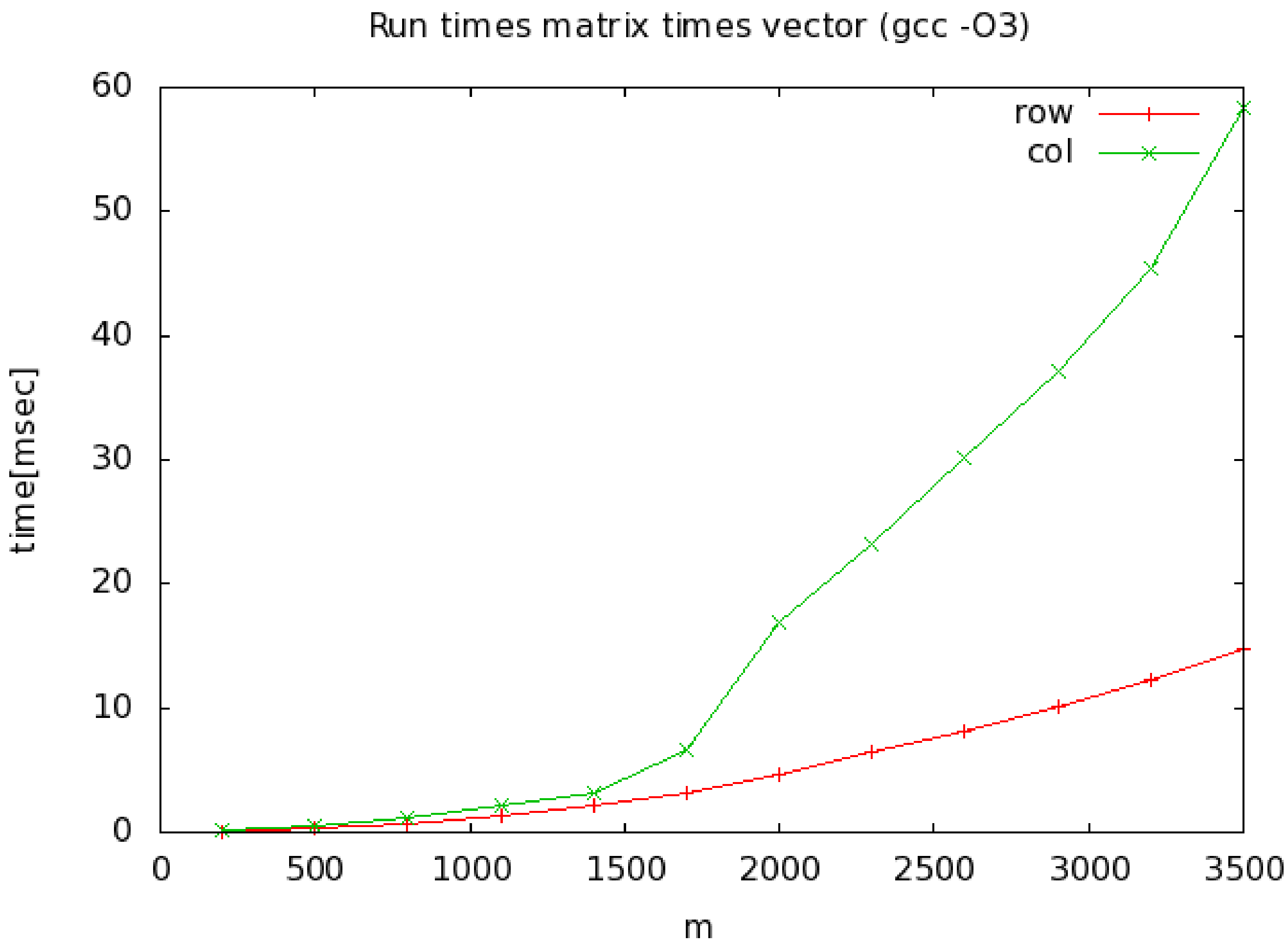
Mathematical Software Programming (02635)

