# Discrete Structures

IIIT Hyderabad

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*Tutorial 2*

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# Introduction

1. Questions
   - Question 0
   - Question 1
   - Question 2

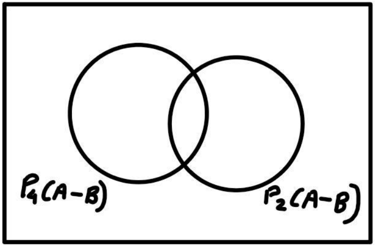
# Question 0

If $|A| = m$ and $|B| = n$ and $A$ and $B$ are not mutually disjoint. Let

$$\mathcal{P}_i(S) = \mathcal{P}(\mathcal{P}(\mathcal{P} \ldots \mathcal{P}(S)))) \ i \text{ times}$$

Comment on the value of $|\mathcal{P}_4(A - B) - \mathcal{P}_2(A - B)|$.

**Sol.**




Let's denote $\mathcal{P}_2(A - B)$ as $C$. We get that, $P_4(A - B)$ and $\mathcal{P}_2(A - B)$ will always the elements $\{\phi, \{\phi\}\}$. Thus the upper limit could be $2^{2^{2^{2^{m-1}}}} - 2$. One of the lower limit could be $2^{2^{2^{2^{m-n}}}} - 2^{2^{m-n}}$

# Question 1

**1.1** Prove the following using basic set identities -

1. $((A - B) - (B - C))' = A' \cup B$

$$
\begin{aligned}
&((A - B) - (B - C))' \\
&= ((A \cap B') - (B \cap C'))' && \ldots\ldots\{\text{De Morgan's Law}\} \\
&= ((A \cap B') \cap (B \cap C')')' && \ldots\ldots\{\text{De Morgan's Law}\} \\
&= ((A \cap B') \cap (B' \cup C))' && \ldots\ldots\{\text{De Morgan's Law}\} \\
&= ((A \cap B') \cap B') \cup ((A \cap B') \cup C))' && \ldots\ldots\{\text{Distributive Law}\} \\
&= ((A \cap (B' \cap B')) \cup ((A \cap B') \cup C))' && \ldots\ldots\{\text{Associative Law}\} \\
&= ((A \cap B') \cup ((A \cap B') \cup C))' && \ldots\ldots\{\text{Idempotent Law}\} \\
&= ((A \cap B') \cap U) \cup ((A \cap B') \cup C))' && \ldots\ldots\{A \cap U = A\} \\
&= ((A \cap B') \cap (U \cup C)))' && \ldots\ldots\{\text{Distributive Law}\} \\
&= ((A \cap B'))' && \ldots\ldots A \cap U = A, A \cup U = A\} \\
&= A' \cup B && \ldots\ldots\{\text{De Morgan's Law}\}
\end{aligned}
$$

2. $(A \cap (A - B)) \cup (A' \cup B)' = A - B$

$$\begin{aligned}
&(A \cap (A - B)) \cup (A' \cup B)' \\
&= (A \cap (A \cap B')) \cup (A \cap B') && \ldots\ldots\{\text{De Morgan's Law}\} \\
&= ((A \cap A) \cap B')) \cup (A \cap B') && \ldots\ldots\{\text{Associative Law}\} \\
&= (A \cap B') \cup (A \cap B') && \ldots\ldots\{\text{Idempotent Law}\} \\
&= (A \cap B') && \ldots\ldots\{\text{Idempotent Law}\} \\
&= (A - B) && \ldots\ldots\{\text{De Morgan's Law}\}
\end{aligned}$$

3. $A \cup (B - C) = (A \cup B) - (C - A)$

$$\begin{aligned}
&A \cup (B - C) \\
&= A \cup (B \cap C') && \ldots\ldots\{\text{De Morgan's Law}\} \\
&= (A \cup B) \cap (A \cup C') && \ldots\ldots\{\text{Distributive Law}\} \\
&= (A \cup B) \cap (A' \cap C)' && \ldots\ldots\{\text{De Morgan's Law}\} \\
&= (A \cup B) - (C - A) && \ldots\ldots\{\text{De Morgan's Law}\}
\end{aligned}$$

4. $(A \Delta B) \Delta C = A \Delta (B \Delta C)$
   **Sol:** Part of Assigment 1, solutions will be posted after it.

