International Institute of Information Technology Hyderabad

Discrete Structures (MA5.101)

Assignment 4
Deadline: November 15, 2020 (Sunday), 23:55 PM
Total Marks: 70
Instructions: Submit ONLY handwritten scanned pdf file in the moodle under Assignments directory.

1. Use the principle of mathemtical induction to show that if $A, B_1, B_2, \cdots, B_n$ are sets, then

$$A \cup (B_1 \cap B_2 \cap \cdots \cap B_n) = (A \cup B_1) \cap (A \cup B_2) \cap \cdots \cap (A \cup B_n).$$

[10]

2. Find the permutation:
$(1\ i) \cdot (1\ 2\ 3 \cdots, n)$.

[5]

3. Given a set of sixteen natural numbers, none of having a prime factor > 7, show that either some number is a perfect square or the product of some two distinct numbers is a perfect square.

[10]

4. Show that if seven numbers from 1 to 12 are chosen, then two of them will add up to 13.

[5]

5. Let

$$\begin{aligned} a_r &= \begin{cases} 1, r = 0 \\ 3, r = 1 \\ 2, r = 2 \\ 0, r \geq 3 \end{cases} \\ c_r &= 5^r \text{ for all } r \end{aligned}$$

Given that $c = a * b$, that is c is the convolution of numeric functions a and b. Show that $b_r = \frac{25}{42}5^r - \frac{1}{6}(-1)^r + \frac{4}{7}(-1)^r 2^r, r \geq 0$.

[10]

6. Using the generating function, show that solution of the following recurrence relation

$$a_k - 7a_{k-1} + 10a_{k-2} = 3^k$$

with initial conditions $a_0 = 0$ and $a_1 = 1$, is

$$a_k = \frac{8}{3}2^k - \frac{9}{2}3^k + \frac{11}{6}5^r$$

[10]

7. Consider an air traffic-control system in which the desired altitude of an aircraft, a_r, is computed by a computer every second and is compared with the actual altitude of the aircraft, b_{r-1}, determined by a tracking radar 1 second earlier. Depending on whether a_r is larger or smaller than b_{r-1}, the altitude of the aircraft will be changed accordingly. Specifically, the change in altitude at the r-th second, $b_r - b_{r-1}$, is proportional to the difference $a_r - b_{r-1}$. That is,

$$b_r - b_{r-1} = K(a_r - b_{r-1})$$

where K is a proportional constant.
(a) Determine b_r, given that $a_r = 1000(\frac{3}{2})^2, K = 3,$ and $b_0 = 0$.
(b) Determine b_r, given that

$$a_r \quad = \quad \begin{cases} 1000(\frac{3}{2})^r, 0 \leq r \leq 9 \\ 1000(\frac{3}{2})^{10}, r \geq 10 \end{cases}$$

$K = 3$, and $b_0 = 0$.

[10 + 10 = 20]

All the best!!!