Assignment 1

Xikrant Dewangan

1.b (a) $A = \begin{pmatrix} 2 & 2 & 2 & 2 \\ 17/10 & 1/10 & -17/10 & -1/10 \\ 3/5 & 9/5 & -3/5 & -9/5 \end{pmatrix}$

Compute AA^T;

$\det(AA^T - \lambda I) = 0 \Rightarrow \lambda = \{4, 3, 2\}.$

compute eigenvectors of $AA^T \rightarrow$

$u_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ $u_2 = \begin{bmatrix} 0 \\ 0.6 \\ 0.8 \end{bmatrix}$ $u_3 = \begin{bmatrix} 0 \\ 0.8 \\ -0.6 \end{bmatrix}$

~~det~~ now, compute eigenvectors of $A^TA \rightarrow$

$v_1 = \begin{bmatrix} 0.5 \\ 0.5 \\ 0.5 \\ -0.5 \end{bmatrix}$ $v_2 = \begin{bmatrix} 0.5 \\ 0.5 \\ -0.5 \\ 0.5 \end{bmatrix}$ $v_3 = \begin{bmatrix} 0.5 \\ -0.5 \\ -0.5 \\ -0.5 \end{bmatrix}$ $v_4 = \begin{bmatrix} 0.5 \\ -0.5 \\ 0.5 \\ 0.5 \end{bmatrix}$

Thus we get

$U = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.6 & 0.8 \\ 0 & 0.8 & -0.6 \end{bmatrix}$ $V = \begin{bmatrix} 0.5 & 0.5 & -0.5 & 0.5 \\ 0.5 & 0.5 & -0.5 & -0.5 \\ 0.5 & -0.5 & -0.5 & 0.5 \\ -0.5 & 0.5 & -0.5 & 0.5 \end{bmatrix}$

$$\Sigma = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

Thus $\boxed{A = U \Sigma V^T}$

(b) Used in –

- solving ill-conditioned least squares problems.
- solving discretized ill-posed problems.
- solving linear systems.
- determining rank of matrix
- determining low-rank approx. to matrix.

2.4

(a)
$$B = \begin{pmatrix} 2 & 0 & 2 \\ 2 & 2 & 0 \\ 0 & 2 & 2 \end{pmatrix}$$

$$B = [\, b_1 \mid b_2 \mid b_3 \,]$$

$u_1 = b_1$, $e_1 = \dfrac{u_1}{\|u_1\|} = \left(\dfrac{1}{\sqrt{2}}, \dfrac{1}{\sqrt{2}}, 0\right)$

$= (2, 2, 0)$

$u_2 = b_2 - (b_2 \cdot e_1) e_1 = (0, 2, 2) - (0, 2, 2).$

$\left(\dfrac{1}{\sqrt{2}}, \dfrac{1}{\sqrt{2}}, 0\right)$

$\left(1/\sqrt{2}, 1/\sqrt{2}, 0\right)$

$$= (0,2,2) - (1,1,0)$$

$$= (-1,1,2)$$

$$e_2 = \frac{u_2}{\|u_2\|} = \left(\frac{-1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}\right)$$

$$u_3 = b_3 - (b_3 \cdot e_1)e_1 - (b_3 \cdot e_2)e_2$$

$$= (2,0,2) - (2,0,2)\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right)\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right) - (2,0,2)\left(-\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}\right)\left(-\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}\right)$$

$$= (2,0,2) - \left[\frac{1}{\cancel{\sqrt{2}}}, 1, 0\right] - \left[\frac{-1}{3}, \frac{1}{3}, \frac{2}{3}\right]$$

$$= \left[\frac{4}{3}, -\frac{4}{3}, \frac{4}{3}\right]$$

$$e_3 = \frac{u_3}{\|u_3\|} = \left(\frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$$

$$Q = [e_1 | e_2 | e_3] = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} \\ 0 & \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} \end{bmatrix}$$

$$R = \begin{bmatrix} b_1 e_1 & b_2 e_1 & b_3 e_1 \\ 0 & b_2 \cdot e_2 & b_3 e_2 \\ 0 & 0 & b_3 e_3 \end{bmatrix} = \begin{bmatrix} 2\sqrt{2} & \sqrt{2} & \sqrt{2} \\ 0 & \sqrt{6} & \frac{2}{\sqrt{6}} \\ 0 & 0 & \frac{4}{\sqrt{3}} \end{bmatrix}$$

