Regularized Optimal Transport with the α-Rényi divergence

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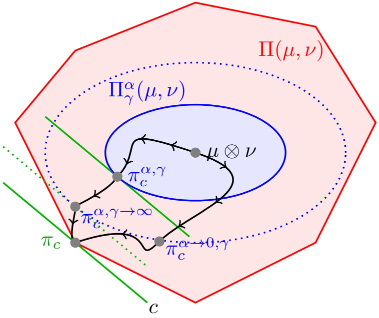
Optimal Transport Workshop - Berlin
TU Berlin, Institut für Mathematik
March 13th, 2024

Rényi regularized Optimal Transport

– Solving the strictly convex problem for $\alpha \in (0,1)$

$$\mathrm{OT}_{\frac{1}{\lambda},\alpha}(\mu,\nu) \coloneqq \min_{\pi \in \Pi(\mu,\nu)} \langle c, \pi \rangle + \frac{1}{\lambda} R_\alpha(\pi \mid \mu \otimes \nu), \quad \mu,\nu \in \mathcal{P}(\mathbb{X}).$$



– Rényi divergence $\notin \{f\text{-divergence}, \text{Bregman divergence}\}$

$$R_\alpha(\mu \mid \nu) \coloneqq \frac{1}{\alpha - 1} \ln \left[\int_{\mathbb{X}} \left(\frac{\mathrm{d}\mu}{\mathrm{d}\tau} \right)^{\alpha} \left(\frac{\mathrm{d}\nu}{\mathrm{d}\tau} \right)^{1-\alpha} \mathrm{d}\tau \right],$$

where τ is a σ-finite reference measure (for instance $\tau = \mu + \nu$).

– If $\mathbb{X} \subset \mathbb{R}^d$ is compact, then

$$\mathrm{OT}(\mu,\nu) \xleftarrow{\alpha \searrow 0} \mathrm{OT}_{\frac{1}{\lambda},\alpha}(\mu,\nu) \xrightarrow{\alpha \nearrow 1} \mathrm{OT}_{\frac{1}{\lambda}}^{\mathrm{KL}}(\mu,\nu)$$