Module 9 – Fall 2025

Instructor: Bernd Dammann

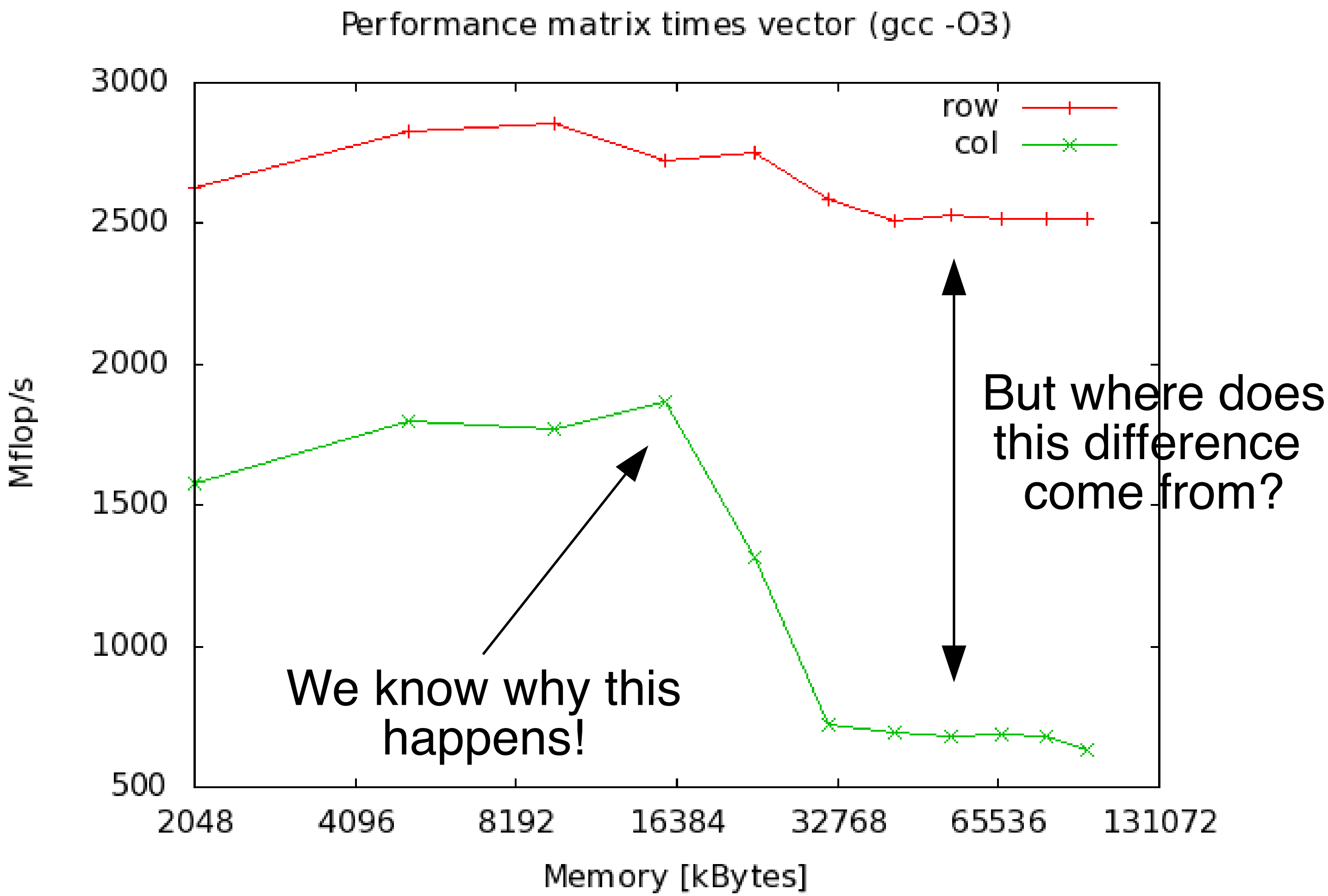
Re-cap from Module 7

Matrix times vector (runtimes):



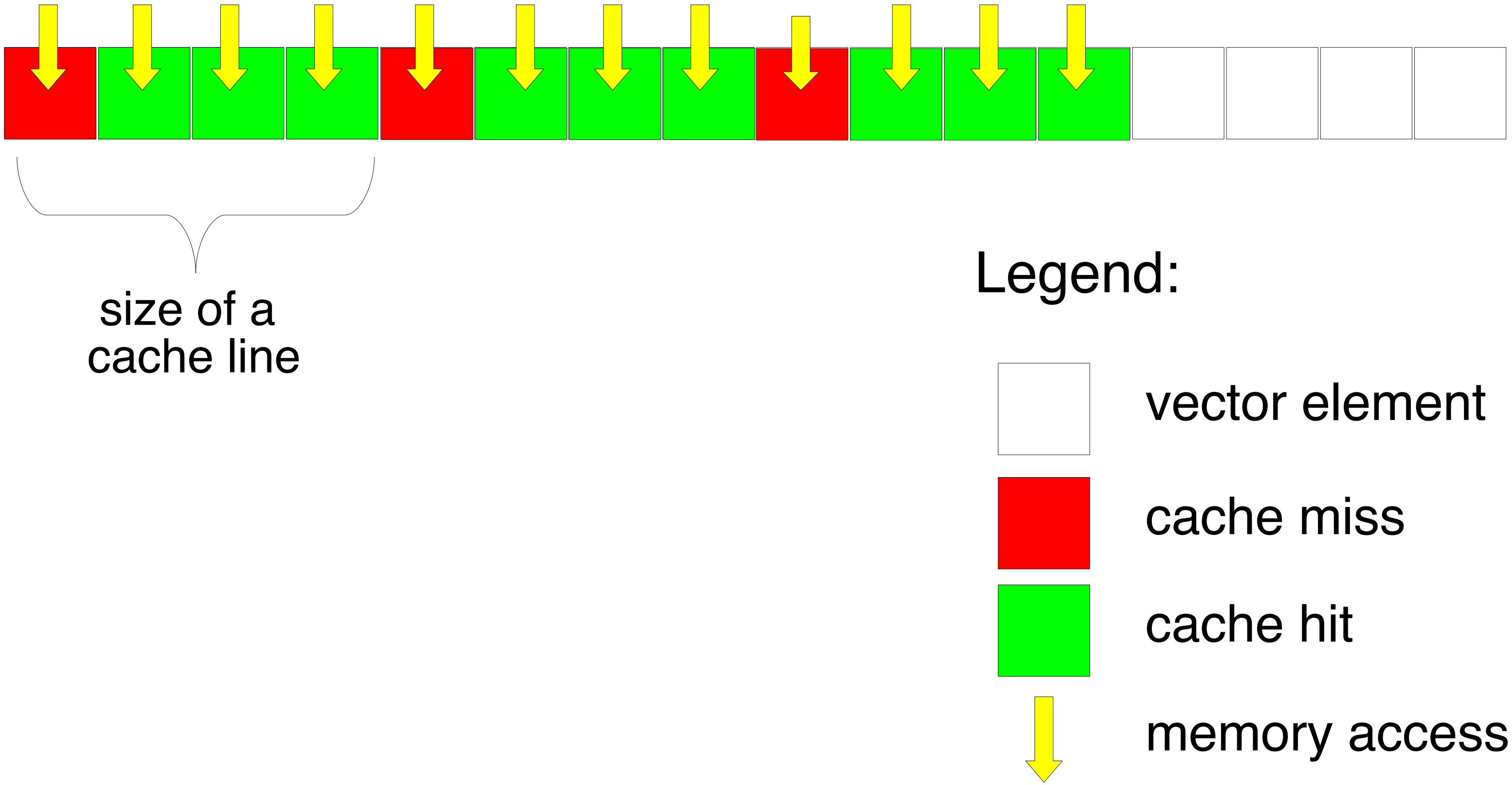
Re-cap from Module 7

Matrix times vector (performance):



