Nonlinear Tracking of Target Submarine Using Extended Kalman Filter (EKF)

S. Vikranth, P. Sudheesh(✉), and M. Jayakumar

Department of Electronics and Communication Engineering,
Amrita School of Engineering, Coimbatore Amrita Vishwa
Vidyapeetham University, Coimbatore 641112, India
viswak.nawin.ks@icloud.com,
{p_sudheesh,m_jayakumar}@cb.amrita.edu

Abstract. This paper presents the effective method for submarine tracking using EKF. EKF is a Bayesian recursive filter based on the linearization of nonlinearities in the state and the measurement system. Here the sonar system is used to determine the position and velocity of the target submarine which is moving with respect to non moving submarine, and sonar is the most effective methods in finding the completely immersed submarine in deep waters. When the target submarines position and velocity is located from the reflected sonar, an extended Kalman filter is used as smoothening filters that describes the position and velocity of the ship with the noisy measurements given by sonar that is reflected back. By using the algorithm of extended Kalman filter we derived to estimate the position and velocity. Here the target motion is defined in Cartesian coordinates, while the measurements are specified in spherical coordinates with respect to sonar location. When the target submarine is located, the alert signal is sent to the own ship. This can be excessively used in military applications for tracking the state of the target submarine. Prediction of the state of the submarine is possible, with Gaussian noise to the input data. The simulation results show that proposed method is able to track the state estimate of the target, this was validated by plotting SNR vs MSE of state estimates. Here in this algorithm regressive iteration method is used to converge to the actual values from the data received.

Keywords: Extended Kalman Filter (EKF) · Active sonar · Passive sonar · Non-linear filters · Prediction methods

1 Introduction

One of the major applications of sonar system is in marine military applications. Sonar is used to find the target submarines location which includes the position and velocity in which target travels. Modern marine applications are becoming very quite and sophisticated. Sonar is an acronym meaning Sound Navigation and Ranging used to identify the potential threats and also to determine their own position. Types of sonar used in marine applications are of two types. They are Active sonar and Passive sonar. In Active sonar, system emits a pulse and where the operator waits for the echoes [1]. Active sonar uses transmitter and receiver. A pulse of sound is called ping. It is electronically generated

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using sonar. Then the sound wave is recorded and by the difference of time from which the pulse emitted to time when it returns back we can locate the location of the object. In Passive sonar, the submarine listens to the sound around the Submarine instead of transmitting the signal and by comparing the recorded sound to the library sound, it will be able to find the position of the approaching submarine [2]. We can use Active sonar to measure the position and velocity of approaching submarine. It is due to Doppler Effect. When a Vehicle approaching moves away, the pitch of the sound decreases. This happens because the crest of the sound waves moves far away as the source moves. This is called Doppler effect. In a similar concept submarines work, when they emit a known frequency and it returns back with different frequency they can calculate the position and velocity of that object which is intercepted by the frequency [3]. Our primary objective is tracking of target submarine position and velocity.

Here the position and velocity of the submarine can be precisely attained by the Extended Kalman Filter. The extended Kalman filter linearizes about current mean and covariance [16]. Extended Kalman filter is the novel method to estimate the state of the system subjected to noise. Extended Kalman filter linearize the estimation using the partial derivatives of process and measurement functions around the current estimate to compute in non linear conditions [4, 5]. Since it is non linearized version of Kalman filter we can use it for many applications unless like Kalman filter. Firstly, prediction strategy is used where the data required for the filter gets predicted. In the second stage Correction of errors will be made [6]. People can design the filter depending on the application. Due to its capacity to handle non linear dynamic problems, Extended Kalman filter is used widely in various fields. This method is used extensively in missile guidance and location tracking systems applications. Here, the application, the own Ship is assumed to be stationary at one place from which any submarine which enters in to the patrol region covered by the own ship will be detected. In this application, the Active sonar is used. Here the Integrated sonars have been placed all along the length of the submarine. Integrated sonar includes both passive and active sonars. During war time only passive sonar is on because of the advantage of the stealth characteristics, it has over active sonar. Active sonar when used, searcher cannot identify the target without revealing its own position. Here the Extended Kalman Filter is used for a particular range and the results are plotted with their respective errors.

