

HW2

February 13, 2024

Problem 1 [Implementation - 2 pts] See the attached notebook, HW2-1 (Cross-Validation).

Problem 2 [Logistic regression - 3 pts] See the attached notebook, HW2-2 (Classification).

Problem 3 [Logistic regression - 2 pts] Complete the following derivation: find the derivatives of the loss term w.r.t. $\boldsymbol{\theta}$:

Given the loss term

$$\ell(\boldsymbol{\theta}) = y \log(h(\boldsymbol{\theta}^\top \mathbf{x})) + (1 - y) \log(1 - h(\boldsymbol{\theta}^\top \mathbf{x}))$$

where

$$h(z) = \frac{1}{1 + e^{-z}}, \quad \boldsymbol{\theta} = \begin{bmatrix} \theta_0 \\ \theta_1 \\ \theta_2 \end{bmatrix}, \quad \text{and} \quad \mathbf{x} = \begin{bmatrix} 1 \\ x_1 \\ x_2 \end{bmatrix},$$

write down the analytical expression of the derivative of $\ell(\boldsymbol{\theta})$ w.r.t. $\boldsymbol{\theta}$:

$$\nabla_{\boldsymbol{\theta}} \ell(\boldsymbol{\theta}) = \begin{bmatrix} \frac{\partial \ell(\boldsymbol{\theta})}{\partial \theta_0} \\ \frac{\partial \ell(\boldsymbol{\theta})}{\partial \theta_1} \\ \frac{\partial \ell(\boldsymbol{\theta})}{\partial \theta_2} \end{bmatrix} =$$

Write down a step-by-step derivation. You will have to use the following mathematical facts. In every step of the derivation, explain how you used these mathematical facts.

- use the chain-rule: $\frac{dy}{dx} = \frac{dy}{dz} \frac{dz}{dx}$
- derivative of the log function: $\frac{d \log(x)}{dx} = \frac{1}{x}$
- derivative of $h(x) = \frac{1}{1+e^{-x}}$: $\frac{dh(x)}{dx} = h(x)(1 - h(x))$