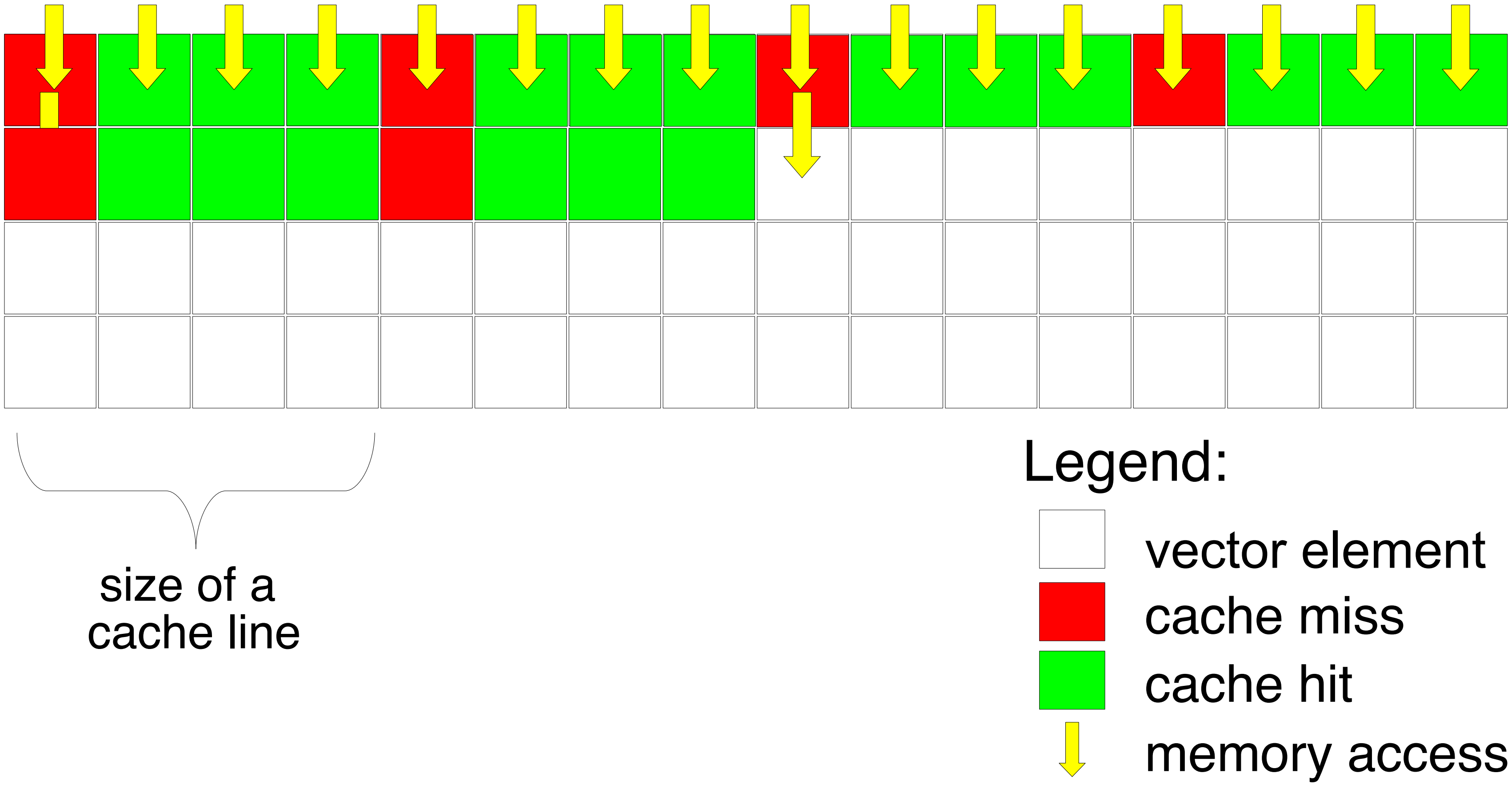
Re-cap from Module 7

Accessing vector elements in C:



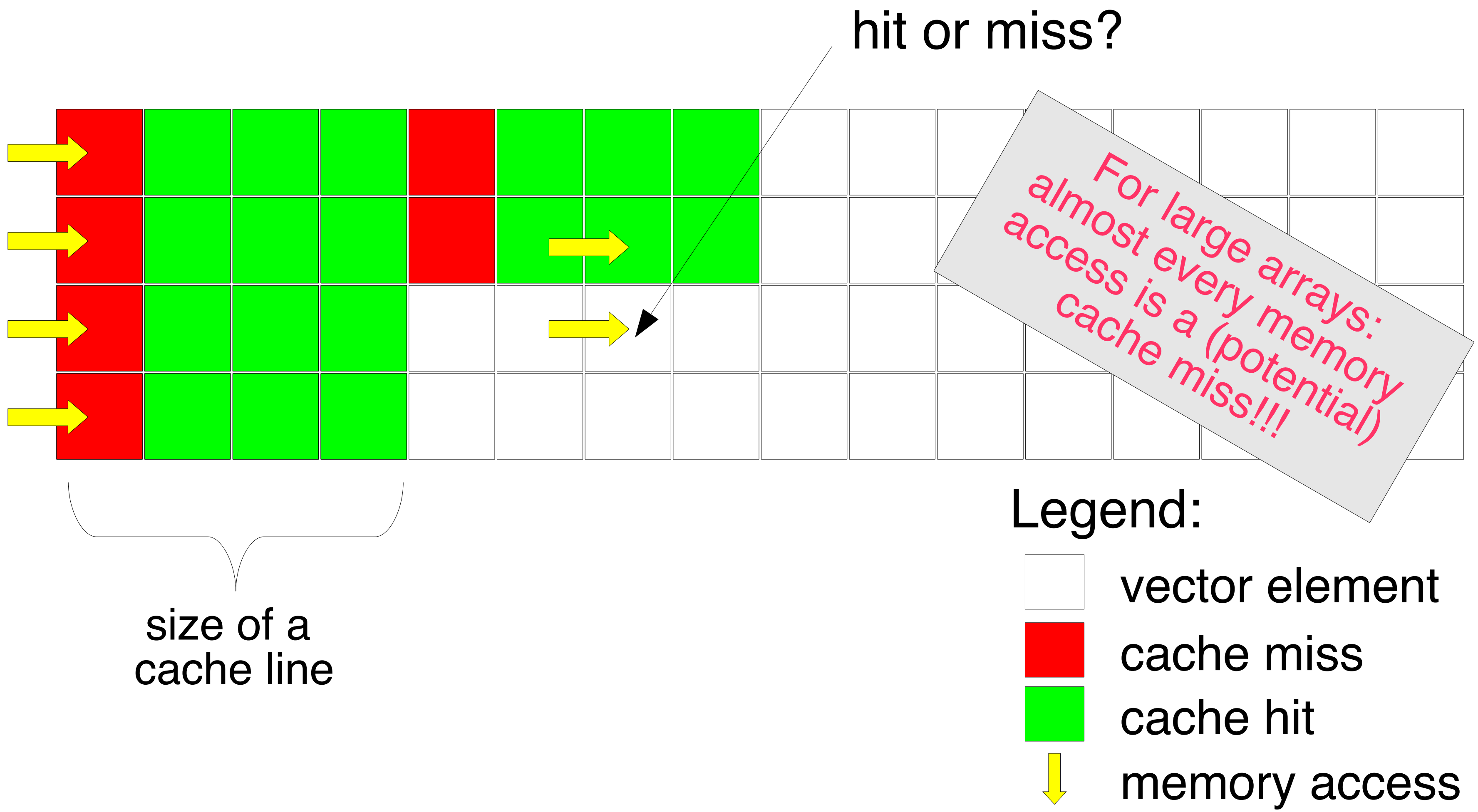
Re-cap from Module 7

Accessing 2d arrays in C (RowMajor) – row wise:



Re-cap from Module 7

Accessing 2d arrays in C (RowMajor) – column wise:



How can we use this to improve the matrix times vector code from the previous session?

