Discrete Structures

IIIT Hyderabad

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Tutorial 11

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Introduction

1. Questions
   - Question 1
   - Question 2
   - Question 3
   - Question 4

Question 1

Let a permutation p be :-

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 2 & 1 & 5 & 6 & 7 & 3 & 4 \end{pmatrix}$$

1. Let q be defined as

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 5 & 7 & 4 & 3 & 1 & 2 & 6 \end{pmatrix}$$

   Find the permutation $q \circ p$.
2. Identify all the cycles in p.
3. How many transpositions are there in p ? p an odd or even permutation ?

Question 2

2.1: Show that the sets $S = \{x \in \mathbb{C}, |x| = 1\}$ and $\mathbb{R}$ have same cardinality.

Question 3

Let $A = \{x \in \mathbb{R} | x \in [0, 1]\}$
$B = \{x \in \mathbb{N} | x \text{ is a perfect square}\}$
$C = \{x \in \mathbb{Z} | x < 10\}$
Which of the following are countable ?

1. $B \cup C$
2. $A \cap B$
3. $\mathcal{P}(B)$

Question 4

4.1: Find left and right inverses of each of them (wherever exist) -

1. $f : \mathbb{N} \to \mathbb{N}$, $f(n) = n + 3$
2. $f : \mathbb{Z} \to \mathbb{E}^*, f(x) = |x| + x$. E^* is the set of even numbers.

4.2: Which of the following is/are projections -

1. $f(x) = e^x$,$f : \mathbb{R} \to \mathbb{R}$
2. $f(x) = \lfloor x \rfloor$, $f : \mathbb{Z} \to \mathbb{Z}$

4.3: Find $\sum_{j=1}^{j=100} e_S(j)$ on $U = \mathbb{Z}$, when $f : \mathbb{R} \to \mathbb{R}, f(x) = x^2$ and $S = Range(f(x))$. Recall what $e_s(j)$ meant.