e+00  3e-01  3e-01
 2: -3.9945e+00 -4.0256e+00  3e-02  2e-15  3e-16
 3: -3.9999e+00 -4.0003e+00  3e-04  1e-15  4e-16
 4: -4.0000e+00 -4.0000e+00  3e-06  2e-15  4e-16
Optimal solution found.

[0. 2. 2. 4.]
[-2. -2.]

3.0
```

In [ ]:

```
x1 = np.arange(-0.01, 1.1, 0.01)

plt.scatter(C1.iloc[:, [0]], C1.iloc[:, [1]], label = 'Class 1', color = '#FF0000')
plt.scatter(C2.iloc[:, [0]], C2.iloc[:, [1]], label = 'Class 2', color = '#00FF00')
plt.legend()

slope, c = -w[0] / w[1], -b / w[1]
plt.plot(x1, slope * x1 + c)
plt.show()
```



Q2. Train a **single perceptron and SVM** to learn the two classes in the following table.

x_1	x_2	ω
2	2	1
-1	-3	0
-1	2	1
0	-1	0
1	3	1
-1	-2	0
1	-2	0
-1	-1	1

where x_1 and x_2 are the inputs and ω is the target class. Assume that all the weights of the perceptron are initialized as 0 with learning rate 0.01 and 0.5 separately. Also, tabulate the number of iterations required to converge the perception algorithm with these two learning rates.

Importing the necessary libraries

In [ ]:

```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
import seaborn as sns
from cvxopt import matrix, solvers
```

Single Perceptron

Defining the necessary functions used

In [ ]:

```
def decision_plot(w, x_min, x_max, y_min, y_max):
    inp_0 = []
    inp_1 = []

    x_min = x_min - np.abs(((x_max - x_min) / 10))
    y_min = y_min - np.abs(((y_max - y_min) / 10))

    x_max = x_max + np.abs(((x_max - x_min) / 10))
    y_max = y_max + np.abs(((y_max - y_min) / 10))

    for i in np.arange(x_min, x_max, np.abs(x_max - x_min) / 100):
        for j in np.arange(y_min, y_max, np.abs(x_max - x_min) / 100):
            y_pred = np.dot(np.array([[1, i, j]]), w)

            if y_pred < 0:
                inp_0.append(np.array((i, j)))

            else:
                inp_1.append(np.array((i, j)))

    return inp_0, inp_1
```

In [ ]:

```
def perceptron_training_alg(X, Y, learning_rate):
```

```
  # Adding Bias to the input and Randomly initializing weights
  num_samples, num_features = X.shape

  X = np.hstack((np.ones((num_samples, 1)), X))
  W = np.zeros(num_features + 1).reshape((-1, 1))

  weight_vector = []
  weight_vector.append(W)

  print(W, "\n")

  # Running the algorithm until all the data points are correctly classified
  samples_misclassified = True
  cnt=0
  while samples_misclassified and cnt < 100000:
    # Initially we'll assume that there are no misclassified samples
    samples_misclassified = False

    for index, x in enumerate(X):
      if cnt >= 100000:
        return W, np.array(weight_vector)

      x = Y[index] * x

      if np.dot(x.reshape((1, -1)), W) <= 0: # if samples misclassified
        samples_misclassified = True
        W = W + (learning_rate * x.reshape((-1, 1)))     # Gradient Descent Step
        weight_vector.append(W)

        print(W, "\n")
        cnt += 1

  return W, np.array(weight_vector)
```

In [ ]:

```
def sorter(X, Y):
  X_new0 = []
  Y_new0 = []

  X_new1 = []
  Y_new1 = []

  for x, y in zip(X, Y):
    if y == 0:
      Y_new0.append(y)
      X_new0.append(x)

    else:
      Y_new1.append(y)
      X_new1.append(x)

  X0 = []
  Y0 = []

  X1 = []
  Y1 = []

  if len(X_new0) > 0:
    X0 = np.stack(X_new0)
    Y0 = np.stack(Y_new0)

  elif len(X_new1) > 0:
    X1 = np.stack(X_new1)
    Y1 = np.stack(Y_new1)

  if len(X0) > 0 and len(X1) > 0:
    X = np.concatenate((X0, X1))
    Y = np.concatenate((Y0, Y1))
```

```
  elif len(X0) > 0:
    X = X0
    Y = Y0

  else:
    X = X1
    Y = Y1

  return X, Y
```

In [ ]:

```
def decision_boundary(W, X, Y):
  X, Y = sorter(X, Y)
  num_samples, num_features = X.shape
  np2 = np.hstack((np.ones((num_samples, 1)), X))

  X=np2

  x1, y1 = decision_plot(W, min(X[:,1]), max(X[:,1]), min(X[:,2]), max(X[:,2]))

  plt.scatter([i[0] for i in x1], [i[1] for i in x1])
  plt.scatter([i[0] for i in y1], [i[1] for i in y1])
  plt.scatter(X[:np.argmax(Y), 1], X[:np.argmax(Y), 2])
  plt.scatter(X[np.argmax(Y):, 1], X[np.argmax(Y):, 2])
  plt.show()
```

Code

In [ ]:

```
X = np.array([[2, 2], [-1, -3], [-1, 2], [0, -1], [1, 3], [-1, -2], [1, -2], [-1, -1]])
Y = [[1], [-1], [1], [-1], [1], [-1], [-1], [1]]
ans = [[1], [0], [1], [0], [1], [0], [0], [1]]

len(X)
```

Out[ ]: 8

Learning Rate = 0.01

In [ ]:

```
W1, weight_vector1 = perceptron_training_alg(X, Y, 0.01)
```

```
[[0.]
 [0.]
 [0.]]

[[0.01]
 [0.02]
 [0.02]]

[[0.02]
 [0.01]
 [0.01]]

[[0.01]
 [0.01]
 [0.02]]

[[0.02]
 [0.  ]
 [0.01]]

[[0.01]
 [0.  ]
 [0.02]]

[[ 0.02]
 [-0.01]
 [ 0.01]]
```

```
[[ 0.01]
 [-0.01]
 [ 0.02]]

[[ 0.02]
 [-0.02]
 [ 0.01]]

[[0.03]
 [0.  ]
 [0.03]]

[[0.02]
 [0.  ]
 [0.04]]

[[ 0.03]
 [-0.01]
 [ 0.03]]

[[ 0.02]
 [-0.01]
 [ 0.04]]

[[ 0.03]
 [-0.02]
 [ 0.03]]

[[ 0.02]
 [-0.02]
 [ 0.04]]

[[ 0.03]
 [-0.03]
 [ 0.03]]

[[ 0.02]
 [-0.03]
 [ 0.04]]
```

In [ ]:

```
W1
```

Out[ ]:

```
array([[ 0.02],
       [-0.03],
       [ 0.04]])
```

In [ ]:

```
weight_vector1
```

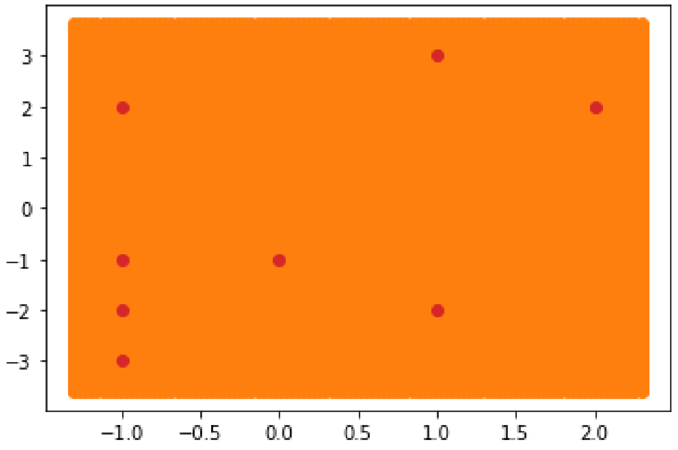
Out[ ]:

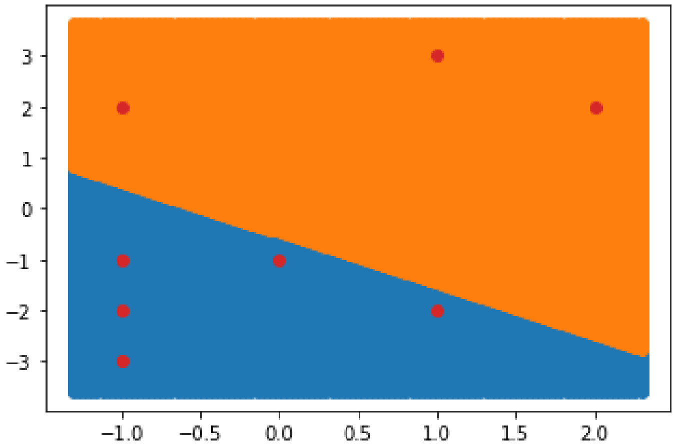
```
array([[[ 0.  ],
        [ 0.  ],
        [ 0.  ]],

       [[ 0.01],
        [ 0.02],
        [ 0.02]],

       [[ 0.02],
        [ 0.01],
        [ 0.01]],

       [[ 0.01],
        [ 0.01],
        [ 0.02]],

       [[ 0.02],
        [ 0.  ],
        [ 0.01]],

       [[ 0.01],
        [ 0.  ],
        [ 0.02]],
```

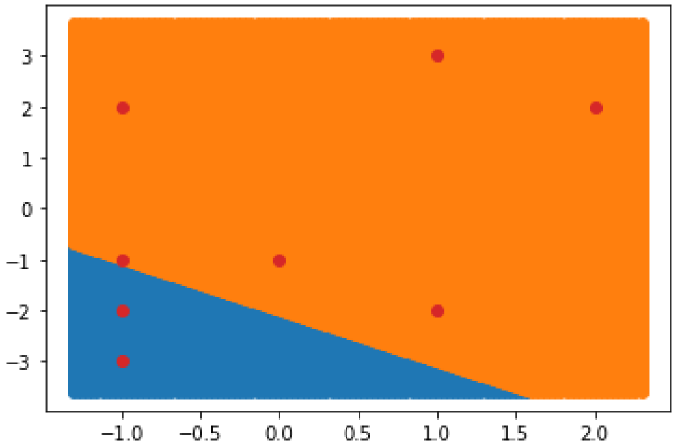
```
[[ 0.02],
 [-0.01],
 [ 0.01]],

[[ 0.01],
 [-0.01],
 [ 0.02]],

[[ 0.02],
 [-0.02],
 [ 0.01]],

[[ 0.03],
 [ 0.  ],
 [ 0.03]],

[[ 0.02],
 [ 0.  ],
 [ 0.04]],

[[ 0.03],
 [-0.01],
 [ 0.03]],

[[ 0.02],
 [-0.01],
 [ 0.04]],

[[ 0.03],
 [-0.02],
 [ 0.03]],

[[ 0.02],
 [-0.02],
 [ 0.04]],

[[ 0.03],
 [-0.03],
 [ 0.03]],

[[ 0.02],
 [-0.03],
 [ 0.04]]]])
```

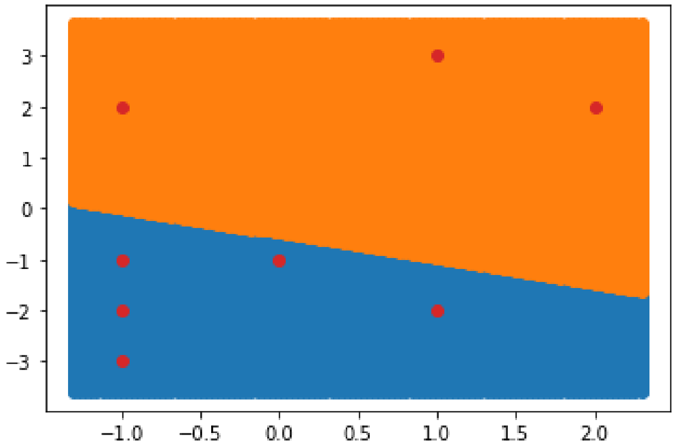
In [ ]:

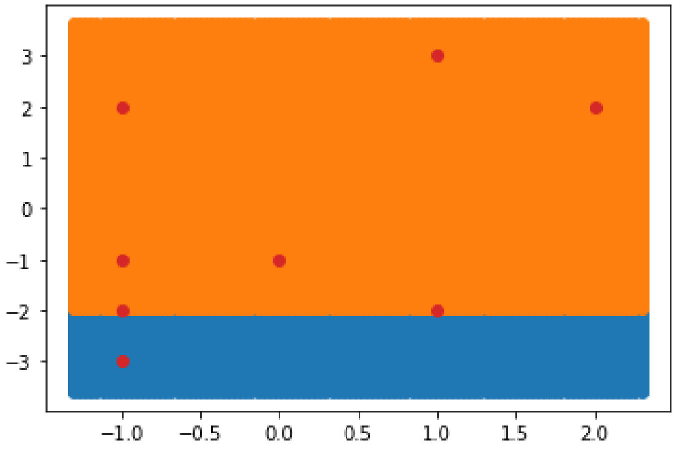
```
for w in weight_vector1:
  decision_boundary(w, X, ans)
```

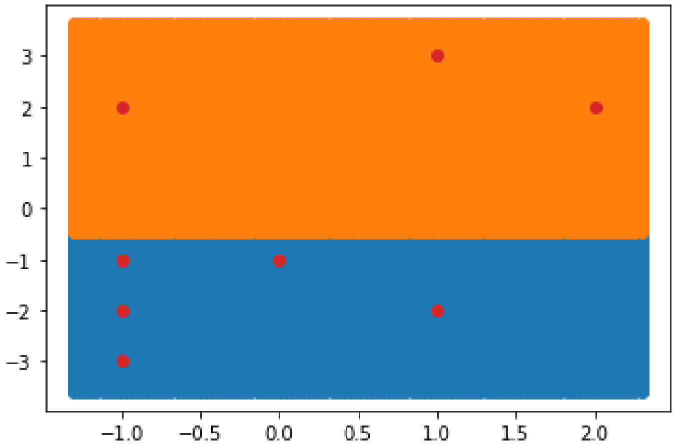



3
2
1
0
−1
−2
−3
−1.0 −0.5 0.0 0.5 1.0 1.5 2.0

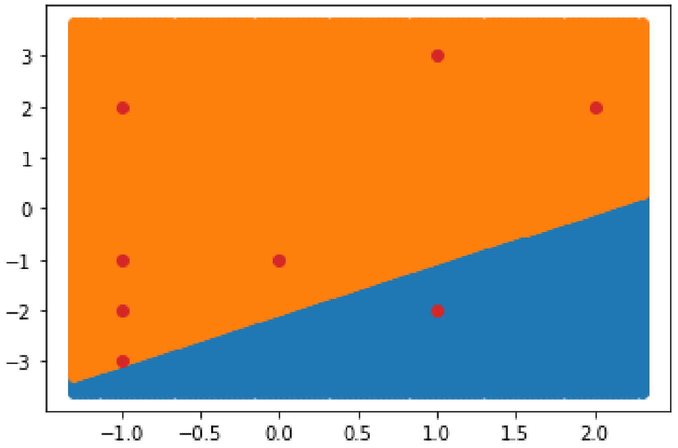

3
2
1
0
−1
−2
−3
−1.0 −0.5 0.0 0.5 1.0 1.5 2.0


3
2
1
0
−1
−2
−3
−1.0 −0.5 0.0 0.5 1.0 1.5 2.0

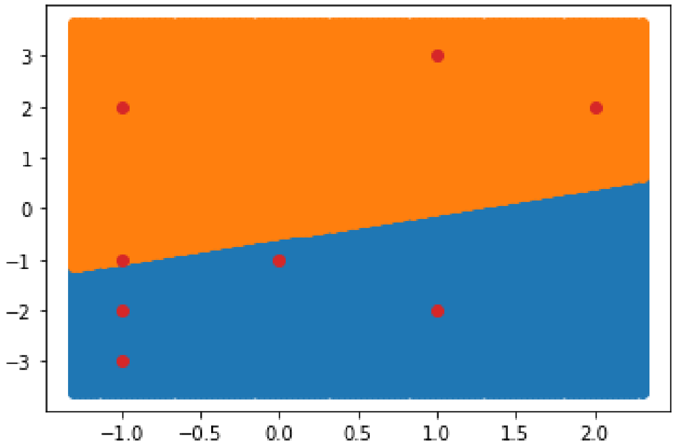

3
2
1
0
−1
−2
−3
−1.0 −0.5 0.0 0.5 1.0 1.5 2.0



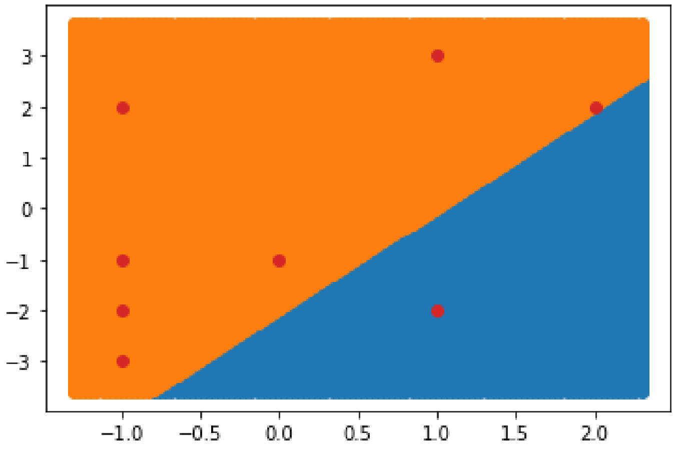
3
2
1
0
−1
−2
−3
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0



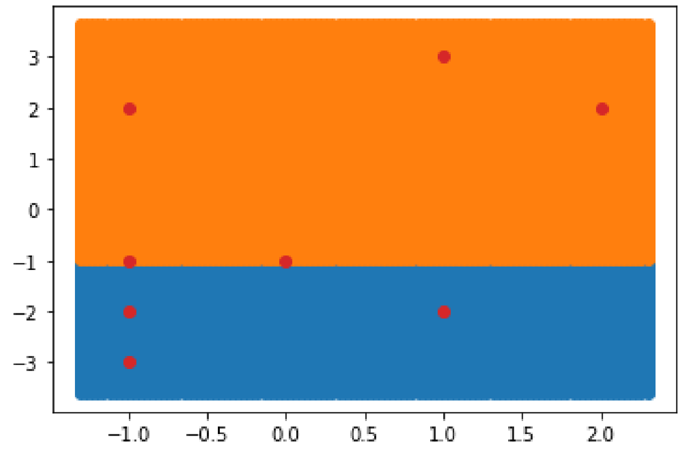
3
2
1
0
−1
−2
−3
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0

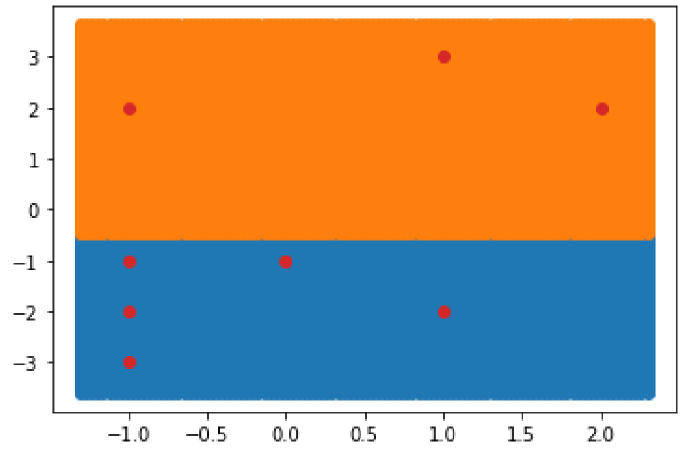


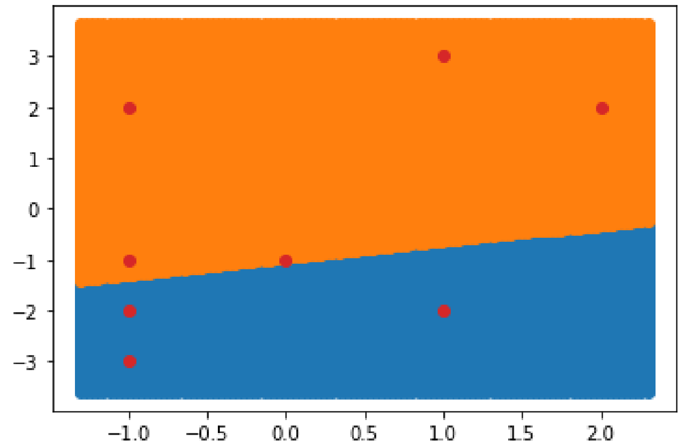
3
2
1
0
−1
−2
−3
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0

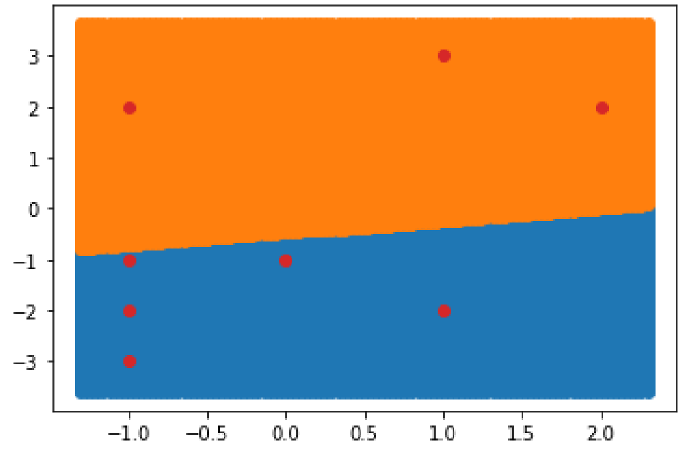


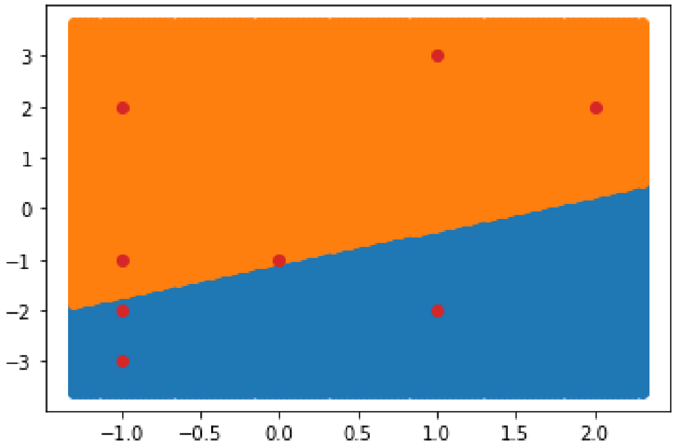
3
2
1
0
−1
−2
−3
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0

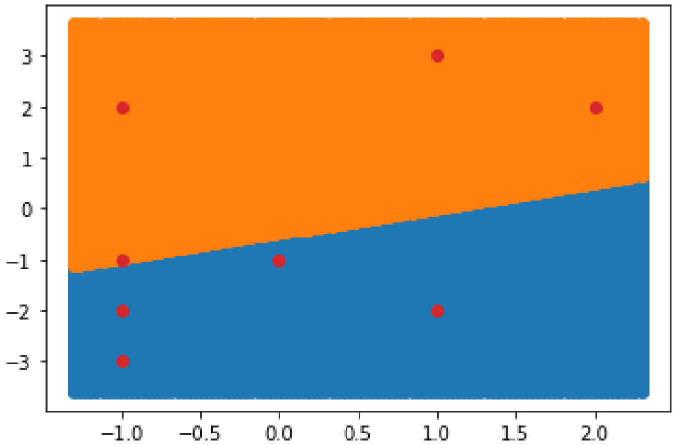


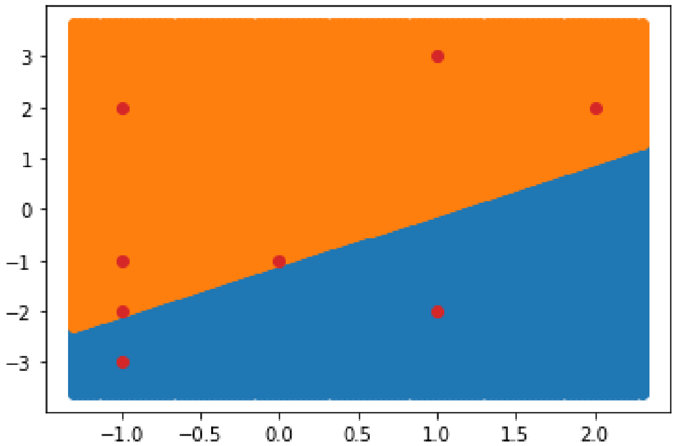


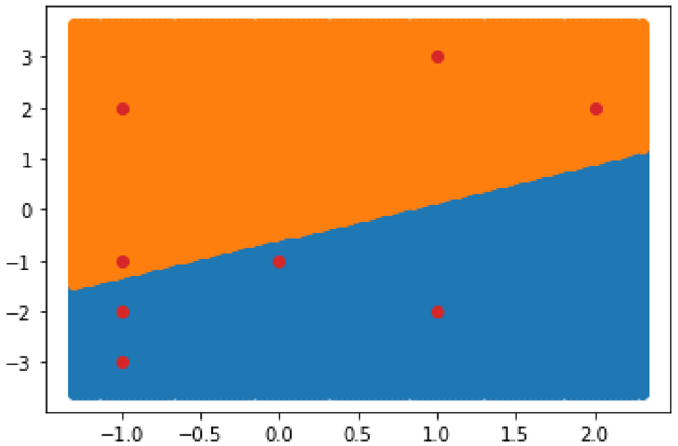












Learning Rate = 0.5

In [ ]:
```
W2, weight_vector2 = perceptron_training_alg(X,Y,0.5)
```

