

```
In[2725]:= Remove[r, theta, x1, x2, mu, omega, sol, sol1, eq1, eq2, T, r0]
sol = Solve[{mu * r == r^3}, {r}]
```

```
r0 = r /. sol[[3]]
```

```
thetaP = omega + nu * r0^2;
```

```
T = 2 * Pi / (thetaP)
```

```
x1 = r * Cos[theta];
```

```
x2 = r * Sin[theta];
```

```
Remove[thetaP, rP, omega, nu, mu]
```

```
eq1 = rP * Cos[theta] - r * Sin[theta] * thetaP ==
1 / 10 * x1 - x2^3 - x1 * x2^2 - x1^2 * x2 - x2 - x1^3;
```

```
eq2 = rP * Sin[theta] + r * Cos[theta] * thetaP ==
x1 + 1 / 10 * x2 + x1 * x2^2 + x1^3 - x2^3 - x1^2 * x2;
```

```
sol1 = Solve[{eq1, eq2}, {rP, thetaP}];
```

```
rP = rP /. sol1[[1]]
```

```
thetaP = thetaP /. sol1[[1]]
```

```
{mu, omega, nu} = {1 / 10, 1, 1}
```

```
Out[2726]= {{r -> 0}, {r -> -sqrt[mu]}, {r -> sqrt[mu]}}
```

```
Out[2727]= sqrt[mu]
```

```
Out[2729]= 
$$\frac{2 \pi}{\mu + \omega}$$

```

```
Out[2736]= 
$$\frac{1}{10} \left( r - 10 r^3 \cos^2[\theta] - 10 r^3 \sin^2[\theta] \right)$$

```

```
Out[2737]= 
$$1 + r^2 \cos^2[\theta] + r^2 \sin^2[\theta]$$

```

```
Out[2738]= 
$$\left\{ \frac{1}{10}, 1, 1 \right\}$$

```

```
In[2680]:= Remove[r, theta, x1, x2, mu, omega, t]
```

```
x1'[t] = 1/10 * x1[t] - x2[t]^3 - x1[t] * x2[t]^2 - x1[t]^2 * x2[t] - x2[t] - x1[t]^3;
```

```
x2'[t] = x1[t] + 1/10 * x2[t] + x1[t] * x2[t]^2 + x1[t]^3 - x2[t]^3 - x1[t]^2 * x2[t];
```

```
Show[
```

```
StreamPlot[{1/10 * x1[t] - x2[t]^3 - x1[t] * x2[t]^2 - x1[t]^2 * x2[t] - x2[t] - x1[t]^3,
```

