02635E25 trail exam

Der anvendes en scoringsalgoritme, som er baseret på "One best answer"

Dette betyder følgende:
Der er altid netop ét svar som er mere rigtigt end de andre
Studerende kan kun vælge ét svar per spørgsmål
Hvert rigtigt svar giver 1 point
Hvert forkert svar giver 0 point (der benyttes IKKE negative point)

The following approach to scoring responses is implemented and is based on "One best answer"
There is always only one correct answer – a response that is more correct than the rest
Students are only able to select one answer per question
Every correct answer corresponds to 1 point
Every incorrect answer corresponds to 0 points (incorrect answers do not result in subtraction of points)

Conditioning and stability

Answer the following questions about conditioning and stability.

The conditioning of a problem is a measure of

Vælg en svarmulighed

- the stability of the solution to perturbations in the data.
- the sensitivity of the algorithm used to solve the problem.
- the sensitivity of the data to perturbations in the solution.
- the stability of the algorithm used to solve the problem.
- the sensitivity of the solution to perturbations in the data.

What is the relative condition number of $f(x) = e^x - 1$ at x?

Vælg en svarmulighed

- $|e^x - 1|/e^x$
- $|xe^x|/|e^x - 1|$
- $|e^x|/|e^x - 1|$
- $|x|$

$$\text{cond}(f, x) = \left| \frac{x f'(x)}{f(x)} \right| = \left| \frac{x e^x}{e^x - 1} \right|$$

Sequential summation of n real numbers $x_1, \ldots, x_n$ is

Vælg en svarmulighed

- ○ forward stable.
- ○ backward stable.
- ● forward and backward stable.
- ○ not stable.

$$y = f(x) = \sum_{i=1}^{n} x_i \qquad \text{cond}(f, x) = \frac{\sum_i |x_i|}{|\sum_i x_i|}$$

Forward stable: $\|\hat{y} - y\| \leq c \cdot \text{cond}(f, x) \|y\|$

c: small constant

Backward stable:

$$\inf_{\Delta x} \left\{ \frac{\|\Delta x\|}{\|x\|} : \hat{y} = f(x + \Delta x) \right\} \quad \text{small}$$

Floating-point arithmetic

Answer the following questions about floating-point numbers and floating-point arithmetic.

Catestrophic cancellation may occur when

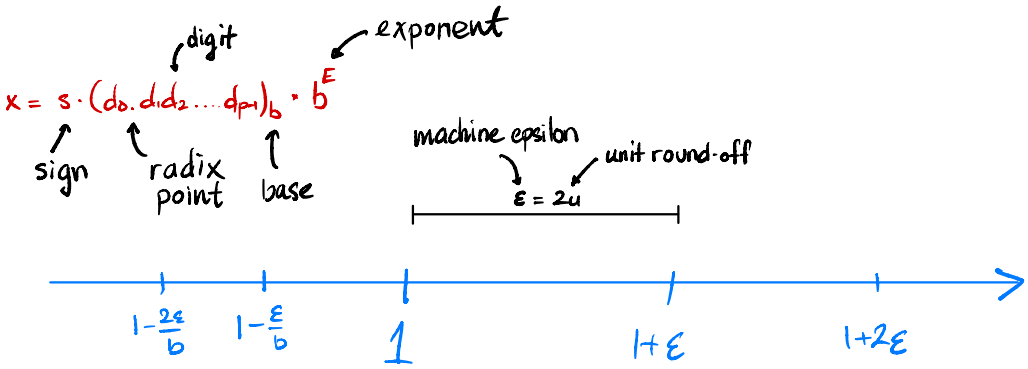
Vælg en svarmulighed

- the sum of two nearly equal numbers is computed.
- the division of two nearly equal numbers is computed.
- the difference between two nearly equal numbers is computed.
- the product of two nearly equal numbers is computed.

What does "machine epsilon" refer to?

Vælg en svarmulighed

- The distance from 1 to the next representable floating-point number that is larger than 1.
- The smallest positive number that can be represented in the floating-point system.
- The largest possible absolute value of round-off errors in the floating-point system.
- The smallest positive number that can be added to 0 in the floating-point system.



Let a, b, and c be real numbers.

The commutative property of multiplication holds in finite precision arithmetic, i.e., $a \cdot b = b \cdot a$.

Vælg en svarmulighed

- ○ False
- ● True

$$fl(a \cdot b) = fl(b \cdot a)$$
$$= (a \cdot b)(1+\delta) \qquad |\delta| \leq u$$

What does "overflow" refer to when working with floating-point numbers?