```
[[0.]
```

```
 [0.]
 [0.]]

[[0.5]
 [1. ]
 [1. ]]

[[1. ]
 [0.5]
 [0.5]]

[[0.5]
 [0.5]
 [1. ]]

[[1. ]
 [0. ]
 [0.5]]

[[0.5]
 [0. ]
 [1. ]]

[[ 1. ]
 [-0.5]
 [ 0.5]]

[[0.5]
 [0. ]
 [2. ]]

[[ 1. ]
 [-0.5]
 [ 1.5]]

[[ 1.5]
 [-1. ]
 [ 1. ]]

[[ 1. ]
 [-1. ]
 [ 1.5]]
```

In [ ]:

```
W2
```

Out[ ]:

```
array([[ 1. ],
       [-1. ],
       [ 1.5]])
```

In [ ]:

```
weight_vector2
```

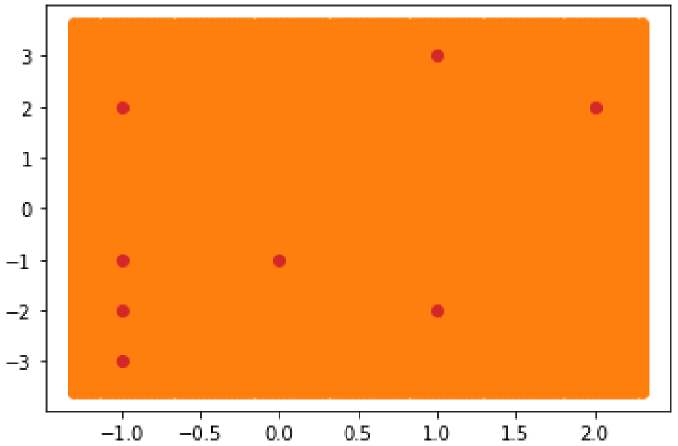
Out[ ]:

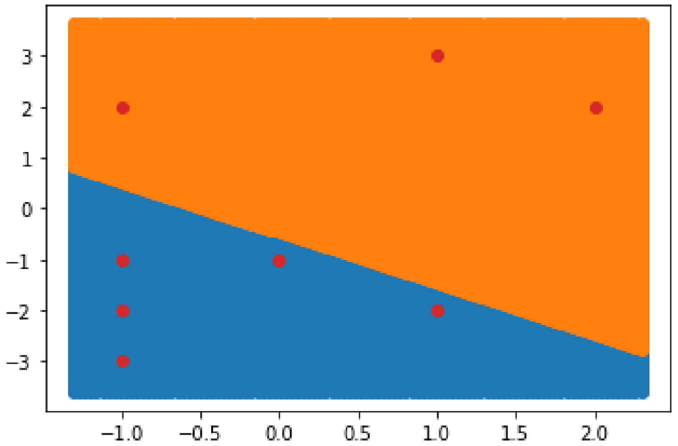
```
array([[[ 0. ],
        [ 0. ],
        [ 0. ]],

       [[ 0.5],
        [ 1. ],
        [ 1. ]],

       [[ 1. ],
        [ 0.5],
        [ 0.5]],

       [[ 0.5],
        [ 0.5],
        [ 1. ]],

       [[ 1. ],
        [ 0. ],
        [ 0.5]],
```

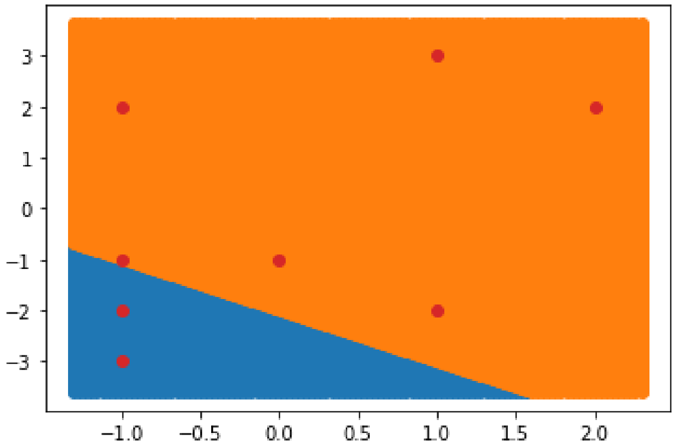
```
 [[ 0.5],
  [ 0. ],
  [ 1. ]],

 [[ 1. ],
  [-0.5],
  [ 0.5]],

 [[ 0.5],
  [ 0. ],
  [ 2. ]],

 [[ 1. ],
  [-0.5],
  [ 1.5]],

 [[ 1.5],
  [-1. ],
  [ 1. ]],

 [[ 1. ],
  [-1. ],
  [ 1.5]]]])
```

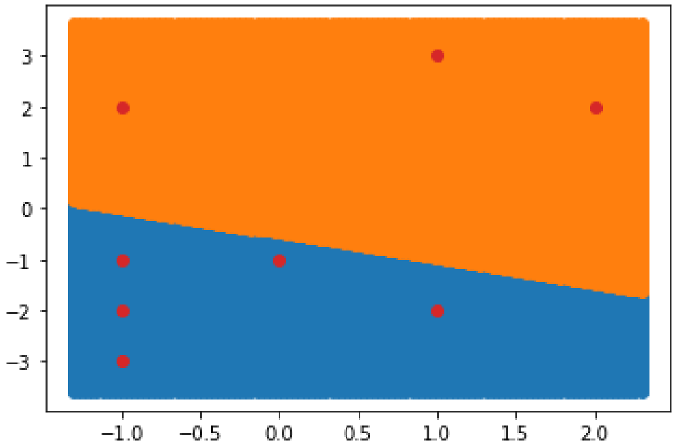
In [ ]:

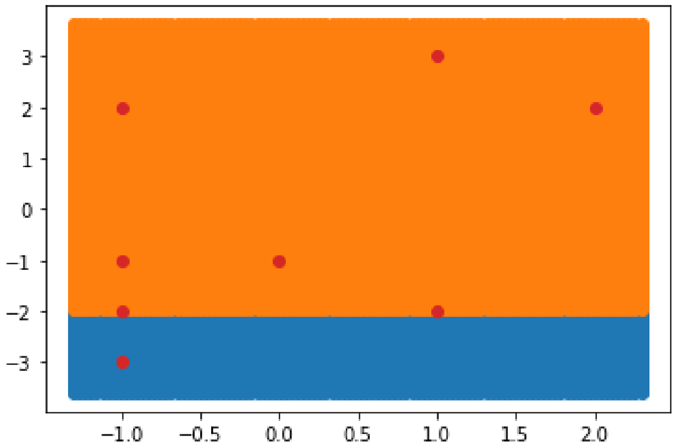
```
for w in weight_vector2:
  decision_boundary(w, X, ans)
```

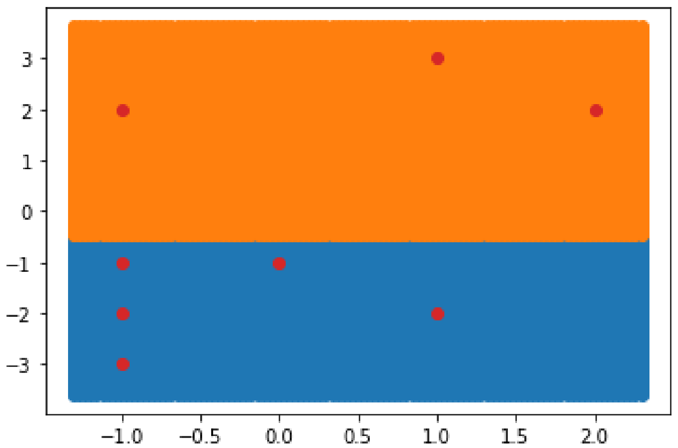


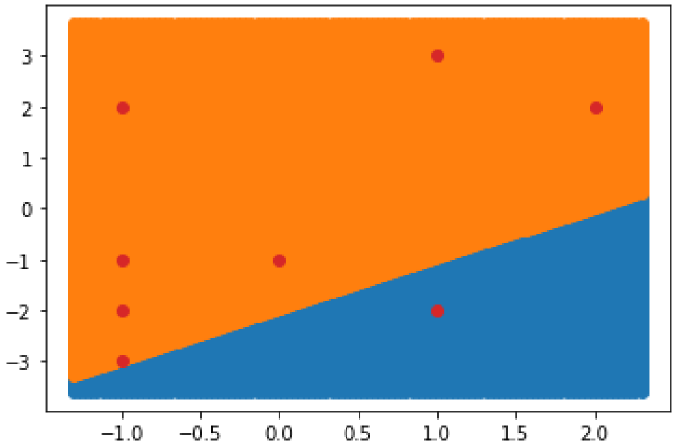



3
2
1
0
−1
−2
−3
−1.0 −0.5 0.0 0.5 1.0 1.5 2.0

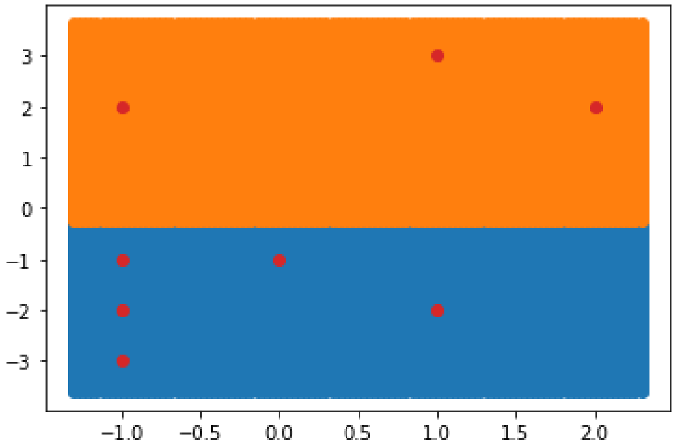

3
2
1
0
−1
−2
−3
−1.0 −0.5 0.0 0.5 1.0 1.5 2.0


3
2
1
0
−1
−2
−3
−1.0 −0.5 0.0 0.5 1.0 1.5 2.0

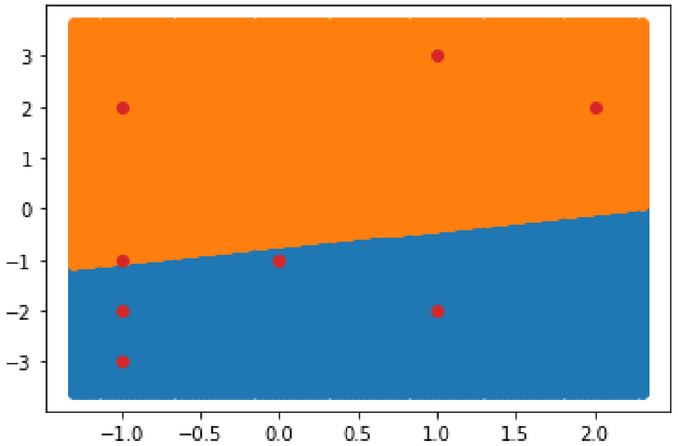

3
2
1
0
−1
−2
−3
−1.0 −0.5 0.0 0.5 1.0 1.5 2.0



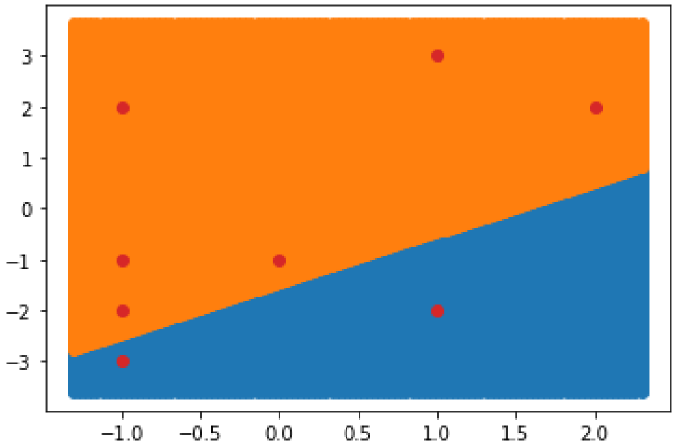
3
2
1
0
−1
−2
−3
−1.0 −0.5 0.0 0.5 1.0 1.5 2.0



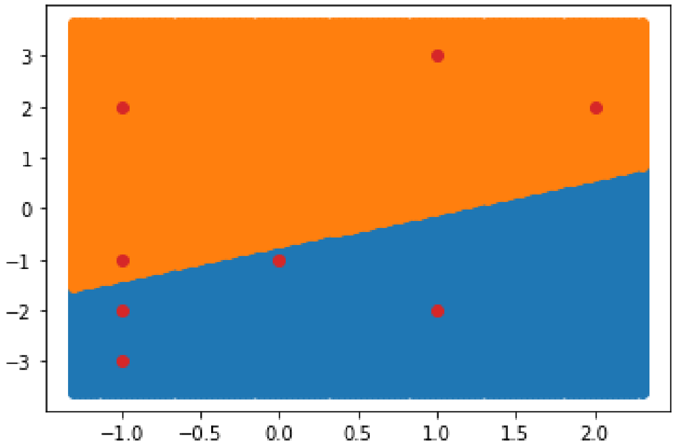
3
2
1
0
−1
−2
−3
−1.0 −0.5 0.0 0.5 1.0 1.5 2.0



3
2
1
0
−1
−2
−3
−1.0 −0.5 0.0 0.5 1.0 1.5 2.0



3
2
1
0
−1
−2
−3
−1.0 −0.5 0.0 0.5 1.0 1.5 2.0



SVM

Defining the necessary functions used

In [ ]:

```
def SVM(C1, C2):
  x1 = np.array(C1.to_numpy())
  x2 = np.array(C2.to_numpy())

  x = np.vstack((x1, x2))
  y = np.hstack((np.ones(len(x1)), np.ones(len(x2)) * -1))

  X = np.dot(x, np.transpose(x))
  Y = np.outer(y, y)

  n = X.shape[0]

  H = matrix(Y * X)
  f = matrix(np.ones(n) * -1)
  A = matrix(np.diag(np.ones(n) * -1))
  a = matrix(np.zeros(n))
  B = matrix(y, (1, n))
  b = matrix(0.0)

  # solve QP problem
  solution = solvers.qp(H, f, A, a, B, b)
  print('')

  # Lagrange multipliers
  t1 = np.ravel(solution['x'])

  # Support vectors have non zero lagrange multipliers
  for i in range(0, len(t1)):
    if(t1[i] < 1e-4):
      t1[i] = 0.0
    else:
      t1[i] = round(t1[i], 4)

  t1 = np.array(t1)
  print(t1)

  # Weight vector
  w = np.zeros(X.shape[1])
  for i in range(X.shape[1]):
    w[i] = t1[i] * y[i]

  w = np.dot(w, x)

  # Intercept
  b = 0
  for i in range(len(t1)):
    if(t1[i] != 0.0):
```

```
            b = round(1 - np.dot(w, np.transpose(x[i])), 4)
            break

    return w, b
```

Code

In [ ]:
```
X = [[-1, -3], [0, -1], [-1, -2], [1, -2]]

C1 = pd.DataFrame(X)
C1
```

Out[ ]:

	0	1
0	-1	-3
1	0	-1
2	-1	-2
3	1	-2

In [ ]:
```
X = [[2, 2], [-1, 2], [1, 3], [-1, -1]]

C2 = pd.DataFrame(X)
C2
```

Out[ ]:

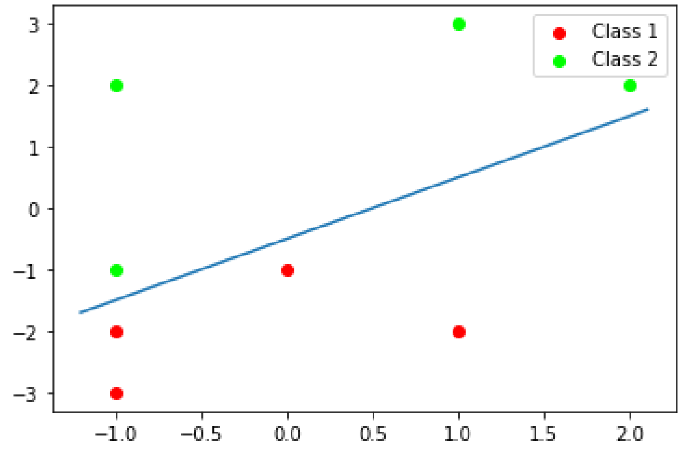
	0	1
0	2	2
1	-1	2
2	1	3
3	-1	-1

In [ ]:
```
w, b = SVM(C1, C2)


print(w)
print('')
print(b)
```

```
     pcost       dcost       gap    pres   dres
 0: -2.2863e+00 -5.2491e+00  2e+01  3e+00  2e+00
 1: -6.6436e+00 -7.0845e+00  6e+00  1e+00  7e-01
 2: -3.2806e+00 -4.6847e+00  1e+00  1e-15  1e-14
 3: -3.9727e+00 -4.0106e+00  4e-02  1e-15  3e-15
 4: -3.9997e+00 -4.0001e+00  4e-04  1e-15  9e-16
 5: -4.0000e+00 -4.0000e+00  4e-06  9e-16  1e-15
Optimal solution found.

