ECE250: Signals and Systems
Assignment 3

Issued on: October 19, 2023 | **Max-Marks : 55** | Due by: October 27,2023 (6:00 pm)

Guidelines for submission

Theory Problems:

- Submit a hard copy of your solutions in the wooden box kept on the 3rd Floor of Old Academic Block (right side of the lift).
- Write your Name, Roll No. on the hard copy of your solutions.
- Do all questions in sequence.
- Use A4 sheets (Plain). Staple your sheets properly
- Submission Policy: Expect no extensions. Late submissions will not be evaluated and hence will be awarded zero marks strictly.
- **Institute Plagiarism Policy** Applicable. This will be subjected to a strict plagiarism check.

Programming Problems:

- Use Matlab or python to solve the programming problems.
- For your solutions, you need to submit a zipped file on Google classroom with the following:
  - program files (.m) or (.ipynb) with all dependencies.
  - a report (.pdf) with your coding outputs and generated plots. The report should be self-complete with all your assumptions and inferences clearly specified.
- Before submission, please name your zipped file as: "A3_RollNo_Name.zip".
- Codes/reports submitted without a zipped file or without following the naming convention will NOT be checked.

Theory Problems (35 points)

[CO3] **Q1.** Consider a causal continuous time LTI system whose input x(t) and output y(t) are related by the following differential equation:**[2x5 Points]**

$$\frac{dy(t)}{dt} + 4y(t) = x(t) \tag{1}$$

Find the Fourier series representation of the output y(t) for each of the following inputs

(a) $x(t) = \frac{1}{3}cos(4\pi t)$

(b) $x(t) = sin(4\pi t) + cos(6\pi t + \pi/4)$

[CO3] **Q2.** Consider a periodic signal $x(t)$ with fourier series coefficient a_k given below with the condition that the fundamental angular frequency w_0 is equal to π for x(t), then Determine the signal $x(t)$.**[6 Points]**

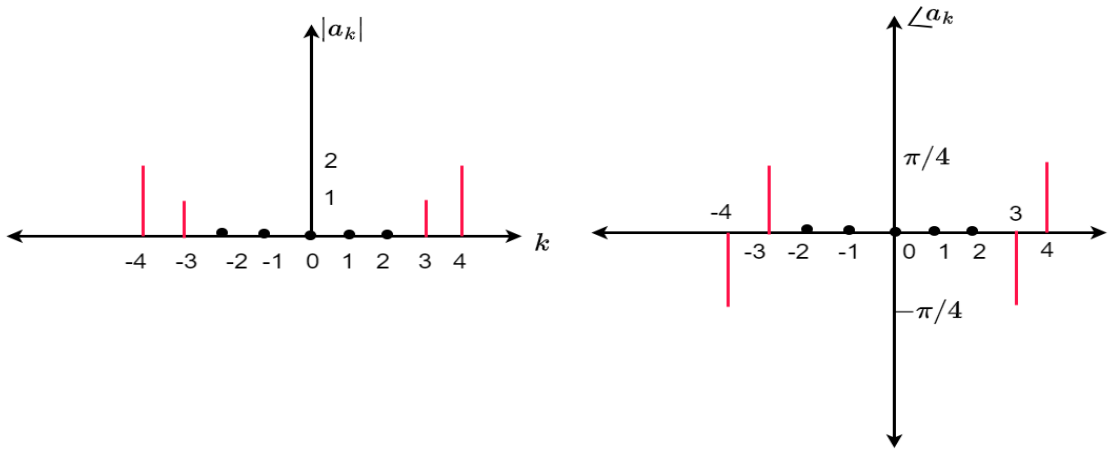

Figure 1: Problem 2

[CO3] **Q3.** Determine the Fourier series coefficients for each of the following discrete-time periodic signa]s. Plot the magnitude and phase of each set of coefficients a_k.**[24 Points]**

(a) Each $x[n]$ depicted in figure 2.**[3x4 Points]**

(b) $x[n] = sin(2\pi n/3)cos(\pi n/2)$**[4 Points]**

(c) $x[n]$ is periodic with period 4 and**[4 Points]**

$$x[n] = 1 - sin(\pi n/4) \quad for \quad 0 \leq n \leq 3 \tag{2}$$

(d) $x[n]$ is periodic with period 12 and **[4 Points]**

$$x[n] = 1 - sin(\pi n/4) \quad for \quad 0 \leq n \leq 11 \tag{3}$$

[CO3] **Q4.** The signal $x(t)$ is defined as follows, with time period $T = 6$:

$$x(t) = \begin{cases} t+2 & \text{for} -2 < t < -1 \\ 1 & \text{for } -1 < t < 1 \\ 2-t & \text{for } 1 < t < 2 \end{cases} \tag{4}$$

(a) Determine the Fourier series representation for the signal x(t). **[4 Points]**
(b) Determine the ratio of power in the 7^{th} harmonic to the power in the 5^{th} harmonic. **[1 Points]**

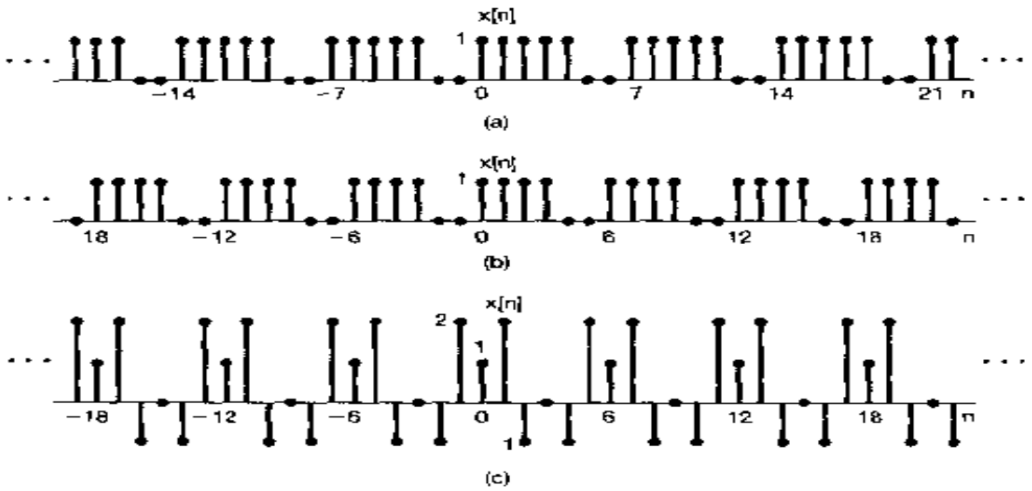

Figure 2: Problem 3

Programming Problems (10 points)

[CO3] **Q1.**

1. Given a discrete-time signal $x[n]$:**[6 Points]**

$$x[n] = \begin{cases} 1 & \text{for } -N_1 \leq n \leq N_1, N_1 = 2 \\ \\ 0 & \text{for } N_1 < |n| \leq \frac{(N-1)}{2} \end{cases} \tag{5}$$

Compute the Fourier series for the following cases and give the inference.

(a) N $=4N_1 + 1$
(b) N $=8N_1 + 1$
(c) N $=10N_1 + 1$

[CO3] **Q2.** A periodic function is defined by:**[4 Points]**

$f(x) = x + \pi, \quad -\pi \leq x < \pi$

$f(x + 2\pi) = f(x)$

(a) Sketch the graph of $f(x)$ for three periods.
(b) Find the Fourier series of $f(x)$ on the interval $-\pi < x < \pi$.

Hint: To simulate continuous signals use appropriate discretization (wherever required).