Generic my_dgemv()

- with the knowledge from the previous slides ...
- plus the information about the storage order, e.g. in the MSPTools data structure for 2d arrays ...
- we can create a “generic” my_dgemv(), that – at runtime – chooses the fastest execution path for the given data layout.
- This is part of your exercises, today!

Today's topics

- Parallelism – what is that?
- Parallel execution models
- Parallel speed-up:
  - what is that?
  - what can we expect?
- Exploiting parallelism with OpenMP

Today's goal

- Basic understanding of parallel computations.
- Implement a parallel version of the Module 7 code, i.e. matrix times vector, using OpenMP.

A quick poll (on Vevox)

A given application, Prog.exe, takes 60 seconds to execute on a single-core of a CPU. What is the execution time on all 4 cores of a quad-core CPU?

A) 15 seconds

B) less than 15 seconds

C) between 15 and 30 seconds

D) between 30 and 60 secs

E) more than 60 seconds

F) I need more information about Prog.exe

What is Parallelization?

An attempt of a definition:

> "*Something*" is parallel, if there is a certain level of independence in the order of operations

"*Something*" can be:

- A collection of program statements
- An algorithm
- A part of your program
- The problem you are trying to solve

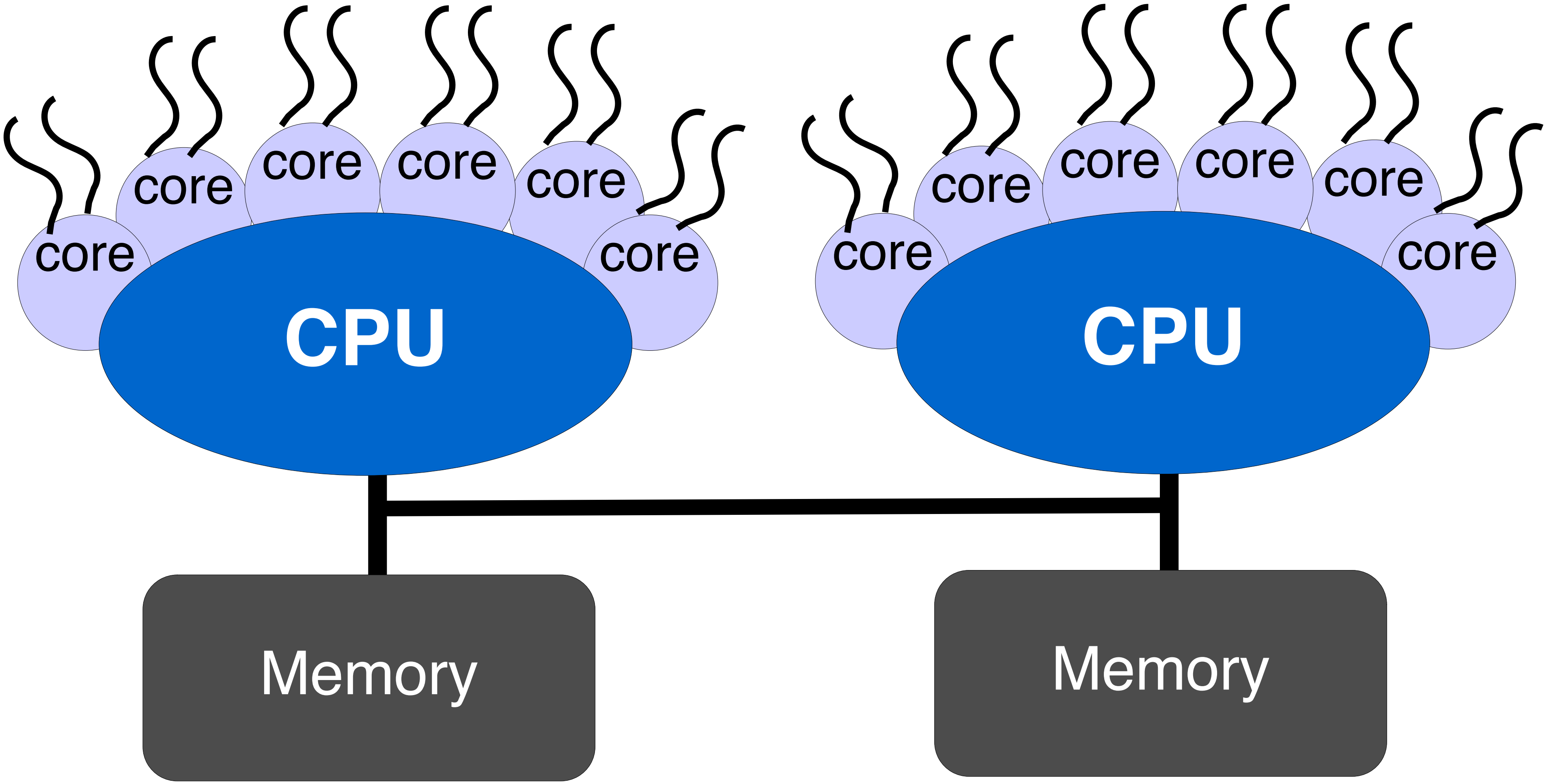
granularity

Parallelism is everywhere

In today's computer installations one has many levels of parallelism:

- Instruction level (ILP)
- Chip level (multi-core, multi-threading)
- System level (multi-socket, i.e. multi-CPU)
- accelerators: GPU, ~~Intel Xeon Phi~~, FPGA
- Cluster: “network of compute nodes”
- ...