[0.     3.3296 0.6704 0.     0.4432 0.     0.     3.5568]
[ 2. -2.]

-1.0
```

In [ ]:
```
x1 = np.arange(-1.2, 2.2, 0.1)

plt.scatter(C1.iloc[:, [0]], C1.iloc[:, [1]], label = 'Class 1', color = '#FF0000')
plt.scatter(C2.iloc[:, [0]], C2.iloc[:, [1]], label = 'Class 2', color = '#00FF00')
plt.legend()

slope, c = -w[0] / w[1], -b / w[1]
```

```
plt.plot(x1, slope * x1 + c)
plt.show()
```



Q3. In the given I set of images from poly1.png to poly14.png, let poly1 to poly 7 belong to class 1 and poly 8 to poly 14 belong to class 2. Assume that all the weights of the perceptron are initialized as 0 with the learning rate of 0.01.

- Identify two discriminant features x_1 and x_2 for the two target classes $\omega = \{\omega_1, \omega_2\}$. Here, ω_1 - class 1 and ω_2 - class 2.
- Generate an input feature vector X for all the images mapping them to a corresponding taget classes ω_i, where $i \in (1, 2)$.
- Train a **single perceptron and SVM** to learn the feature vector X mapping to ω.
- Plot and draw the final decision boundary separating the three classes

Importing the necessary libraries

In [ ]:
```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
import seaborn as sns
import cv2 as cv
from cvxopt import matrix, solvers

from matplotlib.colors import ListedColormap
import matplotlib.patches as mpatches
```

In [ ]:
```
from google.colab import files
uploaded = files.upload()
```

Choose files No file chosen Upload widget is only available when the cell has been executed in the current browser session. Please rerun this cell to enable.

```
Saving poly1.png to poly1.png
Saving poly2.png to poly2.png
Saving poly3.png to poly3.png
Saving poly4.png to poly4.png
Saving poly5.png to poly5.png
Saving poly6.png to poly6.png
Saving poly7.png to poly7.png
Saving poly8.png to poly8.png
Saving poly9.png to poly9.png
Saving poly10.png to poly10.png
Saving poly11.png to poly11.png
Saving poly12.png to poly12.png
Saving poly13.png to poly13.png
Saving poly14.png to poly14.png
```

In [ ]:
```
ls=[]

class1 = ["poly1.png", "poly2.png", "poly3.png", "poly4.png", "poly5.png", "poly6.png", "poly7.png"]

for image in class1:
    img = cv.imread(image)
    img = cv.resize(img, (256, 256))

    b, g, r = cv.split(img)

    img = cv.merge((r, g, b))
    ls.append(img)
```

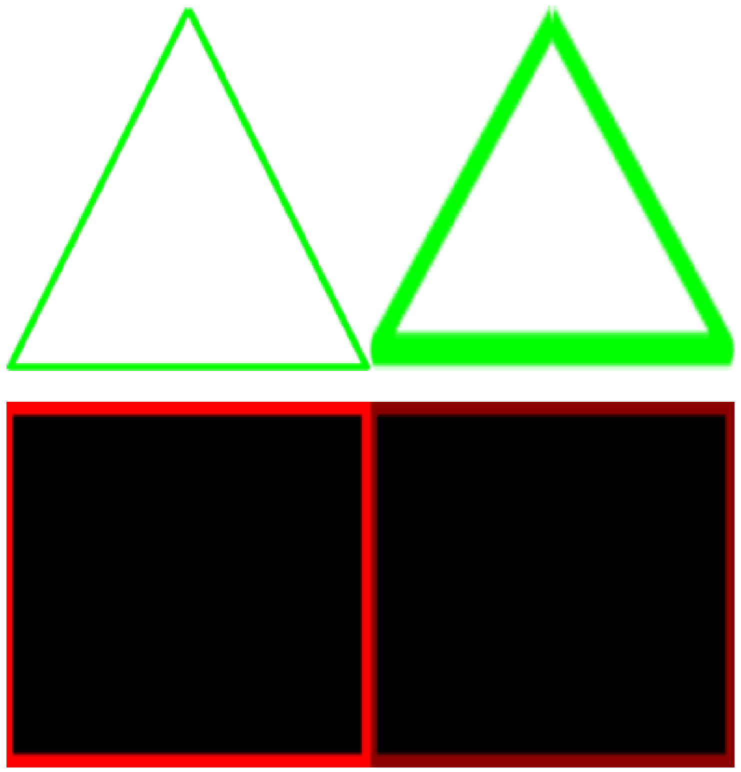
In [ ]:
```
ls2=[]

class2 = ["poly8.png", "poly9.png", "poly10.png", "poly11.png", "poly12.png", "poly13.png", "poly14.png"]

for image in class2:
    img = cv.imread(image)
    img = cv.resize(img, (256, 256))
```

```
    b, g, r = cv.split(img)

    img = cv.merge((r, g, b))
    ls2.append(img)
```

In [ ]:

```
plt.figure(figsize = (7, 7))
plt.imshow(np.concatenate((ls[0], ls[1]), axis = 1))
plt.axis('off')

plt.figure(figsize = (7, 7))
plt.imshow(np.concatenate((ls2[0], ls2[1]), axis = 1))
plt.axis('off')
```

Out[ ]: (-0.5, 511.5, 255.5, -0.5)



Single Perceptron

Defining the necessary functions used

In [ ]:

```
def perceptron_training_alg(X, Y, learning_rate):
  # Adding Bias to the input and Randomly initializing weights
  num_samples, num_features = X.shape

  X = np.hstack((np.ones((num_samples, 1)), X))
  W = np.zeros(num_features + 1).reshape((-1, 1))

  weight_vector = []
  weight_vector.append(W)

  print(W, "\n")

  # Running the algorithm until all the data points are correctly classified
  samples_misclassified = True
  cnt=0
  while samples_misclassified and cnt < 100000:
    # Initially we'll assume that there are no misclassified samples
    samples_misclassified = False
```

```
    for index, x in enumerate(X):
      if cnt >= 100000:
        return W, np.array(weight_vector)

      x = Y[index] * x

      if np.dot(x.reshape((1, -1)), W) <= 0: # if samples misclassified
        samples_misclassified = True
        W = W + (learning_rate * x.reshape((-1, 1)))     # Gradient Descent Step
        weight_vector.append(W)

        print(W, "\n")
        cnt += 1

  return W, np.array(weight_vector)
```

Code

In [ ]:

```
X = []

for img in ls:
    r, g, b = cv.split(img)
    X.append([np.sum(r)/np.sum(r+g+b), np.sum(g)/np.sum(r+g+b)])

for img in ls2:
    r, g, b = cv.split(img)
    X.append([np.sum(r)/np.sum(r+g+b), np.sum(g)/np.sum(r+g+b)])
```

In [ ]:

```
Y=np.concatenate((np.ones(len(ls))*-1,np.ones(len(ls2))))
ans=np.concatenate((np.zeros(len(ls)),np.ones(len(ls2))))
```

In [ ]:

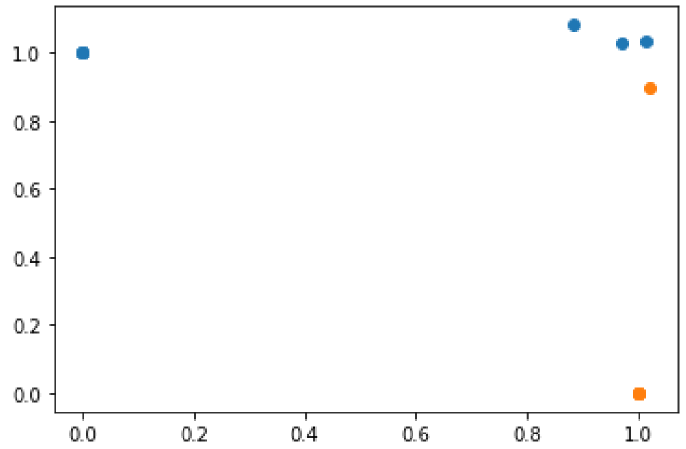
```
X=np.stack(X)
X
```

Out[ ]:

```
array([[0.96962233, 1.02958576],
       [0.88314034, 1.08201861],
       [0.        , 1.        ],
       [0.        , 1.        ],
       [0.        , 1.        ],
       [0.        , 1.        ],
       [1.01528142, 1.03526193],
       [1.        , 0.        ],
       [1.        , 0.        ],
       [1.        , 0.        ],
       [1.        , 0.        ],
       [1.01927362, 0.89945752],
       [1.        , 0.        ],
       [1.        , 0.        ]])
```

In [ ]:

```
plt.scatter(X[:7,0],X[:7,1])
plt.scatter(X[7:,0],X[7:,1])
plt.show()
```



In [ ]:

```
W, weight_vector=perceptron_training_alg(X,Y,0.01)
```

```
[[0.]
 [0.]
 [0.]]

[[-0.01      ]
 [-0.00969622]
 [-0.01029586]]

[[ 0.        ]
 [ 0.00030378]
 [-0.01029586]]

[[ 0.01      ]
 [ 0.01049651]
 [-0.00130128]]

[[ 0.        ]
 [ 0.00080029]
 [-0.01159714]]

[[ 0.01      ]
 [ 0.01099303]
 [-0.00260256]]

[[ 0.        ]
 [ 0.0012968 ]
 [-0.01289842]]

[[ 0.01      ]
 [ 0.01148954]
 [-0.00390385]]

[[ 0.        ]
 [ 0.00179332]
 [-0.0141997 ]]

[[ 0.01      ]
 [ 0.01198605]
 [-0.00520513]]

[[ 0.        ]
 [ 0.00228983]
 [-0.01550099]]

[[ 0.01      ]
 [ 0.01248256]
 [-0.00650641]]

[[ 0.        ]
 [ 0.00278634]
 [-0.01680227]]

[[ 0.01      ]
 [ 0.01297908]
```

```
 [-0.00780769]]

[[ 0.        ]
 [ 0.00328285]
 [-0.01810355]]

[[ 0.01      ]
 [ 0.01347559]
 [-0.00910898]]

[[ 0.        ]
 [ 0.00377937]
 [-0.01940483]]

[[ 0.01      ]
 [ 0.0139721 ]
 [-0.01041026]]

[[ 0.        ]
 [ 0.00427588]
 [-0.02070612]]

[[ 0.01      ]
 [ 0.01446862]
 [-0.01171154]]

[[ 0.        ]
 [ 0.00477239]
 [-0.0220074 ]]

[[ 0.01      ]
 [ 0.01496513]
 [-0.01301282]]

[[ 0.        ]
 [ 0.00526891]
 [-0.02330868]]

[[ 0.01      ]
 [ 0.01546164]
 [-0.01431411]]

[[ 0.        ]
 [ 0.00576542]
 [-0.02460996]]

[[ 0.01      ]
 [ 0.01595816]
 [-0.01561539]]

[[ 0.        ]
 [ 0.00626193]
 [-0.02591125]]

[[ 0.01      ]
 [ 0.01645467]
 [-0.01691667]]

[[ 0.        ]
 [ 0.00675844]
 [-0.02721253]]

[[ 0.01      ]
 [ 0.01695118]
 [-0.01821795]]

[[ 0.        ]
 [ 0.00725496]
 [-0.02851381]]

[[ 0.01      ]
 [ 0.01744769]
 [-0.01951924]]

[[ 0.        ]
```

```
 [ 0.00775147]
 [-0.02981509]]

[[ 0.01      ]
 [ 0.01794421]
 [-0.02082052]]

[[ 0.        ]
 [ 0.00824798]
 [-0.03111638]]

[[ 0.01      ]
 [ 0.01844072]
 [-0.0221218 ]]

[[ 0.        ]
 [ 0.0087445 ]
 [-0.03241766]]

[[ 0.01      ]
 [ 0.01893723]
 [-0.02342308]]

[[ 0.        ]
 [ 0.00924101]
 [-0.03371894]]

[[ 0.01      ]
 [ 0.01943375]
 [-0.02472437]]

[[ 0.        ]
 [ 0.00973752]
 [-0.03502022]]

[[ 0.01      ]
 [ 0.01993026]
 [-0.02602565]]

[[ 0.        ]
 [ 0.01023404]
 [-0.03632151]]

[[ 0.01      ]
 [ 0.02042677]
 [-0.02732693]]

[[ 0.        ]
 [ 0.01073055]
 [-0.03762279]]

[[ 0.01      ]
 [ 0.02092328]
 [-0.02862821]]

[[ 0.        ]
 [ 0.01122706]
 [-0.03892407]]

[[ 0.01     ]
 [ 0.0214198]
 [-0.0299295]]

[[ 0.        ]
 [ 0.01126698]
 [-0.04028211]]

[[ 0.01      ]
 [ 0.02145972]
 [-0.03128754]]
```

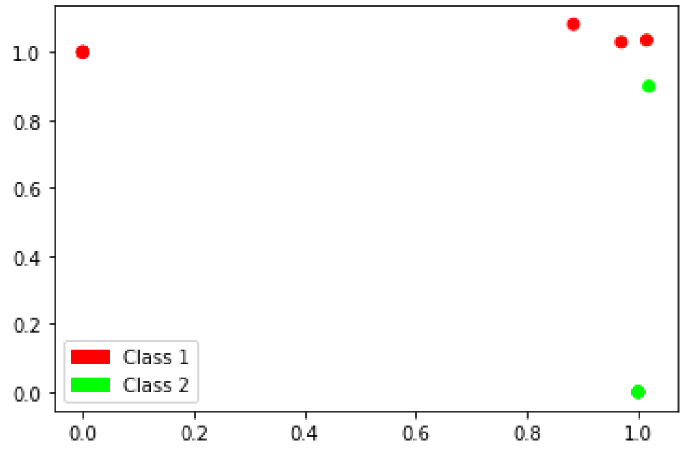
In [ ]:

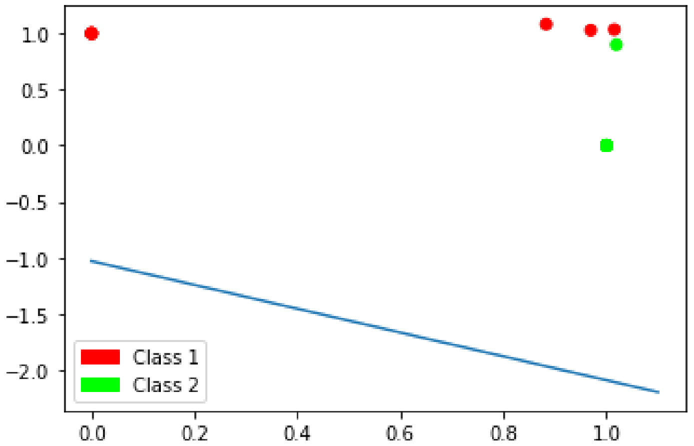
```
W
```

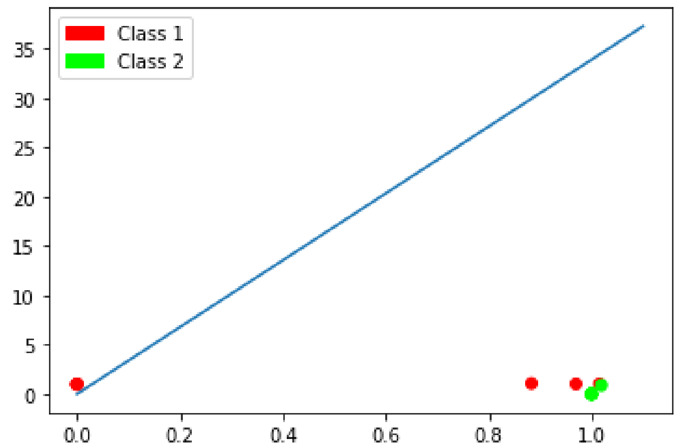
Out[ ]:
```
array([[ 0.01      ],
       [ 0.02145972],
       [-0.03128754]])
```

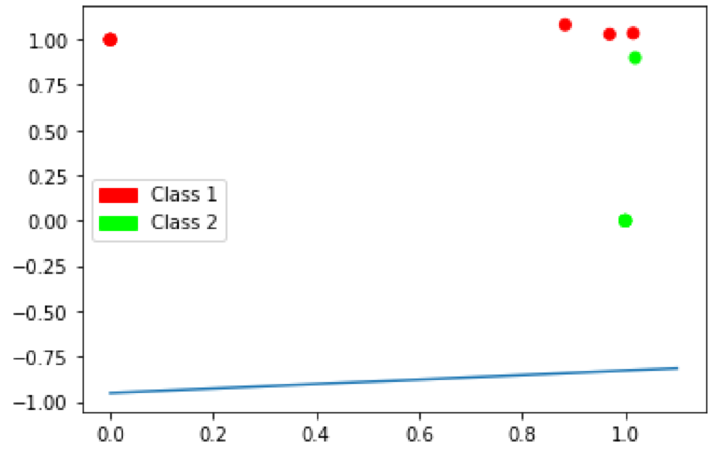
In [ ]:
```
cm  = ListedColormap(['#FF0000', '#00FF00'])
patch0 = mpatches.Patch(color = '#FF0000', label = 'Class 1')
patch1 = mpatches.Patch(color = '#00FF00', label = 'Class 2')

x1 = np.arange(0, 1.2, 0.1)

for w in weight_vector:
  plt.scatter(X[:,0], X[:,1], c = [0] * 7 + [1] * 7, cmap = cm)
  plt.legend(handles = [patch0, patch1])

