Exercise 1.1

1. Which of these statement are propositions what are the truth values of those that are propositions?

(i) Delhi is the capital of India.

(ii) $2 + 3 = 5$

(iii) $5 + 7 = 10$

(iv) $x + 2 = 11$

(v) Answer this question.

(i), (ii) and (iii) are propositions. (i) & (ii) are true but (iii) is False.

(iv) and (v) are not propositions.

3. What is the negation of each of these propositions?

(i) Jennifer and John are friend.

(ii) $2 + 1 = 3$

(iii) There is no pollution in New Jersey.

Soln (i) Jennifer and John are not friend.

(ii) $2 + 1 \neq 3$

(iii) There is pollution in New Jersey.

Ques Let p and q be the propositions

p : I bought a lottery ticket this week.

q : I won the million dollar Jackpot.

Express each of these propositions as an English sentence.

(i) $\neg p$ (ii) $p \vee q$ (iii) $p \to q$

(iv) $p \wedge q$ (v) $p \leftrightarrow q$ (vi) $\neg p \to \neg q$

(vii) $\neg p \vee (p \wedge q)$ (viii) $\neg p \wedge (p \vee \neg q)$

Soln : (i) I did not buy a lottery ticket this week.

(ii) I bought a lottery ticket or I won the million dollar.

(iii) If I bought a lottery ticket, then I won the million dollar.

(iv) I bought a lottery ticket and I won the million dollar.

(v) I bought a lottery ticket if and only if I won the million dollar.

(vi) If I did not buy ticket, then I did not win the million dollar.

(vii) I did not buy a ticket, or I bought a ticket and I won the million dollar.

(viii) I did not buy a ticket, and either I bought a ticket or I did not win the million dollar.

Ques: Let p and q be the propositions

p: It is below freezing.

q: It is snowing.

Write these propositions using p and q and logical connectives (including negations)

(i) It is below freezing and snowing.
(ii) It is below freezing but not snowing.
(iii) It is not below freezing and it is not snowing.
(iv) It is either snowing or below freezing (or both).
(v) If it is below freezing, it is also snowing.
(vi) Either it is below freezing or it is snowing, but it is not snowing if it is below freezing.

(vi) Either it is below freezing or it is snowing, but it is not snowing if it is below freezing.

(vii) That it is below freezing is necessary and sufficient for it ~~to~~ be snowing.

Solⁿ : (i) $p \wedge q$ (ii) $p \wedge \neg q$ (iii) $\neg p \wedge \neg q$

(iv) $p \vee q$ (v) $p \rightarrow q$ (vi) $(p \vee q) \wedge (p \rightarrow \neg q)$

(vii) $p \leftrightarrow q$

Ques : Let p, q and r be the propositions

p : You get an A on the final exam.

q : You do every exercise in this book.

r : You get an A in this class.

~~working~~ Write these propositions using p, q and r and logical connectives (including negation).

(i) You get A in this class, but you do not do every exercise in this book.

(ii) You get an A on the final, you do every exercise in this book, and you get A in this class.

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(iii) To get an A in this class, it is necessary for you to get an A on the final.

(iv) You get an A on the final, but you don't do every exercise in this book; nevertheless, you get an A in this class.

(v) Getting an A on the final and doing every exercise in this book is sufficient for getting an A in this class.

(vi) You will get an A in this class if and only if you either do every exercise in this book or you get an A on the final.

Soln: (i) $r \wedge -q$

(ii) $p \wedge q \wedge r$

(iii) $p \rightarrow r$

(iv) $p \wedge -q \wedge r$

(v) $p \wedge q \rightarrow r$

(vi) $r \leftrightarrow p \vee q$

Ques : Determine whether these biconditionals are true or false.

Soln : (i) $2+2=4$ if and only if $1+1=2$

T T

$\Rightarrow$ T

(ii) $1+1=2$ if and only if $2+3=4$

T F

$\Rightarrow$ F

(iii) $1+1=3$ iff Monkey can fly.

F F

$\Rightarrow$ T

(iv) $0>1$ if and only if $2>1$

F T

$\Rightarrow$ F

Ques : Determine whether each of these conditional statements is true or false.

(i) If $1+1=2$, then $2+2=5$

T F

$\Rightarrow$ F

(ii) If $1+1=3$, then $2+2=4$.

$\quad F \qquad\qquad T$

$\Rightarrow T$

(iii) If $1+1=3$, then $2+2=5$.

$\quad F \qquad\qquad F$

$\Rightarrow T$

(iv) If monkey can fly, then $1+1=3$

$\quad F \qquad\qquad F$

$\Rightarrow T$

(v) If $1+1=3$, then unicorns exist.

$\quad F \qquad\qquad T$

$\Rightarrow T$

(vi) If $1+1=3$, then dogs can fly.

$\quad F \qquad\qquad F$

$\Rightarrow T$

(vii) If $1+1=2$, then dogs can fly.

$\quad T \qquad\qquad F$

$\Rightarrow F$

(viii) If $2+2=4$, then $1+2=3$.

$\quad T \qquad\qquad T$

$\Rightarrow T$

Ques For each of these sentences, determine whether an inclusive or, or an exclusive or, is intended. Explain your answer.

(i) Coffee or tea comes with dinner.

(ii) A password must have at least three digits or be at least eight characters long.

(iii) The prerequisite for the course is a course in number theory or a course in cryptography.

(iv) You can pay using U.S. dollars or euros.

Solⁿ: (i) The dinner gets to choose only one of the beverages.
Therefore, this is exclusive or.

(ii) Long password with many digits are considerable (acceptable)
⇒ Inclusive or

(iii) Student may have both the courses.
⇒ Inclusive or

(iv) The patron would pay a portion of the bill in dollars and remainder in euros.

Ques : Write each of these statements in the form "If p, then q" in English.

(i) It snows whenever the wind blows from the northest.

Ans If the wind blows from the northest, then it snows.

(ii) The apple trees will bloom if it stays warm for a week.

Ans If it stays warm for a week, then the apple trees will bloom.

(iii) That the pistons win the championship implies that they beat the Lakers.

Ans If the pistons win the championship, then they beat the lakers.

(iv) To get tenure as a professor, it is sufficient to be world famous.

Ans If you are world famous, then you will get tenure as a professor.

(v) It is necessary to walk 8 miles to get to the top of Long's Peak.

Ans If you get to the top of Long's Peak, then you must have walked 8 miles.

(vi) If you drive more than 100 miles, you will need to buy gasoline.

Ans If you drive more than 100 miles, then you will need to buy gasoline.

(vii) Your guarantee is good only if you bought your CD player less than 90 days ago.

Ans If your guarantee is good, then you must have bought your CD player less than 90 days ago.

(viii) Jan will go swiming unless the water is too cold.

Ans If water is not too cold, then John will go swiming.

Ques Write each of these propositions in the form of "p if and only if q" in English.

(i) For you to get an A in this course, it is necessary and sufficient that you learn how to solve DMS Problems.

Ans you will get an A in this course if and only if you learn how to solve DMS Problems.

(ii) If you read the newspaper every day, you will be informed and conversely.

Ans You will be informed if and only if you read the newspaper every day.

(iii) It rains if it is a weekend day, and it is a weekend day if it rains.

Ans It rains if and only if it is a weekend day.

(iv) You can see the wizard only if the wizard is not in, and the wizard is not in only if you can see him.

Ans You can see the wizard if and only if the wizard is not in.

Ques State the converse, contrapositive, and inverse of the conditional statements

(i) If it snows today, I will ski tomorrow.

(ii) I come to class whenever there is going to be a quiz.

Ans (i) Converse: If I will ski tomorrow, then it snows today.

Inverse: If it does not snow today, then I will not ski tomorrow.

Contrapositive: If I will not ski tomorrow, then it does not snow today.

(ii) If there will be a quiz, then I come to class.

Converse: If I come to class, then there will be a quiz.

Inverse: If there will not be a quiz, then I do not come to class.

Contrapositive: If I do not come to the class, then there will not be a quiz.

Ques: How many rows appear in a truth table for each of these compound propositions?

(i) $p \to \neg p$ (ii) $(p \vee \neg r) \wedge (q \vee \neg s)$

Ans (i) one proposition, therefore $2^1 = 2$ rows

(ii) 4 propositions, therefore $2^4 = 16$ rows

(iii) $q \vee p \vee \neg s \vee \neg r \vee \neg t \vee u$

6 propositions, therefore 2^6 rows.

(iv) $(p \wedge r \wedge t) \leftrightarrow (q \wedge t)$

4 propositions, therefore 2^4 rows.

<u>Ques</u> Construct a truth table for each of these compound propositions.

