Discrete Structures

IIIT Hyderabad

Monsoon 2020

Tutorial 3

September 23, 2020

Introduction

1. Questions
   - Question 0
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Question 0

0.2 Find the number of primes between 40 - 100 using PRT.

Sol:

Here $n = 100$. Then $\pi(\sqrt{n}) = \pi(\sqrt{100}) = \pi(10) = 4$. The four primes $\leq \sqrt{n} = 10$ are $2, 3, 5$ and 7. Let $p_1 = 2, p_2 = 3, p_3 = 5$ and $p_4 = 7$, $t = 4$. From the previous theorem, we have,

$$\begin{aligned}
\pi(100) &= 100 - 1 + 4 - (\lfloor \frac{100}{2} \rfloor + \lfloor \frac{100}{3} \rfloor + \lfloor \frac{100}{5} \rfloor + \lfloor \frac{100}{7} \rfloor) \\
&\quad + (\lfloor \frac{100}{2.3} \rfloor + \lfloor \frac{100}{2.5} \rfloor + \lfloor \frac{100}{2.7} \rfloor + \lfloor \frac{100}{3.5} \rfloor + \lfloor \frac{100}{3.7} \rfloor + \lfloor \frac{100}{5.7} \rfloor) \\
&\quad - (\lfloor \frac{100}{2.3.5} \rfloor + \lfloor \frac{100}{2.3.7} \rfloor + \lfloor \frac{100}{2.5.7} \rfloor + \lfloor \frac{100}{3.5.7} \rfloor) + \lfloor \frac{100}{2.3.5.7} \rfloor \\
&= 103 - (50 + 33 + 20 + 14) + (16 + 10 + 7 + 6 + 4 + 2) \\
&\quad - (3 + 2 + 1 + 0) + 0 = 25.
\end{aligned}$$

We have $\pi(6) = 3$, thus

$\pi(40) = 40 - 1 + 3 - \lfloor \frac{40}{2} \rfloor - \lfloor \frac{40}{5} \rfloor - \lfloor \frac{40}{3} \rfloor + \lfloor \frac{40}{10} \rfloor + \lfloor \frac{40}{6} \rfloor + \lfloor \frac{40}{15} \rfloor - \lfloor \frac{40}{30} \rfloor$

$= 12$

Thus we get number of primes is $25 - 12 = 13$.

Question 1

Simplify the following -

1. $\left(A - (B \cup C)\right) \cap \left((B \cap C) - A\right)$

   Sol:

   $$\left(A - (B \cup C)\right) \cap \left((B \cap C) - A\right)$$
   $$= \left(A \cap (B' \cap C')\right) \cap \left((B \cap C) \cap A'\right) \quad \ldots\ldots \{ \text{ De Morgan's Laws } \}$$

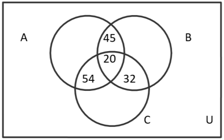
   The above can be re arranged using associative and commutative law as $\left((A \cap A') \cap (B \cap B') \cap (C \cap C')\right) = \phi \cap \phi \cap \phi = \phi$

2. $(A \cap B') \cup (A' \cap B) \cup (A' \cap B')$

   $$(A \cap B') \cup (A' \cap B) \cup (A' \cap B')$$
   $$(A \cap B') \cup (A' \cap (B \cup B') \quad \ldots\ldots \{ \text{ Associative Law } \}$$
   $$(A \cap B') \cup (A' \cap (U)) \quad \ldots\ldots \{ \text{ A' } \cup \text{ A = U} \}$$
   $$(A \cap B') \cup (A') \quad \ldots\ldots \{ \text{ A } \cap \text{ U = A } \}$$
   $$(A \cup A') \cap (B' \cup A') \quad \ldots\ldots \{ \text{ Since commutative } \}$$
   $$U \cup (B' \cap A') = (B' \cap A') \quad \ldots\ldots \{ \text{ A } \cup \text{ A' = U, U } \cup \text{ A = A } \}$$

Question 2

2.1 A is the set of people who go to resort area for vacation, B is the set of people who take cruise for vacation, C is the set of people who go to national park for vacation. Suppose $|A| = 150, |B| = 100$ and $|C| = 300$



a. How many people only go to a resort area for vacation?