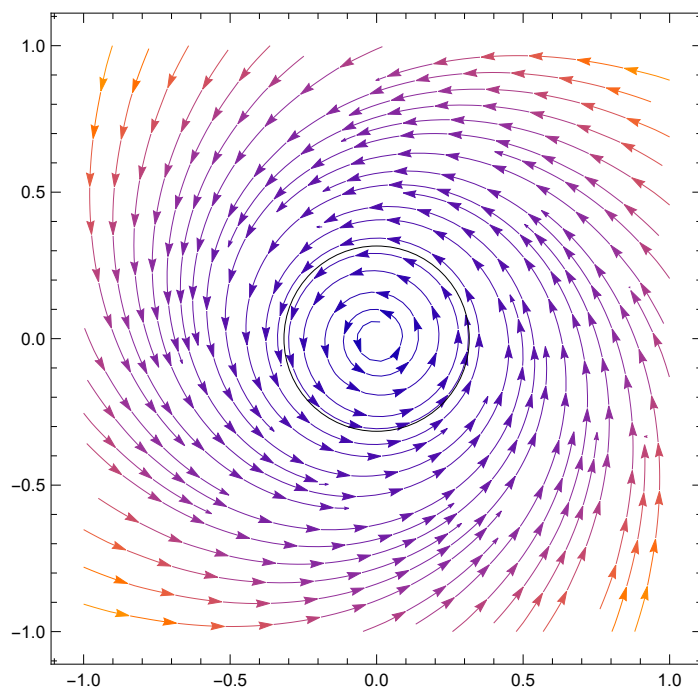
```
x1[t] + 1/10 * x2[t] + x1[t] * x2[t]^2 + x1[t]^3 - x2[t]^3 - x1[t]^2 * x2[t]},
```

```
{x1[t], -1, 1}, {x2[t], -1, 1}],
```

```
Graphics[Circle[{0, 0}, Sqrt[1/10]]]
```

```
]
```

```
Out[2683]=
```



```

In[2684]:= Remove[x1, x2, m11, m21, m22, m12]
T = 2 * Pi / 1.1;
s = NDSolve[
  {x1'[t] == x1[t] / 10 - x2[t]^3 - x1[t] * x2[t]^2 - x1[t]^2 * x2[t] - x2[t] - x1[t]^3,
   x2'[t] == x1[t] + x2[t] / 10 + x1[t] * x2[t]^2 +
   x1[t]^3 - x2[t]^3 - x1[t]^2 * x2[t],
   m11'[t] == m11[t] * (1 / 10 - x2[t]^2 - 2 * x1[t] * x2[t] - 3 * x1[t]^2) +
   m21[t] * (-3 * x2[t]^2 - 2 * x1[t] * x2[t] - x1[t]^2 - 1),
   m12'[t] == m12[t] * (1 / 10 - x2[t]^2 - 2 * x1[t] * x2[t] - 3 * x1[t]^2) +
   m22[t] * (-3 * x2[t]^2 - 2 * x1[t] * x2[t] - x1[t]^2 - 1),
   m21'[t] == m11[t] * (1 + x2[t]^2 + 3 * x1[t]^2 - 2 * x1[t] * x2[t]) +
   m21[t] * (1 / 10 + 2 * x1[t] * x2[t] - 3 * x2[t]^2 - x1[t]^2),
   m22'[t] == m12[t] * (1 + x2[t]^2 + 3 * x1[t]^2 - 2 * x1[t] * x2[t]) +
   m22[t] * (1 / 10 + 2 * x1[t] * x2[t] - 3 * x2[t]^2 - x1[t]^2),
   x1[0] == Sqrt[0.1], x2[0] == 0, m11[0] == 1, m12[0] == 0,
   m21[0] == 0, m22[0] == 1}, {x1, x2, m11, m12, m21, m22}, {t, 0, T}];
{x1[T] /. s, x2[T] /. s, m11[T] /. s, m12[T] /. s, m21[T] /. s, m22[T] /. s}

Show[ParametricPlot[Evaluate[{t, x1[t]} /. s], {t, 0, T}, PlotLegends -> {x1}],
ParametricPlot[Evaluate[{t, x2[t]} /. s],
{t, 0, T}, PlotStyle -> Gray, PlotLegends -> {x2}],
ParametricPlot[Evaluate[{t, m11[t]} /. s], {t, 0, T},
PlotStyle -> Red, PlotLegends -> {m11}],
ParametricPlot[Evaluate[{t, m12[t]} /. s], {t, 0, T},
PlotStyle -> Blue, PlotLegends -> {m12}],
ParametricPlot[Evaluate[{t, m21[t]} /. s], {t, 0, T},
PlotStyle -> Green, PlotLegends -> {m21}],
ParametricPlot[Evaluate[{t, m22[t]} /. s], {t, 0, T},
PlotStyle -> Yellow, PlotLegends -> {m22}],
PlotRange -> All
]

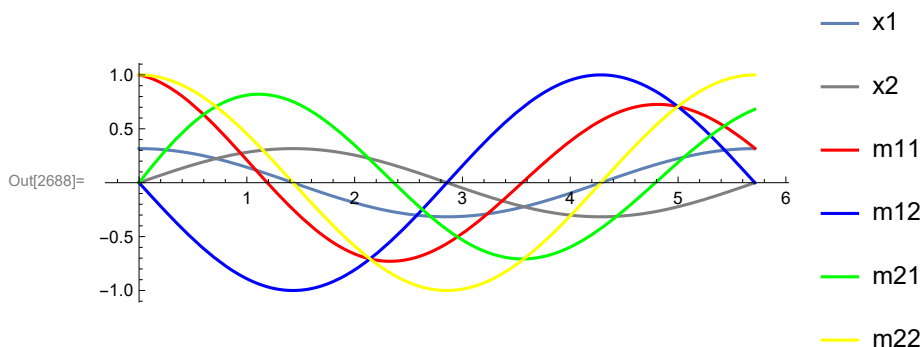
M = {{m11[T] /. s[[1]], m12[T] /. s[[1]]}, {m21[T] /. s[[1]], m22[T] /. s[[1]]}}
lambda = Eigenvalues[M]
Log[lambda[[1]]] / T
Log[lambda[[2]]] / T

```

```

Out[2687]= {{0.316228}, {-6.71405 × 10-9}, {0.319053}, {2.12317 × 10-8}, {0.680947}, {1.}}

```



```

Out[2689]= {{0.319053, 2.12317 × 10-8}, {0.680947, 1.}}

```

Out[2690]= {1., 0.319053}

Out[2691]= 5.78753×10^{-9}

Out[2692]= -0.2