(i) $(p \vee q) \rightarrow (p \oplus q)$

(ii) $(p \oplus q) \rightarrow (p \vee q)$

(iii) $(p \vee q) \oplus (p \wedge q)$

(iv) $(p \leftrightarrow q) \oplus (\neg p \leftrightarrow q)$

(v) $(p \leftrightarrow q) \oplus (p \wedge q)$

<u>Soln</u>

p	q	$p \vee q$	$p \oplus q$	$p \vee q \rightarrow p \oplus q$
T	T	T	F	F
T	F	T	T	T
F	T	T	T	T
F	F	F	F	T

(ii)

p	q	$p \oplus q$	$p \vee q$	$p \oplus q \to p \vee q$
T	T	F	T	T
T	F	T	T	T
F	T	T	T	T
F	F	F	F	T

(iii)

p	q	$p \vee q$	$p \wedge q$	$(p \vee q) \oplus (p \wedge q)$
T	T	T	T	F
T	F	T	F	T
F	T	T	F	T
F	F	F	F	F

(iv)

p	q	$\neg p$	$p \leftrightarrow q$	$\neg p \leftrightarrow q$	$(p \leftrightarrow q) \oplus (\neg p \leftrightarrow q)$
T	T	F	T	F	T
T	F	F	F	T	T
F	T	T	F	T	T
F	F	T	T	F	T

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(v)

p	q	$p \leftrightarrow q$	$p \wedge q$	$(p \leftrightarrow q) \oplus (p \wedge q)$
T	T	T	T	F
T	F	F	F	F
F	T	F	F	F
F	F	T	F	T

Ques
Construct a truth table for each of these compound propositions.

(i) $(p \vee q) \vee r$

(ii) $p \rightarrow (\neg q \vee r)$

(iii) $(p \rightarrow q) \wedge (\neg p \rightarrow r)$

p	q	r	$p \vee q$	$(p \vee q) \vee r$
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	T	T
F	T	T	T	T
F	T	F	T	T
F	F	T	F	T
F	F	F	F	F

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(ii)

p	q	r	$-q$	$-q \vee r$	$p \to (-q \vee r)$
T	T	T	F	T	T
T	T	F	F	F	F
T	F	T	T	T	T
T	F	F	T	T	T
F	T	T	F	T	T
F	T	F	F	F	T
F	F	T	T	T	T
F	F	F	T	T	T

(iii)

p	q	r	$-p$	$p \to q$	$-p \to r$	$(p \to q) \wedge (-p \to r)$
T	T	T	F	T	T	T
T	T	F	F	T	T	T
T	F	T	F	F	T	F
T	F	F	F	F	T	F
F	T	T	T	T	T	T
F	T	F	T	T	F	F
F	F	T	T	T	T	T
F	F	F	T	T	F	F

Ques Find the bitwise OR, bitwise AND, and bitwise XOR of each of these pair of strings.

(i) 101 1110, 010 0001

(ii) 1111 0000, 1010 1010

Soln:

(i)

```
 101 1110    bitwise OR
 010 0001
 --------
 111 1111
```

```
 101 1110    bitwise AND
 010 0001
 --------
 000 0000
```

```
 101 1110    bitwise XOR
 010 0001
 --------
 111 1111
```

(ii)

```
 1111 0000
 1010 1010
 ---------
 1111 1010    OR

 1010 0000    AND

 0101 1010    XOR
```

Propositional Equivalences

Exercise 1.3

(1) Use truth tables to verify these equivalences

(a) $p \wedge T \equiv p$ (b) $p \vee F \equiv p$ (c) $p \vee p \equiv p$ (d) $p \wedge p \equiv p$

(a)

p	T	$p \wedge T$
T	T	T
F	T	F

$\Rightarrow p \wedge T \equiv p$

(b)

p	F	$p \vee F$
T	F	T
F	F	F

$\Rightarrow p \vee F \equiv p$

(c) (d)

p	p	$p \vee p$	$p \wedge p$
T	T	T	T
F	F	F	F

$\Rightarrow p \vee p \equiv p$

$p \wedge p \equiv p$

(2) Show that $-(-p)$ and p are logically equivalent.

p	$-p$	$-(-p)$
T	F	T
F	T	F

$\Rightarrow p \equiv -(-p)$

(4) Use truth table to verify the associative laws.

(a) $(p \vee q) \vee r \equiv p \vee (q \vee r)$

(b) $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$

p	q	r	$p \vee q$	$q \vee r$	$(p \vee q) \vee r$	$p \vee (q \vee r)$
T	T	T	T	T	T	T
T	T	F	T	T	T	T
T	F	T	T	T	T	T
T	F	F	T	F	T	T
F	T	T	T	T	T	T
F	T	F	T	T	T	T
F	F	T	F	T	T	T
F	F	F	F	F	F	F

Similarly, we can verify

$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$.

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(7) Use De Morgan's Law to find the negation of each of the following statements.

(i) Jan is rich and happy.

(ii) Mei walks or takes the bus to class.

Ans (i) Jan is rich and happy.

Let p = Jan is rich

q = Jan is happy

$\Rightarrow$ $p \wedge q$ = Jan is rich and happy.

We know that, By De Morgan's Law

$$-(p \wedge q) \equiv -p \vee -q$$

Therefore $-(p \wedge q) \equiv -p \vee -q$

= Jan is not rich or not happy.

(ii) Mei walks or takes the bus to the class.

Let p = Mei walks

q = Mei takes the bus to the class.

Therefore Mei walks or takes the bus to the class = $p \vee q$

So, Negation of $p \vee q = -(p \vee q)$

By De Morgan's Law $-(p \vee q) \equiv -p \wedge -q$

$-(p \vee q)$ = Mei does not walk and Mei does not take the bus.

Q Show that each of these conditional statements is a tautology by using truth tables

(i) $(p \wedge q) \rightarrow p$ (ii) $\neg p \rightarrow (p \rightarrow q)$

(iii) $(p \wedge q) \rightarrow (p \rightarrow q)$.

we know that

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

(i)

p	q	$p \wedge q$	$p \wedge q \rightarrow p$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

(ii)

p	q	$\neg p$	$p \rightarrow q$	$\neg p \rightarrow (p \rightarrow q)$
T	T	F	T	T
T	F	F	F	T
F	T	T	T	T
F	F	T	T	T

(iii)

p	q	$p \wedge q$	$p \rightarrow q$	$(p \wedge q) \rightarrow (p \rightarrow q)$
T	T	T	T	T
T	F	F	F	T
F	T	F	T	T
F	F	F	T	T

Ques Show that each of these conditional statements is a tautology ~~by~~ without using truth tables.

(i) $(p \wedge q) \rightarrow p$ (ii) $p \rightarrow (p \vee q)$

(iii) $-p \rightarrow (p \rightarrow q)$ (iv) $(p \wedge q) \rightarrow (p \rightarrow q)$

(v) $-(p \rightarrow q) \rightarrow p$ (vi) $-(p \rightarrow q) \rightarrow -q$

Sol^n (i) $(p \wedge q) \rightarrow p \equiv -(p \wedge q) \vee p$

$\equiv (-p \vee -q) \vee p$ by DeMorgan's Law

$\equiv (-q \vee -p) \vee p$ by commutative

$\equiv -q \vee (-p \vee p)$ by Associative law

$\equiv -q \vee (T)$ (by Negation law)

$\equiv -q \vee T$

$\equiv T$ (by Domination law)

(ii) $p \rightarrow (p \vee q) \equiv -p \vee (p \vee q)$

$\equiv (-p \vee p) \vee q$ (By associative law)

$\equiv T \vee q$ by negation law

$\equiv T$ by Domination law

(iii) $-p \rightarrow (p \rightarrow q) \equiv -(-p) \vee (p \rightarrow q)$ By logical conditional statement $p \rightarrow q \equiv -p \vee q$

$\equiv +p \vee (-p \vee q)$

$\equiv p \vee (-p \vee q)$

$\equiv (p \vee -p) \vee q$ by associative law

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$\equiv T \vee q$ (By negation law)

$\equiv T$ by Domination law.

(iv) $(p \wedge q) \rightarrow (p \rightarrow q)$ By $p \rightarrow q \equiv \neg p \vee q$

$\equiv \neg(p \wedge q) \vee (p \rightarrow q)$

$\equiv \neg(p \wedge q) \vee (\neg p \vee q)$

$\equiv (\neg p \vee \neg q) \vee (\neg p \vee q)$

$\equiv (\neg p \vee \neg q) \vee (\neg p \vee q)$

$\equiv (\neg p \vee \neg q) \vee (q \vee \neg p)$ by commutative

$\equiv ((\neg p \vee \neg q) \vee q) \vee \neg p$ by associative

$\equiv (\neg p \vee (\neg q \vee q)) \vee \neg p$ Again by associative

$\equiv (\neg p \vee T) \vee \neg p$ by negation law

$\equiv (T \vee \neg p) \vee \neg p$ by commutative law

$\equiv T \vee \neg p$ by domination laws

$\equiv T$ by Domination law.

