Discrete Structures (MA5.101)

Instructor: Dr. Ashok Kumar Das — **Assignment 1 Solutions**
IIIT Hyderabad — **Total Marks**: 50

Problem 1 We first define

$$\begin{aligned}
&(A\Delta B)'\\
&=((A\cup B)\cap(A'\cup B'))' && \ldots\{\text{by definition}\}\\
&=((A\cup B)'\cup(A'\cup B')') && \ldots\{\text{by De-Morgan's Laws}\}\\
&=((A'\cap B')\cup(A\cap B)) && \ldots\{\text{by De-Morgan's Laws}\}
\end{aligned}$$

We simplify LHS first -

$$\begin{aligned}
&(A\Delta B)\Delta C\\
&=[(A\Delta B)\cap C')]\cup[(A\Delta B)'\cap C] && \ldots\{\text{by definition.}\}\\
&=[((A\cap B')\cup(A'\cap B))\cap C]\cup[((A'\cap B')\cup(A\cap B))\cap C] && \ldots\{\text{by definition.}\}\\
&=[(A\cap B'\cap C')\cup(A'\cap B\cap C')]\cup[(A'\cap B'\cap C)\cup(A\cap B\cap C)] && \ldots\{\text{distributive property}\}\\
&=(A\cap B'\cap C')\cup(A'\cap B\cap C')\cup(A'\cap B'\cap C)\cup(A\cap B\cap C) && \ldots\{\text{associative property}\}
\end{aligned}$$

We simplify RHS next -

$$\begin{aligned}
&A\Delta(B\Delta C)\\
&=[A\cap(B\Delta C)']\cup[A'\cap(B\Delta C)] && \ldots\{\text{by definition}\}\\
&=[A\cap((B'\cap C')\cup(B\cap C))]\cup[A'\cap((A'\cap B)\cup(A\cap B'))] && \ldots\{\text{by definition}\}\\
&=[((A\cap(B'\cap C'))\cup(A\cap(B\cap C))]\cup[(A'\cap(B'\cap C))\cup(A'\cap(B\cap C'))] && \ldots\{\text{by distributive property}\}\\
&=[((A\cap B'\cap C')\cup(A\cap B\cap C)]\cup[(A'\cap B'\cap C)\cup(A'\cap B\cap C')] && \ldots\{\text{by associative property}\}\\
&=(A\cap B'\cap C')\cup(A'\cap B\cap C')\cup(A'\cap B'\cap C)\cup(A\cap B\cap C) && \ldots\{\text{by commutative property}\}
\end{aligned}$$

Hence proved.

Problem 2 Given:

$$A=\{n|n \text{ is a multiple of } 12\}$$
$$B=\{n|n \text{ is a multiple of } 18\}$$

1. $A\cup B=\{n|n \text{ is a multiple of 12 or 18}\}$
2. $A\cap B=\{n|n \text{ is a multiple of 12 and 18}\}=\{n|n \text{ is a multiple of 36}\}$
3. $(A-B)\cup(B-A)=A\Delta B$ ...{Using Property}
   $A\Delta B=\{n|n \text{ is a multiple of 12 or } n \text{ is a multiple of 18 but not both}\}$
4. $A\times B=\{(a,b)|a \text{ is a multiple of 12 and } b \text{ is a multiple of 18}\}$

5. Multiple answers accepted here — either listing out the answer in roster form, or providing a proper set builder definition.

$$
\begin{aligned}
P(A \cup B) = \Big\{ &\phi && \ldots \text{(Null/Empty Set)} \\
&\{12\}, \{18\}, \{24\}, \{36\}, && \ldots \text{(Taken one at a time)} \\
&\{12, 24\}, \{18, 36\}, \{12, 18\}, && \ldots \text{(Taken two at a time)} \\
&\{12, 18, 24\}, \{12, 18, 36\}, && \ldots \text{(Taken three at a time)} \\
&\vdots \\
&\Big\}
\end{aligned}
$$

Problem 3 The Venn Diagram for the problem is:

- U = Set of all quadrilaterals
- T = Set of rectangles
- R = Set of rhombuses
- S = Set of squares