2 Tracking Basics

2.1 Algorithm

The Extended Kalman filter is the Non optimal estimator for the sequence of states produced by the model. It has two parts Time update and measurement update. Before going to the next step we have to initialize the values of position and velocity in a range. First Time update is done which gives the initial values. The general predicted state estimate equation is,

$$x_{k+1} = f(x_k) + w_k \tag{1}$$

Measurement model is,

$$z_k = h(x_k) + v_k \tag{2}$$

Where

x_k - State vector representing a particular model at the time instant k,
$f(x_k)$ - State transition function,
w_k - Random Gaussian noise with mean zero with covariance matrix q_k
$h(x_k)$ - Transformation matrix that maps the state vector parameters to measurement vector domain,
z_k - Measurement vector,
v_k is the random Gaussian noise vector with mean zero and covariance matrix r_k. Here for this application the EKF algorithm approximates (2) by taking the linear terms in the function at the predicted state estimate. In this algorithm the State transition matrix and measurement vector are modelled depending upon the application situation,

The state vector in (1) is made up of three components of position and velocity.

$$x_k = [Xp_k, Yp_k, Zp_k, \dot{X}v_k, \dot{Y}v_k, \dot{Z}v_k]^T \tag{3}$$

$$x_{k+1} = F_k x_k + w_k$$

Where F_k is the state transition matrix. This F_k can be linearized about the particular instant k.

Where Xp_k, Yp_k, Zp_k are the coordinates of the submarine at the time instant k and $\dot{X}v_k, \dot{Y}v_k, \dot{Z}v_k$ are the component of velocity of the submarine at time instant k.

We know state transition matrix can be modelled by taking Jacobian of the dynamic equation,

$$F_k = \left.\frac{\partial f}{\partial x}\right|_{\hat{x}_k} \tag{4}$$

and the state transition matrix can be modelled as,

$$F = \begin{bmatrix} 1 & 0 & 0 & T & 0 & 0 \\ 0 & 1 & 0 & 0 & T & 0 \\ 0 & 0 & 1 & 0 & 0 & T \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{5}$$

Here $T = t - \Delta t$, where dt is the time taken by the consecutive sonar ping to get reflected back to the sonar. In other words, time elapse between measurements.

The consecutive sonar pings can be reflected back to onboard sonar sensor on different time interval depending on the ocean condition.

Measurement equation is given by the matrix

$$h(.) = \begin{bmatrix} \sqrt{(Xp - Xp_0)^2 + (Yp - Yp_0)^2 + (Zp - Zp_0)^2} \\ \tan^{-1}\frac{(Zp - Zp_0)}{(Xp - Xp_0)} \\ \tan^{-1}\frac{(\sqrt{(Xp - Xp_0)^2 + (Yp - Yp_0)^2})}{(Zp - Zp_0)} \end{bmatrix} \tag{6}$$

Here the measurement equation is nonlinear. So it is necessary to take the Jacobian for the h(.)

$$H_k = \left.\frac{\partial h}{\partial X}\right|_{\hat{x}_k} \tag{7}$$

Here the measurement matrix is consisting of Range, Bearing angle and Azimuthal angle. Figure 1 shows the position of the target as in h(.).

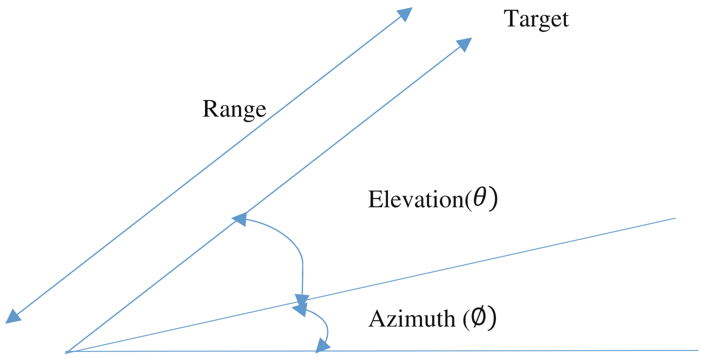

Fig. 1. Location of target submarine.

Range is the area in which the submarine can be found from the own submarine. Bearing angle is the angle in degrees with respect to the true east. Elevation angle gives the altitude of the target submarine from the altitude of the own ship. Here the position and velocity of the target submarine is given with respect to own submarines position were the sonar receiver is placed. That is what is used in the above measurement equation were the Cartesian coordinates are subtracted by values. These x_0, y_0, z_0 denote the location of the receiver in the submarine [7–9]. However, this nonlinear measurement equation can be linearized [10, 11]. Position and velocity of target at time k can be calculated by linearizing it.

$$z_{k+1} - z_k = \Delta(z_k) = H\Delta x_k + v_k \tag{8}$$

From Eq. (8) it is clear that it can be linearized.