(v) $\neg(p \rightarrow q) \rightarrow p \equiv \neg(\neg p \vee q) \rightarrow p$

$\equiv (p \wedge \neg q) \rightarrow p$ by Demorgan's Law

$\equiv \neg(p \wedge \neg q) \vee p$

$\equiv (\neg p \vee q) \vee p$

$\equiv (q \vee \neg p) \vee p$ by commutative law

$\equiv q \vee (\neg p \vee p)$ by associative law

$\equiv q \vee T$ by negation law

$\equiv T$ by Domination law.

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(vi) $\neg(p \to q) \to \neg q \equiv \neg(\neg p \vee q) \to \neg q$

$\equiv (p \wedge \neg q) \to \neg q$ by De Morgan's law

$\equiv \neg(p \wedge \neg q) \vee \neg q$

$\equiv (\neg p \vee q) \vee \neg q$ by De Morgan's law

$\equiv \neg p \vee (q \vee \neg q)$ by associative law

$\equiv \neg p \vee T$ by negation law

$\equiv T$ by domination law.

__Ques__ Show that each of these conditional statements is a tautology by using Truth Table.

(i) $[(p \to q) \wedge (q \to r)] \to (p \to r)$

(ii) $[p \wedge (p \to q)] \to q$

__Ans__

p	q	r	$(p \to r)$ c	$p \to q$ a	$q \to r$ b	$(p \to q) \wedge (q \to r)$ $a \wedge b$	$(a \wedge b) \to c$
T	T	T	T	T	T	T	T
T	T	F	F	T	F	F	T
T	F	T	T	F	T	F	T
T	F	F	F	F	T	F	T
F	T	T	T	T	T	T	T
F	T	F	T	T	F	F	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

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(iii) $[p \wedge (p \rightarrow q)] \rightarrow q$

p	q	$p \rightarrow q$	$p \wedge (p \rightarrow q)$	$[p \wedge (p \rightarrow q)] \rightarrow q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

Ques Show that $(p \leftrightarrow q)$ and $(p \wedge q) \vee (-p \wedge -q)$ are logically equivalent.

Solⁿ:

$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

$\equiv (-p \vee q) \wedge (-q \vee p)$

$\equiv ((-p \vee q) \wedge -q) \vee ((-p \vee q) \wedge p)$

by distributive law

$\equiv ((-p \wedge -q) \vee (q \wedge -q)) \vee ((-p \wedge p) \vee (q \wedge p))$

$\equiv ((-p \wedge -q) \vee F) \vee (F \vee (q \wedge p))$

by negation law

$\equiv ((-p \vee F) \wedge (-q \vee F)) \vee ((F \vee q) \wedge (F \vee p))$

by Distributive law

$\equiv (-p \wedge -q) \vee (q \wedge p)$ by Identity law

$\equiv (-p \wedge -q) \vee (p \wedge q)$ by commutative

$p \leftrightarrow q \equiv (p \wedge q) \vee (-p \wedge -q)$ by commutative law.

Ques Show that $-(p \leftrightarrow q)$ and $p \leftrightarrow -q$ are logically equivalent.

Sol^n

$$-(p \leftrightarrow q) \equiv -\{(p \to q) \wedge (q \to p)\}$$

$$\equiv -((-p \vee q) \wedge (-q \vee p))$$

$$\equiv -(-p \vee q) \vee -(-q \vee p)$$ by DeMorgan's law

$$\equiv (p \wedge -q) \vee (q \wedge -p)$$

By DeMorgan's law

$$\equiv ((p \wedge -q) \vee q) \wedge ((p \wedge -q) \vee -p)$$

By Distributive law

$$\equiv ((p \vee q) \wedge (-q \vee q)) \wedge ((p \vee -p) \wedge (-q \vee -p))$$

$$\equiv ((p \vee q) \wedge T) \wedge (T \wedge (-q \vee -p))$$

By negation law

$$\equiv (p \vee q) \wedge (-q \vee -p)$$

By Identity law

$$\equiv (-q \to p) \wedge (p \to -q) \quad \left\{ \begin{array}{l} \text{by} \\ p \to q \equiv -p \vee q \end{array} \right.$$

$$\equiv -q \leftrightarrow p$$

$$\equiv p \leftrightarrow -q$$ Proved

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Ques Show that $(p \to q) \wedge (p \to r)$ and $p \to (q \wedge r)$ are logically equivalent.

Soln : $(p \to q) \wedge (p \to r) \equiv (\neg p \vee q) \wedge (\neg p \vee r)$

$\equiv \neg p \vee (q \wedge r)$

By Distributive law

$\equiv p \to (q \wedge r)$

Proved

Ques : Show that $(p \to q) \vee (p \to r)$ and $p \to (q \vee r)$ are logically equivalent.

Soln : $(p \to q) \vee (p \to r) \equiv (\neg p \vee q) \vee (\neg p \vee r)$

$\equiv (\neg p \vee q) \vee (\neg p \vee r)$

$\equiv (q \vee \neg p) \vee (\neg p \vee r)$

By commutative law

$\equiv q \vee (\neg p \vee (\neg p \vee r))$

By Associative law

$\equiv q \vee ((\neg p \vee \neg p) \vee r)$ By asso. law

$\equiv q \vee (\neg p \vee r)$ By Idempotent

$\equiv (q \vee \neg p) \vee r$ By asso.

$\equiv (\neg p \vee q) \vee r$ By commutative law

$\equiv \neg p \vee (q \vee r)$ By Associative law

$\equiv p \to q \vee r$

Proved

Ques: Show that $(p \to q) \wedge (q \to r) \to (p \to r)$ is a tautology.

Sol^n: $(p \to q) \wedge (q \to r) \to (p \to r)$

$\equiv ((-p \vee q) \wedge (-q \vee r)) \to (p \to r)$

$\equiv -((-p \vee q) \wedge (-q \vee r)) \vee (p \to r)$

$\equiv -((-p \vee q) \wedge (-q \vee r)) \vee (-p \vee r)$

$\equiv \{-(-p \vee q) \vee -(-q \vee r)\} \vee (-p \vee r)$ By De Morgan's law

$\equiv ((p \wedge -q) \vee (q \wedge -r)) \vee (-p \vee r)$ By De Morgan's law

$\equiv \{((p \wedge -q) \vee q) \wedge ((p \wedge -q) \vee -r)\} \vee (-p \vee r)$ By Distributive law

$\equiv \{((p \vee q) \wedge (-q \vee q)) \wedge ((p \vee -r) \wedge (-q \vee -r))\} \vee (-p \vee r)$ By Distributive law

$\equiv \{((p \vee q) \wedge T) \wedge ((p \vee -r) \wedge (-q \vee -r))\} \vee (-p \vee r)$ By Negation law

$\equiv \{(p \vee q) \wedge ((p \vee -r) \wedge (-q \vee -r)\} \vee (-p \vee r)$ By Identity

$\equiv ((p \vee q) \vee (-p \vee r)) \wedge \{((p \vee -r) \wedge (-q \vee -r)) \vee (-p \vee r)\}$ By Distributive law

$\equiv \{(q \vee p) \vee (-p \vee r)\} \wedge \{(p \vee -r) \wedge (-q \vee -r)) \vee (-p \vee r)\}$ By Commutative law

$\equiv \{q \vee ((p \vee -p) \vee r)\} \wedge \{((p \vee -r) \wedge (-q \vee -r)) \vee (-p \vee r)\}$ By Associative.

$\equiv (q \vee (T \vee r)) \wedge \{(p \vee \neg r) \wedge (\neg q \vee \neg r)\} \vee (\neg p \vee r)$

by Negation

$\equiv T \wedge \{(p \vee \neg r) \wedge (\neg q \vee \neg r)\} \vee (\neg p \vee r)$

By domination law

$\equiv T \wedge \{((p \vee \neg r) \vee (\neg p \vee r)) \wedge ((\neg q \vee \neg r) \vee (\neg p \vee r))\}$

By Distributive law

$\equiv T \wedge \{((p \vee \neg r) \vee (r \vee \neg p)) \wedge ((\neg q \vee \neg r) \vee (r \vee \neg p))\}$

By commutative law

$\equiv T \wedge \{(p \vee ((\neg r \vee r) \vee \neg p)) \wedge (\neg q \vee ((\neg r \vee r) \vee \neg p))\}$

$\equiv T \wedge \{(p \vee (T \vee \neg p)) \wedge (\neg q \vee (T \vee \neg p))\}$

By Negation law

$\equiv T \wedge \{(p \vee (T \vee \neg p)) \wedge \neg q \vee (T \vee \neg p)\}$

$\equiv T \wedge \{(p \vee T) \wedge (\neg q \vee T)\}$ By Domination law

$\equiv T \wedge \{T \wedge T\}$ By domination law

$\equiv T \wedge T$

$\equiv T$

Proved

Ques Show that $(p \wedge q) \rightarrow r$ and $(p \rightarrow r) \wedge (q \rightarrow r)$ are not logically equivalent.