  slope, c = -w[2] / w[1], -w[0] / w[1]
  plt.plot(x1, slope * x1 + c)
  plt.show()
```

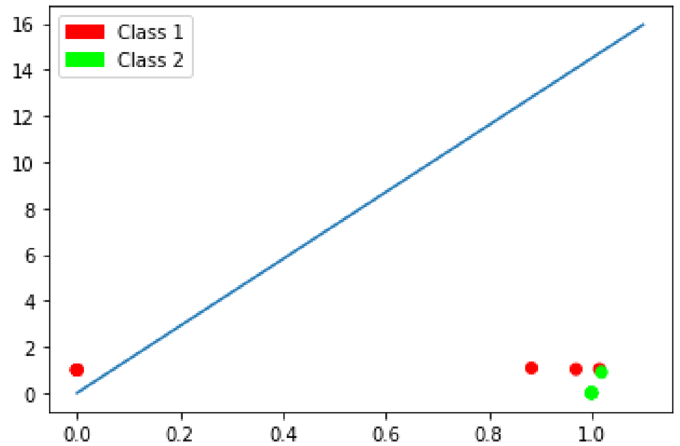
```
/usr/local/lib/python3.7/dist-packages/ipykernel_launcher.py:11: RuntimeWarning: invalid value encountered in true_divide
  # This is added back by InteractiveShellApp.init_path()
```

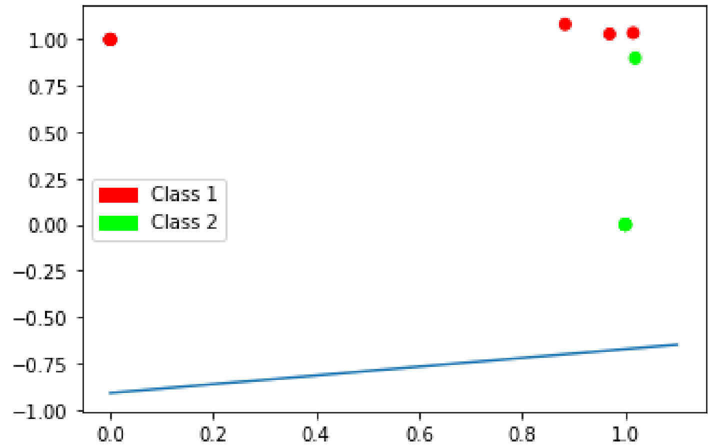


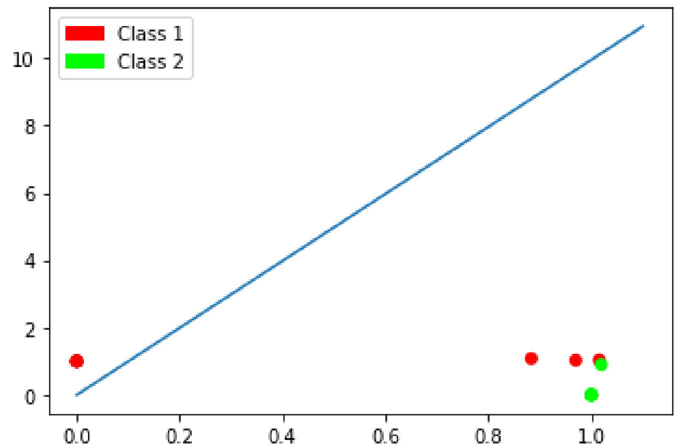


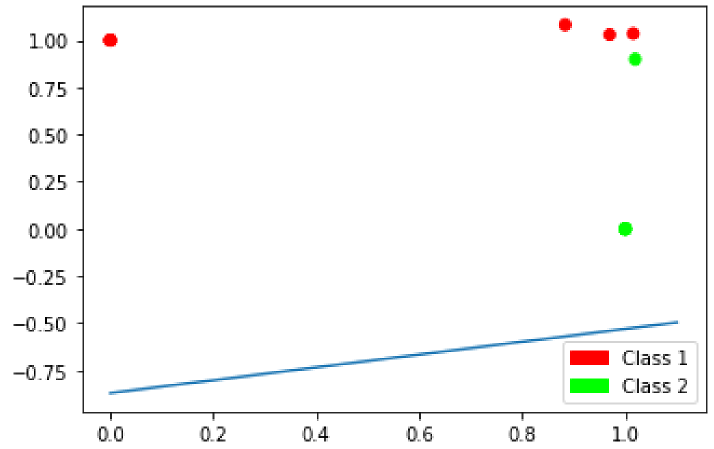


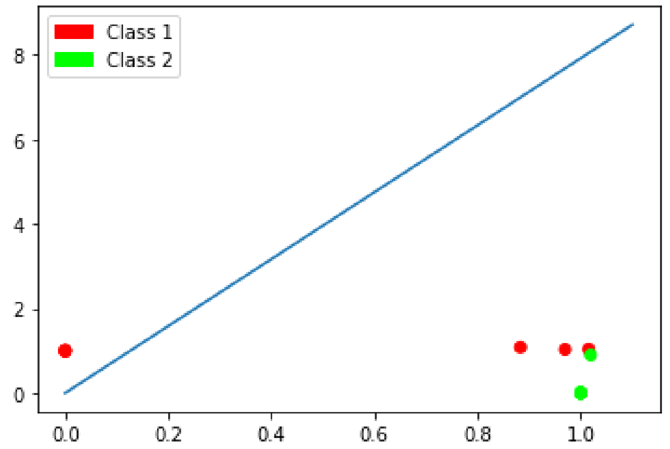

Class 1
Class 2

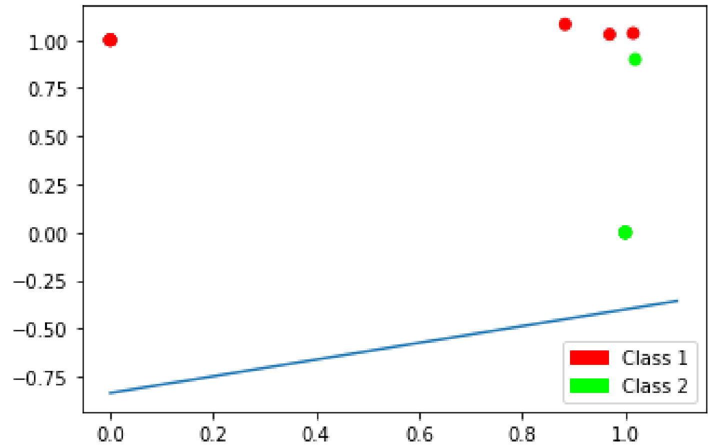

Class 1
Class 2

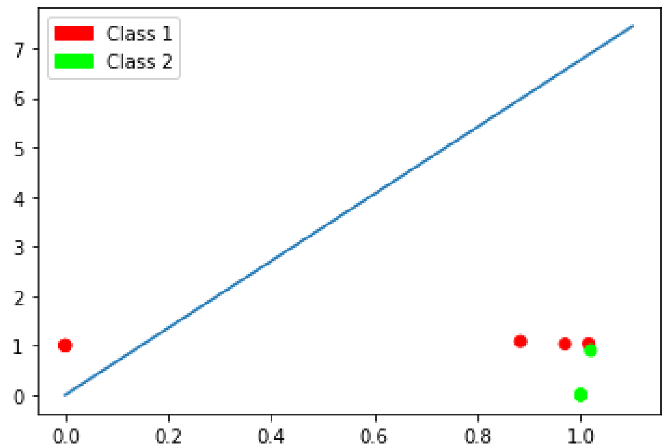

Class 1
Class 2

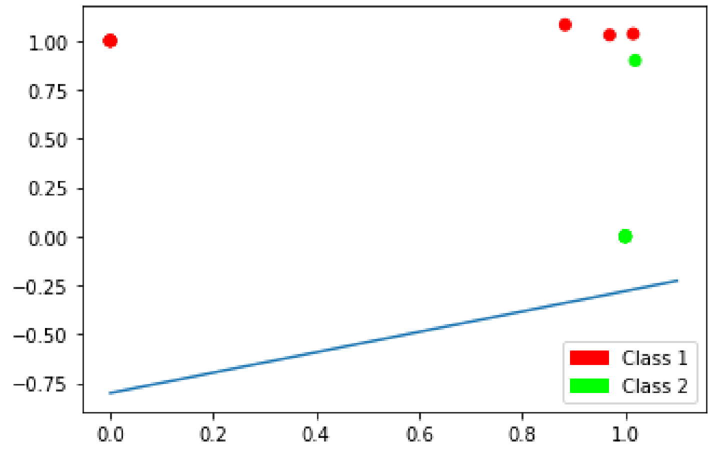

Class 1
Class 2


1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
0.0
0.2
0.4
0.6
0.8
1.0
Class 1
Class 2


8
6
4
2
0
0.0
0.2
0.4
0.6
0.8
1.0
Class 1
Class 2


1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
0.0
0.2
0.4
0.6
0.8
1.0
Class 1
Class 2


7
6
5
4
3
2
1
0
0.0
0.2
0.4
0.6
0.8
1.0
Class 1
Class 2


Class 1
Class 2
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0.50
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0.00
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−0.75
0.0
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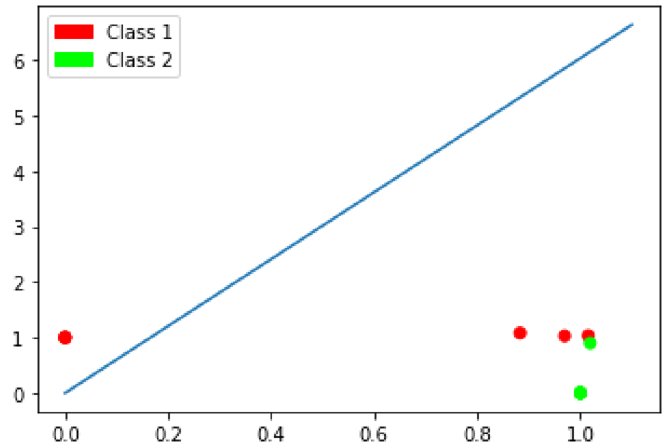
0.8
1.0


Class 1
Class 2
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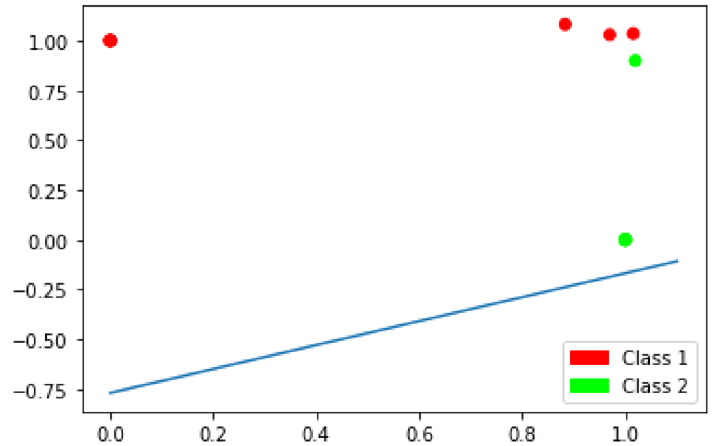
Class 1
Class 2
1.00
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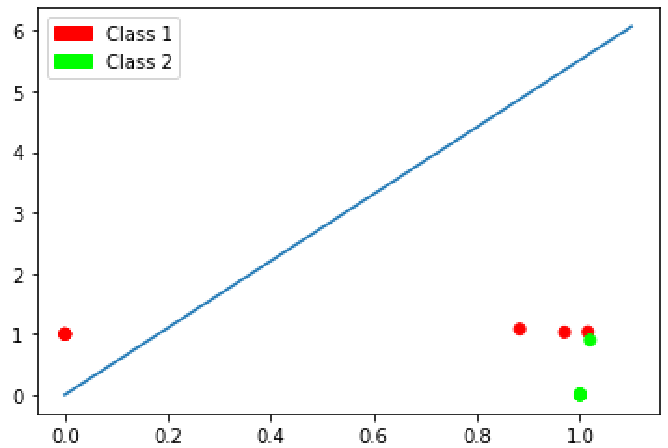
Class 1
Class 2
6
5
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0.4
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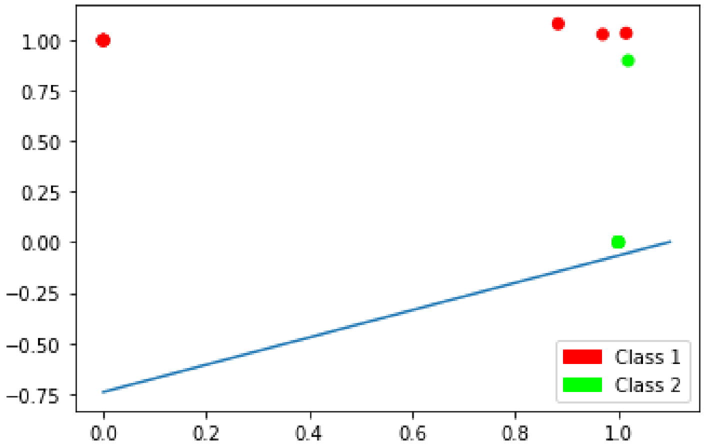
0.8
1.0


1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
0.0
0.2
0.4
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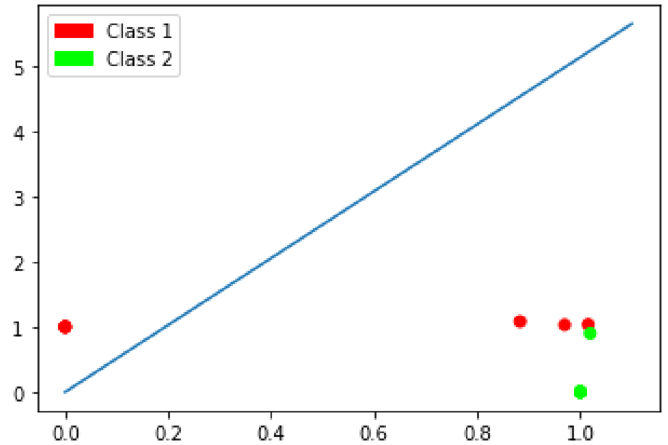
0.8
1.0
Class 1
Class 2


Class 1
Class 2
5
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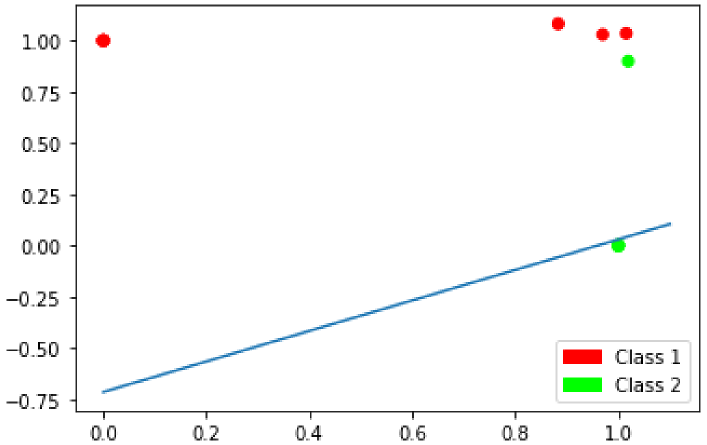
1.0


1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