In next step project the error covariance ahead,

$$P_{k+1} = F_k P_k F_k^T + q_k$$

Other equations which are used for estimation and are used in measurement update equations are,

(a) Kalman Gain

$$k_k = P_{k+1} H^T (H P_{k+1} H^T + r_k) \tag{9}$$

(b) Update error with estimate

$$x_{k+1}^{up} = x_{k+1} + k_k (z_k - H(x_{k+1})) \tag{10}$$

(c) Update error covariance

$$P_{k+1}^{up} = (I - k_k H) P_{k+1} \tag{11}$$

All these steps are repeated to get the fine tuned values.

Once these values are formulated next step is to use it to the next set of vector received from the sensor.

2.2 Modelling Noise

Any unwanted signal in the desired signal is called as 'noise'. All the practical system has some noise. All the noise from nature is modelled as white noise because it has a constant power spectral density for all frequency bands. Here we also add the Gaussian noise to the already present signal which has noise present in the system. So system can be modelled as real world applications with noise present in it. We know Gaussian distribution has Zero mean and variance which is finite. Given random process Y $\sim N(\mu, \sigma^2)$ where Y is continuous random process with mean μ and variance σ^2. The probability density function of Y is,

$$f(y) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(y-\mu)^2}{2\sigma^2}} \text{ for } -\infty < y < \infty \tag{12}$$

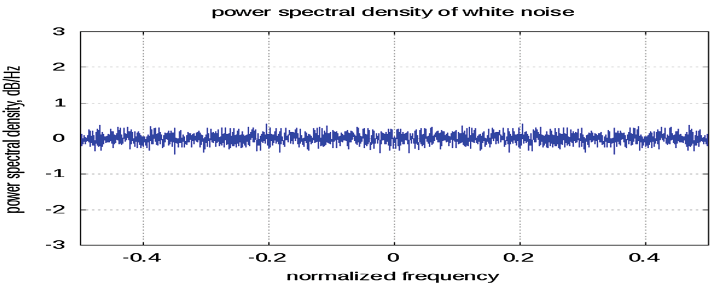

Fig. 2. Power spectral density of white noises

Hence both the process and measurement noise are considered as additive white Gaussian noise [13, 17] (Fig. 2).

Here in the Submarine target location, we have factors like P_k and q_k and r_k. The P (k) is the covariance associated with the state vector. The terms along the main diagonal of the P matrix gives the variance associated with the corresponding terms in state vector.

$$\mathrm{P(k)} = 10^{-5}\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{13}$$

Where noises cannot be modelled completely because it depends on complete randomness. Here the values of measurement noise and process noise are assumed.

Here t1 and t2 are sampling time interval.

The covariance matrix w_k is given by q_k, is given by the main diagonal elements of matrix.

$$\mathrm{q_k} = \begin{bmatrix} t1\sigma_x^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & t1\sigma_y^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & t1\sigma_z^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & t2\sigma_{\dot{x}}^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & t2\sigma_{\dot{y}}^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & t2\sigma_{\dot{z}}^2 \end{bmatrix} \tag{14}$$

$$t1 = \frac{\Delta t^2}{2}, \quad t2 = \Delta t^2$$

The covariance matrix of V is given by,

$$r_k = \begin{bmatrix} \sigma_r^2 & 0 & 0 \\ 0 & \sigma_\theta^2 & 0 \\ 0 & 0 & \sigma_\varphi^2 \end{bmatrix} \tag{15}$$

The values of variances in above matrices can be changed depending upon the application. In the following application the values taken are depending upon the scenario as mentioned. Here Δt is assumed as 1 and the values are incorporated in the algorithm. Δt is the sampling rate. The values of q_k and r_k can be varied to match the real world environment conditions.

2.3 Results and Discussions

In the Graphs Following, we note that the submarine is assumed to be found from the own ship in a particular distance and velocity. From that we can interpret the efficiency of Tracking algorithm by comparing the tracking value to the original value and the error obtained by following method is also proposed. In the case of position, the values are considered to be detected in a particular range. The Velocity is approximated to the normal scenario of submarines in which it usually travels. In Figs. 3, 4, 5, 6, 7, 8 Original values and True value are plotted for both position and velocity which shows that error values are much reduced, in which the maximum value of error was found to be limited to 0.25 km. In case of Velocity also it is reduced to 0.3 km/h.