Solⁿ:

p	q	r	$p \wedge q$	$p \rightarrow r$	$q \rightarrow r$	$p \wedge q \rightarrow r$	$(p \rightarrow r) \wedge q \rightarrow r$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F
T	F	T	F	T	T	T	T
T	F	F	F	F	T	T	F
F	T	T	F	T	T	T	T
F	T	F	F	T	F	T	F
F	F	T	F	T	T	T	T
F	F	F	F	T	T	T	T

By truth table, it is clear the truth values of $(p \wedge q) \rightarrow r$ and $(p \rightarrow r) \wedge (q \rightarrow r)$ are not same.
ie Not identical.

So $(p \wedge q) \rightarrow r$ and $(p \rightarrow r) \wedge (q \rightarrow r)$ are not logically equivalent.

Ques : Show that $(p \rightarrow q) \rightarrow (r \rightarrow s)$ and $(p \rightarrow r) \rightarrow (q \rightarrow s)$ are not logically equivalent.

Q ~~Nos~~ %: Soln:

p	q	r	s	$p \to q$	$r \to s$	$p \to r$	$q \to s$	$(p \to q) \to (r \to s)$	$(p \to r) \to (q \to s)$
T	T	T	T	T	T	T	T	T	T
T	T	T	F	T	F	T	F	F	F
T	T	F	T	T	T	F	T	T	T
T	T	F	F	T	T	F	F	T	T
T	F	T	T	F	T	T	T	T	T
T	F	T	F	F	F	T	T	T	T
T	F	F	T	F	T	F	T	T	T
T	F	F	F	F	T	F	T	T	T
F	T	T	T	T	T	T	T	T	T
F	T	T	F	T	F	T	F	F	F
F	T	F	T	T	T	T	T	T	T
F	T	F	F	T	T	T	F	T	F
F	F	T	T	T	T	T	T	T	T
F	F	T	F	T	F	T	T	F	T
F	F	F	T	T	T	T	T	T	T
F	F	F	F	T	T	T	T	T	T

Thus $(p \to q) \to (r \to s)$ and $(p \to r) \to (q \to s)$ are not logically equivalent.

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Ques: Show that each of these conditional Statements is a tautology without using Truth Table.

Sol^n: (i) $[-p \wedge (p \vee q)] \to q$ $\quad \left\{ \because p \to q \equiv -p \vee q \right.$

$\equiv -(-p \wedge (p \vee q)) \vee q$

$\equiv -(-p \wedge +(p \vee q)) \vee q$

$\equiv (-(-p) \vee -(p \vee q)) \vee q$ By DeMorgan's law

$\equiv (p \vee (-p \wedge -q)) \vee q$

$\equiv ((p \vee -p) \wedge (p \vee -q)) \vee q$, By distributive law

$\equiv (T \wedge (p \vee -q)) \vee q$ by Negation

$\equiv ((T \wedge p) \vee (T \wedge -q)) \vee q$ by Distributive law

$\equiv (p \vee -q) \vee q$ by Identity

$\equiv p \vee (-q \vee q)$ by Associative law

$\equiv p \vee T$ by Negation law.

$\equiv T$ by Domination law.

(ii) $[p \wedge (p \to q)] \to q$ $\quad \because p \to q \equiv -p \vee q$

$\equiv (p \wedge (-p \vee q)) \to q$

$\equiv -(p \wedge (-p \vee q)) \vee q$

$\equiv (\neg p \vee \neg(\neg p \vee q)) \vee q$

$\equiv (\neg p \vee (p \wedge \neg q)) \vee q$

$\equiv ((\neg p \vee p) \wedge (\neg p \vee \neg q)) \vee q$ by Distributive law

$\equiv (T \wedge (\neg p \vee \neg q)) \vee q$ By Negation law

$\equiv ((T \wedge \neg p) \vee (T \wedge \neg q)) \vee q$

$\equiv ((\neg p \vee \neg q) \vee q)$ by Identity.

$\equiv \neg p \vee (\neg q \vee q)$ By associative law

$\equiv \neg p \vee T$ by negation law

$\equiv T$ By Domination law.

Proved

(iii) Show that $(p \wedge q) \rightarrow (p \vee q)$ is a Tautology.

$(p \wedge q) \rightarrow (p \vee q) \equiv \neg(p \wedge q) \vee (p \vee q)$

$\equiv (\neg p \vee \neg q) \vee (p \vee q)$ by De Morgan's law

$\equiv (\neg p \vee \neg q) \vee (q \vee p)$ by commutative law

$\equiv ((\neg p \vee \neg q) \vee q) \vee p$ by associative law

$\equiv (\neg p \vee (\neg q \vee q)) \vee p$ by associative law

$\equiv (\neg p \vee T) \vee p$ by negation law

$\equiv (T \vee p)$ by Domination law

$\equiv T$ by domination law.

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Exercise 1.4

Q. (1) Let P(x) denote the statement "$x \leq 4$". What are these truth values?

(i) P(0) , (ii) P(4) (iii) P(6)

(i) P(0): $0 \leq 4$ is a proposition.
It is True.

(ii) P(4): $4 \leq 4$ is a proposition.
It is True.

(iii) P(6): $6 \leq 4$ is a proposition.
It is False.

(2) Let P(x) be the statement "the word x contains the letter a." What are these truth values?

(i) P(orange), It is a proposition & T.

(ii) P(lemon), It is a proposition & False

(iii) P(true), It is a proposition & False.

(iv) P(False), It is a proposition & T.

Ques Let P(x) be the statement "x spends more than 5 hours every weekday in class" where the domain for x consists of all students. Express each of these quantification in English.

(i) $\exists x\ P(x)$ (ii) $\forall x\ P(x)$

(iii) $\exists x\ -P(x)$ (iv) $\forall x\ -P(x)$

Solⁿ: (i) $\exists x\ P(x)$

= There exist a student that spends more than 5 hours every weekday in class.

(ii) $\forall x\ P(x)$ = Every students spend more than 5 hours every weekday in class.

(iii) There exist a student that does not spend more than 5 hours every weekday in class.

(iv) Every student do not spend more than 5 hours every weekday in class.

Ques Translate these statements into English, where $C(x)$ is "x is a comedian" and $F(x)$ is "x is funny" and domain consist of all people.

Solⁿ: (i) $\forall x\ (C(x) \rightarrow F(x))$

Ans Every comedian are funny.

(ii) $\forall x\ (C(x) \wedge F(x))$

Every person is a comedian and funny.

(iii) $\exists x \, (C(x) \rightarrow F(x))$

There exist a person, such that if he is a comedian, then he is funny.

(iv) $\exists x \, (C(x) \wedge F(x))$

There exists a person that is comedian and Funny.

<u>Ques</u> ~~[illegible]~~ Let $P(x)$ be the statement "x can speak Russian" and let $Q(x)$ be the statement "x knows the computer language C^{++}". Express each of these sentences in term of $P(x)$, $Q(x)$, quantifiers and logical connectives. The domain for quantifiers consists of all students at your school.

<u>Soln</u>: (i) There is a student at your school who can speak Russian but who doesn't know C++.

<u>Ans</u> $\exists x \, (P(x) \wedge \neg Q(x))$

(ii) There is a student at your school who can speak Russian and who knows C++.

<u>Ans</u> $\exists x \, (P(x) \wedge Q(x))$.

(iii) Every student at your school either can speak Russian or knows C++.

Ans $\forall x\,(C(x) \vee Q(x))$

(iv) No student at your school can speak Russian or knows C++.

Ans $\neg \exists x\,(C(x) \vee Q(x))$

Ques Let $P(x)$ be the statement "$x = x^2$". If the domain consists of integers, what are these truth values?