0.0
0.2
0.4
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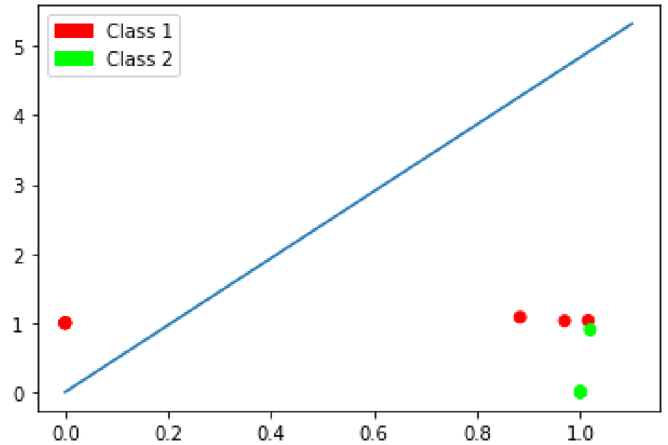
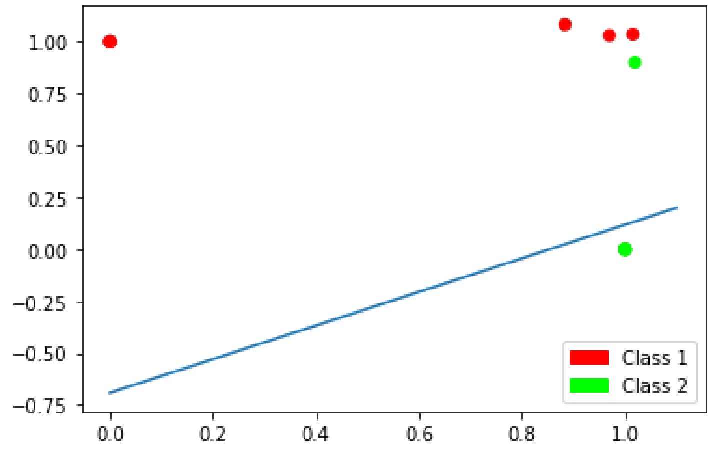
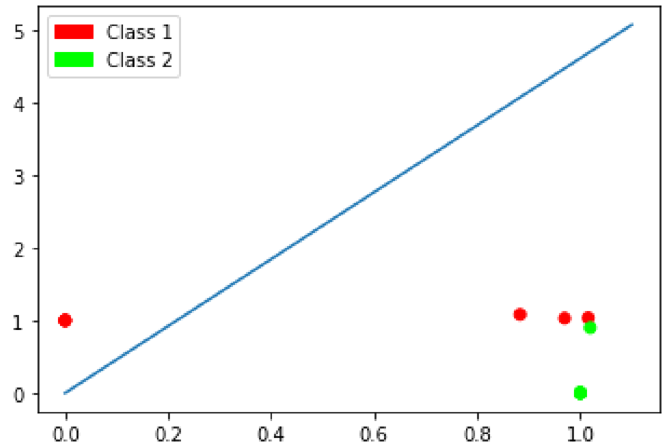
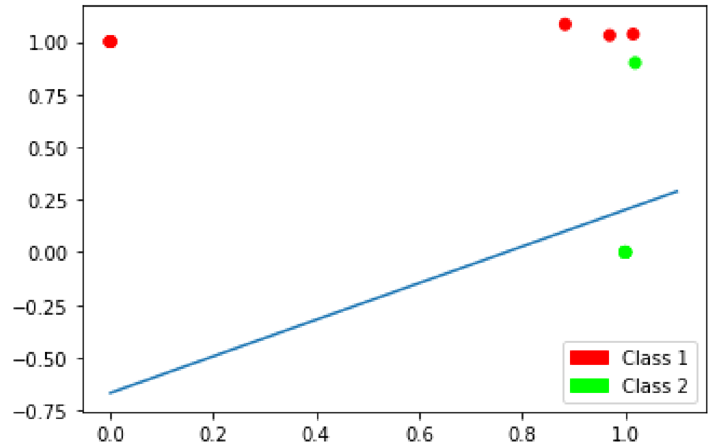
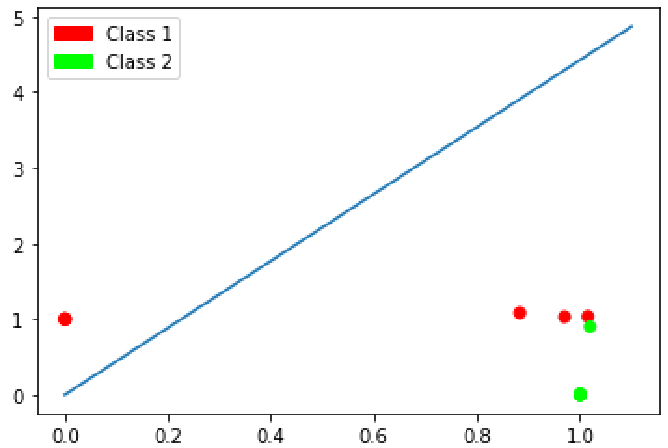
0.8
1.0
Class 1
Class 2

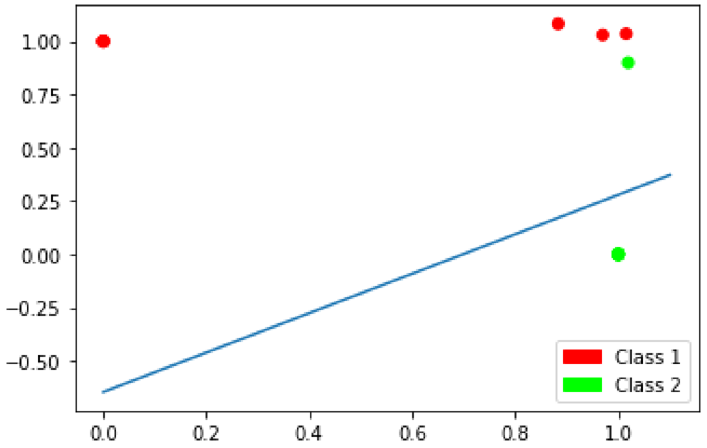

Class 1
Class 2
5
4
3
2
1
0
0.0
0.2
0.4
0.6
0.8
1.0

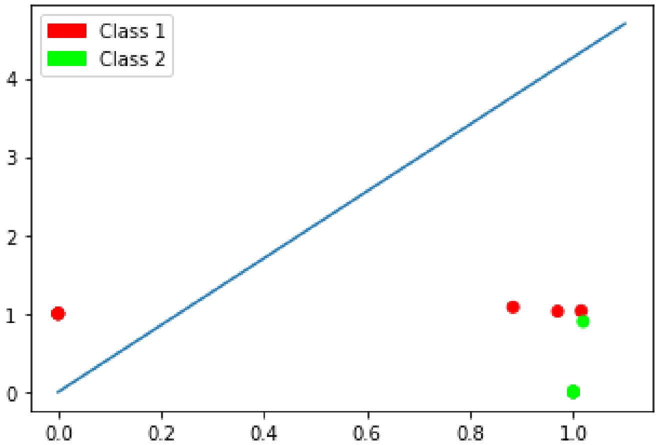

Class 1
Class 2

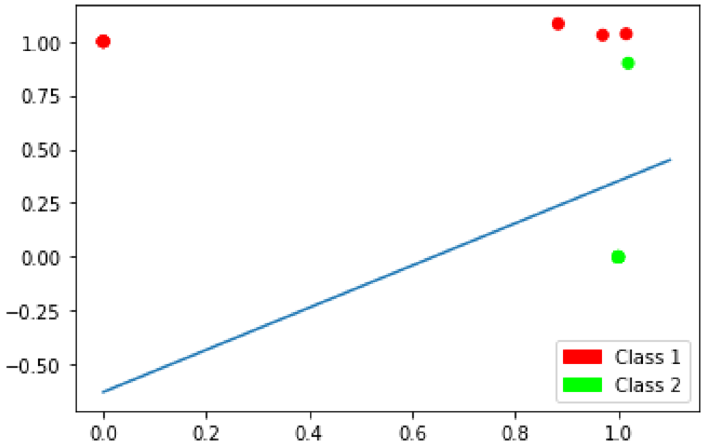

Class 1
Class 2

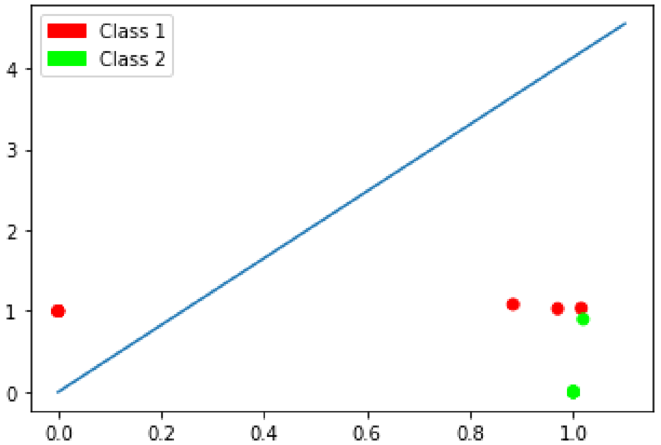

Class 1
Class 2

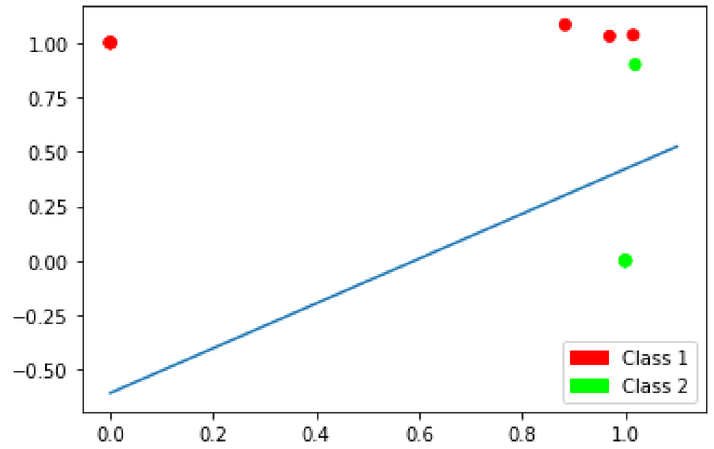

Class 1
Class 2

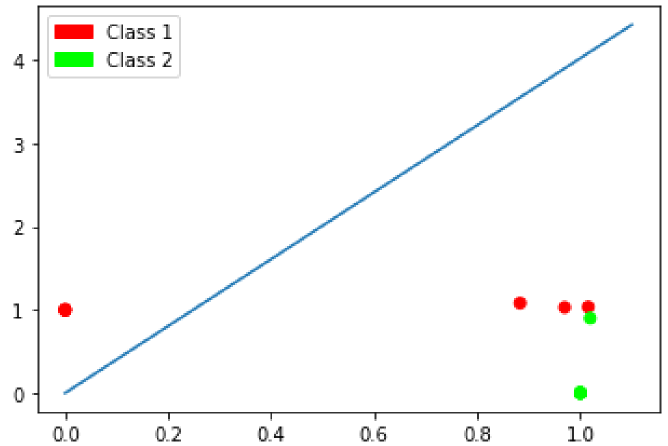

Class 1
Class 2

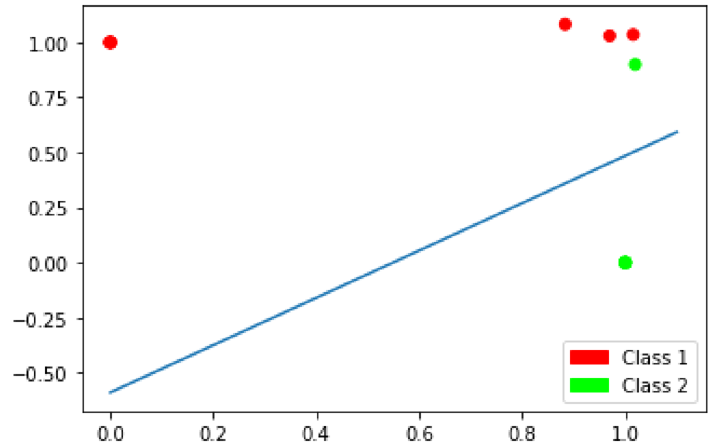

Class 1
Class 2

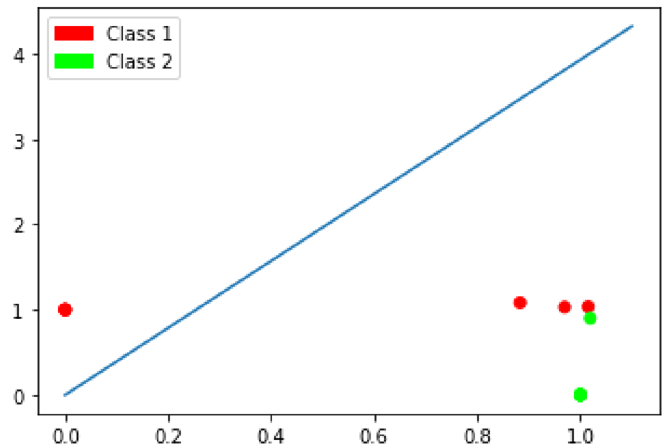

Class 1
Class 2

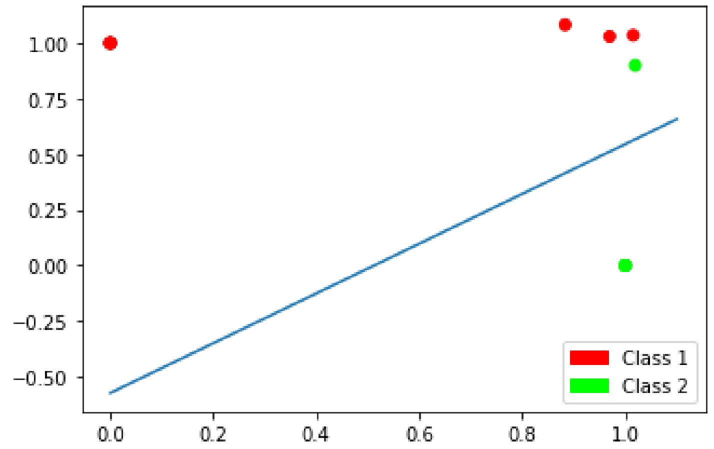

Class 1
Class 2

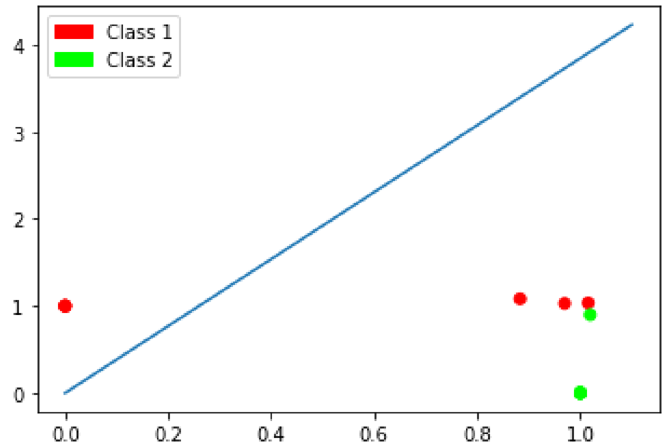

Class 1
Class 2

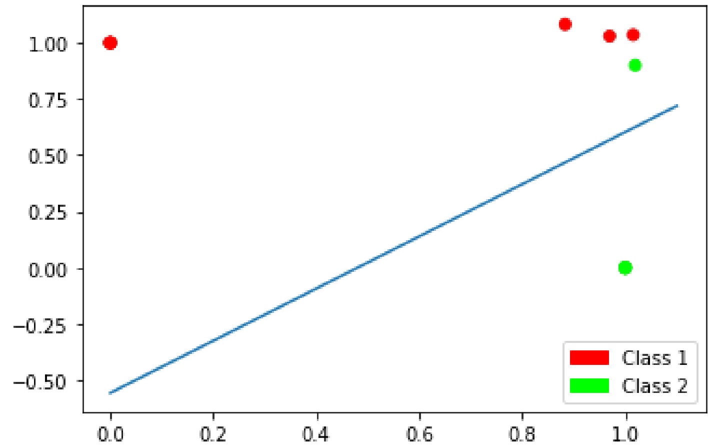

Class 1
Class 2

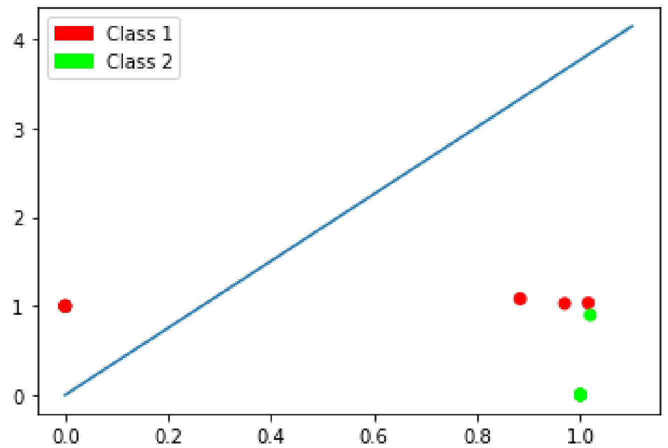

Class 1
Class 2

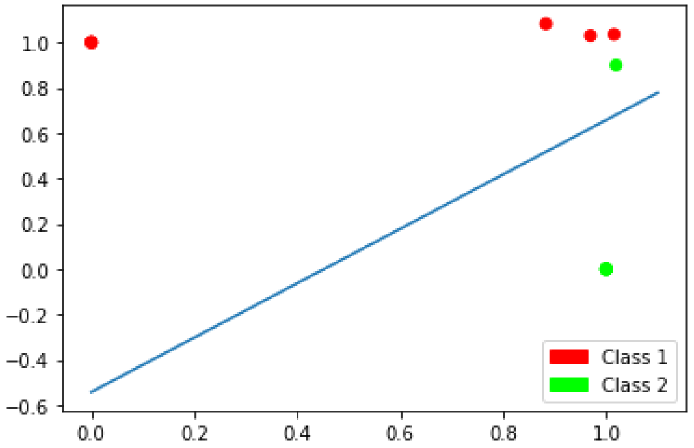

Class 1
Class 2

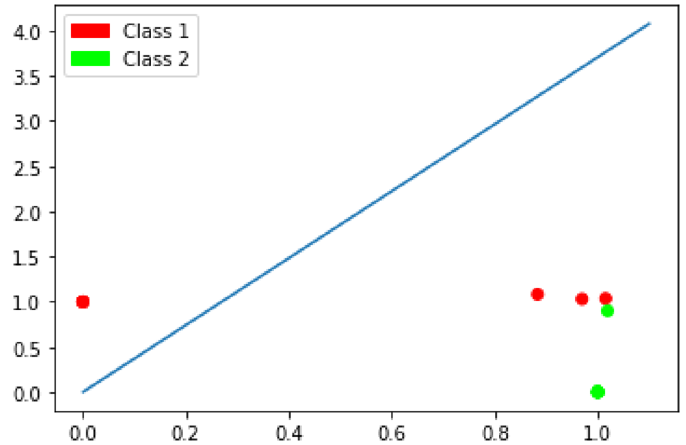

Class 1
Class 2

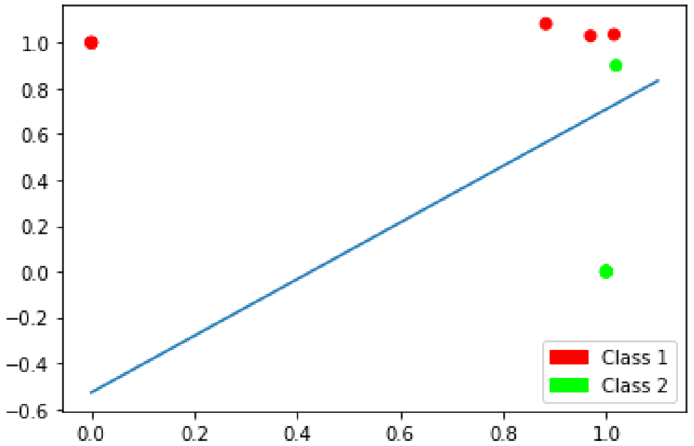

Class 1
Class 2

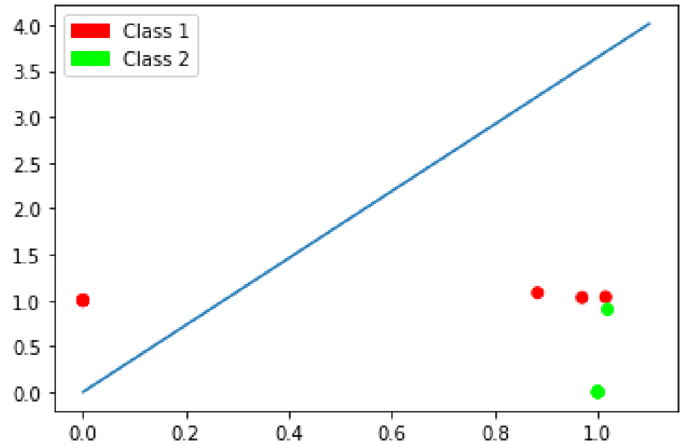

Class 1
Class 2

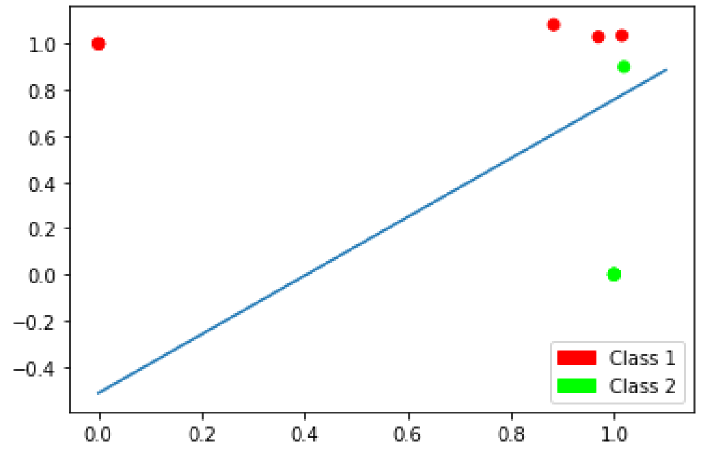

Class 1
Class 2

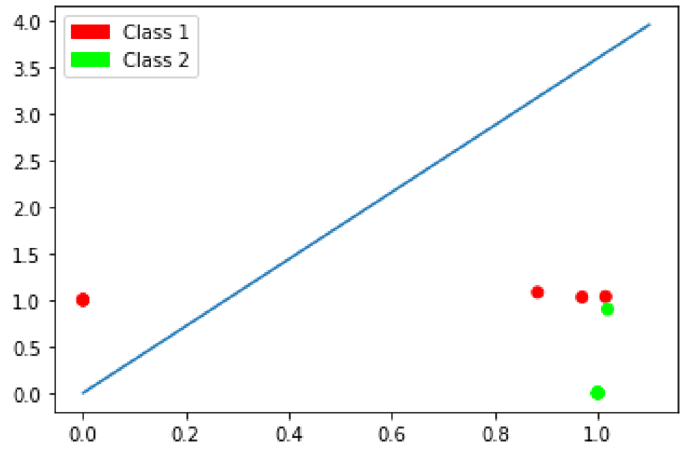

Class 1
Class 2

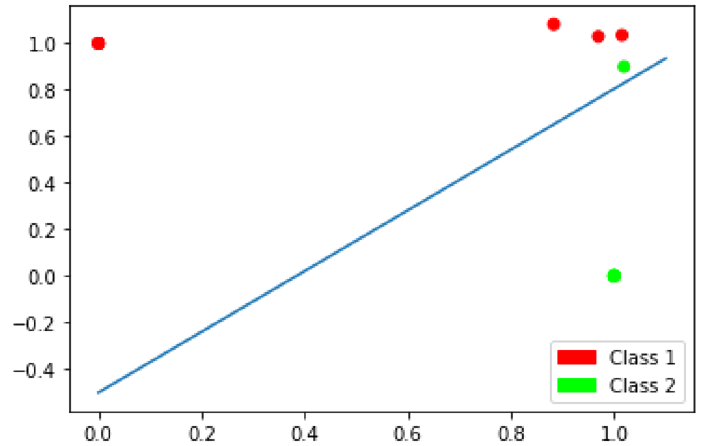

Class 1
Class 2

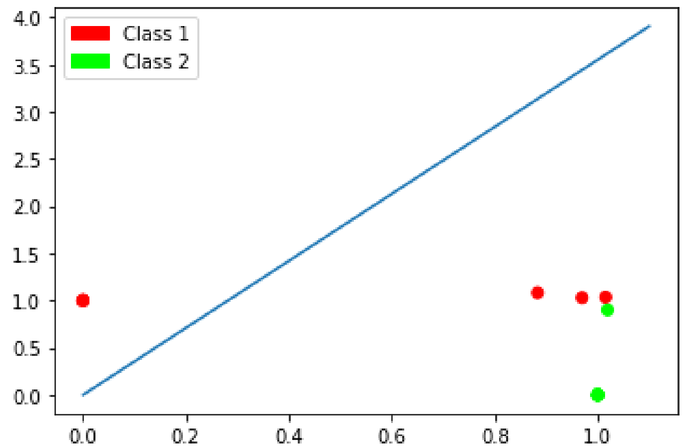

Class 1
Class 2

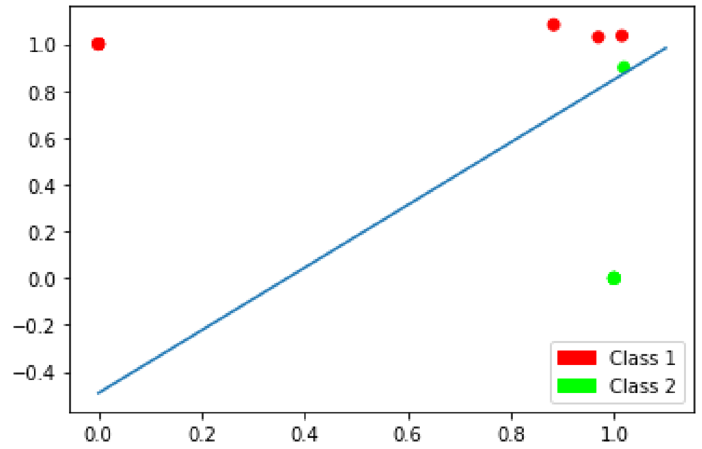

Class 1
Class 2

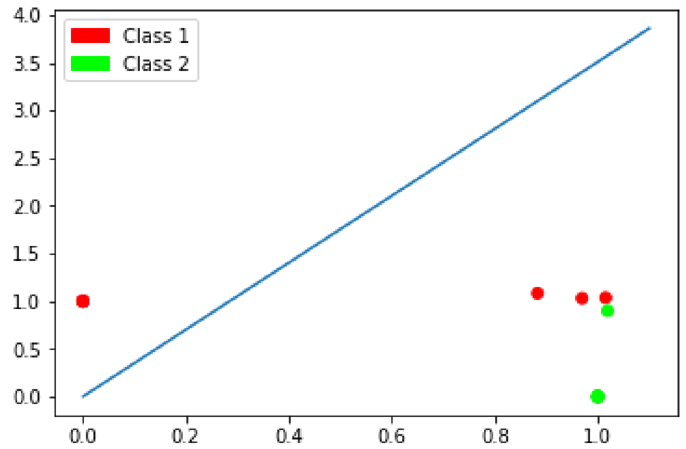

Class 1
Class 2

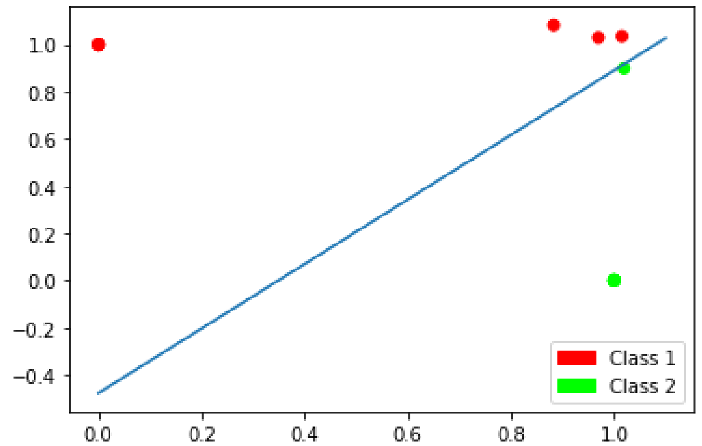

Class 1
Class 2

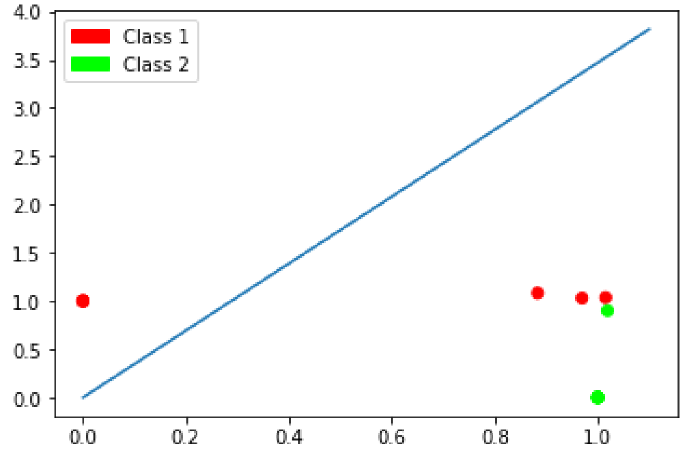

Class 1
Class 2

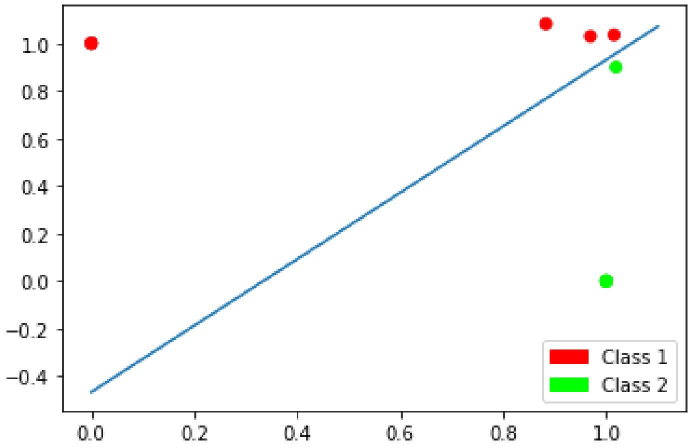

Class 1
Class 2

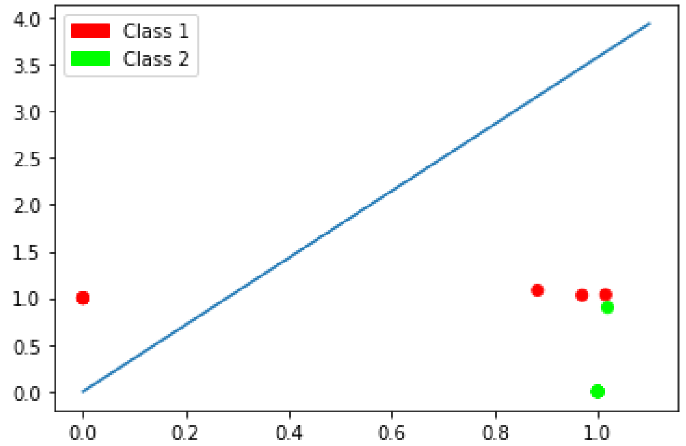

Class 1
Class 2

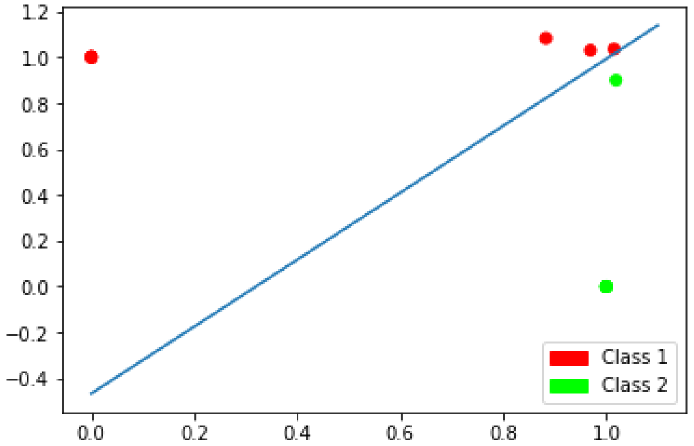

Class 1
Class 2


Class 1
Class 2


Class 1
Class 2







SVM

Defining the necessary functions used

In [ ]:

```
def SVM(C1, C2):
  x1 = np.array(C1.to_numpy())
  x2 = np.array(C2.to_numpy())

  x = np.vstack((x1, x2))
  y = np.hstack((np.ones(len(x1)), np.ones(len(x2)) * -1))

  X = np.dot(x, np.transpose(x))
  Y = np.outer(y, y)

  n = X.shape[0]

  H = matrix(Y * X)
  f = matrix(np.ones(n) * -1)
  A = matrix(np.diag(np.ones(n) * -1))
```

```
    a = matrix(np.zeros(n))
    B = matrix(y, (1, n))
    b = matrix(0.0)

    # solve QP problem
    solution = solvers.qp(H, f, A, a, B, b)
    print('')

    # Lagrange multipliers
    t1 = np.ravel(solution['x'])

    # Support vectors have non zero lagrange multipliers
    for i in range(0, len(t1)):
      if(t1[i] < 1e-4):
        t1[i] = 0.0
      else:
        t1[i] = round(t1[i], 4)

    t1 = np.array(t1)
    print(t1)

    # Weight vector
    w = np.zeros(X.shape[1])
    for i in range(X.shape[1]):
      w[i] = t1[i] * y[i]

    w = np.dot(w, x)

    # Intercept
    b = 0
    for i in range(len(t1)):
      if(t1[i] != 0.0):
        b = round(1 - np.dot(w, np.transpose(x[i])), 4)
        break

    return w, b
```

Code

In [ ]:

```
X = []

for img in ls:
    r, g, b = cv.split(img)
    X.append([np.sum(r)/np.sum(r+g+b), np.sum(g)/np.sum(r+g+b)])

C1 = pd.DataFrame(X)
C1
```

Out[ ]:

	0	1
0	0.969622	1.029586
1	0.883140	1.082019
2	0.000000	1.000000
3	0.000000	1.000000
4	0.000000	1.000000
5	0.000000	1.000000
6	1.015281	1.035262

In [ ]:

```
Y = []

for img in ls2:
```

```
    r, g, b = cv.split(img)
    Y.append([np.sum(r)/np.sum(r+g+b), np.sum(g)/np.sum(r+g+b)])

C2 = pd.DataFrame(Y)
C2
```

Out[ ]:

	0	1
0	1.000000	0.000000
1	1.000000	0.000000
2	1.000000	0.000000
3	1.000000	0.000000
4	1.019274	0.899458
5	1.000000	0.000000
6	1.000000	0.000000

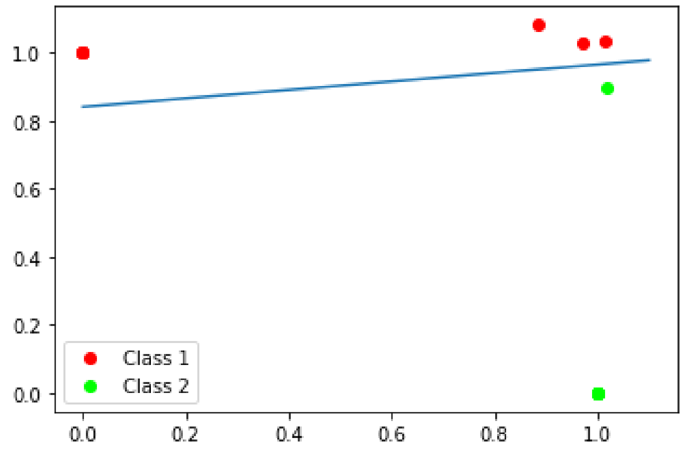
In [ ]:

```
w, b = SVM(C1, C2)


print(w)
print('')
print(b)
```

```
     pcost       dcost       gap    pres   dres
 0: -3.5318e+00 -1.0519e+01  7e+00  8e-16  2e+00
 1: -6.4286e+00 -8.1498e+00  2e+00  8e-17  9e-01
 2: -3.7324e+01 -4.2474e+01  5e+00  4e-15  9e-01
 3: -7.6169e+01 -9.3636e+01  2e+01  4e-15  7e-01
 4: -1.0565e+02 -1.2390e+02  2e+01  2e-14  3e-01
 5: -1.0930e+02 -1.1091e+02  2e+00  2e-14  2e-02
 6: -1.0932e+02 -1.0934e+02  2e-02  2e-14  2e-04
 7: -1.0932e+02 -1.0932e+02  2e-04  1e-14  2e-06
 8: -1.0932e+02 -1.0932e+02  2e-06  3e-14  2e-08
Optimal solution found.

[ 30.3932   0.       0.       0.       0.       0.      78.9254   0.
   0.       0.       0.     109.3187   0.       0.    ]
[-1.82424974 14.67334115]

-12.3386
```

In [ ]:

```
x1 = np.arange(0, 1.2, 0.1)

plt.scatter(C1.iloc[:, [0]], C1.iloc[:, [1]], label = 'Class 1', color = '#FF0000')
plt.scatter(C2.iloc[:, [0]], C2.iloc[:, [1]], label = 'Class 2', color = '#00FF00')
plt.legend()

slope, c = -w[0] / w[1], -b / w[1]
plt.plot(x1, slope * x1 + c)
plt.show()
```


Class 1
Class 2
0.0
0.2
0.4
0.6
0.8
1.0

Q4. From the iris dataset, choose the 'petal length', 'sepal width' for setosa, versicolor and virginica flowers. Learn a decision boundary for the two features using a **single perceptron and SVM**. Assume that all the weights of the perceptron are initialized as 0 with the learning rate of 0.01. Draw the decision boundary.

Importing the necessary libraries

In [ ]:

```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
import seaborn as sns
from cvxopt import matrix, solvers

from matplotlib.colors import ListedColormap
import matplotlib.patches as mpatches
```

Iris Dataset

In [ ]:

```
from google.colab import files
uploaded = files.upload()
```

Choose files No file chosen Upload widget is only available when the cell has been executed in the current browser session. Please rerun this cell to enable.

```
Saving Iris_dataset.csv to Iris_dataset.csv
```

In [ ]:

```
df = pd.read_csv('Iris_dataset.csv')
df
```

Out[ ]:

	sepal.length	sepal.width	petal.length	petal.width	variety
0	5.1	3.5	1.4	0.2	Setosa
1	4.9	3.0	1.4	0.2	Setosa
2	4.7	3.2	1.3	0.2	Setosa
3	4.6	3.1	1.5	0.2	Setosa
4	5.0	3.6	1.4	0.2	Setosa
...	...	...	...	...	...
145	6.7	3.0	5.2	2.3	Virginica
146	6.3	2.5	5.0	1.9	Virginica
147	6.5	3.0	5.2	2.0	Virginica
148	6.2	3.4	5.4	2.3	Virginica
149	5.9	3.0	5.1	1.8	Virginica

150 rows × 5 columns

In [ ]:

```
df = df.drop(['sepal.length', 'petal.width'], axis = 1)
df
```

Out[ ]:

	sepal.width	petal.length	variety
0	3.5	1.4	Setosa
1	3.0	1.4	Setosa

	sepal.width	petal.length	variety
2	3.2	1.3	Setosa
3	3.1	1.5	Setosa
4	3.6	1.4	Setosa
...	...	...	...
145	3.0	5.2	Virginica
146	2.5	5.0	Virginica
147	3.0	5.2	Virginica
148	3.4	5.4	Virginica
149	3.0	5.1	Virginica

150 rows × 3 columns

Single Perceptron

Defining the necessary functions used

In [ ]:

```
def perceptron_training_alg(X, Y, learning_rate):
  # Adding Bias to the input and Randomly initializing weights
  num_samples, num_features = X.shape

  X = np.hstack((np.ones((num_samples, 1)), X))
  W = np.zeros(num_features + 1).reshape((-1, 1))

  weight_vector = []
  weight_vector.append(W)

  print(W, "\n")

  # Running the algorithm until all the data points are correctly classified
  samples_misclassified = True
  cnt=0
  while samples_misclassified and cnt < 100000:
    # Initially we'll assume that there are no misclassified samples
    samples_misclassified = False

    for index, x in enumerate(X):
      if cnt >= 100000:
        return W, np.array(weight_vector)

      x = Y[index] * x

      if np.dot(x.reshape((1, -1)), W) <= 0: # if samples misclassified
        samples_misclassified = True
        W = W + (learning_rate * x.reshape((-1, 1)))     # Gradient Descent Step
        weight_vector.append(W)

        print(W, "\n")
        cnt += 1

  return W, np.array(weight_vector)
```

In [ ]:

```
def sorter(X, Y):
  X_new0 = []
  Y_new0 = []

  X_new1 = []
  Y_new1 = []
```

```
  for x, y in zip(X, Y):
    if y == 0:
      Y_new0.append(y)
      X_new0.append(x)

    else:
      Y_new1.append(y)
      X_new1.append(x)

  X0 = []
  Y0 = []

  X1 = []
  Y1 = []

  if len(X_new0) > 0:
    X0 = np.stack(X_new0)
    Y0 = np.stack(Y_new0)

  elif len(X_new1) > 0:
    X1 = np.stack(X_new1)
    Y1 = np.stack(Y_new1)

  if len(X0) > 0 and len(X1) > 0:
    X = np.concatenate((X0, X1))
    Y = np.concatenate((Y0, Y1))

  elif len(X0) > 0:
    X = X0
    Y = Y0

  else:
    X = X1
    Y = Y1

  return X, Y
```

In [ ]:

```
def solve_perceptron(clas, df):
    df_4 = df[['sepal.width', 'petal.length']]
    X1 = np.array(df_4)

    ls = []
    ls2 = []

    for x in df.itertuples():
        print(x)

        if x[-1] == clas:
            ls.append([1])
            ls2.append([1])

        else:
            ls.append([-1])
            ls2.append([0])

    Y1=np.array(ls)
    ans1=np.array(ls2)

    W, weight_vector = perceptron_training_alg(X1, Y1, 0.01)
    return W, X1, ans1, weight_vector
```

Code

In [ ]:

```
df_1 = df.loc[df.variety == 'Setosa']
print(df_1.head(5))
```

```
print(df_1.tail(5))
df_1.shape
```

```
   sepal.width  petal.length variety
0          3.5           1.4  Setosa
1          3.0           1.4  Setosa
2          3.2           1.3  Setosa
3          3.1           1.5  Setosa
4          3.6           1.4  Setosa
    sepal.width  petal.length variety
45          3.0           1.4  Setosa
46          3.8           1.6  Setosa
47          3.2           1.4  Setosa
48          3.7           1.5  Setosa
49          3.3           1.4  Setosa
```

Out[ ]: (50, 3)

In [ ]:

```
df_2 = df.loc[df.variety == 'Virginica']
print(df_2.head(5))
print(df_2.tail(5))
df_2.shape
```

```
     sepal.width  petal.length    variety
100          3.3           6.0  Virginica
101          2.7           5.1  Virginica
102          3.0           5.9  Virginica
103          2.9           5.6  Virginica
104          3.0           5.8  Virginica
     sepal.width  petal.length    variety
145          3.0           5.2  Virginica
146          2.5           5.0  Virginica
147          3.0           5.2  Virginica
148          3.4           5.4  Virginica
149          3.0           5.1  Virginica
```

Out[ ]: (50, 3)

In [ ]:

```
df_3 = df.loc[df.variety == 'Versicolor']
print(df_3.head(5))
print(df_3.tail(5))
df_3.shape
```

```
    sepal.width  petal.length     variety
50          3.2           4.7  Versicolor
51          3.2           4.5  Versicolor
52          3.1           4.9  Versicolor
53          2.3           4.0  Versicolor
54          2.8           4.6  Versicolor
    sepal.width  petal.length     variety
95          3.0           4.2  Versicolor
96          2.9           4.2  Versicolor
97          2.9           4.3  Versicolor
98          2.5           3.0  Versicolor
99          2.8           4.1  Versicolor
```

Out[ ]: (50, 3)

In [ ]:

```
W1, X1, ans1, weight_vector1 = solve_perceptron('Setosa', df)
```

```
Pandas(Index=0, _1=3.5, _2=1.4, variety='Setosa')
Pandas(Index=1, _1=3.0, _2=1.4, variety='Setosa')
Pandas(Index=2, _1=3.2, _2=1.3, variety='Setosa')
Pandas(Index=3, _1=3.1, _2=1.5, variety='Setosa')
Pandas(Index=4, _1=3.6, _2=1.4, variety='Setosa')
Pandas(Index=5, _1=3.9, _2=1.7, variety='Setosa')
Pandas(Index=6, _1=3.4, _2=1.4, variety='Setosa')
Pandas(Index=7, _1=3.4, _2=1.5, variety='Setosa')
Pandas(Index=8, _1=2.9, _2=1.4, variety='Setosa')
Pandas(Index=9, _1=3.1, _2=1.5, variety='Setosa')
Pandas(Index=10, _1=3.7, _2=1.5, variety='Setosa')
```

```
Pandas(Index=11, _1=3.4, _2=1.6, variety='Setosa')
Pandas(Index=12, _1=3.0, _2=1.4, variety='Setosa')
Pandas(Index=13, _1=3.0, _2=1.1, variety='Setosa')
Pandas(Index=14, _1=4.0, _2=1.2, variety='Setosa')
Pandas(Index=15, _1=4.4, _2=1.5, variety='Setosa')
Pandas(Index=16, _1=3.9, _2=1.3, variety='Setosa')
Pandas(Index=17, _1=3.5, _2=1.4, variety='Setosa')
Pandas(Index=18, _1=3.8, _2=1.7, variety='Setosa')
Pandas(Index=19, _1=3.8, _2=1.5, variety='Setosa')
Pandas(Index=20, _1=3.4, _2=1.7, variety='Setosa')
Pandas(Index=21, _1=3.7, _2=1.5, variety='Setosa')
Pandas(Index=22, _1=3.6, _2=1.0, variety='Setosa')
Pandas(Index=23, _1=3.3, _2=1.7, variety='Setosa')
Pandas(Index=24, _1=3.4, _2=1.9, variety='Setosa')
Pandas(Index=25, _1=3.0, _2=1.6, variety='Setosa')
Pandas(Index=26, _1=3.4, _2=1.6, variety='Setosa')
Pandas(Index=27, _1=3.5, _2=1.5, variety='Setosa')
Pandas(Index=28, _1=3.4, _2=1.4, variety='Setosa')
Pandas(Index=29, _1=3.2, _2=1.6, variety='Setosa')
Pandas(Index=30, _1=3.1, _2=1.6, variety='Setosa')
Pandas(Index=31, _1=3.4, _2=1.5, variety='Setosa')
Pandas(Index=32, _1=4.1, _2=1.5, variety='Setosa')
Pandas(Index=33, _1=4.2, _2=1.4, variety='Setosa')
Pandas(Index=34, _1=3.1, _2=1.5, variety='Setosa')
Pandas(Index=35, _1=3.2, _2=1.2, variety='Setosa')
Pandas(Index=36, _1=3.5, _2=1.3, variety='Setosa')
Pandas(Index=37, _1=3.6, _2=1.4, variety='Setosa')
Pandas(Index=38, _1=3.0, _2=1.3, variety='Setosa')
Pandas(Index=39, _1=3.4, _2=1.5, variety='Setosa')
Pandas(Index=40, _1=3.5, _2=1.3, variety='Setosa')
Pandas(Index=41, _1=2.3, _2=1.3, variety='Setosa')
Pandas(Index=42, _1=3.2, _2=1.3, variety='Setosa')
Pandas(Index=43, _1=3.5, _2=1.6, variety='Setosa')
Pandas(Index=44, _1=3.8, _2=1.9, variety='Setosa')
Pandas(Index=45, _1=3.0, _2=1.4, variety='Setosa')
Pandas(Index=46, _1=3.8, _2=1.6, variety='Setosa')
Pandas(Index=47, _1=3.2, _2=1.4, variety='Setosa')
Pandas(Index=48, _1=3.7, _2=1.5, variety='Setosa')
Pandas(Index=49, _1=3.3, _2=1.4, variety='Setosa')
Pandas(Index=50, _1=3.2, _2=4.7, variety='Versicolor')
Pandas(Index=51, _1=3.2, _2=4.5, variety='Versicolor')
Pandas(Index=52, _1=3.1, _2=4.9, variety='Versicolor')
Pandas(Index=53, _1=2.3, _2=4.0, variety='Versicolor')
Pandas(Index=54, _1=2.8, _2=4.6, variety='Versicolor')
Pandas(Index=55, _1=2.8, _2=4.5, variety='Versicolor')
Pandas(Index=56, _1=3.3, _2=4.7, variety='Versicolor')
Pandas(Index=57, _1=2.4, _2=3.3, variety='Versicolor')
Pandas(Index=58, _1=2.9, _2=4.6, variety='Versicolor')
Pandas(Index=59, _1=2.7, _2=3.9, variety='Versicolor')
Pandas(Index=60, _1=2.0, _2=3.5, variety='Versicolor')
Pandas(Index=61, _1=3.0, _2=4.2, variety='Versicolor')
Pandas(Index=62, _1=2.2, _2=4.0, variety='Versicolor')
Pandas(Index=63, _1=2.9, _2=4.7, variety='Versicolor')
Pandas(Index=64, _1=2.9, _2=3.6, variety='Versicolor')
Pandas(Index=65, _1=3.1, _2=4.4, variety='Versicolor')
Pandas(Index=66, _1=3.0, _2=4.5, variety='Versicolor')
Pandas(Index=67, _1=2.7, _2=4.1, variety='Versicolor')
Pandas(Index=68, _1=2.2, _2=4.5, variety='Versicolor')
Pandas(Index=69, _1=2.5, _2=3.9, variety='Versicolor')
Pandas(Index=70, _1=3.2, _2=4.8, variety='Versicolor')
Pandas(Index=71, _1=2.8, _2=4.0, variety='Versicolor')
Pandas(Index=72, _1=2.5, _2=4.9, variety='Versicolor')
Pandas(Index=73, _1=2.8, _2=4.7, variety='Versicolor')
Pandas(Index=74, _1=2.9, _2=4.3, variety='Versicolor')
Pandas(Index=75, _1=3.0, _2=4.4, variety='Versicolor')
Pandas(Index=76, _1=2.8, _2=4.8, variety='Versicolor')
Pandas(Index=77, _1=3.0, _2=5.0, variety='Versicolor')
Pandas(Index=78, _1=2.9, _2=4.5, variety='Versicolor')
Pandas(Index=79, _1=2.6, _2=3.5, variety='Versicolor')
Pandas(Index=80, _1=2.4, _2=3.8, variety='Versicolor')
Pandas(Index=81, _1=2.4, _2=3.7, variety='Versicolor')
Pandas(Index=82, _1=2.7, _2=3.9, variety='Versicolor')
Pandas(Index=83, _1=2.7, _2=5.1, variety='Versicolor')
Pandas(Index=84, _1=3.0, _2=4.5, variety='Versicolor')
Pandas(Index=85, _1=3.4, _2=4.5, variety='Versicolor')
```

```
Pandas(Index=86, _1=3.1, _2=4.7, variety='Versicolor')
Pandas(Index=87, _1=2.3, _2=4.4, variety='Versicolor')
Pandas(Index=88, _1=3.0, _2=4.1, variety='Versicolor')
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In [ ]:
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W2, X2, ans2, weight_vector2 = solve_perceptron('Virginica', df)
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W3, X3, ans3, weight_vector3 = solve_perceptron('Versicolor', df)
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Streaming output truncated to the last 5000 lines.

