Discrete Structures

IIIT Hyderabad

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Tutorial 1 Solutions

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Introduction

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Question 1

1.1. Find the power set (or $\mathcal{P}(S)$) and it's order (or $|\mathcal{P}(S)|$) for the following sets -

- Set $S = \{\phi\}$
  Sol: $\mathcal{P}(S) = \{\phi, \{\phi\}\}$, $|\mathcal{P}(S)| = 2$
- Set $S = \{\text{a}\}$
  Sol: $\mathcal{P}(S) = \{\phi, \{\text{a}\}\}$, $|\mathcal{P}(S)| = 2$
- Set $S = \{\text{a}, \phi\}$
  Sol: $\mathcal{P}(S) = \Big\{\phi, \{\text{a}\}, \{\phi\}, \{\text{a}, \phi\}\Big\}$, $|\mathcal{P}(S)| = 4$
- Set $S = \{\text{a}, \{\phi\}\}$
  Sol: $\mathcal{P}(S) = \Big\{\phi, \{\text{a}\}, \{\{\phi\}\}, \{\text{a}, \{\phi\}\}\Big\}$, $|\mathcal{P}(S)| = 4$
- Set $S = \{\phi, \{\phi\}, \{\{\phi\}\}\}$
  Sol: $$\mathcal{P}(S) = \Big\{\phi, \{\phi\}, \{\{\phi\}\}, \{\{\{\phi\}\}\}, \{\phi, \{\phi\}\}, \{\phi, \{\{\phi\}\}\}, \{\phi, \{\{\phi\}\}\}, \{\phi, \{\phi\}, \{\{\phi\}\}\}\Big\}$$, $|\mathcal{P}(S)| = 8$

Question 1

1.2. If $|A| = m$ and $|B| = n$ and A and B are not mutually disjoint. Let

$$\mathcal{P}_i(S) = \mathcal{P}(\mathcal{P}(\mathcal{P} \dots \mathcal{P}(S)))) \ i \text{ times}$$

(Given m $\geq$ n) then what are the bounds of the value of $|\mathcal{P}_4(A - B)|$, $|\mathcal{P}_2(A - B)|$.
*[On the basis of above, can you tell about $|\mathcal{P}_4(A - B) - \mathcal{P}_2(A - B)|$?]
Sol: We have that $A - B = A - (A \cap B)$. Now $A \cap B$ can have maximum n and minimum 1 elements (since not disjoint). Thus

$$m - n \leq |A - (A \cap B)| \leq m - 1$$
$$2^{m-n} \leq |P(A - (A \cap B))| \leq 2^{m-1}$$
$$2^{2^{m-n}} \leq |P_2(A - (A \cap B))| \leq 2^{2^{m-1}}$$
$$\vdots$$
$$2^{2^{2^{2^{m-n}}}} \leq |P_4(A - (A \cap B))| \leq 2^{2^{2^{2^{m-1}}}}$$

Question 2

For answering this question, go to: tinyurl.com/dstut1

True or False:

1. $\phi \subseteq \{\phi\}$ - **True**
2. $\{x^2 | x^2 = 1\} = \{x | x^2 = x\}$ - **False**
3. $\mathcal{P}(\{x, y, \{x\}, \{y\}) = \mathcal{P}(\{x, y, \{x, y\})$ - **False**
4. $\{a, \phi\} \in \{a\}$ - **False**
5. $\{a, \phi\} \subseteq \{a, \{a, \phi\}\}$ - **False**
6. If $a \in \mathcal{P}(A)$, then $a \in A$ always - **False**
7. For any set A, $A \subseteq A$ - **True**
8. For any set A, $A \in A$ - **False**
9. Every nonempty set has at least two subsets - **True**

Question 2 explanations

True or False:

1. **Sol:** $\subseteq$ means "subset of", and ϕ is a subset of every set.
2. **Sol:** LHS describes the set $\{1\}$. RHS describes the set $\{0,1\}$.
3. **Sol:** The cardinality of both the setsitself is different.
4. **Sol:** $\phi \notin \{a\}$. However $\phi \subseteq \{a\}$.
5. **Sol:** $\phi \notin \{a, \{a, \phi\}\}$
6. **Sol:** Take $A = \{4\}$. $\mathcal{P}(A) = \{\phi, \{4\}\}$, now both $\phi \notin A$ and $\{4\} \notin A$
7. **Sol:** Take any arbitrary set A. Now by the definition of subset for $A \subseteq A : \forall x \in A,\ x \in A$ — which is always **true**
8. **Sol:** This is not always true. Set S = $\{a\}$, $\{a\} \notin \{a\}$
9. **Sol:** Power set of a set S with cardinality $|S|$ has $2^{|S|}$ elements, and thus this is true.

Question 3

3.1 Given $A = \{a, b, \{a, c\}, \phi\}$. Determine the following:

- $A - \phi$
  Sol: $A - \phi = A$
- $A - \{a, c\}$
  Sol: $A - \{a, c\} = \{b, \{a, c\}, \phi\}$
- $A - \{\{a, c\}\}$
  Sol: $A - \{\{a, c\}\} = \{a, b, \phi\}$
- $\{a, c\} - A$
  Sol: $\{a, c\} - A = \{c\}$

Question 3

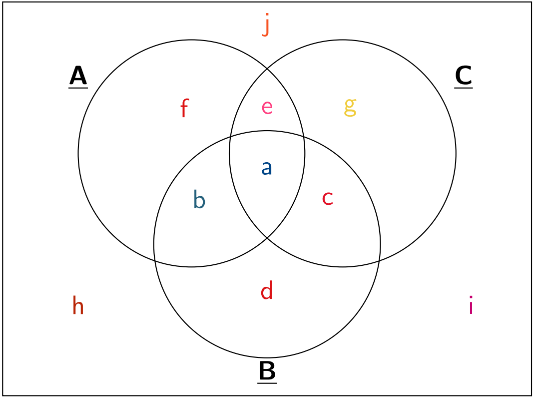
3.2 One of the following set is different, which one ? (Choose one, MCQ)($\wedge$ is **AND** and $\vee$ is **OR**)

1. Set $S = \{x | (x^2 = 1) \vee (x^2 = 4) \vee (x \text{ is prime} < 10)\}$
2. Set $S = \{x | (x^2 = 4) \vee (x^2 = 1) \vee (x \text{ is odd} < 10)\}$
3. Set $S = \{x | (x < 9) \wedge (x > -3) \wedge ((x \text{ is odd}) \vee (x^2 = 4))\}$
4. Set $S = \{-2, -1, 1, 2, 3, 5, 7\}$

<u>Sol:</u> Option 2

Option 2 describes the set $\{-2, -1, 1, 2, 3, 5, 7, 9\}$ whereas all others describe the set $\{-2, -1, 1, 2, 3, 5, 7\}$.

Question 4



1. (U - (A ∩ B))' ∪ ((C - B) ∩ A') — **Sol:** $\{a, b\} \cup \{g\} = \{a, b, g\}$
2. (A ∩ C') ∪ (A ∪ B ∪ C)' — **Sol:** $\{b, f\} \cup \{h, i, j\} = \{b, f, h, i, j\}$
3. (A - (B ∩ C)) ∩ (U' - (C ∩ B))' — **Sol:** $\{b, e, f\} \cap \{U\} = \{b, e, f\}$