Solⁿ: (i) $P(0)$, Ans $0 = 0^2 \Rightarrow 0 = 0$ True

(ii) $P(1)$, Ans $1 = 1^2 \Rightarrow 1 = 1$ T

(iii) $P(2)$, Ans $2 \neq 2^2 \Rightarrow 2 \neq 4$ False

(iv) $P(-1)$, Ans $-1 = (-1)^2 \Rightarrow -1 = 1$ Not possible

False

(V) $\exists x\, P(x)$

$P(0)$ is True. i.e. there exist an element $0 \in \mathbb{Z}$ such that $P(0)$ is true.

$\Rightarrow \exists x\, P(x)$ is True.

(vi) $\forall x\, P(x)$

$P(2)$, $P(-1)$ is not true.

This means $P(x)$ is not true for every element of set of integers.

$\Rightarrow \forall x\, P(x)$ is False.

Q: Let $Q(x)$ be the statement '$x+1>2x$'. If the domain consists of all integers, what are these truth values?

(i) $Q(0)$, (ii) $Q(-1)$ (iii) $Q(1)$ (iv) $\exists x\, Q(x)$

(v) $\forall x\, Q(x)$ (vi) $\exists x\, \neg Q(x)$ (vii) $\forall x\, \neg Q(x)$.

Ans (i) $\because$ $Q(0)$ is $0+1>2\times 0$

$\Rightarrow$ $1>0$ True.

$Q(0)$ is True.

(ii) $Q(-1)$ is $-1+1>2\times -1$ $\Rightarrow$ $0>-2$ True.

$\Rightarrow$ $Q(-1)$ True.

(iii) $Q(1)$ is $1+1>2\cdot 1$ $\Rightarrow$ $2>2$ False
Not possible.

$\therefore$ $Q(1)$ is False.

(iv) $Q(0)$ is true. This means there exist an element $0 \in \mathbb{Z}$ such that $Q(x)$ is true.

$\Rightarrow$ $\exists x\, Q(x)$ is True

(v) $Q(1)$ is False.

i.e. $Q(x)$ is not true for every integer.

$\Rightarrow$ $\forall x\, Q(x)$ is False.

(vi)
$Q(x) : x+1 > 2x$

$\neg Q(x) : x+1 \not> 2x \Rightarrow x+1 \leq 2x$

i.e. $\neg Q(x)$ is $x+1 \leq 2x$.

$\neg Q(2)$ is $2+1 \leq 2\times 2 \Rightarrow 3 \leq 4$ is true

$\Rightarrow \neg Q(2)$ is true.

$\Rightarrow \exists x \neg Q(x)$ is True.

(vii) $\neg Q(x)$ is $x+1 \leq 2x$.

$\neg Q(0)$ is $0+1 \leq 2\times 0 \Rightarrow 1 \leq 0$ is not possible.

$\Rightarrow \neg Q(0)$ is False.

$\Rightarrow \forall x \neg Q(x)$ is False.

__Ques__ Determine the truth value of each of these statements if the domain for all variables consists of all integers.

__Sol^n__: (i) $\forall n (n^2 \geq 0)$, $n \in \mathbb{Z}$

__Ans__ As we know square of any integer is always +ve (positive). Therefore square of integer is always greater than or equal zero

i.e $n^2 \geq 0 \quad \forall n \in \mathbb{Z} \Rightarrow \forall n (n^2 \geq 0)$ is True.

(ii) $\exists n\ (n^2 = 2)$, $n \in \mathbb{Z}$ = set of integers

This is not possible for set of integers.

Note we have $\sqrt{2}, -\sqrt{2} \in \mathbb{R}$ (set of real number)

such that $(\sqrt{2})^2 = (-\sqrt{2})^2 = 2$

ie $\nexists\ n \in \mathbb{Z}$ such that $n^2 = 2$.

$\Rightarrow$ $\exists n\ (n^2 = 2)$ is False.

(iii) $\forall n\ (n^2 \geq n)$

square of any integer is always greater than or equal to the same number.

So $\forall n\ (n^2 \geq n)$ is True.

(iv) $\exists n\ (n^2 < 0)$

The square of any integer is greater than or equal to zero.

So $n^2 < 0$ is not Possible for any $n \in \mathbb{Z}$.

$\Rightarrow$ $\exists n\ (n^2 < 0)$ is False.

Ques Determine the truth value of each of these statements of the domain of each variables consists of all integers.

Soln: (i) $\exists x\ (x^2 = 2)$ (ii) $\exists x\ (x^2 = -1)$ (iii) $\forall x\ (x^2 + 2 \geq 1)$

(iv) $\forall x\ (x^2 \neq x)$

(i) $\exists x\ (x^2 = 2)$, $x \in \mathbb{R}$ (set of real numbers)

$\sqrt{2} \in \mathbb{R}$ such that $(\sqrt{2})^2 = 2$.

$\Rightarrow$ $\exists x\ (x^2 = 2)$ is True.

(ii) $\exists x\ (x^2 = -1)$

This is not possible in real numbers.

square of any real number $\neq -1$

$\Rightarrow$ $\exists x\ (x^2 = -1)$ is False.

(iii) $\forall x\ (x^2 + 2 \geq 1)$

we know $2 \geq 1$ and x^2 is always positive.

So, $x^2 + 2 \geq 1$

$\Rightarrow$ $\forall x\ (x^2 + 2 \geq 1)$ is True.

(iv) $\forall x\ (x^2 \neq x)$, $x \in \mathbb{R}$.

If we take $1 \in \mathbb{R}$, then we get $1^2 = 1$

$\Rightarrow$ $1 = 1$ True.

ie we have $1 \in \mathbb{R}$ which not satisfies $(x^2 \neq x)$.

$\Rightarrow$ $\forall x\ (x^2 \neq x)$ is False.

Ques: Suppose that the domain of the propositional function $P(x)$ consists of the integers 0, 1, 2, 3 and 4. Write out each of these propositions using disjunctions, conjunctions, and negations.

(i) $\exists x\, P(x)$ (ii) $\forall x\, P(x)$ (iii) $\exists x - P(x)$

(iv) $\forall x - P(x)$ (v) $-\exists x\, P(x)$ (vi) $-\forall x\, P(x)$.

(vii) $\exists x (-P(x)) \wedge \forall x ((x \neq 3) \rightarrow P(x))$

Soln: (i) $\exists x\, P(x) = P(0) \vee P(1) \vee P(2) \vee P(3) \vee P(4)$

(ii) $\forall x\, P(x) = P(0) \wedge P(1) \wedge P(2) \wedge P(3) \wedge P(4)$

(iii) $-P(0) \vee -P(1) \vee -P(2) \vee -P(3) \vee -P(4) = \exists x - P(x)$

(iv) $\forall x - P(x) = -P(0) \wedge -P(1) \wedge -P(2) \wedge -P(3) \wedge -P(4)$

(v) $-\exists x\, P(x) = -(P(0) \vee P(1) \vee P(2) \vee P(3) \vee P(4))$

or $-\exists x\, P(x) = -P(0) \wedge -P(1) \wedge -P(2) \wedge -P(3) \wedge -P(4)$

(vi) $-\forall x\, P(x) = -(P(0) \wedge P(1) \wedge P(2) \wedge P(3) \wedge P(4))$

or $-\forall x\, P(x) = -P(0) \vee -P(1) \vee -P(2) \vee -P(3) \vee -P(4)$.

(vii) $\exists x(-P(x)) \wedge \forall x ((x \neq 3) \rightarrow P(x))$

$\exists x (-P(x)) = -P(0) \vee -P(1) \vee -P(2) \vee -P(3) \vee -P(4)$

$\forall x ((x \neq 3) \rightarrow P(x)) = P(0) \wedge P(1) \wedge P(2) \wedge P(4)$

Therefore

$\exists x (-P(x)) \wedge \forall x ((x \neq 3) \rightarrow P(x)) = (-P(0) \vee -P(1) \vee -P(2) \vee -P(3) \vee -P(4))$
$\wedge P(0) \wedge P(1) \wedge P(2) \wedge P(4)$ Ans

Ques Translate in two ways each of these statements into logical expressions using predicates, quantifiers, and logical connectives. First, let the domain consist of the students in your class and second let, the ~~the~~ domain consist of all people.

(i) Someone in your class can speak Hindi.

I Domain D = students of the class.

Let $H(x)$: x can speak Hindi.

$x \in D$ = student of the class.

So, $\exists x\, H(x)$

II Domain D' = all people.

Let $A(x)$ = x is in your class.

& $H(x)$ = x can speak Hindi.

So, $\exists x\, (A(x) \wedge H(x))$

(ii) Everyone in your class is friendly.

I Domain D = student of your class.

Let $F(x)$ = x is friendly.

$x \in D$ = student of your class.

So $\forall x\, F(x)$.