Vælg en svarmulighed

- The situation when the magnitude of the result of an arithmetic operation is too small to be represented in the floating-point system.
- The loss of precision due to the subtraction of two nearly equal numbers.
- The situation when the magnitude of the result of an arithmetic operation is too large to be represented in the floating-point system.
- The round-off errors caused by the limited precision of the floating-point system.

Let a, b, and c be real numbers.

The associative property of multiplication holds in finite precision arithmetic, i.e., $(a \cdot b) \cdot c = a \cdot (b \cdot c)$.

Vælg en svarmulighed

- False
- True

Recursion

Consider the following recursive function:

```
#include <math.h>
double legendre(double x, int n) {
    if (n < 0 || isnan(x))
      return NAN;
    else if (n >= 2)
      return ((2.0*n-1)*x*legendre(x,n-1) - (n-1)*legendre(x,n-2))/n;
    else if (n == 1)
      return x;
    else  // n == 0
      return 1.0;
}
```

Let $T(n)$ denote the runtime of the function legendre for a given n. Which of the following best characterizes the time complexity?

Vælg en svarmulighed

- $T(n) = O(\log n)$
- $T(n) = O(n)$
- $T(n) = O(n^2)$
- $T(n) = O(2^{\text{poly}(n)})$

Which kind of recursion is used in the function?

Vælg en svarmulighed

- (●) Multiple recursion
- ( ) Single recursion

Parallel computing

Answer the following questions about parallelization.

Does Amdahl's law account for parallel overhead?

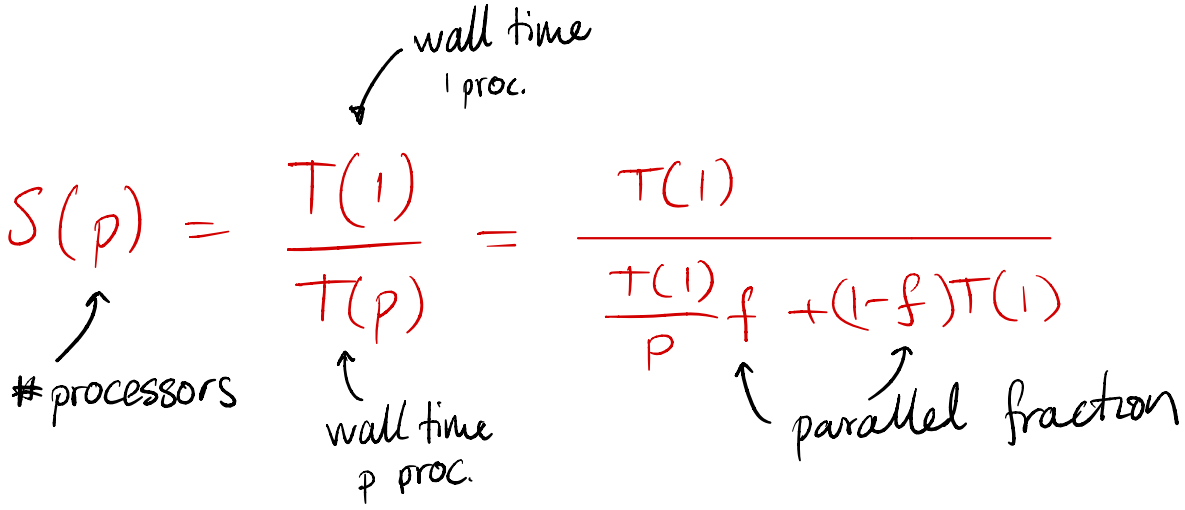
Vælg en svarmulighed

- [x] No
- [ ] Yes

According to Amdahl's law, what is the maximum speedup that can be achieved by parallelizing a program with parallel fraction f?

Vælg en svarmulighed

- $f/(1-f)$
- $1/(1-f)$
- $(1-f)/f$
- $1/f$



$$S(p) \to \frac{1}{1-f} \quad \text{as} \quad p \to \infty$$

Conditioning

Let $x = (x_1, \ldots, x_n)$ be a given vector of n samples. The sample mean is then

$$\mu(x) = \frac{1}{n} \sum_{i=1}^{n} x_i$$

and the sample variance is

$$v(x) = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \mu(x))^2.$$

When is the problem of evaluating μ at x ill-conditioned?

Vælg en svarmulighed

- ○ When $|\mu(x)|$ is small.
- ○ When $|\mu(x)|$ is large.
- ● When $|\mu(x)| \ll \|x\|_2$.
- ○ When $|\mu(x)| \gg \|x\|_2$.
- ○ When $|\mu(x)| \approx \|x\|_2$.