## 1.2 Prove the following using element of argument -

1. $(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$

$$
\begin{aligned}
&(x, y) \in (A \times B) \cap (C \times D) \\
&\implies ((x, y) \in (A \times B)) \wedge ((x, y) \in (C \times D)) \\
&\implies (x \in A \wedge y \in B) \wedge (x \in C \wedge y \in D) \\
&\implies (x \in A \wedge x \in C) \wedge (y \in B \wedge y \in D) \\
&\implies (x \in (A \cap C)) \wedge (y \in (B \cap D)) \\
&\implies (x, y) \in (A \cap C) \times (B \cap D)
\end{aligned}
$$

Thus we have $(A \times B) \cap (C \times D) \subseteq (A \cap C) \times (B \cap D)$

$$
\begin{aligned}
&(x, y) \in (A \cap C) \times (B \cap D) \\
&\implies (x \in A \wedge x \in C) \wedge (y \in B \wedge y \in D) \\
&\implies (x \in A \wedge y \in B) \wedge (x \in C \wedge y \in D) \\
&\implies (x, y) \in (A \times B) \wedge (x, y) \in (C \times D) \\
&\implies (x, y) \in (A \times B) \cap (C \times D)
\end{aligned}
$$

Thus we have $(A \cap C) \times (B \cap D) \subseteq (A \times B) \cap (C \times D)$

2. $(A \times B) \cup (C \times D) \subseteq (A \cup C) \times (B \cup D)$

$$\begin{aligned}
&(x, y) \in (A \times B) \cup (C \times D) \\
&\implies ((x, y) \in (A \times B)) \vee ((x, y) \in (C \times D)) \\
&\implies (x \in A \wedge y \in B) \vee (x \in C \wedge y \in D) \\
&\implies (x \in A \wedge x \in C) \wedge (y \in B \wedge y \in D) \\
&\implies (x \in (A \cap C)) \wedge (y \in (B \cap D) \\
&\implies (x, y) \in (A \cap C) \times (B \cap D)
\end{aligned}$$

Thus we have $(A \times B) \cup (C \times D) \subseteq (A \cup C) \times (B \cup D)$

# Question 2

**2.1**: Odd one out (ONLY ONE)

1. $\{(a, b) | (a^2 = 1) \wedge (b^2 = 4)\}$
2. $\{-1, -2, 2, 4\}$
3. $(\{-1, 1\} \times \{-2, 2\}) - \phi$
4. $(\{-1, 1\} \times \{-2, 2\}) \cup \phi$

**Sol: Option 2**

All others denote the set {(-1,-2),(-1,2),(1,-2),(1,2)}.

**2.2**: Select the false statements:

1. $\mathcal{P}(A \times B) = \mathcal{P}(A) \times \mathcal{P}(B)$
2. $\mathcal{P}(A \cap B) = \mathcal{P}(A) \cap \mathcal{P}(B)$
3. $\mathcal{P}(A \cup B) = \mathcal{P}(A) \cup \mathcal{P}(B)$
4. $\mathcal{P}(A - B) = \mathcal{P}(A) - \mathcal{P}(B)$

**Sol: Option 1,3 and 4**

For options 1 and 3, take $A = \{a, b\}, B = c$. For option 3, take $A = \{a, b\}, B = \{b\}$ to disprove. It can be noted that in option 4, the element $\phi$ will get subtracted and will thus never be equal to LHS. For option 2 -

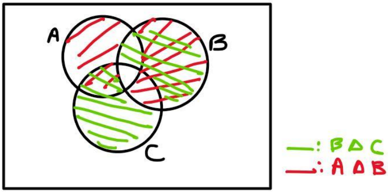
We will show both sides.

$$X \in P(A \cap B)$$
$$(\forall x \in X)(x \in A \wedge x \in B)$$
$$(\forall x \in X)(x \in P(A) \wedge x \in P(B))$$
$$X \in P(A) \cap P(B)$$

$$X \in P(A) \cap P(B)$$
$$(X \in P(A)) \wedge (X \in P(B))$$
$$(\forall x \in X)((x \in P(A)) \wedge (x \in P(B))$$
$$(\forall x \in X)(x \in A \wedge x \in B)$$
$$(\forall x \in X)(x \in (A \cap B))$$
$$X \in P(A \cap B)$$

**2.4**: $|U| = 20, |A| = 10, |B| = 5, |C| = 5, |A \cap B| = 2, |A \cap C| = 4, |B \cap C| = 1, |A \cap B \cap C| = 1.$

1. $|A \cap B' \cap C|$
   <u>**Sol:**</u> $|A \cap C| - |A \cap B \cap C| = 3$
2. $|((A - B) \times C)|$
   <u>**Sol:**</u> $|A - B| \cdot |C| = |A \cap B'| \cdot |C| = (|A| - |A \cap B|) \cdot |C| = 40$
3. $|(A \Delta B) \cup (B \Delta C)|$
   <u>**Sol:**</u>




From Venn Diagram it can be seen that it includes all regions within A, B and C except $A \cap B \cap C$. Thus, it would be equal to $|A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| = 13$

**2.5**: Use inclusion exclusion principle to find numbers less than 1000 divisible by none of 2,3 or 5.

**Sol:** We first find out the numbers divisible by 2,3 or 5.

$$\lfloor \frac{999}{2} \rfloor + \lfloor \frac{999}{5} \rfloor + \lfloor \frac{999}{3} \rfloor - \lfloor \frac{999}{10} \rfloor - \lfloor \frac{999}{15} \rfloor - \lfloor \frac{999}{6} \rfloor + \lfloor \frac{999}{30} \rfloor$$
$$= 499 + 199 + 333 - 99 - 66 - 166 + 33$$
$$= 733$$

So we would have rest as 999 - 733 = 266.