II domain D' = all people

Let $F(x)$ = x is Friendly
$A(x)$ = x is in your class.

So $\forall x\,(A(x) \rightarrow F(x))$ Ans

(iii) There is a person in your class who was not born in California.

I Domain D = student of your Class

Let $C(x)$ = x is born in California

So, $\exists x\,(-C(x))$ Ans

II Domain = all people.

Let $A(x)$ = x is in your class.
$C(x)$ = x is born in California.

$\exists x\,(A(x) \wedge -C(x))$ Ans

(iv) A student in your class has been in a movie.

I Domain D = student of your Class

Let $M(x)$ = x has been in a movie.

$\exists x(M(x))$ Ans

II Domain D' = all People

Let $M(x)$ = x has been in a movie.
$A(x)$ = x is in your class

$\exists x\,(A(x) \wedge M(x))$ Ans

(v) No student in your class has taken a course in logic programming.

I D = student of your class

Let $L(x)$ = x has taken a course in logic Programming.

No student of your class = $\nexists x$ = $\neg(\exists x)$

$\neg(\exists x \, L(x))$ Ans

II Domain D' = all people.

Let $L(x)$ = x has taken a course in logic programming

$A(x)$ = x is in your class.

So, $\neg(\exists x \; A(x) \wedge L(x))$ Ans

Ques Translate each of these statements into logical expressions using perdicates, quantifiers, and logical connectives.

(i) No one is perfect.

Ans Let D = All people

& $P(x)$ = x is perfect

Noone = $\nexists x$ = $\neg(\exists x)$ = There does not exist a person

So, $\neg \exists x \, P(x)$

(ii) Not everyone is perfect.
Let D = all people
$P(x)$ = x is perfect
everyone = $\forall x$
Not everyone = $\neg \forall x$ = ~~[illegible]~~ Not all people

So, $\neg \forall x \, P(x)$ Ans

(iii) All your friend is perfect.
Let D = all people
$P(x)$ = x is perfect.
$F(x)$ = x is your friend
So, $\forall x \, (F(x) \rightarrow P(x))$ Ans

(iv) At least one of your friend is perfect.
D = all people
$P(x)$ = x is perfect
$F(x)$ = x is your friend
At least one = $\exists x$
So, $\exists x \, (F(x) \wedge P(x))$ Ans

(v) Everyone is perfect and your friend.
Domain D, $P(x)$ & $F(x)$ are same as (iv).
So, $\forall x \, (F(x) \wedge P(x))$ Ans

(vi) Not everybody is your friend or some one is not perfect.
$\neg \forall x F(x)$ = Not everybody is your friend
$\exists x \neg P(x)$ = someone is not perfect, So, $(\neg \forall x F(x)) \vee (\exists x \neg P(x))$
Ans

Ques Translate each of these statements into logical expressions in three different ways by varying the domain and by using predicates with one and with two variables.

(i) Someone in your school has visited Germany.

I Domain = people of your school

$G(x)$ = x has visited Germany.

So, $\exists x\, G(x)$ Ans

II Domain = all people of world.

$S(x)$ = x is in your school.

$G(x)$ = x has visited Germany.

So, $\exists x\, (S(x) \wedge G(x))$ Ans

III Domain = all people.

$S(x)$ = x is in your school.

$P(x, y)$ = x has visited y.

So, $\exists x\, (S(x) \wedge P(x, \text{Germany}))$

(ii) Everyone in your class has studied calculus and C++.

I D = people of your class.

$C(x)$ = x has studied calculus.

$T(x)$ = x has studied C++

So, $\forall x\, (C(x) \wedge T(x))$ Ans

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II

Domain = all people of world.

$A(x)$ = x is in your class

$C(x)$ = x has studied Calculus.

$T(x)$ = x has studied C++.

So, $\forall x \, (A(x) \rightarrow (C(x) \wedge T(x)))$ Ans

III

Domain = all people.

$A(x)$ = x is in your class.

$P(x, y)$ = x has studied y

So, $\forall x \, (A(x) \rightarrow (P(x, \text{Calculus}) \wedge P(x, \text{C++})))$

Ans

Exercise 1.6

Ques Find the argument form for the following argument and determine whether it is valid. Can we conclude that the conclusion is true if the premises are true.

(i) If Socrates is human, then Socrates is mortal.

Socrates is human

$\therefore$ Socrates is mortal.

Soln: Let p = Socrates is human

q = Socrates is mortal.

Then form of the argument is

$$\begin{array}{l} p \to q \\ p \\ \hline \therefore q \end{array}$$

This argument is a Modus Ponens.

$\Rightarrow$ This is valid.

& By Modus Ponens, we know that if $p \to q$ and p are true then q is True.

$\Rightarrow$ Conclusion q is True.

(ii) If George does not have eight legs, then he is not a spider.
George is a spider

$\therefore$ George has eight legs

Let p = George have eight legs
q = He is a spider

Then form of argument is

$$\begin{array}{l} -p \rightarrow -q \\ q \\ \hline \therefore p \end{array}$$

we can rewrite

$$\begin{array}{l} -p \rightarrow -q \\ -(-q) \\ \hline -(-p) \end{array}$$

This argument is Modus tollens.

$\Rightarrow$ This is valid.

& By Modus tollens, we know that if premises are true, then conclusion must be true.

$\Rightarrow$ $-(-p) = p$ is true.

Ques What rule of inference is used in each of these arguments?

(i) Alice is a mathematics major. Therefore, Alice is either mathematics major or a computer science major.

Solⁿ: Let p = Alice is a maths major
q = Alice is C.S. major.

So, form of Argument is

$$\frac{p}{\therefore p \vee q}$$

This is addition rule.

(ii) Jerry is a mathematics major and a c.s. major. Therefore, Jerry is a mathematics major.

Solⁿ: Let p = Jerry is a mathematics major
q = Jerry is a C.S. major

Then form of a argument $\frac{p \wedge q}{\therefore p}$

This is simplification rule.

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(iii) If it is raining, then pool will be closed. It is raining. Therefore, the pool is closed.

Soln : Let p = It is raining
q = pool is closed

Then form of argument is

$$\begin{array}{l} p \to q \\ p \\ \hline \therefore q. \end{array}$$

This argument is Modus Ponens.

(iv) If I go swimming, then I will stay in the sun too long. If I stay in the sun too long, then I will sunburn. Therefore, if I go swimming, then I will sunburn.

Soln Let p = I go swimming
q = I will stay in the sun too long.
r = I will sunburn.

Then form of argument is

$$\begin{array}{l} p \to q \\ q \to r \\ \hline \therefore p \to r. \end{array}$$

This argument is Hypothetical syllogism.

Ques 5 Use rule of inference to show that the hypotheses "Randy work hard", "If Randy work hard, then he is a dull boy," and "If Randy is a dull boy, then he will not get the job" imply the conclusion "Randy will not get the job".

Soln Let p = Randy work hard

q = Randy is a dull boy

r = Randy will ~~not~~ get the job.

So form of the argument

① p Premise

$p \to q$ Premise

$q \to \neg r$ Premise

$\therefore \neg r$

① p premise

② $p \to q$ premise

③ q by ① & ② with modus Ponens

④ $q \to \neg r$ Premise

⑤ $\neg r$ By ③ & ④ with modus Ponens

This the conclusion.

i.e. $\neg r$ = Randy will not get the job.

Ques: What rules of inference are used in this famous argument? All men are mortal. Socrates is a man. Therefore, Socrates is mortal.

Soln: Let $M(x)$ = x is man
$N(x)$ = x is mortal.

Then we have

1. $\forall x\,(M(x) \rightarrow N(x))$ Premise
2. $M(\text{Socrates})$ Premise.

1. $\forall x\,(M(x) \rightarrow N(x))$ Premise
2. $M(\text{Socrates}) \rightarrow N(\text{Socrates})$ Universal instantiation of 1
3. $M(\text{Socrates})$ Premise
4. $N(\text{Socrates})$ Modus Ponens of 2 & 3

or

1. $\forall x\,(M(x) \rightarrow N(x))$ Premise
2. $M(\text{Socrates})$ Premise

$\therefore N(\text{Socrates})$ By universal Modus Ponens.

Ques For each of these collections of premises, what relevant conclusion or conclusions can be drawn? Explain the rules of inference used to obtain each conclusion from the premises.

(i) "If I take the day off, it either rains or snows." I took Tuesday off or I took Thursday off. It was sunny on Tuesday. It did not snow or Thursday.

Sol^n:

Let $P(x)$ = I take the day x off.
$Q(x)$ = It rains on day x.
$R(x)$ = It snow on day x.

