# Discrete Structures

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# Introduction



1. Questions
   - Question 1
   - Question 2
   - Question 3

# Question 1

**1.1** : Construct an addition and multiplication table for the following. Subsequently conclude whether they are rings, fields or integral domains -

1. $< Z_4, +_4, \cdot_4 >$
2. $< Z_2[i], +_2, \cdot_2 >$ here $Z_n[i] = a + bi$, where $a, b \in Z_n$.

**1.2** : Let

$$R = \left\{ \begin{bmatrix} a & -b \\ b & a \end{bmatrix}, a, b \in Z_7 \right\}$$

Prove that R is a commutative ring under $+_7$ and $\cdot_7$. How many elements are in R? Is R a field? What happens when $Z_7$ is replaced by $Z_5$ ?

# Question 2

**2.1** : Show that a ring is commutative if it has the property that $ab = ca$ implies $b = c$ when $a \neq 0$.
**2.2** : Show that if $m, n$ are integers and $a, b$ are elements in a ring. Then $(ma)(nb) = (mn)(ab)$, here if $m$ is an integer and $a$ is an element, then $ma$ means $a + a + \ldots + a$ ($m$ times).
**2.2** : Show that a ring that is cyclic under addition is commutative.

# Question 3

**3.1** : Which of the following are irreducible -

1. $x^4 + x^2 + 4$ over G.F(11).
2. $x^3 + 6$ over G.F(7).
3. $x^3 + x^2 + x + 1$ over GF(2).

**3.2** : Find multiplicative inverse of 343 in $Z_{821}$ using Extended Euclidean Algorithm.