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[[ 0.08 ]
 [-0.014]
 [ 0.005]]

[[ 0.07 ]
 [-0.047]
 [-0.055]]

[[ 0.08 ]
 [-0.015]
 [-0.008]]

[[0.09 ]
 [0.017]
 [0.037]]

[[ 0.08 ]
 [-0.016]
 [-0.023]]

[[ 0.07 ]
 [-0.045]
 [-0.037]]

[[ 0.08 ]
 [-0.013]
 [ 0.01 ]]

[[ 0.07 ]
 [-0.046]
 [-0.05 ]]

[[ 0.08 ]
 [-0.014]
 [-0.003]]

[[ 0.07 ]
 [-0.047]
 [-0.063]]

[[ 0.08 ]
 [-0.015]
 [-0.016]]

[[0.09 ]
 [0.017]
 [0.029]]

[[ 0.08 ]
 [-0.016]
 [-0.031]]
```
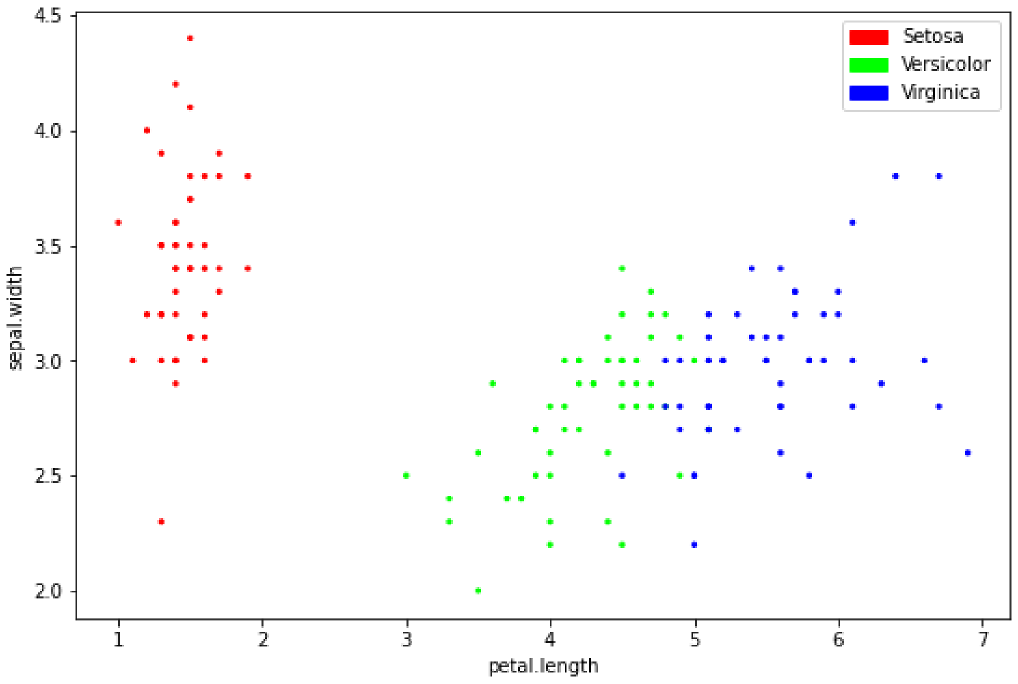
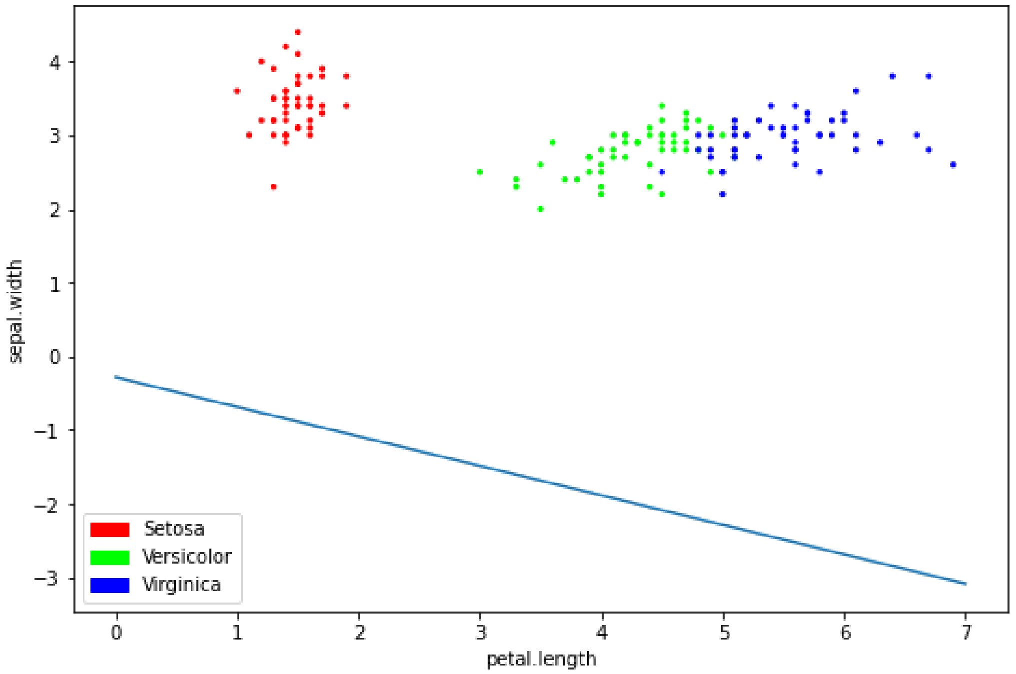

```
[[ 0.07 ]
 [-0.039]
 [-0.044]]

[[ 0.08 ]
 [-0.007]
 [ 0.003]]

[[ 0.07 ]
 [-0.04 ]
 [-0.057]]

[[ 0.08 ]
 [-0.008]
 [-0.01 ]]

[[ 0.07 ]
 [-0.035]
 [-0.061]]

[[ 0.08 ]
 [-0.003]
 [-0.014]]

[[ 0.07 ]
 [-0.03 ]
 [-0.065]]

[[ 0.08 ]
 [ 0.002]
 [-0.018]]

[[0.09 ]
 [0.033]
 [0.031]]

[[ 8.00000000e-02]
 [-3.31401573e-14]
 [-2.90000000e-02]]

[[ 0.07 ]
 [-0.035]
 [-0.043]]

[[ 0.08 ]
 [-0.003]
 [ 0.004]]
```

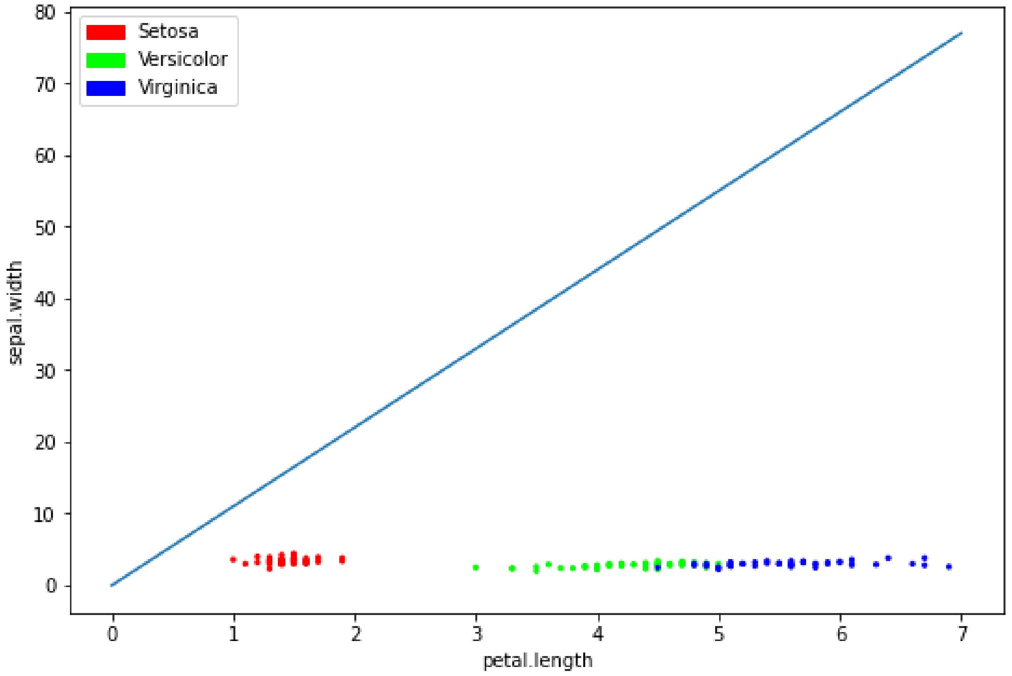
In [ ]:

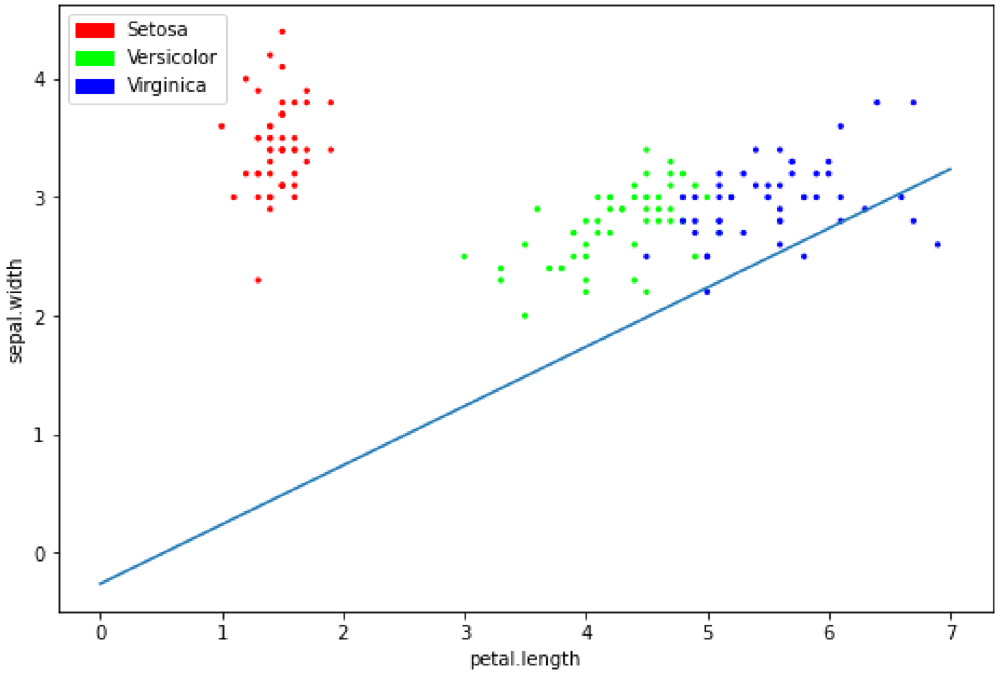
```
for w in weight_vector1:
  cm  = ListedColormap(['#FF0000', '#00FF00', '#0000FF'])
  patch0 = mpatches.Patch(color = '#FF0000', label = 'Setosa')
  patch1 = mpatches.Patch(color = '#00FF00', label = 'Versicolor')
  patch2 = mpatches.Patch(color = '#0000FF', label = 'Virginica')

  x1 = np.arange(8)

  plt.figure(figsize = (9, 6))
  plt.scatter(df['petal.length'], df['sepal.width'], c = [0] * 50 + [1] * 50 + [2] * 50, cmap = cm, s = 5)
  plt.xlabel('petal.length')
  plt.ylabel('sepal.width')
  plt.legend(handles = [patch0, patch1, patch2])

  slope, c = -w[2] / w[1], -w[0] / w[1]
  plt.plot(x1, slope * x1 + c)
  plt.show()
```

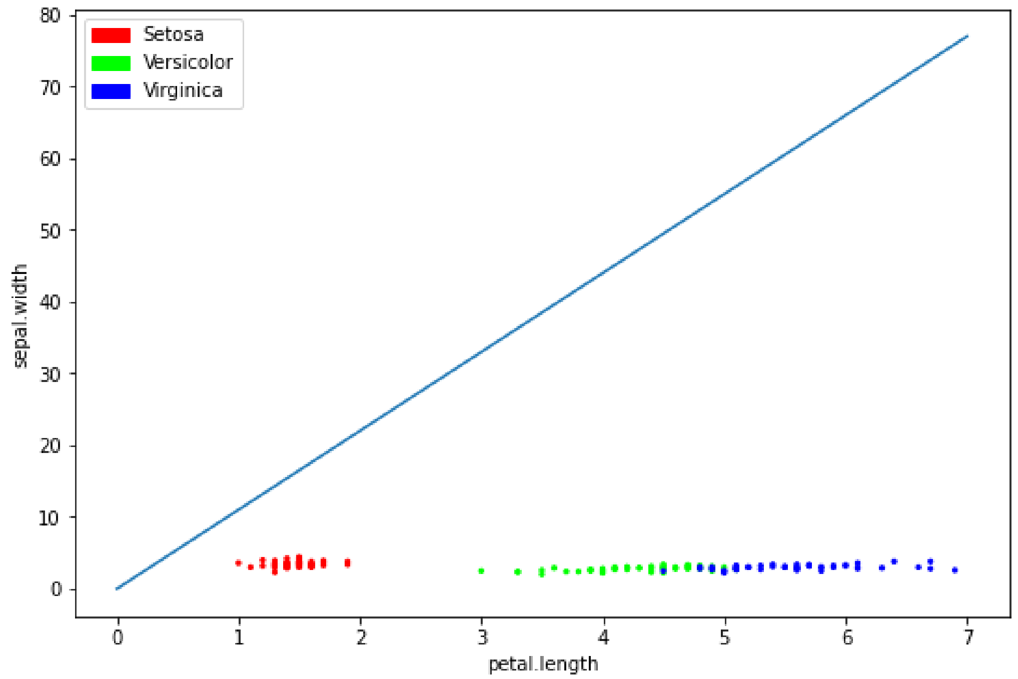
```
/usr/local/lib/python3.7/dist-packages/ipykernel_launcher.py:15: RuntimeWarning: invalid value encountered in true_divide
  from ipykernel import kernelapp as app
```

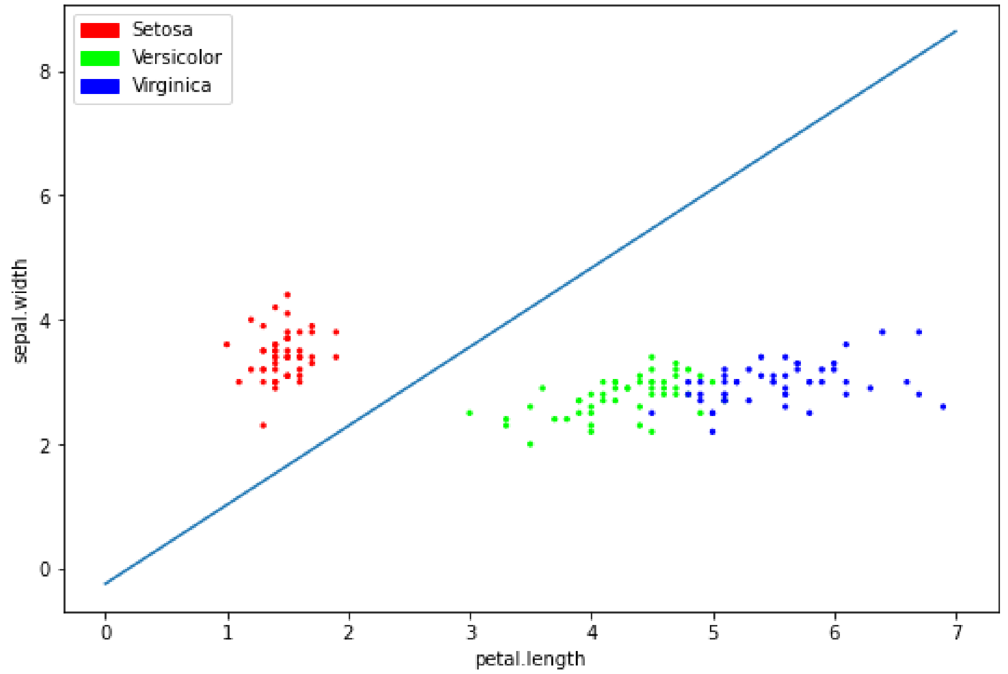

Setosa
Versicolor
Virginica
sepal.width
petal.length
4.5
4.0
3.5
3.0
2.5
2.0
1
2
3
4
5
6
7


Setosa
Versicolor
Virginica
sepal.width
petal.length
4
3
2
1
0
−1
−2
−3
0
1
2
3
4
5
6
7


Setosa
Versicolor
Virginica
sepal.width
petal.length
0
10
20
30
40
50
60
70
80
0
1
2
3
4
5
6
7

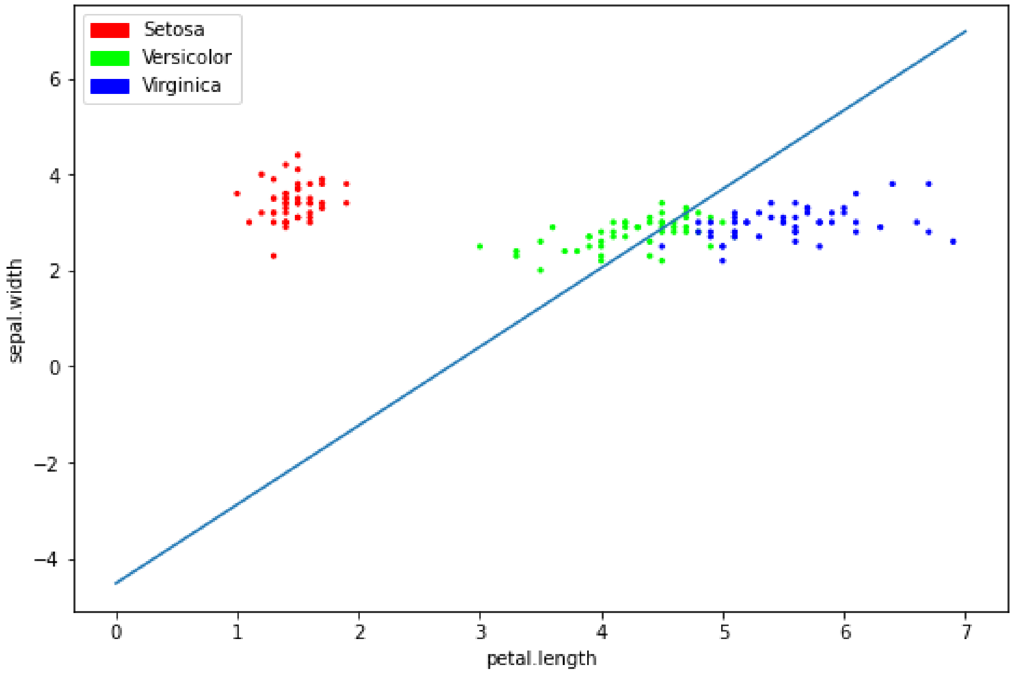

Setosa
Versicolor
Virginica
sepal.width
petal.length
0
1
2
3
4
0
1
2
3
4
5
6
7





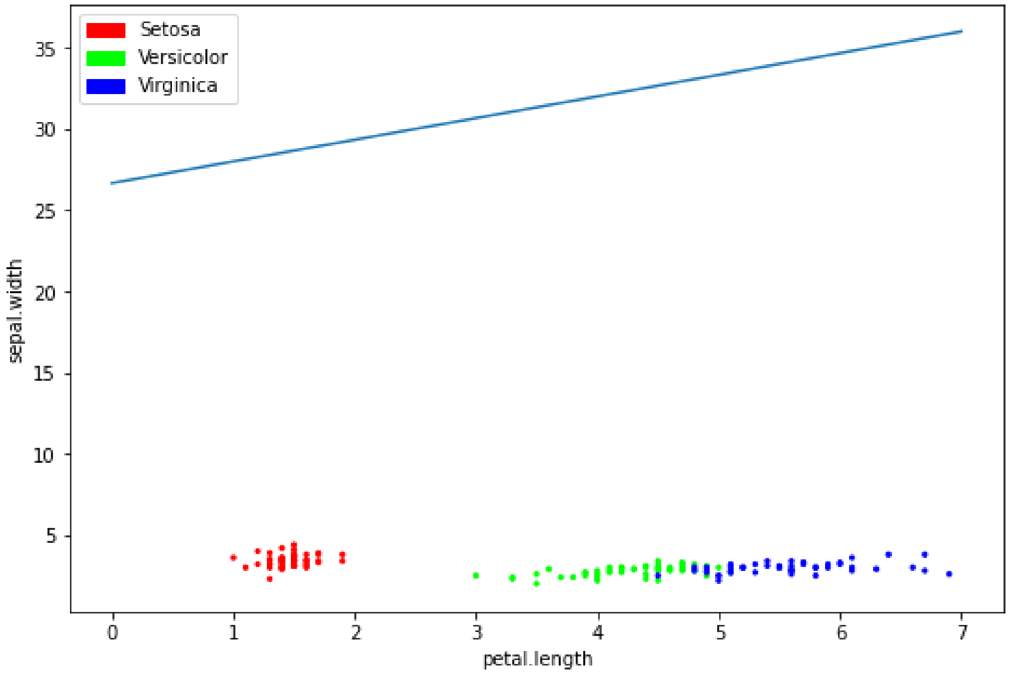
In [ ]:

```
w = W2
cm  = ListedColormap(['#FF0000', '#00FF00', '#0000FF'])
patch0 = mpatches.Patch(color = '#FF0000', label = 'Setosa')
patch1 = mpatches.Patch(color = '#00FF00', label = 'Versicolor')
patch2 = mpatches.Patch(color = '#0000FF', label = 'Virginica')

x1 = np.arange(8)

plt.figure(figsize = (9, 6))
plt.scatter(df['petal.length'], df['sepal.width'], c = [0] * 50 + [1] * 50 + [2] * 50, cmap = cm, s = 5)
plt.xlabel('petal.length')
plt.ylabel('sepal.width')
plt.legend(handles = [patch0, patch1, patch2])

slope, c = -w[2] / w[1], -w[0] / w[1]
plt.plot(x1, slope * x1 + c)
plt.show()
```



In [ ]:

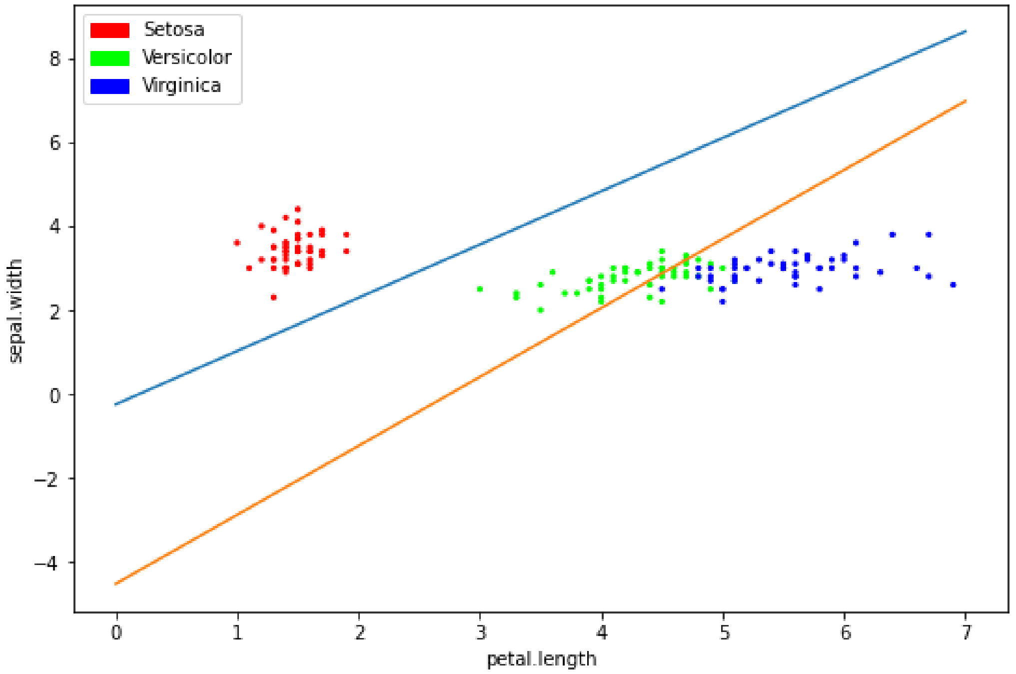
```
w = W3
cm  = ListedColormap(['#FF0000', '#00FF00', '#0000FF'])
patch0 = mpatches.Patch(color = '#FF0000', label = 'Setosa')
patch1 = mpatches.Patch(color = '#00FF00', label = 'Versicolor')
patch2 = mpatches.Patch(color = '#0000FF', label = 'Virginica')

x1 = np.arange(8)

plt.figure(figsize = (9, 6))
plt.scatter(df['petal.length'], df['sepal.width'], c = [0] * 50 + [1] * 50 + [2] * 50, cmap = cm, s = 5)
plt.xlabel('petal.length')
plt.ylabel('sepal.width')
plt.legend(handles = [patch0, patch1, patch2])

slope, c = -w[2] / w[1], -w[0] / w[1]
plt.plot(x1, slope * x1 + c)
plt.show()
```



In [ ]:

```
cm  = ListedColormap(['#FF0000', '#00FF00', '#0000FF'])
patch0 = mpatches.Patch(color = '#FF0000', label = 'Setosa')
```

```
patch1 = mpatches.Patch(color = '#00FF00', label = 'Versicolor')
patch2 = mpatches.Patch(color = '#0000FF', label = 'Virginica')

x1 = np.arange(8)

plt.figure(figsize = (9, 6))
plt.scatter(df['petal.length'], df['sepal.width'], c = [0] * 50 + [1] * 50 + [2] * 50, cmap = cm, s = 5)
plt.xlabel('petal.length')
plt.ylabel('sepal.width')
plt.legend(handles = [patch0, patch1, patch2])

w = W1
slope, c = -w[2] / w[1], -w[0] / w[1]
plt.plot(x1, slope * x1 + c)

w = W2
slope, c = -w[2] / w[1], -w[0] / w[1]
plt.plot(x1, slope * x1 + c)
plt.show()
```



SVM

Defining the necessary functions used

In [ ]:

```
def SVM(C1, C2):
  x1 = np.array(C1.to_numpy())
  x2 = np.array(C2.to_numpy())

  x = np.vstack((x1, x2))
  y = np.hstack((np.ones(len(x1)), np.ones(len(x2)) * -1))

  X = np.dot(x, np.transpose(x))
  Y = np.outer(y, y)

  n = X.shape[0]

  H = matrix(Y * X)
  f = matrix(np.ones(n) * -1)
  A = matrix(np.diag(np.ones(n) * -1))
  a = matrix(np.zeros(n))
  B = matrix(y, (1, n))
  b = matrix(0.0)
```

```
  # solve QP problem
  solution = solvers.qp(H, f, A, a, B, b)
  print('')

  # Lagrange multipliers
  t1 = np.ravel(solution['x'])

  # Support vectors have non zero lagrange multipliers
  for i in range(0, len(t1)):
    if(t1[i] < 1e-4):
      t1[i] = 0.0
    else:
      t1[i] = round(t1[i], 4)

  t1 = np.array(t1)
  print(t1)

  # Weight vector
  w = np.zeros(X.shape[1])
  for i in range(X.shape[1]):
    w[i] = t1[i] * y[i]

  w = np.dot(w, x)

  # Intercept
  b = 0
  for i in range(len(t1)):
    if(t1[i] != 0.0):
      b = round(1 - np.dot(w, np.transpose(x[i])), 4)
      break

  return w, b
```

In [ ]:

```
def decision_boundary(labels, C1, W1, b1, C2, W2, b2, C3, W3, b3):
  x = np.arange(8)

  plt.figure(figsize = (9, 6))
  plt.scatter(C1['petal.length'], C1['sepal.width'], label = 'Setosa', color = '#FF0000')
  plt.scatter(C2['petal.length'], C2['sepal.width'], label = 'Versicolor', color = '#00FF00')
  plt.scatter(C3['petal.length'], C3['sepal.width'], label = 'Virginica', color = '#0000FF')

  plt.suptitle("Iris Dataset Classification", fontweight = "bold")
  plt.ylabel("Petal length (in cm)", fontweight = "bold")
  plt.xlabel("Sepal width (in cm)", fontweight = "bold")

  w = W1
  b = b1
  print(w, "\n")
  slope, c = -w[0] / w[1], -b / w[1]
  plt.plot(x, slope * x + c, label = "Decision Boundary - Setosa and Versicolor")

  w = W3
  b = b3
  print(w, "\n")
  slope, c = -w[0] / w[1], -b / w[1]
  plt.plot(x, slope * x + c, label = "Decision Boundary - Setosa and Virginica")
  plt.legend()
  plt.show()

  plt.figure(figsize = (9, 6))
  plt.scatter(C1['petal.length'], C1['sepal.width'], label = 'Setosa', color = '#FF0000')
  plt.scatter(C2['petal.length'], C2['sepal.width'], label = 'Versicolor', color = '#00FF00')
  plt.scatter(C3['petal.length'], C3['sepal.width'], label = 'Virginica', color = '#0000FF')

  plt.suptitle("Iris Dataset Classification", fontweight = "bold")
  plt.ylabel("Petal length (in cm)", fontweight = "bold")
  plt.xlabel("Sepal width (in cm)", fontweight = "bold")
```

```
    w = W1
    b = b1
    print(w, "\n")
    slope, c = -w[0] / w[1], -b / w[1]
    plt.plot(x, slope * x + c, label = "Decision Boundary - Setosa and Versicolor")

    w = W3
    b = b3
    print(w, "\n")
    slope, c = -w[0] / w[1], -b / w[1]
    plt.plot(x, slope * x + c, label = "Decision Boundary - Setosa and Virginica")

    w = W2
    b = b2
    print(w, "\n")
    slope, c = -w[0] / w[1], -b / w[1]
    plt.plot(x, slope * x + c, label = "Decision Boundary - Versicolor and Virginica")
    plt.legend()
    plt.show()
```

Code

In [ ]:

```
C1 = df.loc[df.variety == 'Setosa'].drop(['variety'], axis = 1)
print(C1.head(5))
print(C1.tail(5))
C1.shape
```

```
   sepal.width  petal.length
0          3.5           1.4
1          3.0           1.4
2          3.2           1.3
3          3.1           1.5
4          3.6           1.4
    sepal.width  petal.length
45          3.0           1.4
46          3.8           1.6
47          3.2           1.4
48          3.7           1.5
49          3.3           1.4
```

Out[ ]: (50, 2)

In [ ]:

```
C2 = df.loc[df.variety == 'Versicolor'].drop(['variety'], axis = 1)
print(C2.head(5))
print(C2.tail(5))
C2.shape
```

```
    sepal.width  petal.length
50          3.2           4.7
51          3.2           4.5
52          3.1           4.9
53          2.3           4.0
54          2.8           4.6
    sepal.width  petal.length
95          3.0           4.2
96          2.9           4.2
97          2.9           4.3
98          2.5           3.0
99          2.8           4.1
```

Out[ ]: (50, 2)

In [ ]:

```
C3 = df.loc[df.variety == 'Virginica'].drop(['variety'], axis = 1)
print(C3.head(5))
print(C3.tail(5))
C3.shape
```

```
     sepal.width  petal.length
```

```
100          3.3           6.0
101          2.7           5.1
102          3.0           5.9
103          2.9           5.6
104          3.0           5.8
     sepal.width  petal.length
145          3.0           5.2
146          2.5           5.0
147          3.0           5.2
148          3.4           5.4
149          3.0           5.1
```

Out[ ]:
```
(50, 2)
```

In [ ]:
```python
w1, b1 = SVM(C1, C2)

print(w1)
print('')
print(b1)
```

```
     pcost       dcost       gap    pres   dres
 0: -4.3867e+00 -8.1716e+00  3e+02  1e+01  2e+00
 1: -2.3798e+00 -2.5258e+00  2e+01  1e+00  2e-01
 2: -4.3479e-01 -1.7538e+00  2e+00  5e-02  6e-03
 3: -6.7595e-01 -1.0544e+00  5e-01  1e-02  1e-03
 4: -8.1435e-01 -1.1260e+00  4e-01  4e-03  5e-04
 5: -1.0085e+00 -1.0271e+00  2e-02  8e-05  1e-05
 6: -1.0251e+00 -1.0253e+00  2e-04  9e-07  1e-07
 7: -1.0253e+00 -1.0253e+00  2e-06  9e-09  1e-09
 8: -1.0253e+00 -1.0253e+00  2e-08  9e-11  1e-11
Optimal solution found.

[0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.8098 0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.2155 0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     1.0253 0.    ]
[ 0.68572 -1.25713]

1.0571
```

In [ ]:
```python
w2, b2 = SVM(C2, C3)

print(w2)
print('')
print(b2)
```

```
     pcost       dcost       gap    pres   dres
 0: -3.7394e+01 -8.7261e+01  5e+02  2e+01  2e+00
 1: -1.0739e+02 -1.7754e+02  3e+02  1e+01  2e+00
 2: -5.7593e+02 -8.7585e+02  4e+02  1e+01  2e+00
 3: -1.9989e+03 -2.2838e+03  3e+02  1e+01  1e+00
 4: -6.2946e+03 -6.9640e+03  7e+02  1e+01  1e+00
 5: -3.5722e+04 -3.7828e+04  2e+03  1e+01  1e+00
 6: -1.6095e+05 -1.6859e+05  8e+03  1e+01  1e+00
 7: -2.7869e+06 -2.8212e+06  3e+04  9e+00  1e+00
 8: -1.2140e+08 -1.2212e+08  7e+05  9e+00  1e+00
 9: -1.9228e+10 -1.9236e+10  8e+06  9e+00  1e+00
10: -1.9394e+10 -1.9402e+10  8e+06  9e+00  1e+00
11: -2.1446e+10 -2.1454e+10  8e+06  9e+00  1e+00
12: -3.4380e+10 -3.4393e+10  1e+07  9e+00  1e+00
13: -5.4784e+10 -5.4805e+10  2e+07  9e+00  1e+00
14: -8.7577e+10 -8.7607e+10  3e+07  9e+00  1e+00
15: -1.0218e+11 -1.0222e+11  3e+07  9e+00  1e+00
Terminated (singular KKT matrix).

[2.71521160e+07 4.51595280e+06 1.29133612e+08 1.00265211e+08
```

```
 3.09216898e+08 1.42980907e+08 4.25115472e+06 1.24416108e+07
 1.96826025e+08 2.17717942e+07 1.22555173e+08 8.75290549e+06
 7.67649435e+07 1.58491041e+08 2.83907018e+05 2.47893925e+05
 3.68052772e+07 4.61959347e+07 1.00816888e+09 8.83969315e+07
 1.09546196e+07 2.12476560e+07 2.32690275e+09 5.51639458e+09
 1.57936433e+06 1.95865133e+07 1.61922907e+08 2.70030424e+08
 8.40604842e+07 1.41692613e+07 5.99425006e+07 2.47557397e+07
 2.40241351e+07 3.66390422e+10 2.30503621e+07 7.90295844e+06
 4.95320836e+07 4.32821956e+08 1.63526678e+07 2.19136307e+08
 3.34342989e+08 4.28761312e+07 2.65803368e+08 2.20079195e+07
 5.62148082e+07 1.59587525e+08 5.83123718e+07 3.85951082e+06
 1.63510780e+09 1.03242757e+08 8.28841307e+07 2.61588909e+08
 2.35253914e+07 6.27638197e+07 3.70348945e+07 8.94810821e+07
 2.68823443e+10 1.80964094e+07 3.07022922e+08 3.14754482e+07
 3.52233325e+08 2.51425758e+09 5.19114959e+07 2.13739902e+07
 1.19027991e+08 1.44055382e+08 5.18845100e+07 8.34998928e+07
 1.26029271e+06 4.46210403e+07 4.89155288e+08 4.28792523e+08
 4.12928047e+04 2.17695331e+08 1.59905611e+08 1.80865194e+07
 4.76400125e+09 1.83710018e+09 6.17354594e+07 3.24900781e+07
 5.78027985e+09 1.06941424e+08 6.17370471e+07 1.19094824e+08
 1.37617004e+08 8.42083709e+07 8.68930538e+07 5.50808776e+07
 1.26831614e+09 2.95476071e+08 1.01801411e+09 8.46133030e+08
 2.20642492e+08 6.11327938e+07 1.59593590e+08 6.12728556e+08
 8.96855032e+06 6.14535479e+08 1.36892256e+08 2.36350952e+08]
[0.00036621 0.00012207]

0.9983
```

In [ ]:

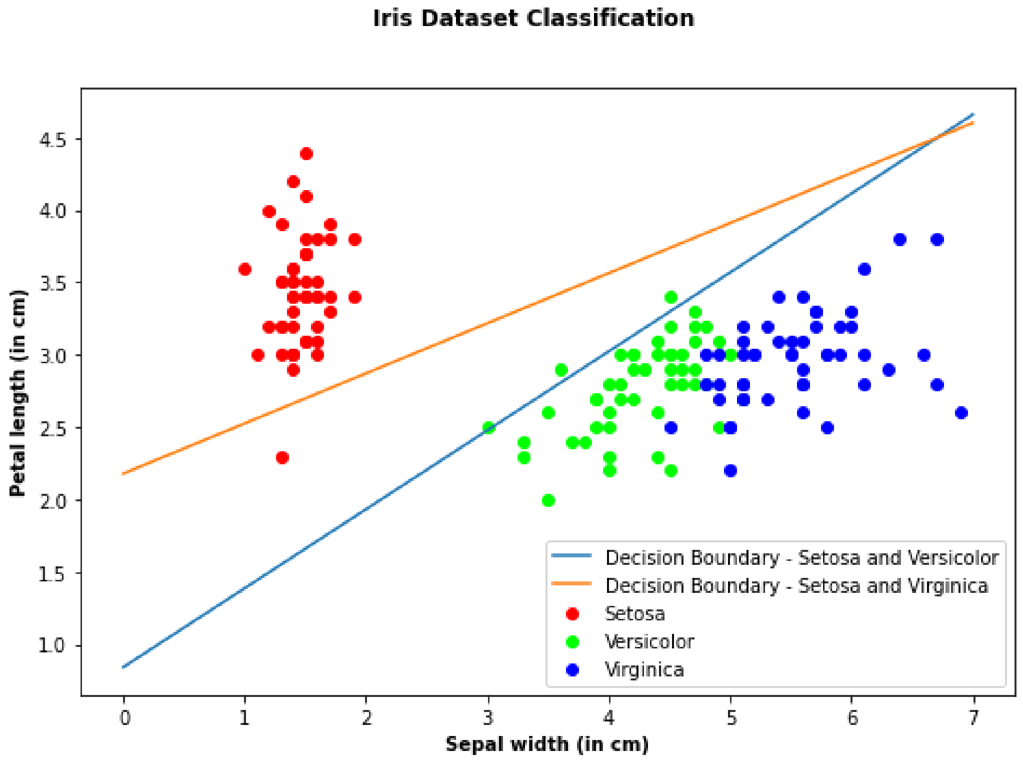
```python
w3, b3 = SVM(C1, C3)

print(w3)
print('')
print(b3)
```

```
     pcost       dcost       gap    pres   dres
 0: -3.3137e+00 -4.9765e+00  2e+02  2e+01  1e+00
 1: -1.6126e-01 -8.1531e-01  1e+01  9e-01  8e-02
 2:  1.9106e-02 -6.5205e-01  9e-01  1e-02  1e-03
 3: -1.5487e-01 -2.7974e-01  1e-01  7e-04  6e-05
 4: -2.2572e-01 -2.8972e-01  6e-02  1e-04  1e-05
 5: -2.5983e-01 -2.6459e-01  5e-03  1e-05  8e-07
 6: -2.6415e-01 -2.6421e-01  5e-05  1e-07  9e-09
 7: -2.6420e-01 -2.6420e-01  5e-07  1e-09  9e-11
 8: -2.6420e-01 -2.6420e-01  5e-09  1e-11  9e-13
Optimal solution found.

[0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.2642 0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.2642 0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.    ]
[ 0.23778 -0.68692]

1.4967
```

In [ ]:

```python
decision_boundary(["Setosa", "Versicolor", "Virginica"], C1, w1, b1, C2, w2, b2, C3, w3, b3)
```

```
[ 0.68572 -1.25713]

[ 0.23778 -0.68692]
```




```
[ 0.68572 -1.25713]

[ 0.23778 -0.68692]

[0.00036621 0.00012207]
```