Hand in problem 3 in Information Theory (EITN45)

Spring 2020

Problem

In the course material, error correcting encoding and decoding is described through a Hamming code. There are many other codes defined over the years. The one treated in this problem is called Reed-Muller code.

For a positive integer m and a non-negative integer r, the generator matrix for the Reed-Muller code $\mathrm{RM}(r,m)$ is defined as

$$G(r,m) = \begin{pmatrix} G(r,m-1) & G(r,m-1) \\ 0 & G(r-1,m-1) \end{pmatrix}$$

To start the induction, define $G(0,m)$ as a row vector of 2^m ones, i.e. the generator matrix for a length 2^m repetition code, and $G(m,m)$ the unit matrix of size 2^m. In this problem the $\mathrm{RM}(1,3)$ code will be treated.

- Use the above induction formula to derive the generator matrix $G(1,3)$.

- List all codewords and determine $d_{\min}$. How many errors can be corrected? How many errors can be detected?

- Verify that $H = G$, i.e. that $GG^T = 0$. That means the generator matrix is its own parity check matrix, and such code is said to be self dual.[1]

- Write the syndrome table for all single errors that can occur on the channel.

- Choose a codeword and introduce a correctable error in it. Decode using the syndrome table.

Hand in details

You should hand in your solution of the problem as pdf online at the Canvas page. The line of reasoning should be clear from the solution. If you use computer to solve parts of the problem you should also hand in the code for these scripts as appendix as part of the pdf for the solutions.

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[1]This is a special case for this RM code. In general, the parity check matrix for an $\mathrm{RM}(r,m)$ code is given by the generator matrix $G(m-r-1,m)$.