Danmarks Tekniske Universitet

Skriftlig prøve/dato: / Written examination date:	9. December 2024
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Kursus navn: / Course title:	Mathematical Software Programming
Kursus nummer: / Course number:	02635
Hjælpemidler: / Aids allowed:	All aids allowed.
Varighed: / Exam duration:	4 hours
Vægtning: / Weighting:	Part 1: 35/75 Part 2: 40/75

Please note that the exam consists of two parts: part 1 is a set of multiple-choice questions, and part 2 is a set of programming questions.

This is part 2 of the exam. Use the templates in the ZIP file for your answers.

Question 1

Consider the real-valued function

$$f(x) = \begin{cases} 1, & x = 0, \\ (e^{2x} - e^{x})/x, & x \neq 0. \end{cases}$$

Implement a function that takes x as input and returns $f(x)$.

The function must have the following prototype:

```
double feval(double x);
```

The function should return `NAN` if the input is `NAN`.

Use the template `exam_e24_q1.c` for your implementation.

Question 2

Consider the polynomial

$$p(x) = \sum_{k=0}^{n} c_k T_k(x), \quad x \in [-1, 1],$$

where $(c_0, c_1, \ldots, c_n)$ are $n+1$ real-valued coefficients and $T_k(x)$ is the so-called kth order Chebyshev polynomial of the first kind, which is defined recursively as

$$T_0(x) = 1, \qquad T_1(x) = x, \qquad T_{k+1}(x) = 2xT_k(x) - T_{k-1}(x),\ k \geq 1.$$

Write a function that evaluates $p(x)$ given x and the coefficients $(c_0, c_1, \ldots, c_n)$.

The function must have the following prototype:

```c
double chebseries(double x, const double *c, int n);
```

- The function should return `NAN` if the input is invalid or if $x \notin [-1, 1]$.
- The input `c` is a pointer to an array of length $n+1$ containing the coefficients $(c_0, c_1, \ldots, c_n)$.
- The input `n` is the degree of the polynomial $p(x)$.

Use the template `exam_e24_q2.c` for your implementation.

Question 3

Consider the log-sum-exp function, defined as

$$f(x) = \ln\left(\sum_{i=1}^{n} e^{x_i}\right),$$

where $x = (x_1, x_2, \ldots, x_n)$ is a real-valued vector of length n. The gradient of f is given by

$$\nabla f(x) = \begin{bmatrix} \frac{\partial f}{\partial x_1} \\ \frac{\partial f}{\partial x_2} \\ \vdots \\ \frac{\partial f}{\partial x_n} \end{bmatrix} = \frac{1}{\sum_{i=1}^{n} e^{x_i}} \begin{bmatrix} e^{x_1} \\ e^{x_2} \\ \vdots \\ e^{x_n} \end{bmatrix}$$

Question 3a

Implement a function that evaluates $f(x)$.

The function must have the following prototype:

```
double lse(const double * x, int n);
```

The function should return `NAN` if the input is invalid, e.g., if `x` is a NULL pointer or if `n` is not positive.

Use the template `exam_e24_q3a.c` for your implementation.

Question 3b

Implement a function that overwrites x by $\nabla f(x)$.

The function must have the following prototype:

```
int lsegrad(double * x, int n);
```

The function should return `0` if the inputs are valid, and otherwise it should return `1`.

Use the template `exam_e24_q3b.c` for your implementation.