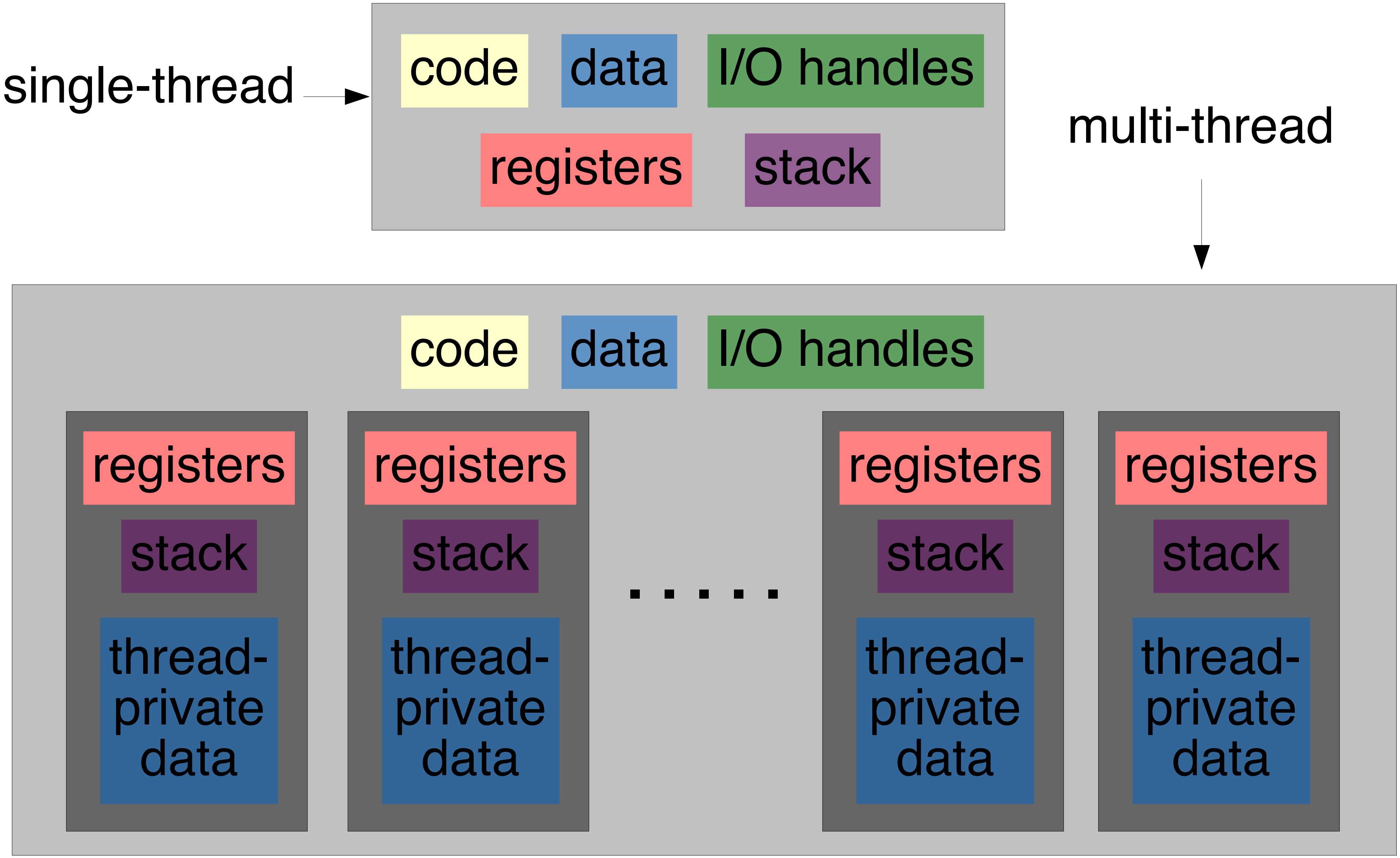
A typical multi-core setup



a 2-socket, 12-core, 24-(hyper-)threads server ⇒ 24 logical CPUs

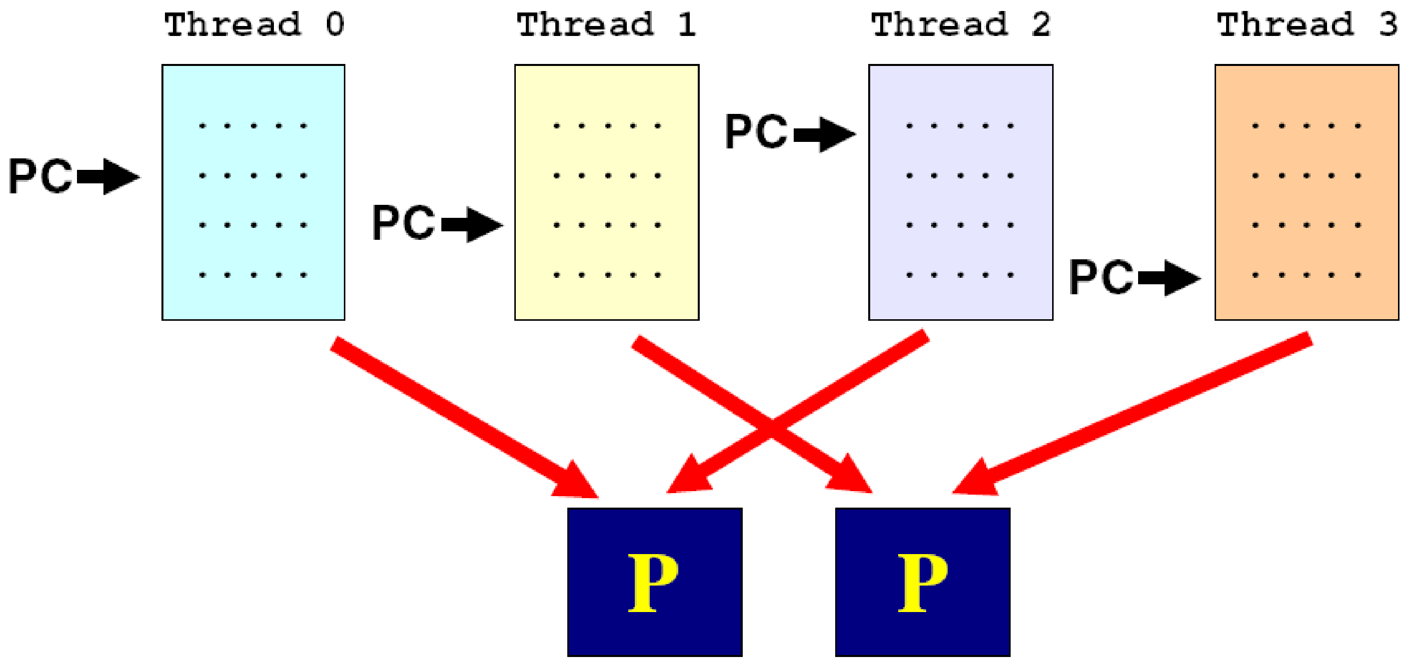
Note: we do not use hyperthreading in our setup!