So, it is sunny $\Rightarrow$ it does not rain or snow
$\Rightarrow -(Q(x) \vee R(x))$

So, Premises are
① $\forall x \, (P(x) \rightarrow (Q(x) \vee R(x)))$
② $P(\text{Tuesday}) \vee P(\text{Thursday})$
③ $-(Q(\text{Tuesday}) \vee R(\text{Tuesday}))$
④ $-R(\text{Thursday})$

So, Premise ① $\forall x \, (P(x) \rightarrow (Q(x) \vee R(x)))$
② $P(\text{Tuesday}) \rightarrow Q(\text{Tuesday}) \vee R(\text{Tuesday})$
By Universal instantiation from ①
③ $P(\text{Thursday}) \rightarrow Q(\text{Thursday}) \vee R(\text{Thursday})$
Premise ④ $P(\text{Tuesday}) \vee P(\text{Thursday})$
Premise ⑤ $-(Q(\text{Tuesday}) \vee R(\text{Tuesday}))$
⑥ $-P(\text{Tuesday})$ By Modus tallens from ② & ⑤
⑦ $P(\text{Thursday})$ By Disjunctive syllogism from ④ & ⑥
⑧ $Q(\text{Thursday}) \vee R(\text{Thursday})$ By Modus Ponens from ③ & ⑦

Premise (9) $-R(\text{Thursday})$

(10) $Q(\text{Thursday})$

By Disjunctive syllogism with (8) and (9)

So, conclusions are

$-P(\text{Tuesday})$, $P(\text{Thursday})$ and $Q(\text{Thursday})$.

i.e. I did not take Tuesday off.

I took Thursday off.

It rain on Thursday.

Ans

(ii) If I eat spicy foods, then I have strange dreams", "I have strange dreams if there is thunder while I sleep", "I did not have strange dreams.

Let p = I eat spicy food.

q = I have strange dream

r = There is a thunder while I sleep.

So, premises are (1) $p \to q$

(2) $r \to q$

(3) $-q$

(1) $p \to q$

(2) $r \to q$

(3) $-q$

(4) $-p$ by Modus Tollens with (1) & (3)

(5) $-r$ by Modus Tollens with (2) & (3)

Therefore conclusions are $\neg p$ & $\neg r$.

i.e. I did not eat spicy food.

There is no thunder while I sleep.

(iii) I am either clever or lucky. I am not lucky. If I am lucky, then I will win the lottery.

Let p = I am clever

q = I am lucky

r = I will win the lottery

So, Premises are ① $p \vee q$
② $\neg q$
③ $q \to r$

① $p \vee q$
② $\neg q$
③ $q \to r$
④ p Disjunctive syllogism from ① & ②

Therefore conclusion is p.

⇒ I am clever.

(iv) Every computer science major has a personal computer. Ralph does not have a personal computer. Ann has a personal computer.

Let $P(x)$ = x is computer science major

$Q(x)$ = x has a personal computer

So premises are

$$\forall x\ (P(x) \to Q(x))$$
$$\neg Q(\text{Ralph})$$
$$Q(\text{Ann})$$

① $\forall x\ (P(x) \to Q(x))$
② $\neg Q(\text{Ralph})$
③ $Q(\text{Ann})$

$\neg P(\text{Ralph})$ By universal Modus tollens from ① & ②

or

premise ① $\forall x\ (P(x) \to Q(x))$
② $P(\text{Ralph}) \to Q(\text{Ralph})$ By universal instantiation
premise from ①
③ $\neg Q(\text{Ralph})$
④ $Q(\text{Ann})$

$\neg P(\text{Ralph})$ From ② & ③ with Modus tollens

⇒ So Conclusion is $\neg P(\text{Ralph})$
⇒ Ralph is not a computer science major.

(v) What is good for corporations is good for the United states. What is good for the United states is good for you. What is good for corporations is for you to buy lots of stuff.

Let p = good for corporation
q = good for United states
r = good for you
s = buy lots of stuff

Scanned with CamScanner

So premises are $p \to q$
$q \to r$
$s \to p$

Premise ① $p \to q$
Premise ② $q \to r$
③ $p \to r$ Hypothetical syllogism from ① & ②
Premise ④ $s \to p$
⑤ $s \to q$ Hypothetical syllogism from ① & ④
⑥ $s \to r$ Hypothetical syllogism from ③ & ④

So, conclusion are $p \to r$, $s \to q$ & $s \to r$.

i.e. What is good for corporations is good for you.
you buying lots of stuff is good for the U.S.
you buying lots of stuff is good for you.

(vi) All rodents gnaw their food. Mice are rodents. Rabbits do not gnaw their food. Bats are not rodents.

Soln Let $P(x) = x$ is rodent
$Q(x) = x$ gnaws their food.

So, Premises are $\forall x \left(P(x) \to Q(x) \right)$
$P(\text{Mice})$
$\neg Q(\text{Rabbit})$
$\neg P(\text{Bats})$

Premise ① $\forall x\,(P(x) \to Q(x))$

Premise ② $P(\text{Mice})$

③ $Q(\text{Mice})$ by universal modus Ponens from ① & ②

Premise ④ $\neg Q(\text{Rabbit})$

⑤ $\neg P(\text{Rabbit})$ by universal modus Tollens from ① & ④

⑥ $\neg P(\text{Bats})$

So conclusions are $+Q(\text{Mice})$ & $\neg P(\text{Rabbit})$.

i.e Mice gnaw their food

Rabbits are not rodents.

Ques Show that the argument form with premises $p_1, p_2, \ldots p_n$ and conclusion $q \to r$ is valid if the argument form with premises $p_1, p_2, \ldots p_n, q$ and conclusion r is valid.

Proof The argumen

$$\begin{array}{l} p_1 \\ p_2 \\ \vdots \\ p_n \\ q \\ \hline \therefore r \end{array}$$

is valid.

$\Rightarrow$ $p_1, p_2, \ldots, p_n, q,$ premises are valid and

Conclusion ~~conclusion~~ r is valid.

we have to prove

$$\begin{array}{l} p_1 \\ p_2 \\ \vdots \\ p_n \\ \hline \therefore q \to r \end{array}$$

valid.

$p_1, p_2, \ldots p_n$ are premises.

I If we take q is false, the $q \to r$ is true.

Thus we can say

$$\begin{array}{l} p_1 \\ p_2 \\ \vdots \\ p_n \\ \hline q \to r \end{array}$$

is valid.

II If we take q is true.

It is given, $p_1, p_2, \ldots p_n, q$ are true,

then conclusion is true.

$\Rightarrow$ $q \to r$ is True.

$\Rightarrow$

$$\begin{array}{l} p_1 \\ p_2 \\ \vdots \\ p_n \\ \hline q \to r \end{array}$$

is valid.

Ques Show that the argument form with premises $(p \wedge t) \to (r \vee s)$, $q \to (u \wedge t)$, $u \to p$ and $\neg s$ and conclusion $q \to r$ is valid.

Soln : By previous question, we know that

$$\begin{array}{l} p_1 \\ p_2 \\ \vdots \\ p_n \\ \hline \therefore q \to r \end{array} \quad \text{is valid if} \quad \begin{array}{l} p_1 \\ p_2 \\ \vdots \\ p_n \\ q \\ \hline \therefore r \end{array} \quad \text{is valid.}$$

By using this result and modus ponens, simplification, conjunction, disjunctive syllogism we can prove (solve) the problem.

To prove the result, we need

$$\begin{array}{l} p \wedge q \to r \vee s \\ q \to r \wedge t \\ u \to p \\ -s \\ q \\ \hline \therefore r \end{array}$$

i.e. one premise is.

premise	1	$p \wedge t \to (r \vee s)$	
premise	2	$q \to u \wedge t$	
premise	3	$u \to p$	
premise	4	$-s$	
premise	5	q	
	6	$u \wedge t$	By 2 & 5 using modus ponens
	7	u	By 6 with simplification.
	8	p	By 3 & 7 with modus ponens
	9	t	simplification by 6
	10	$p \wedge t$	Conjunction from 8 & 9

(11) $r \vee s$ By (1) & (10) with modus ponens

(12) r By (11) & (4) using disjunctive syllogism.

$\Rightarrow$ r is true i.e. (valid).

Thus,

$$\begin{array}{l} p \wedge t \rightarrow r \vee s \\ q \rightarrow u \wedge t \\ u \rightarrow p \\ \neg s \\ q \\ \hline \therefore r \end{array}$$

So, by Previous Problem, we get

$$\begin{array}{l} p \wedge t \rightarrow r \vee s \\ q \rightarrow u \wedge t \\ u \rightarrow p \\ \neg s \\ q \\ \hline \therefore q \rightarrow r \end{array}$$

$\Rightarrow$ $q \rightarrow r$ is valid.

