

# Discrete Structures

IIIT Hyderabad

Monsoon 2020

*Tutorial 5*

September 30, 2020

## 1 Questions

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# Question 0

**0.1:** Prove that if  $R, S$  are transitive  $\implies R \cap S$  is transitive.

**0.2:** Let  $R$  be a reflexive relation on set  $A$ . Show that  $R$  is an equivalence relation if and only if  $(a, b)$  and  $(a, c)$  are in  $R$  implies that  $(b, c)$  is in  $R$ .

# Question 1

**1.1:** Find  $\phi(120)$ .

**1.2:** Find a number  $a < p$  such that  $a \cdot p = 1 \bmod(p)$  (modular inverse) without using any online tools. ( $p = 31, a = 12$ ).

[Hint: Use Extended Euclid's Division Algorithm]

## Question 2

An encoding scheme is defined as follows -

$A = 00$ ,  $B = 01$ ,  $C = 02$  and so on, and 00 for space. Take  $p = 13$ ,  $q = 17$  and the public key ( $e$ ) as 5.

- ① Find the private key.
- ② Find the ciphertext of the message "HI ALL".
- ③ Decrypt the ciphertext to verify the same.

- ① If  $A, B$  and  $C$  are any three sets, then prove that  $(A \Delta B) \Delta C = A \Delta (B \Delta C)$  without apply the Venn-Euler's diagram, where  $\Delta$  denotes the symmetric difference operation between two sets.
- ② If  $A = \{n \in \mathbb{N} : n \text{ is a multiple of } 12\}$  and  $B = \{n \in \mathbb{N} : n \text{ is a multiple of } 18\}$ , find (i)  $A \cup B$ , (ii)  $A \cap B$ , (iii)  $(A \cup B) \setminus (B \cap A)$ , (iv)  $A \times B$  and (v)  $P(A \cup B)$
- ③ Let  $U$  be the set of all quadrilaterals in a plane, and  $P, R, T$  and  $S$  are the subsets of  $U$  defined as follows:
  - $P$  = set of all parallelograms
  - $R$  = set of all rhombus
  - $T$  = set of all rectangles
  - $S$  = set of all squares

Find the relationships between  $P, R, T$  and  $S$  in terms of containment.

- 4 In a survey of 100 delegates attending a conference on Discrete Structures held at IIIT Hyderabad, the number of delegates who knew one or more of the 3 languages: English, French and Germany, was as follows: English 28, French 30, Germany 42; English and Germany 10; English and French 8; French and Germany 5. Only 3 people know all the three languages.
- How many did not know any language at all?
  - How many knew only Germany?
- 5 Prove or disprove the following statements.
- $(A - B) \times C = (A \times C) - (B \times C)$
  - $(A \Delta B) \times C = (A \times C) \Delta (B \times C)$