Sol:
$|A| - |A \cap B| - |A \cap C| + |A \cap B \cap C| = 31$

b. How many people only take a cruise for vacation?

Sol:
$|B| - |B \cap A| - |B \cap C| + |A \cap B \cap C| = 3$

c. How many people only go to national park for vacation?

Sol:
$|C| - |C \cap A| - |C \cap B| + |A \cap B \cap C| = 194$

d. How many people either go to a resort area or take a cruise for vacation but not national park?

Sol: $|A| + |B| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C| = 79$

e. How many people use any of the 3 methods to take a vacation?

Sol: $|A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C| = 379$

Question 2 (contd.)

2.2 Observe the values of the table and find the number of students who:

		Pudding			
		Chocolate	**Tapioca**	**Neither**	**Total**
Ice Cream	Vanilla	68	53	12	133
	Strawberry	59	48	9	116
	Neither	23	21	7	51
	Total	150	122	28	300

a. Like strawberry ice cream and tapioca pudding

Sol: 48

b. Do not like pudding

Sol: 28

c. Like at least one of the ice cream flavours

Sol: $133 + 116 = 249$

d. Like neither ice cream nor pudding

Sol: 7

2.3 Mark the following as true or false:

a. $26 \in \mathbb{Z}$ – **True**

b. $-5 \in \mathbb{N}$ – **False**

c. $\sqrt{2} \notin \mathbb{Q} \cap \mathbb{R}$ – **True**

d. $\mathbb{Z} \cup \mathbb{Q} = \mathbb{R}$ – **False**

e. $\mathbb{R} \cap \mathbb{C} = \mathbb{R}$ – **True**

Quiz conducted on 22 Sept 2020

Any doubts ?

3. In a hostly fought battle at least 70% of the combatants lost an eye , at least 75% an ear , at least 80% an arm and at least 85% a leg. What is the least number of combatants who lost all four members?
Sol: Let's call the sets A, B, C and D corresponding to eye, ear, arm or leg respectively. Denote $|U| = x$,
$|A| \geq 07.x, |B| \geq 0.75x, |C| \geq 0.8x, |D| \geq 0.85x$.
Now we have,

$$|A \cap B| = |A| + |B| - |A \cap B|$$
$$|A \cap B \cap C| = |A \cup B \cup C| - |A| - |B| - |C|$$
$$+ |A \cap B| + |B \cap C| + |A \cap C|$$

Similarly, we can write for other expansions too. We use the property that $|A \cup B| \leq x, |A \cup B \cup C| \leq x$ and $|A \cup B \cup C \cup D| \leq x$ (with all combinations).

Similarly, we write , substituting, we get $|A \cap B \cap C| \geq 0.25x$, and for all, $|A \cap B \cap C|$ Using the Principle of Inclusion Exclusion (PIE), we get -

$$\begin{aligned}|A \cap B \cap C \cap D| = |A| + |B| + |C| + |D| - |A \cap B| - |A \cap C| \\ - |A \cap D| - |B \cap C| - |B \cap D| - |C \cap D| \\ + |A \cap B \cap C| + |A \cap B \cap D| + \\ + |A \cap C \cap D| + |B \cap C \cap D| \\ - |A \cup B \cup C \cup D|\end{aligned}$$

Denote $|A| + |B| + |C| + |D|$ as r and $|A \cup B| + |A \cup C||A \cup D| + |B \cup C| + |B \cup D| + |C \cup D|$ as s, and $|A \cup B \cup C| + |A \cup B \cup D| + +|A \cup C \cup D| + |B \cup C \cup D|$ as t and $|A \cup B \cup C \cup D|$ as u, then we can simplify above as -

$$\begin{aligned}& r - 3r + s + t - 2s + 3r - u \\ & = r - s + t - u \\ & \geq 3.1x - 6x + 4x - x \\ & \geq 0.1x\end{aligned}$$

Thus, atleast 10% lost eye, ear, arm and leg.