

B5: WASSERSTEIN GRADIENT FLOWS OF MOREAU ENVELOPES OF f -DIVERGENCES IN REPRODUCING KERNEL HILBERT SPACES

joint work with



Sebastian Neumayer, TU Chemnitz



Gabriele Steidl, TU Berlin



Nicolaj Rux, TU Berlin

Goal. Recover $\nu \in \mathcal{P}(\mathbb{R}^d)$ by minimizing f -divergence $D_{f,\nu}$ to ν (e.g. VI, GANs)

Problem. $\mu \ll \nu \implies D_{f,\nu}(\mu) = \infty$.

weak convergence

Our Solution. Regularize $D_{f,\nu}: \mathcal{M}(\mathbb{R}^d) \rightarrow [0, \infty]$.



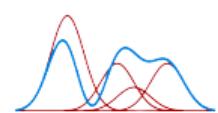
“Kernel trick”

$$m: \mu \mapsto m_\mu := \int_{\mathbb{R}^d} K(x, \cdot) d\mu(x) \in \mathcal{H}_K$$



pointwise convergence

$$\text{“}D_{f,\nu} \circ m^{-1}\text{”} = G_{f,\nu}: \mathcal{H}_K \rightarrow [0, \infty]$$



Moreau envelope regularization



$${}^\lambda G_{f,\nu}(m_\mu) = \min_{\sigma \in \mathcal{M}_+(\mathbb{R}^d)} D_{f,\nu}(\sigma) + \frac{1}{2\lambda} \|m_\sigma - m_\mu\|_{\mathcal{H}_K}^2$$

We prove existence & uniqueness of W_2 gradient flows of $\mu \mapsto {}^\lambda G_{f,\nu}(m_\mu)$.

Simulate particle flows = W_2 gradient flows starting at empirical measure