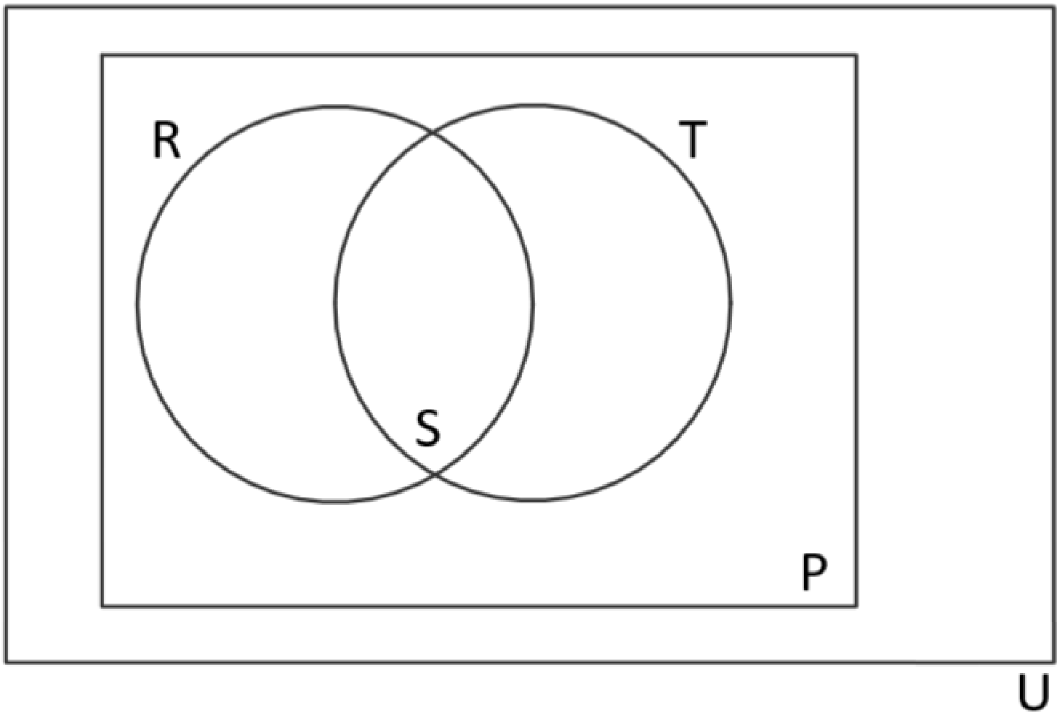

Figure 1: Venn Diagram of the problem

P is defined as a quadrilateral having opposite sides parallel to each other
T is defined as a quadrilateral having opposite sides parallel and each interior angle = $90°$

R is defined as a quadrilateral having opposite sides parallel to each other and all sides equal
S is defined as a quadrilateral having opposite sides parallel, each sides equal and each interior angle $= 90°$

The required relations are:

1. $P \subseteq U, T \subseteq U, R \subseteq U, S \subseteq U$
2. $T \subseteq P, R \subseteq P$
3. $S = R \cap T \implies (S \subseteq R) \wedge (S \subseteq T)$

Problem 4 Let

$$\begin{aligned} E &= \text{ Set of all people who speak English} \\ F &= \text{ Set of all people who speak French} \\ G &= \text{ Set of all people who speak German} \\ U &= \text{ Set of all people who attended the conference} \end{aligned}$$

Given:

$$|E| = 28, |F| = 30, |G| = 42, |U| = 100$$
$$|E \cap F| = 8, |F \cap G| = 5, |E \cap G| = 10$$
$$|E \cap F \cap G| = 3$$

1. We have to calculate $|\overline{E \cup F \cup G}|$ which is given by:

$$|\overline{E \cup F \cup G}| = |U| - |E \cup F \cup G|$$

Now using PIE (Principle of Inclusion-Exclusion) we get:

$$\begin{aligned} |E \cup F \cup G| &= |E| + |F| + |G| - |E \cap F| - |E \cap G| - |F \cap G| + |E \cap F \cap G| \\ &= 28 + 30 + 42 - 8 - 5 - 10 + 3 \\ &= 80 \end{aligned}$$

Thus the answer is:

$$\begin{aligned} |\overline{E \cup F \cup G}| &= |U| - |E \cup F \cup G| \\ &= 100 - 80 \\ &= 20 \end{aligned}$$

2. We have to find the number of people who only speak German. The answer here is given by:

$$\begin{aligned} \text{Ans} &= |G| - |E \cap G| - |F \cap G| + |E \cap F \cap G| \\ &= 42 - 10 - 5 + 3 \\ &= 30 \end{aligned}$$

Problem 5

1. We first prove that $(A - B) \times C \subseteq (A \times C) - (B \times C)$

$$
\begin{aligned}
&(x, y) \in (A - B) \times C \\
&\implies (x \in (A - B)) \wedge (y \in C) && \ldots\{\text{by definition}\} \\
&\implies ((x \in A) \wedge (x \notin B)) \wedge (y \in C) && \ldots\{\text{by defintion of A - B}\}. \\
&\implies ((x \in A) \wedge (y \in C)) \wedge ((x \notin B)) \wedge (y \in C)) && \ldots\{\text{by distributivity of } \wedge\} \\
&\implies ((x, y) \in (A \times C)) \wedge ((x, y) \notin (B \times C)) && \ldots\{\text{by definition}\} \\
&\implies (x, y) \in (A \times C) - (B \times C) && \ldots\{\text{by definition}\}
\end{aligned}
$$

