International Institute of Information Technology Hyderabad

Discrete Structures (MA5.101)

Assignment 2
Deadline: October 16, 2020 (Friday), 23:55 PM
Total Marks: 70
Instructions: Submit ONLY handwritten scanned pdf file in the moodle under Assignments directory.

1. Let R be a symmetric and transitive relation on a set A. Show that if for every a in A there exists b in A such that (a, b) is in R, then R is an equivalence relation.

[5]

2. Let A a set with 10 distinct elements.

   (a) How many different binary relations on A are there?

   (b) How many of them are reflexive?

   (c) How many of them are anti-symmetric?

   (d) How many of them are reflexive and symmetric?

   (e) How many of them are equivalence relations?

[5 × 2 = 10]

3. Let R be a binary relation fro A to B. The *converse* of R, denoted by R^{-1}, is a binary relation from B to A such that

$$R^{-1} = \{(b, a)|(a, b) \in R\}$$

   (a) Let R_1 and R_2 be binary relations from A to B. Is it true that

$$(R_1 \cup R_2)^{-1} = R_1^{-1} \cup R_2^{-1}?$$

   (b) Let R be a binary relation on A.
   (i) If R is reflexive, is R^{-1} necessarily reflexive?
   (ii) If R is symmetric, is R^{-1} necessarily symmetric?
   (iii) If R is transitive, is R^{-1} necessarily transitive?

[4 + 3 × 2 = 10]

4. Let $\mathcal{R}$ be a set of the real numbers. On $\mathcal{R}^2$, the following relations are defined. Check whether they are equivalence relations or not. If not, give reasons. If yes, find the equivalence classes.

   (a) A relation ρ is defined on $\mathcal{R}^2$ as ${}_{(a,b)}\rho_{(c,d)}$ if and only if (iff) both the points (a, b) and (c, d) lie on the same curve $4x + 5y = k$, for some $k \in \mathcal{R}$.

   (b) Another relation ψ is defined on $\mathcal{R}^2$ ${}_{(a,b)}\psi_{(c,d)}$ iff both the points (a, b) and (c, d) lie on the same curve $9x^2 + 16y^2 = k^2$, for some $k \in \mathcal{R}$.

[10 + 10 = 20]

5. Let A be a non-empty set and R be an equivalence relation defined in A. Let $a \in A$ and $b \in A$ be two arbitrary elements. Then, prove that

   (i) $[a] = [b] \Leftrightarrow (a, b) \in R$

   (ii) either $[a] = [b]$ or $[a] \cap [b] = \emptyset$, that is, either two equivalence classes are identical or disjoint.

[5 + 5 = 10]

6. Let R be a partial order relation on set A. Two elements $a, b \in A$ are **comparable** if either ${}_aR_b$ or ${}_bR_a$. If all elements of A are comparable with each other, then the partially ordered set A (with respect to (w.r.t.) R) is said to be a **totally ordered set**.

   Give an example of a **totally ordered set**.

[5]

7. Let R be an equivalence relation on a set A. Let $\{A_1, A_2, \ldots .A_k\}$ be a collection of subsets of A such that $A_i \cap A_j = \emptyset$ for $i \neq j$ such that a and b are contained in one of the subsets if and only if the ordered pair (a, b) is in R. Show that $\{A_1, A_2, \ldots .A_k\}$ is a partition of A.

[10]

All the best!!!