Single- vs. multi-threaded



What is a thread?

- Loosely said, a thread consists of a series of instructions with it's own program counter (“PC”) and state
- A parallel program will execute threads in parallel
- These threads are then scheduled onto processing units (P), e.g. CPU cores, by the OS



Parallel execution models

- Multi-threaded:
  - one process
  - multiple threads
  - “communication” (implicit) via shared-memory (shm)
  - limited to one node (computer)
- Multi-process:
  - multiple processes (usually single threaded)
  - communication via interconnect (network or shm)
  - can run on “any” number of nodes
- Hybrid: multiple multi-threaded processes

Timings in parallel programs

- So far, we have used clock() to time the speed of our programs, i.e. the CPU time
- In parallel programs:
  - the CPU time will very likely go up (parallel overhead)
  - clock() measures the accumulated time of all threads(!)
  - we need another measure: wallclock time, i.e. the time the user has to wait to get the result
- All parallel programming models provide a function to get the wallclock time.
- On the next slides: wall-time = wallclock time

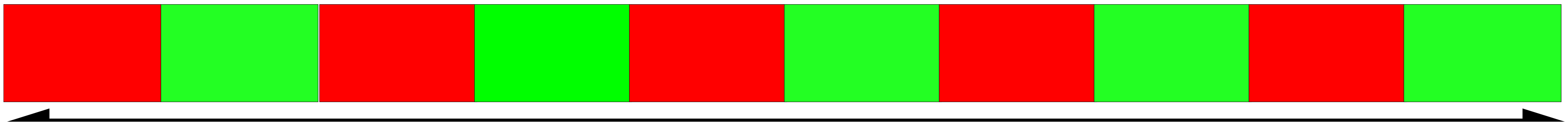
Parallelism: speed-up

- What is this "speed-up"?
  - S(p) := (wall-time on 1 core) / (wall-time on p cores)

    := T(1) / T(p)
- ideal case: linear speed-up, e.g. S(p) = p
  - but: the world is not ideal!
  - parallel overhead: extra instructions, communication, synchronization, etc
  - not all parts of your code can run in parallel – there will always be sequential code
  - in general: wall-time goes down – but CPU time goes up!

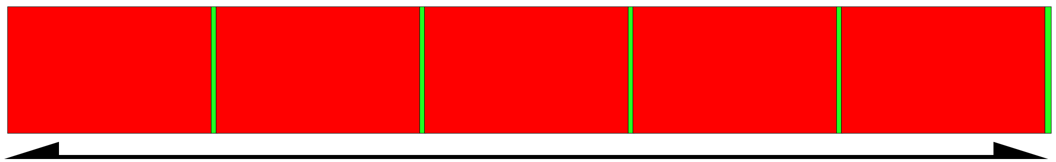
Parallelism: speed-up

- let f be the parallel fraction of your code, and (1-f) the sequential part, e.g. f = 0.5



T(1)

- What is the max. speed-up, if we had an infinite number of cores ($p = \infty$), and no communication costs, etc?

