SAMSUNG CHIP DESIGN FOR HIGH SCHOOL

WEEK 5

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Advanced Simplification and Optimization in Boolean Algebra

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#1. Simplification vs. Short Notation in Boolean Logic

1.1 Simplification:

- **Definition**: Simplification reduces a Boolean expression to its most basic form without altering its functionality.
- **Purpose**: To reduce the number of gates and inputs in a circuit, enhancing efficiency and cost-effectiveness.
- **Example**:
  Original Expression: `Y = A’B + AB + AB’`
  Simplified Expression: `Y = A + B`

1.2 Short Notation:
- **Definition**: A compact way to represent Boolean expressions, using minterms, maxterms, or canonical forms.
- **Application**: Useful for Karnaugh Map (KMap) plotting and Boolean algebra calculations.
- **Example**:
  Expression: `Y = Σm(1, 3, 5, 7)` (Minterm representation)

#2. KMap Revision

2.1 What is a KMap?
- A Karnaugh Map (KMap) is a graphical tool used to simplify Boolean expressions.

- It organizes truth table outputs into a visual grid, where adjacent cells differ by one variable.

2.2 Basics of KMap Simplification:
1. Plot `1s` for minterms of the given expression.
2. Group adjacent `1s` into powers of 2 (1, 2, 4, 8, etc.).
3. Derive the simplified Boolean expression from grouped terms.

Example:
- Function: `Y = A'B + AB`
- KMap: Groups `1s` for rows corresponding to `A'B` and `AB`.
- Simplified Expression: `Y = B`

#3. Optimization Beyond KMap

While KMaps are effective, they have limitations, particularly for complex circuits with more than 4-6 variables. Advanced methods include:

3.1 Quine-McCluskey Method:
- Tabular approach for minimizing Boolean expressions.
- Suitable for systematic computation with a large number of variables.

3.2 Heuristic Methods:
- Genetic algorithms or simulated annealing to find near-optimal solutions.
- Used in complex circuit design for cost and delay optimization.

#4. Introduction to Four-Variable KMap

4.1 Structure:
- A 4-variable KMap has 16 cells, each representing a unique combination of four

variables (A, B, C, D).
- Rows and columns are labeled to ensure adjacency conditions.

4.2 Grouping in Four-Variable KMap:
1. Groups of `1s` must be powers of 2.
2. Larger groups yield simpler expressions.

Example:
- Function: `Y = A′B′C + AB′C′`
- KMap:

AB/CD	00	01	11	10
00	1	0	0	0
01	0	1	0	0
11	0	0	0	0
10	0	0	0	0

Simplified Expression:
`Y = C(A + B′)`

#5. Four-Variable KMap Conclusion with Example

5.1 Key Takeaways:
- KMaps simplify Boolean expressions by visualizing adjacency.
- Four-variable KMaps expand capabilities but require systematic grouping.

Example:
- Function: `Y = Σm(0, 2, 5, 7, 8, 10, 13, 15)`
- Simplification:
  1. Plot minterms on the KMap.

2. Group adjacent `1s`.
3. Derive simplified expression: `Y = AC + B’D`