Sample mean: $\mu(x) = \frac{1}{n}\mathbb{1}^T x$

$$\text{Cond}_2(\mu, x) = \frac{\|\frac{1}{n}\mathbb{1}\|_2 \|x\|_2}{|\mu(x)|} = \frac{1}{\sqrt{n}} \frac{\|x\|_2}{|\mu(x)|}$$

Centering matrix: $C = I - \frac{1}{n}\mathbb{1}\mathbb{1}^T$

Sample variance: $V(x) = \frac{1}{n-1}\|Cx\|_2^2$

$$\text{cond}_2(V, x) = \frac{\|x\|_2 \cdot \|\frac{2}{n-1} Cx\|_2}{\frac{1}{n-1}\|Cx\|_2^2} = \frac{2\|x\|_2}{\|Cx\|_2^2}$$

Write $x = \mu(x)\mathbb{1} + Cx$:

$$\|x\|_2^2 = \|Cx\|_2^2 + \mu(x)^2 \cdot n + \underbrace{2\mu(x)\mathbb{1}^T Cx}_{=0}$$

$$\text{cond}_2(V, x) = 2\frac{\sqrt{\|Cx\|_2^2 + n\mu(x)^2}}{\|Cx\|_2} = 2\sqrt{1 + \frac{n}{n-1}\frac{\mu(x)^2}{V(x)}}$$

When is the problem of evaluating v at x ill-conditioned?

Vælg en svarmulighed

- When $v(x) \ll \mu(x)^2$.
- When $v(x)$ is small.
- When $v(x) \approx \mu(x)^2$.
- When $|\mu(x)|$ is small.
- When $v(x) \gg \mu(x)^2$.

Object-oriented programming

Answer the following questions about object-oriented programming.

What is an object in object-oriented programming?

Vælg en svarmulighed

- An instance of a method.
- An instance of a class.
- A function that performs a specific task.
- A blueprint for creating classes.
- An abstract data type.

What is a class in object-oriented programming?

Vælg en svarmulighed

- A function that performs a specific task.
- A method for automatic memory allocation.
- An instance of a variable.
- A blueprint for creating objects.

Memory

Answer the following questions about memory.

Dynamic memory allocation is used in C but not in C++.

Vælg en svarmulighed

- True
- False

Which of the following formats is the most memory efficient for storing a sparse matrix with a large number of rows but few columns?

Vælg en svarmulighed

- ◯ Coordinate List (COO) (if $N \leq n+1$)
- ◉ Compresed Sparse Column (CSC)
- ○ Compresed Sparse Row (CSR)

$m \times n$ matrix with N nonzero entries

COO: N triplets (i,j,v)

CSC: N pairs (i,v)

$n+1$ col. "offsets"

CSR: N pairs (j,v)

$m+1$ row "offsets"

What is the primary purpose of cache memory in a computer system?

Vælg en svarmulighed

- [x] To reduce the latency of memory access.
- [ ] To increase the storage capacity of the main memory.
- [ ] To store data that is too large to fit in the main memory.
- [ ] To parallelize computations.

A cache miss occurs when

Vælg en svarmulighed

- the cache is full.
- the cache is empty.
- the requested data is found in the cache.
- the requested data is not found in the cache.

Recall the BLAS function **dscal**, which scales a vector by a scalar and has the following prototype:

```
void dscal(int n, double alpha, double *x, int incx);
```

cblas_

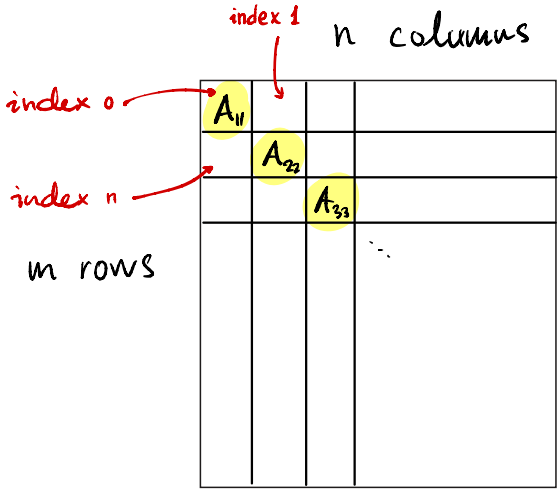
Let **A** be a pointer to the first element of a two-dimensional array of length mn, representing an $m \times n$ matrix A stored in row-major order.

Assuming that $m \geq n$, which of the following BLAS calls scales the diagonal entries of the matrix (i.e., the elements $A_{11}, \ldots, A_{nn}$) by a scalar α?

Vælg en svarmulighed

- dscal(n, alpha, A, n+1)
- dscal(n, alpha, A, n)
- dscal(n, alpha, A, m+1)
- dscal(n, alpha, A, m)

Row major storage



A11, A22, A33, ...

stride n+1