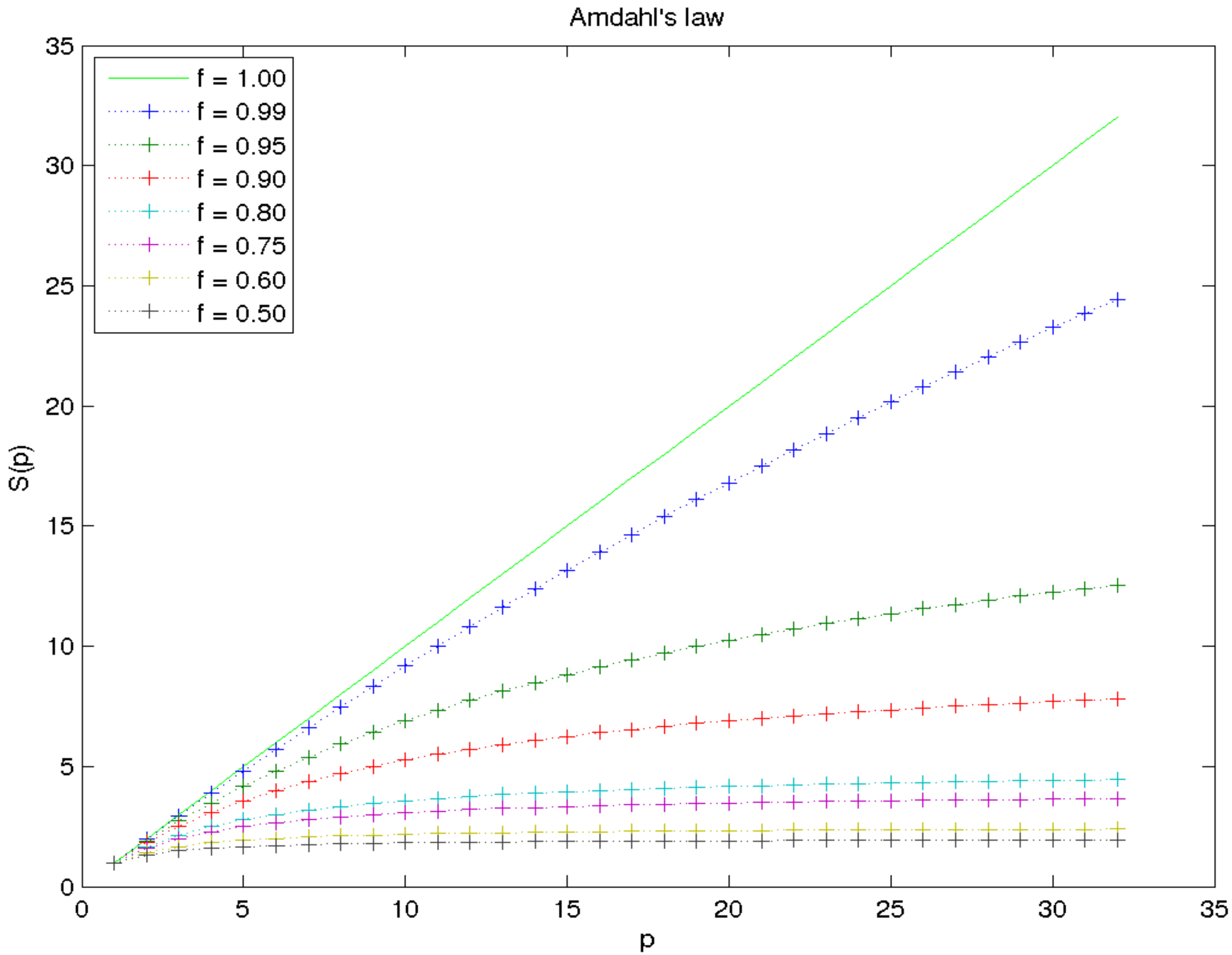


$T(p=\infty)$

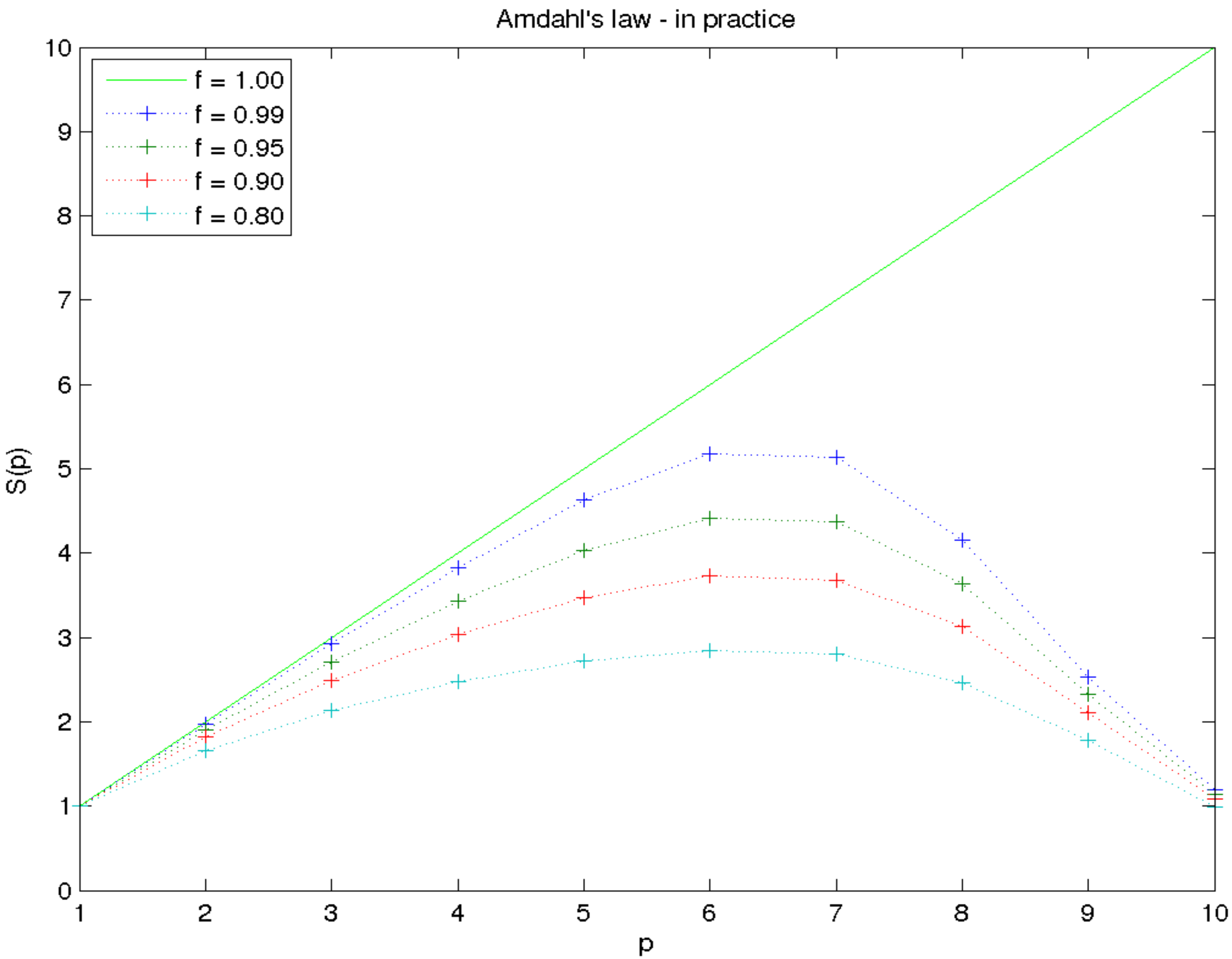
$S = T(1) / T(p=\infty) < 2$

Parallelism: Amdahl's law

In general: T(p) = T(1)*(1-f) + T(1)*f/p, i.e. S(p) = p/((1-f)*p + f))



Parallelism: Amdahl's law in practice



Exploiting parallelism

using OpenMP

What is OpenMP?

From openmp.org:

"The OpenMP API supports multi-platform shared-memory parallel programming in C/C++ and Fortran. The OpenMP API defines a portable, scalable model with a simple and flexible interface for developing parallel applications on platforms from the desktop to the supercomputer."

- OpenMP is a "kind of add-on" to C/C++, Fortran
- it is not a programming language
- it requires a compiler that supports OpenMP

OpenMP components

- Directives
  - in your source code
  - e.g. parallel for-loop
- Runtime library
  - support functions
  - e.g. wallclock timer, etc
- Environment variables
  - control program behaviour at runtime
  - e.g. number of threads to be used

OpenMP: Hello world

OpenMP version of “Hello world”:

```
#include <stdio.h>

int main(int argc, char *argv[]) {
    #pragma omp parallel
    {
    printf("Hello parallel world!\n");
    } /* end parallel */
    return(0);
}
```

OpenMP: Hello world

Compile and run ...

```
$ gcc -o hello_omp hello_omp.c

$ ./hello_omp
Hello parallel world!

$ OMP_NUM_THREADS=2 ./hello_omp
Hello parallel world!
```

OpenMP: Hello world

Compile with OpenMP enabled – and run ...

```
$ gcc -fopenmp -o hello_omp hello_omp.c

$ ./hello_omp
Hello parallel world!

$ OMP_NUM_THREADS=2 ./hello_omp
Hello parallel world!
Hello parallel world!
```

OpenMP: Hello world v2

```
#include <stdio.h>
#ifdef _OPENMP
#include <omp.h>
#endif

int main(int argc, char *argv[]) {
    int t_id = 0;
    #pragma omp parallel private(t_id)
    {
    #ifdef _OPENMP
    t_id = omp_get_thread_num();
    #endif
    printf("Hello world from %d!\n", t_id);
    } /* end parallel */
    return(0);
}
```