In Fig. 9 MSE vs Iterations is plotted. It is evident from the graph that the SNR value increases from 5 to 35 in step of 10 MSE vs SNR curve becomes so smooth. This shows that, as SNR increases the error decreases. The value of the error decreases as the iteration goes on increasing [14]. When the noise in the surroundings is less the error is minimized to minimum value. This also shows Fig. 10 that the filter is able to adapt to the surroundings. We are going see about NMSE VS SNR. Normalized mean square error is calculated by the following formula,

$$NMSE = \frac{\sum square\ of\ error}{Average\ of\ estimate \times Average\ of\ True\ Value} \tag{16}$$

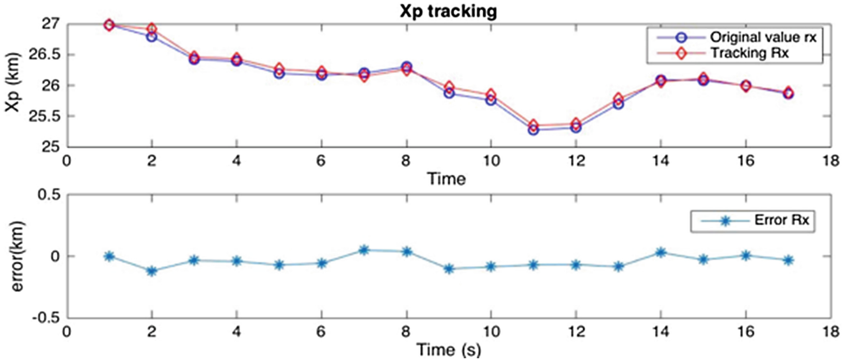

Fig. 3. Position of (x coordinate) of submarine and the corresponding error

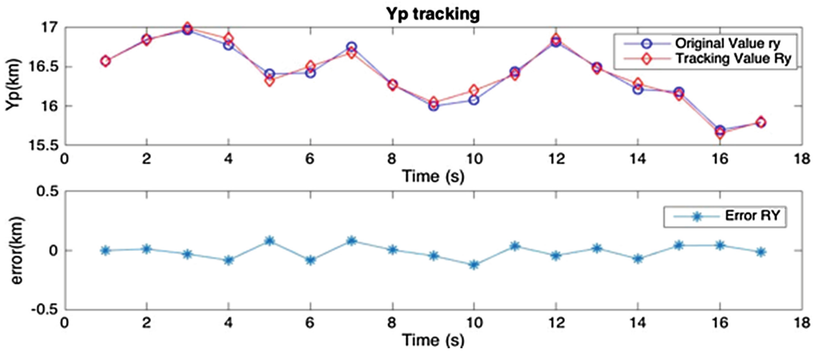

Fig. 4. Position of (y coordinate) of submarine and the corresponding error

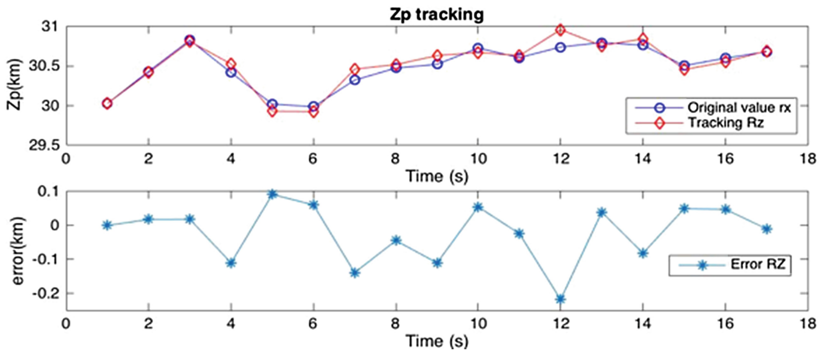

Fig. 5. Position of (z coordinate) of submarine and the corresponding error

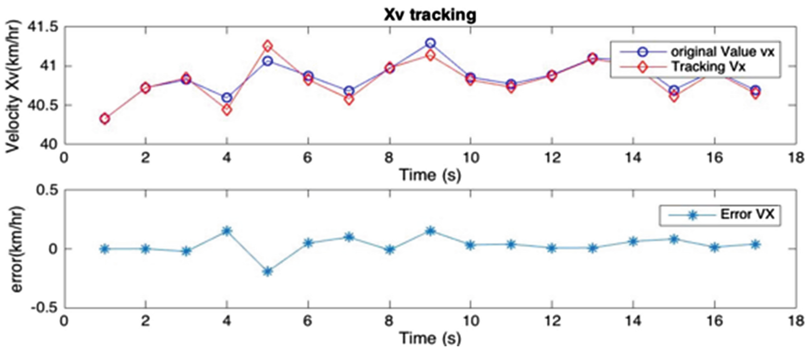

Fig. 6. Velocity of (x coordinate) of submarine and the corresponding error

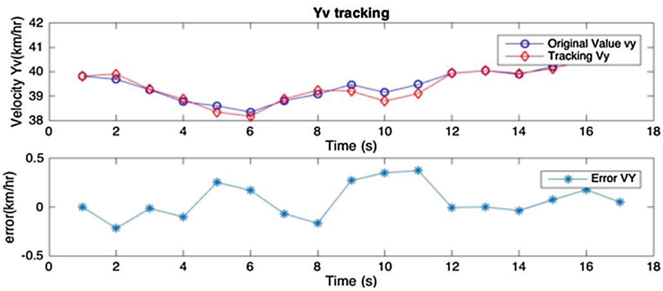

Fig. 7. Velocity of (y coordinate) of submarine and the corresponding error

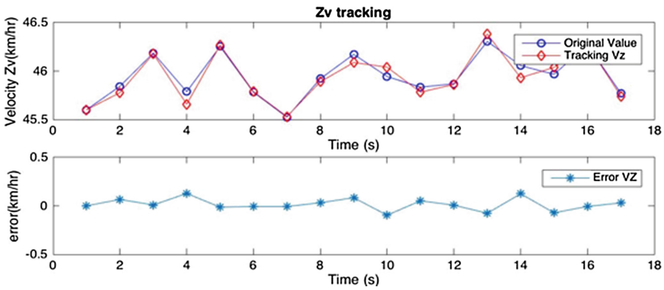

Fig. 8. Velocity of (z coordinate) of submarine and the corresponding error

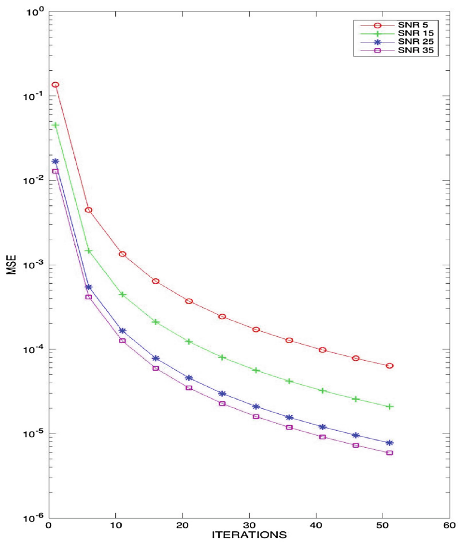

Fig. 9. Plot of mean squared error vs iteration

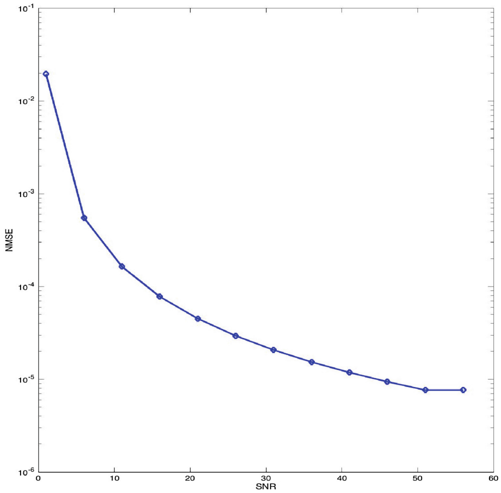

Fig. 10. Plot of NMSE vs SNR for position and velocity (Iterations = 30)

Table 1. Prediction accuracy for different SNR

Case no.	No. of samples	SNR	Accuracy
Case 1	501	5	92.5
