

Discrete Structures

IIIT Hyderabad

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Tutorial 15

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1 Questions

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Question 1

Prove the following -

- 1 In an group, $bc = ac \implies b = a$, and that $cb = ca \implies b = a$. And for an abelian group, $ab = ca \implies b = c$.
- 2 Prove that in an abelian group, $(ab)^n = a^n b^n$.

Question 2

Prove the following -

- ① If H and K are subgroups of G , show that $H \cap K$ is a subgroup of G .
 - ② Show that sub-group of cyclic group is cyclic.
 - ③ Let H be a subgroup of a group G . Let $N = \{x | x \in G, xHx^{-1} = H\}$. Show that N is a sub-group of G .
 - ④ Let $(A, *)$ be a group and B be a subset of A . If B is a finite set, then $(B, *)$ must be a subgroup of $(A, *)$ if $*$ is closed under B .
- * (1969 Putnam Competition) Prove that no group is the union of two proper subgroups. Does the statement remain true if “two” is replaced by “three”?

Question 3

Find all sub-groups of the following -

① $\langle Z_{12}, \times \rangle$

② $\langle Z_8, \times \rangle$

③ $\langle Z_{11}, \times \rangle$

Generalise for $\langle Z_{p^2q}, \times \rangle$ and $\langle Z_{p^n}, \times \rangle$