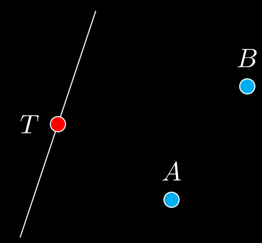
1. Prove that among all triangles with a fixed perimeter P, the equilateral triangle is the one that maximizes the area.

2. Generalize the above for a n-sided polygon and prove it.

3. Consider an acute angled triangle ABC and a point D on BC. Find points (by construction) E on AC and F on AB such that the perimeter of the triangle DEF is minimum.

4. Prove (via geometry) that the area of a cyclic quadrilateral is the maximum possible for any quadrilateral with given side lengths.

5. **Bus terminus location problem**: Geometrically obtain the location of the bus terminus T on the road segment PQ such that the lengths of the roads linking T with the two cities A and B is minimum.



6. Given two points A and B in a vertical plane (with A above B), find the curve which the object must follow so that starting from A, it reaches B in the shortest possible time under gravity.

7. Given three cities A, B and C, geometrically find the location where an artificial water reservoir must be constructed so that the sum of the length of the pipeline from the reservoir to the three cities is minimum.

8. Prove the arithmetic mean geometric mean inequality, i.e.,

$$\frac{a_1 + a_2 + \cdots + a_n}{n} \geq \sqrt[n]{a_1 a_2 \cdots a_n}$$

where $a_i \geq 0$ and $n \in \mathbb{Z}^+$.

9. Prove that in any triangle ABC, we have

$$\sin(A) + \sin(B) + \sin(C) \leq \frac{3\sqrt{3}}{2}$$

10. It is desired to construct a bridge across a river so that the two cities, A and B, on either side of the river are connected. Due to construction constraints it is desired to have the bridge to be perpendicular to the flow of the river. See Figure 1 for the details. Geometrically identify where the bridge needs to be located, i.e., the points C and D, so that the distance of the path from A to B is minimized.

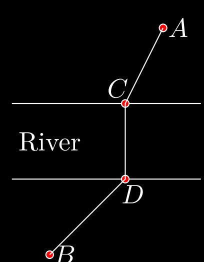


Figure 1: Bridge construction