(b) ~~Input = ( )~~

~~for i~~

~~u = 0 [0, 0, ... 0]~~

~~v = 0 [0, 0, ... 0]~~

(b)

```
function(x):
    n = number_of_cols(x)
    m = number_of_rows(x)

    Q = [[0 for i in range(0,m)] for j in range(0,n)]
    R = [[0 for i in range(0,n)] for j in range(0,n)]

    for i = 1 → n:
        col = get_col(i, x)  # to get column.
        for (j = 1 → (i-1)):
/* inner product */   R[j, i] ← get_col(Q, j) ** col
/* subtract projection */   col ← col - R[j, i] * get_col(Q, j)
/* find L2 norm */   R[i, i] ← sqrt(sum(col^2))
/* put this in i th col */   set_col(Q, i, v/R[i, i])

    output(Q, R).
```

3.

$$[x_1 \quad x_2 \quad x_3]\begin{bmatrix} s_{11} & s_{12} & s_{13} \\ s_{21} & s_{22} & s_{23} \\ s_{31} & s_{32} & s_{33} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$= 4\left(x_1^2 + 4x_2^2 + x_3^2 - 4x_1x_2 + 2x_1x_3 - 4x_2x_3\right)$$

$$= \left[\sum x_i s_{i1} \quad \sum x_i s_{i2} \quad \sum x_i s_{i3}\right]\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$= x_1 \sum x_i s_{i1} + x_2 \sum x_i s_{i2} + x_3 \sum x_i s_{i3}$$

from this we get;

$s_{11} = 4$; $s_{12}+s_{21} = -16$ $s_{13}+s_{31} = 8$

$s_{22} = 16$

$s_{23}+s_{32} = -16$ $s_{33} = 4$

(a)
$$\begin{bmatrix} 4 & -16 & 8 \\ 0 & 16 & -16 \\ 0 & 0 & 4 \end{bmatrix}$$

(b) rank = 3, $\begin{vmatrix} 4-\lambda & -16 & +8 \\ 0 & 16-\lambda & -16 \\ 0 & 0 & 4-\lambda \end{vmatrix} \Rightarrow \lambda = (4, 16)$.

$4x_1$ $-16(x_1)+16(x_2)$ $8x_1 - 16x_2 + 4x_3$ det $\Rightarrow$ 256.

(c) Yes (+ve eigen values).

Hessian = $J(\nabla f)$

(3) 4.b) $f = 3x^2yz$.

$$\nabla f = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \\ \frac{\partial f}{\partial z} \end{bmatrix} = \begin{bmatrix} 6xyz \\ 3x^2z \\ 3x^2y \end{bmatrix} \quad \nabla^2 f = \begin{bmatrix} 6yz & 6xz & 6xy \\ 6xz & 0 & 3x^2 \\ 6xy & 3x^2 & 0 \end{bmatrix}$$

5.b) 3)

$$\nabla f = \lambda \nabla g \Leftrightarrow \quad 3 = 8\lambda x$$
$$-6 = 4\lambda y$$
$$4x^2 + 2y^2 = 25.$$

$$\Rightarrow \frac{1}{\lambda^2}\left[\frac{9}{16} + \frac{9}{2}\right] = 25$$

$$\Rightarrow \lambda = \pm\sqrt{\frac{9}{25}\left[\frac{9}{16}\right]} = \pm\frac{9}{4\cdot 5} = \pm 9/20$$

(1)

$$x = 3/8\lambda = \frac{3}{8\cdot\frac{9}{20}} = 5/6$$

$$y = \frac{-3}{2\lambda} = \frac{-3}{2\cdot 9/20} = -\frac{20}{6}.$$

$$\therefore f = 3x - 6y = \frac{15}{6} + 20 = 22.5 \leftarrow \text{max}$$

(2)

$$x = 3/8\lambda = -5/6$$

$$y = +20/6$$

$$f = 3x - 6y = -15/6 - 20 = -22.5 \leftarrow \text{min}$$

$\nabla f = \lambda \nabla g$

Scanned with CamScanner