OpenMP: Hello World v2

```
$ ./hello_omp2
Hello world from 0!

$ OMP_NUM_THREADS=4 ./hello_omp2
Hello world from 0!
Hello world from 3!
Hello world from 1!
Hello world from 2!
```

- Note: The order of execution will be different from run to run!
- The default no. of threads depends on the OpenMP implementation

OpenMP: Parallel for-loop

Work-sharing – Loop parallelism:

- OpenMP implements parallel for-loops only!

```
int i;
float a[N], b[N], c[N];

for (i=0; i < N; i++)
  a[i] = b[i] = i * 1.0;

#pragma omp parallel shared(a,b,c) private(i)
{
  #pragma omp for
  for (i=0; i < N; i++)
    c[i] = a[i] + b[i];
}  /* end of parallel region */
```

for-loop *has to follow the pragma – no {... }!*

OpenMP: Parallel for-loop

Work-sharing – Loop parallelism:

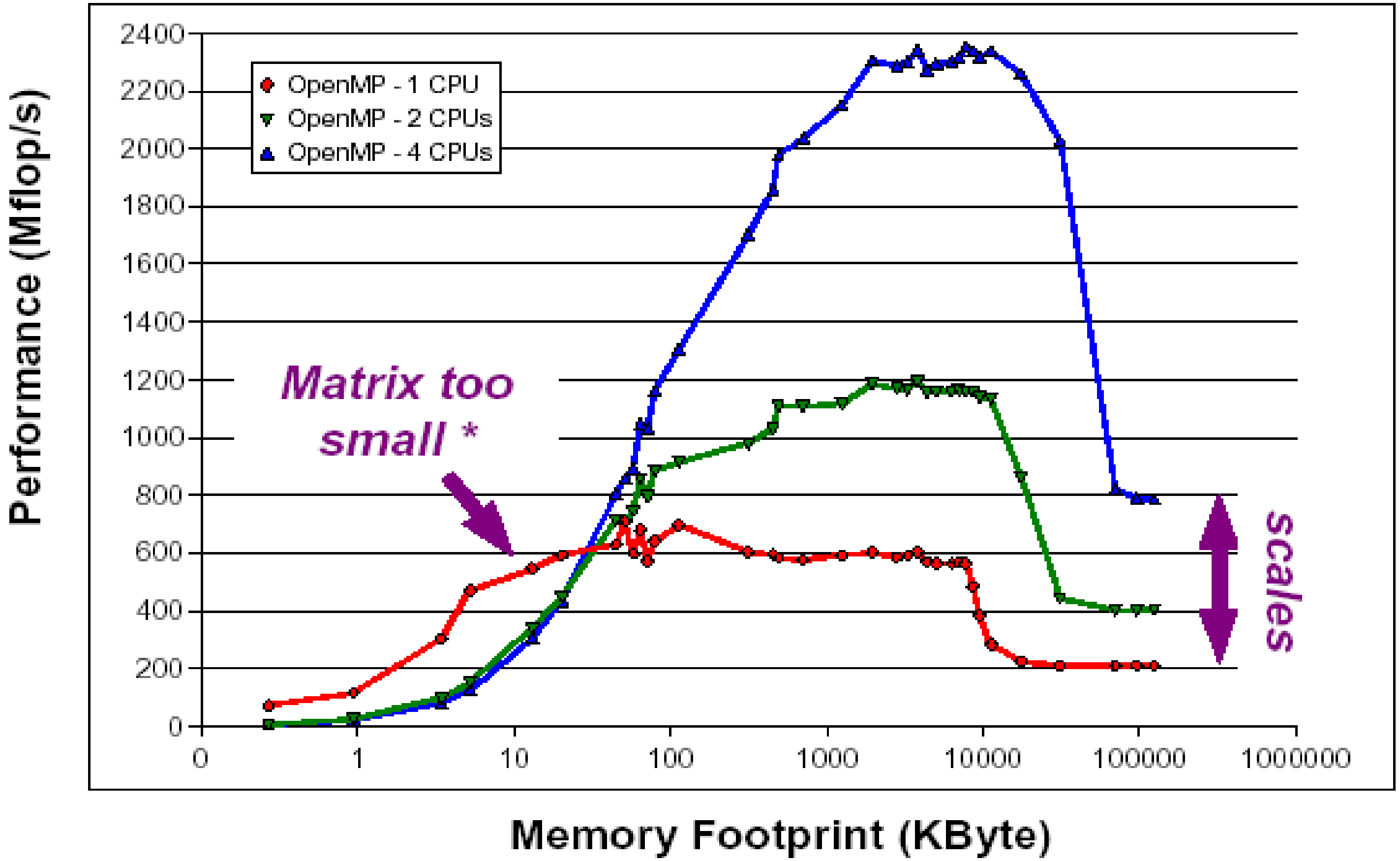
- Another version: combined “parallel for”
- C99 loop-style

```
float a[N], b[N], c[N];

for (int i=0; i < N; i++)
  a[i] = b[i] = i * 1.0;

#pragma omp parallel for shared(a,b,c)
for (int i=0; i < N; i++)
  c[i] = a[i] + b[i];
```

OpenMP: Matrix times vector



SunFire 6800
UltraSPARC III Cu @ 900 MHz
8 MB L2-cache

courtesy: Ruud van der Pas, Oracle

Summary

Summary: Parallelism

- Parallel execution can speed up your code
- Wallclock time goes down – but the CPU time goes usually up (more resources, parallel overhead!)
- Don't expect magic ...
  - remember Amdahl's law!
  - is your problem too small?
  - don't use too many threads!
- Always check your results – compare to serial version!

Recap: the quick poll

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A given application, Prog.exe, takes 60 seconds to execute on a single-core of a CPU. What is the execution time on all 4 cores of a quad-core CPU?

15 seconds — 11.11%

less than 15 seconds — 11.11%

between 15 and 30 seconds — 33.33%

between 30 and 60 secs — 0%

more than 60 seconds — 0%

I need more information about Prog.exe — 44.44%

Correct responses
44.44%

Correct answer
I need more information about Prog.exe

what would you answer now ...

Today's exercises

- Make your first parallel steps:
  - implement the “Hello World” example from the lecture
  - this should help you to understand how OpenMP works with your compiler
- Make a parallel version of last week's examples
  - first create a “generic” version
  - then use OpenMP to make it parallel
  - Note: no Autolab this week, but use the provided tools to check your results!
  - There is a ZIP file with all files (templates) needed!