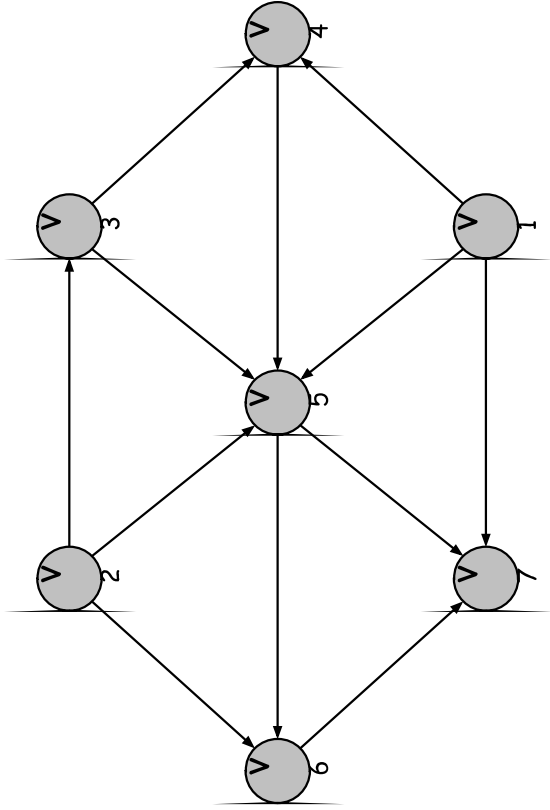


# Topological Sort

Topological Sort is an ordering of vertices in a Directed Acyclic Graph (DAG), such that if there is a path from  $V_i$  to  $V_j$  then  $V_j$  appears after  $V_i$  in the ordering

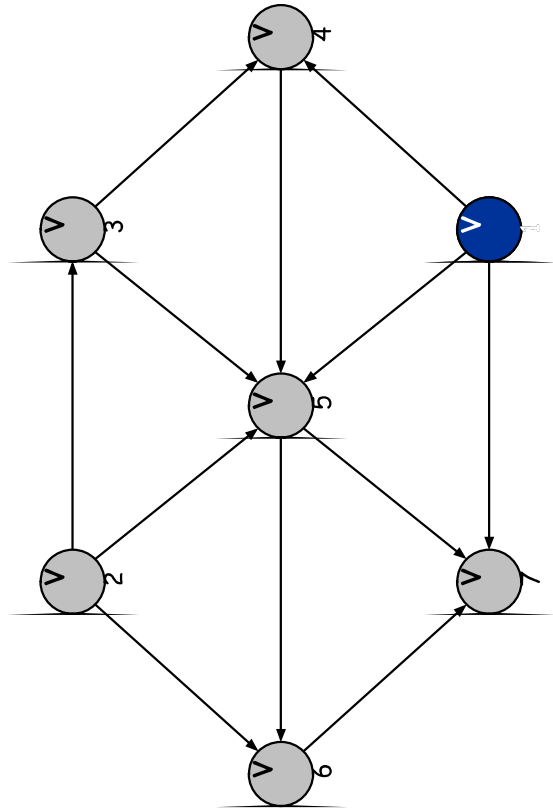
Topological Ordering Algorithm: Example



Vertex	Indegree
$V_1$	0
$V_2$	0
$V_3$	1
$V_4$	2
$V_5$	4
$V_6$	2
$V_7$	3

Enqueue  $V_1, V_2$   
QUEUE:  $V_1, V_2$   
Topological order:

