International Institute of Information Technology Hyderabad

Discrete Structures (MA5.101)

Assignment 3

Deadline: October 25, 2020 (Sunday), 23:55 PM

Total Marks: 70

Instructions: Submit ONLY handwritten scanned pdf file
in the moodle under Assignments directory.

1. State whether the following statements are true or false. Justify the false ones.

    (a) If $f : A \to B$ be any function and $X \subseteq A$, then $f^{-1}(f(X)) = X$.

    (b) If $f : A \to B$ be any function and Y be any subset of B, then $f^{-1}(f(Y)) = Y$.

    [5 + 5 = 10]

2. Let $A = \{1, 2, 3, 4, 5\}$, $B = \{x, y, z, t\}$, $C = \{x, y, t\}$, $D = \{2, 4\}$. Define a function $f : A \to B$ and $g : B \to C$ as follows:

    $$f = \{(2, x), (3, x), (4, t), (1, t), (5, x)\}$$

    $$g = \{(x, 1), (y, 3), (z, 4), (t, 1)\}$$

    Find the following:

    (a) $f \circ g, g \circ f$

    (b) $(f \circ g) \circ f$

    (c) $f^{-1}(f(D))$

    (d) $f(f^{-1}(C))$

    (e) $Im(f), f^{-1}(Im(f))$

    [5 * 2 = 10]

3. Let $\mathcal{R}$ be the set of real numbers. Then, for some $c \in \mathcal{R}$, prove that $(0, 1)$ and (c, ∞) have the same cardinality.

    [10]

4. Let $\mathcal{R}$ be the set of real numbers. Define a function $f : \mathcal{R} \to \mathcal{R}$ such that $f(x) = x^3 + 3ax^2 + 3bx + c$. Find the condition under which f is bijective.

    [5]

5. Let $\mathcal{R}$ be the set of real numbers. Define a function $f : \mathcal{R} \to \mathcal{R}$ such that

$$f(x) \;=\; \begin{cases} x & \text{if } x \text{ is rational} \\ 1 - x & \text{if } x \text{ is irrational} \end{cases}$$

   Prove that f is invertible and find f^{-1}.

   [10]

6. Prove that the number of points on a sphere is the same as those on a plane.

   [10]

7. Let $f : S \to T$ and $g : T \to U$ be two functions such that $g \circ f$ is defined. If both f and g have left inverses, prove that $g \circ f$ has a left inverse.

   [10]

8. Determine whether or not the following set is countable:

   The set of positive integers divisible by 5.

   [5]

All the best!!!