Now we prove that $(A \times C) - (B \times C) \subseteq (A - B) \times C$

$$
\begin{aligned}
&(x, y) \in (A \times C) - (B \times C) \\
&\implies ((x, y) \in (A \times C)) \wedge ((x, y) \notin (B \times C)) && \ldots\{\text{by definition}\} \\
&\implies ((x \in A) \wedge (y \in C)) \wedge ((x \notin B)) \wedge (y \in C)) && \ldots\{\text{by definition}\} \\
&\implies ((x \in A) \wedge (x \notin B)) \wedge (y \in C) && \ldots\{\text{by distributivity of } \wedge\}. \\
&\implies (x \in (A - B)) \wedge (y \in C) && \ldots\{\text{by definition}\} \\
&\implies (x, y) \in (A - B) \times C && \ldots\{\text{by definition}\}
\end{aligned}
$$

2. We first prove that $(A \Delta B) \times C \subseteq (A \times C) \Delta (B \times C)$

$$
\begin{aligned}
&(x, y) \in (A \Delta B) \times C \\
&\implies (x \in (A \Delta B)) \wedge (y \in C) && \ldots\{\text{by definition}\} \\
&\implies [((x \in A) \wedge (x \notin B)) \vee ((x \notin A) \wedge (x \in B))] \wedge (y \in C) && \ldots\{\text{by definition of } A \Delta B\}. \\
&\implies [((x \in A) \wedge (x \notin B) \wedge (y \in C)) \vee ((x \notin A) \wedge (x \in B) \wedge (y \in C))] && \ldots\{\text{by distributivity of } \vee\}. \\
&\implies [(((x \in A) \wedge (y \in C)) \wedge ((x \notin B) \wedge (y \in C))) \\
&\qquad \vee (((x \notin A) \wedge (y \in C)) \wedge ((x \in B) \wedge (y \in C)))] && \ldots\{\text{by distributivity of } \vee\}. \\
&\implies [((x, y) \in (A \times C) - (B \times C)) \\
&\qquad \vee ((x, y) \in (B \times C) - (A \times C))] && \ldots\{\text{by definition}\} \\
&\implies (x, y) \in [(A \times C) - (B \times C)) \cup (B \times C) - (A \times C))] && \ldots\{\text{by definition}\} \\
&\implies (x, y) \in (A \times C) \Delta (B \times C) && \ldots\{\text{by definition}\}
\end{aligned}
$$

Now we prove that $(A \times C)\Delta(B \times C) \subseteq (A\Delta B) \times C$

$$
\begin{aligned}
&(x,y) \in (A \times C)\Delta(B \times C) \\
&\implies (x,y) \in [(A \times C) - (B \times C)) \cup (B \times C) - (A \times C))] && \ldots\{\text{by definition}\} \\
&\implies [((x,y) \in (A \times C) - (B \times C)) \\
&\qquad \vee ((x,y) \in (B \times C) - (A \times C))] && \ldots\{\text{by definition}\} \\
&\implies [(((x \in A) \wedge (y \in C)) \wedge ((x \notin B) \wedge (y \in C))) \\
&\qquad \vee (((x \notin A) \wedge (y \in C)) \wedge ((x \in B) \wedge (y \in C)))] && \ldots\{\text{by definition}\}. \\
&\implies [((x \in A) \wedge (x \notin B)) \wedge (y \in C)) \vee (((x \notin A) \wedge (x \in B)) \wedge (y \in C))] && \ldots\{\text{by distributivity of } \vee\}. \\
&\implies [((x \in A) \wedge (x \notin B)) \vee ((x \notin A) \wedge (x \in B))] \wedge (y \in C) && \ldots\{\text{by distributivity of } \vee\}. \\
&\implies [((x \in (A - B)) \vee ((x \in (B - A)))] \wedge (y \in C) && \ldots\{\text{by definition}\} \\
&\implies [(x \in (A - B) \cup (B - A)] \wedge (y \in C) && \ldots\{\text{by definition}\} \\
&\implies (x \in (A\Delta B)) \wedge (y \in C) && \ldots\{\text{by definition of } A\Delta B\} \\
&\implies (x,y) \in (A\Delta B) \times C && \{\text{by definition}\}
\end{aligned}
$$