Ques Determine whether each of these arguments is valid. If an argument is correct, what rule of inference is being used? If it is not, what logical error occurs?

(i) If n is real number such that $n > 1$, then $n^2 > 1$. Suppose $n^2 > 1$. Then $n > 1$.

Sol^n Let $p = n > 1$, $q = n^2 > 1$

Then form of argument

$$\begin{array}{l} p \rightarrow q \\ q \\ \hline \therefore p \end{array}$$

Invalid

Fallacy case of Modus Ponens.

(ii) If n is a real number with $n>3$, then $n^2>9$. Suppose that $n^2 \leq 9$. Then $n \leq 3$.

Let $p = n>3$, $q = n^2>9$

Then form of argument

$$\begin{array}{l} p \to q \\ \neg q \\ \hline \neg p \end{array}$$

$\because q = n^2>9$
$\therefore \neg q = n^2 \leq 9$
$p = n>3$
$\neg p = n \leq 3$

This is Modus Tollens. <u>Valid</u>

(iii) If n is a real number with $n>2$, then $n^2>4$. Suppose $n \leq 2$. Then $n^2 \leq 4$.

Let $p = n>2$, $q = n^2>4$

then $\neg p = n \leq 2$ & $\neg q = n^2 \leq 4$

So form of argument

$$\begin{array}{l} p \to q \\ \neg p \\ \hline \neg q \end{array}$$

<u>Invalid</u>

This is Fallacy.

<u>Ques</u>: Determine whether these are valid argument

(i) If x is a +ve real number, then x^2 is a +ve real number. Therefore, if a^2 is positive, where a is a real number, then a is a +ve real number.

Sol^n Let $p(x)$ = x is a +ve real number
$q(x)$ = x^2 is a +ve real number

So form of argument

(1) Premise
$$\frac{\forall x\,(p(x) \rightarrow q(x))}{\therefore\ q(a) \rightarrow p(a)}$$ Invalid

By (1) we can conclude $p(a) \rightarrow q(a)$. So it is not $q(a) \rightarrow p(a)$. So it is invalid.

(ii) If $x^2 \neq 0$, where x is a real number, then $x \neq 0$. Let a be a real number with $a^2 \neq 0$, then $a \neq 0$

Let $p(x) = x^2 \neq 0$
$q(x) = x \neq 0$

Then form of argument

(1) Premise
$$\frac{\forall x\,(p(x) \rightarrow q(x))}{\therefore\ p(a) \rightarrow q(a)}$$ By universal instantuation

Ques Identify the error or errors in this argument that suposedly show that if $\exists x P(x) \wedge \exists x Q(x)$ is true then $\exists x (P(x) \wedge Q(x))$ is true.

1. $\exists x\, P(x) \wedge \exists x\, Q(x)$ Premise
2. $\exists x\, P(x)$ Simplification from 1
3. $P(c)$ Existential instantiation from 2
4. $\exists x\, Q(x)$ Simplification from 1
5. $Q(c)$ Existential instantiation from 4
6. $P(c) \wedge Q(c)$ conjunction from 3 & 5
7. $\exists x\, (P(x) \wedge Q(x))$ Existential generalization

Error occurs in 5.
It is not necessary, for same c the statement $P(x)$ & $Q(x)$ will be true
It may be different.

<u>Ques</u> Identify the error or errors in this argument that supposedly show that if $\forall x\, (P(x) \vee Q(x))$ is true then $\forall x\, P(x) \vee \forall x\, Q(x)$ is true.

Soln: 1. $\forall x (P(x) \vee Q(x))$ Premise
2. $P(c) \vee Q(c)$ Universal instantiation from 1
3. $P(c)$ Simplification from 2
4. $\forall x\, P(x)$ Universal generalization from 3
5. $Q(c)$ Simplification from 2
6. $\forall x\, Q(x)$ Universal generalization from 5
7. $\forall x\, (P(x)) \vee \forall x (Q(x))$ conjunction from 4 and 6

Error occur in 3 & 5
we can not use simplification.
Because for simplification $\wedge$ not $\vee$.

Ques For each of these arguments, explain which rules of inference are used for each step.

Soln: (i) Doug, a student in this class, Knows how to write programs in Java. Everyone who know how to write programs in Java can get a high-paying job. Therefore, someone in this class can get a high-paying job.

The form of argument

Let $P(x)$ = x knows how to write programs in Java.

$Q(x)$ = x can get a high-paying job.

Premise $P(\text{Doug})$

Premise $\forall x\,(P(x) \rightarrow Q(x))$

So ① $P(\text{Doug})$ Premise

② $\forall x\,(P(x) \rightarrow Q(x))$ Premise

③ $Q(\text{Doug})$ By universal modus Ponens with ① & ②

Conclusion ④ $\exists x\, Q(x)$ Existential generalization from ③.

(ii) Somebody in this class enjoys whale watching. Every person who enjoys whale watching cares about ocean pollution. Therefore, there is a person in this class who cares about ocean pollution.

Let $P(x)$ = x enjoys whale watching,
$Q(x)$ = x cares about ocean pollution.

So, form of argument

① $\exists x\, P(x)$ Premise

② $\forall x\, (P(x) \rightarrow Q(x))$ Premise

$\therefore \exists x\, Q(x)$

① $\exists x\, P(x)$ Premise

② $\forall x\, (P(x) \rightarrow Q(x))$ Premise

③ $P(c)$ Existential ~~generalization~~ instantiation from ①

④ $P(c) \rightarrow Q(c)$ Universal instantiation from ②

⑤ $Q(c)$ Modus Ponens from ③ & ④

⑥ $\exists x\, Q(x)$ Existential generalization from ⑤

$\exists x\, Q(x)$ = There is a person in this class who cares about ocean pollution.

Ques For each of these arguments determine whether the argument is correct or incorrect and explain why:

(i) All student in this class understand logic. Xavier is a student in this class. Therefore, Xavier understands logic.

Let $P(x)$ = x is student in this class.

$Q(x)$ = x understands logic.

So form of argument.

① $\forall x\,(P(x) \rightarrow Q(x))$ Premise

② $P(\text{Xavier})$ Premise

$\therefore\ Q(\text{Xavier})$

① $\forall x\,(P(x) \rightarrow Q(x))$

② $P(\text{Xavier})$

③ $Q(\text{Xavier})$ By universal modus Ponens with ① & ②

$\Rightarrow$ This argument is correct.

(iii) Every computer science major takes DMS. Natasha is taking DMS. Therefore, Natasha is computer science major.

Let $P(x)$ = x is computer science major

$Q(x)$ = x takes DMS.

So form of argument

① $\forall x\,(P(x) \rightarrow Q(x))$ Premise

② $Q(\text{Natasha})$ Premise

$\therefore\ P(\text{Natasha})$

There is no rule to calculate $P(\text{Natasha})$. This argument is not correct.

(iii) All parrots like fruit. My pet bird is not a parrot. Therefore, my pet bird does not like fruit.

Let $P(x)$ = x is parrot

$Q(x)$ = x likes fruit.

So form of Argument

① $\forall x\,(P(x) \rightarrow Q(x))$

② $-P(\text{Pet bird})$

$\therefore\ -Q(\text{Pet bird})$

Premise ① $\forall x\,(P(x) \rightarrow Q(x))$

② $P(\text{Pet}) \rightarrow Q(\text{Pet})$ Universal instantiation from ①

Premise ③ $-P(\text{Pet})$

But there is no ~~xxx~~ rule to calculate the conclusion $-Q(\text{Pet})$.

$\Rightarrow$ This is incorrect argument.

~~Que~~ (iv) Everyone who eats granola every day is healthy. Linda is not healthy. Therefore, Linda does not eat granola every day.

Let $P(x)$ = x eat granola every day

$Q(x)$ = x is healthy.

So form of argument

(1) $\forall x \, (P(x) \rightarrow Q(x))$ Premise

(2) $\neg Q(\text{Linda})$ Premise

$\therefore \neg P(\text{Linda})$

(1) $\forall x \, (P(x) \rightarrow Q(x))$

(2) $\neg Q(\text{Linda})$

(3) $\therefore \neg P(\text{Linda})$ By universal modus tollens with (1) & (2).

$\Rightarrow$ It is correct argument.

or

Premise (1) $\forall x \, (P(x) \rightarrow Q(x))$

(2) $P(\text{Linda}) \rightarrow Q(\text{Linda})$ Universal Instantiation from (1)

Premise (3) $\neg Q(\text{Linda})$

(4) $\neg P(\text{Linda})$ Modus tollens from (2) & (3)