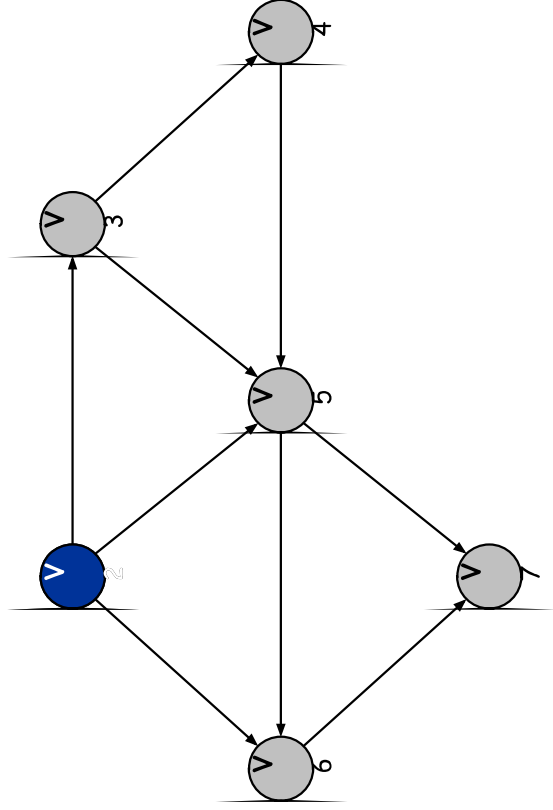
Topological Ordering Algorithm: Example



Vertex	Indegree
$V_1$	0
$V_2$	0
$V_3$	1
$V_4$	2
$V_5$	4
$V_6$	2
$V_7$	3

Enqueue  $V_1, V_2$   
QUEUE:  $V_1, V_2$   
Topological order:

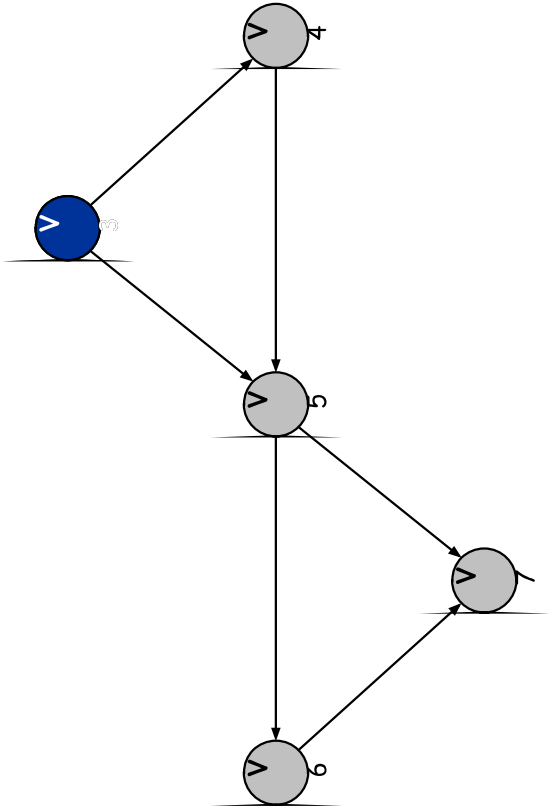
# Topological Ordering Algorithm: Example



