

Discrete Structures

IIIT Hyderabad

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Tutorial 7

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1 Questions

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Question 0

Consider elliptic curve $E_5(2, 1)$ or in equation form as -

$$y^2 = x^3 + 2x + 1$$

If $P = (1, 3)$ and $Q = (3, 2)$ lie on the above curve, find $-2P + Q$.

Question 1

What are the sets in the partition of the integers arising from congruence modulo 4?

Question 2

Determine which one of these are equivalence and list their equivalence classes (if applicable). Also if it isn't an equivalence relation tell which property does it lack:

- 1 $\{(a, b) | a \text{ and } b \text{ are the same age}\}$
- 2 $\{(a, b) | a \text{ and } b \text{ have the same parent}\}$
- 3 $\{(a, b) | a \text{ and } b \text{ have met}\}$
- 4 $\{(a, b) | a \text{ and } b \text{ speak a common language}\}$

Question 3

3.1 Which of the following are partitions of $S = \{1, 2, 3, 4, 5, 6\}$:

- ① $\{1, 2\}, \{2, 3, 4\}, \{4, 5, 6\}$
- ② $\{1\}, \{2, 3, 6\}, \{5\}, \{4\}$

3.2 Which of the following are partitions of the set of real numbers:

- ① Set of intervals $[k, k + 1], k = \dots - 2, -1, 0, 1, \dots$
- ② Set of intervals $(k, k + 1), k = \dots - 2, -1, 0, 1, \dots$
- ③ The sets $\{x + n | n \in \mathbb{Z}\}$ for all $x \in [0, 1)$

3.3 Verify whether it is a partitions of the set $\mathbb{Z} \times \mathbb{Z}$ of ordered pair of integers:

The set of pairs (x, y) , where $3 \mid x$ and $3 \mid y$; the set of pairs (x, y) , where $3 \mid x$ and $3 \nmid y$; the set of pairs (x, y) where $3 \nmid x$ and $3 \mid y$; the set of pairs (x, y) where $3 \nmid x$ and $3 \nmid y$

Question 4

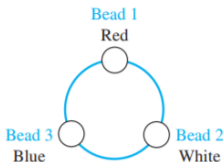
4.1 Let R be the relation on the set of all people who have visited a particular Web page such that xRy if and only if person x and person y have followed the same set of links starting at this Web page (going from Web page to Web page until they stop using the Web). Find out the properties of the relation R .

4.2 a Let n be a positive integer. Show that the relation R on the set of all polynomials with real-valued coefficients of all pairs (f, g) such that $f^{(n)}(x) = g^{(n)}(x)$ is an equivalence relation. (Here $f^{(n)}(x)$ is the n th derivative of $f(x)$)

4.2 b Which functions are in the same equivalence class as the function $f(x) = x^4$, where $n = 3$?

*Question 5

Each bead on a bracelet with three beads is either red, white or blue:



Define the relation R between bracelets as: (B_1, B_2) where B_1 and B_2 are bracelets, belongs to R if and only if B_2 can be obtained from B_1 by rotating it or rotating it and then reflecting it.

- Show that R is an equivalence relation
- What are the equivalence classes for R