Case 2	501	15	95.66
Case 3	501	25	96
Case 4	501	35	97.5
Case 5	501	45	99.35

To verify the extended Kalman filter for the given algorithm, a graph between normalized mean square error and signal to noise ratio over a range of 0 to 60 is plotted. The values in Table 1 which provide necessary details on increase in accuracy as SNR increases.

Table 1 represents the accuracy value increasing over the changes in SNR. Figure 10 shows how precise the extended Kalman filter tracks the position and velocity of the submarine as signal becomes more precise. The continuous drop in the value shows that filter is able to adapt to surroundings.

3 Conclusion

In this paper we presented the idea of finding the position and velocity of submarine using EKF and how to get the knowledge out of the data received from the sonar waves. An extended Kalman filter is successfully applied in determining State estimates in spite of noise present in the system. We also see that in the SNR vs MSE graph the estimated error is in the order of 0.1 m/s and as we increase the SNR the error in the system reduces to the minimum value of 0.00001 m/s which shows that algorithm becomes more regressive as the SNR increases. The important factor which the determination of state estimates position and velocity is meticulously analyzed, verified and plotted. The idea can be extended to the Unscented Kalman Filter (UKF) where the equations are calculated without linearization [15]. It gives more accurate information about the target but it can be achieved with certain tradeoff conditions like increase in the computational complexities.

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