Vertex	Indegree
$V_1$	-
$V_2$	0
$V_3$	1
$V_4$	1
$V_5$	3
$V_6$	2
$V_7$	2

Deque  $V_1$   
Topological order:  $v_1$   
Enque -  
QUEUE :  $V_2$

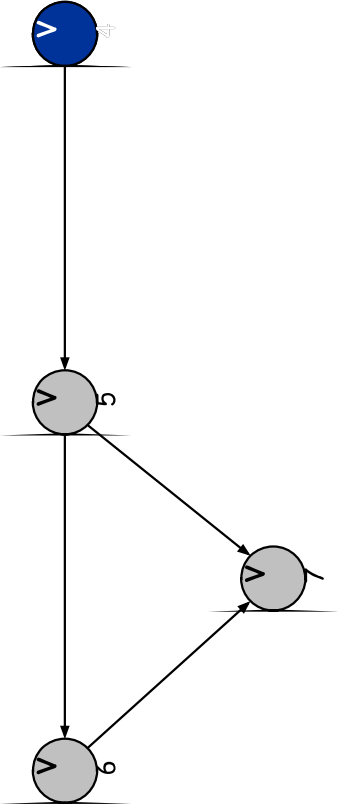
Topological Ordering Algorithm: Example



Vertex	Indegree
$V_1$	-
$V_2$	-
$V_3$	0
$V_4$	1
$V_5$	2
$V_6$	1
$V_7$	2

Deque  $V_2$   
Topological order:  $V_1 V_2$   
Enque  $V_3$   
QUEUE :  $V_3$

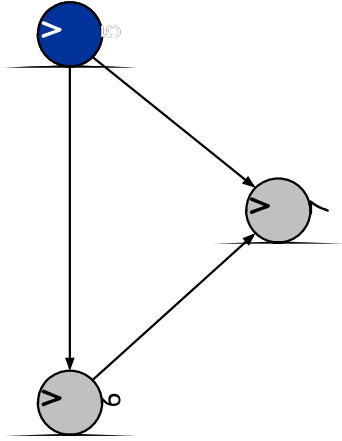
Topological Ordering Algorithm: Example



<i>Vertex</i>	<i>Indegree</i>
$V_1$	-
$V_2$	-
$V_3$	-
$V_4$	0
$V_5$	1
$V_6$	1
$V_7$	2

Deque  $V_3$   
Topological order:  $V_1 V_2 V_3$   
Enque  $V_4$   
QUEUE :  $V_4$

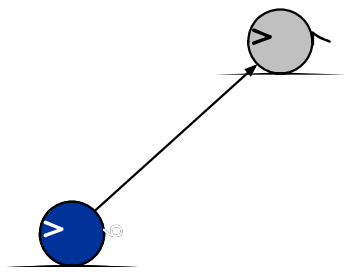
# Topological Ordering Algorithm: Example



Deque  $V_4$   
Topological order:  $V_1 V_2 V_3 V_4$   
Enque  $V_5$   
QUEUE :  $V_5$

<i>Vertex</i>	<i>Indegree</i>
$V_1$	-
$V_2$	-
$V_3$	-
$V_4$	-
$V_5$	0
$V_6$	1
$V_7$	2

# Topological Ordering Algorithm: Example

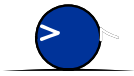


Deque  $V_5$   
Topological order:  $V_1 V_2 V_3 V_4 V_5$   
Enque  $V_6$   
QUEUE :  $V_6$

Vertex	Indegree
$V_1$	-
$V_2$	-
$V_3$	-
$V_4$	-
$V_5$	-
$V_6$	0
$V_7$	1



# Topological Ordering Algorithm: Example



<i>Vertex</i>	<i>Indegree</i>
$V_1$	-
$V_2$	-
$V_3$	-
$V_4$	-
$V_5$	-
$V_6$	-
$V_7$	0

Deque  $V_6$   
Topological order:  $V_1 V_2 V_3 V_4 V_5 V_6$   
Enque  $V_7$   
QUEUE :  $V_7$

## Topological Ordering Algorithm: Example

<i>Vertex</i>	<i>Indegree</i>
$V_1$	-
$V_2$	-
$V_3$	-
$V_4$	-
$V_5$	-
$V_6$	-
$V_7$	-

Deque  $V_7$   
Topological order:  $V_1 V_2 V_3 V_4 V_5 V_6 V_7$

# Topological Sort Algorithm

Algorithm to find a Topological Sort T of a Directed Acyclic Graph, G

Step 1: Find the indegree  $\text{INDEG}(N)$  of every node in the graph  
Step 2: Enqueue all the nodes with a zero in-degree  
Step 3: Repeat Steps 4 and 5 until the QUEUE is empty  
Step 4: Remove the front node N of the QUEUE by setting  $\text{FRONT} = \text{FRONT} + 1$   
Step 5: Repeat for each neighbor M of node N:  
    a) Delete the edge from N to M by setting  $\text{INDEG}(M) = \text{INDEG}(M) - 1$   
    b) IF  $\text{INDEG}(M) = 0$ , then Enqueue M, that is, add M to the rear of the queue  
        [END OF INNER LOOP]  
    [END OF LOOP]  
Step 6: Exit