# Funktioner

sin	o/h	$\sin \theta = \cos \left(\frac{\pi}{2} - \theta\right) = \frac{1}{\csc \theta}$
cos	a/h	$\cos \theta = \sin \left(\frac{\pi}{2} - \theta\right) = \frac{1}{\sec \theta}$
tan	o/a	$\tan \theta = \frac{\sin \theta}{\cos \theta} = \cot \left(\frac{\pi}{2} - \theta\right) = \frac{1}{\cot \theta}$
cot	a/o	$\cot \theta = \frac{\cos \theta}{\sin \theta} = \tan \left(\frac{\pi}{2} - \theta\right) = \frac{1}{\tan \theta}$
		$\sec \theta = \csc \left(\frac{\pi}{2} - \theta\right) = \frac{1}{\cos \theta}$
csc	h/a	$\csc \theta = \sec \left(\frac{\pi}{2} - \theta\right) = \frac{1}{\sin \theta}$

#### Satser

Sinussatsen

$$\frac{\sin \alpha}{a} = \frac{\sin \beta}{b} = \frac{\sin \gamma}{c}$$

Cosinus sats en

$$a^{2} = b^{2} + c^{2} - 2bc \cdot \cos(\alpha)$$
$$b^{2} = a^{2} + c^{2} - 2ac \cdot \cos(\beta)$$
$$c^{2} = a^{2} + b^{2} - 2ab \cdot \cos(\gamma)$$

Areas at sen

$$\frac{ab\sin\gamma}{2}=\frac{ac\sin\beta}{2}=\frac{bc\sin\gamma}{2}$$

 $Hj\ddot{a}lpvinkelmetoden$ 

$$\begin{split} a\sin x + b\cos x &= \\ &= \sqrt{a^2 + b^2} \sin \left( x + \arcsin \left( \frac{b}{\sqrt{a^2 + b^2}} \right) \right) \\ &= \sqrt{a^2 + b^2} \sin \left( x + \arccos \left( \frac{a}{\sqrt{a^2 + b^2}} \right) \right) \end{split}$$

Identiteter

$$(\sin x)^{2} + (\cos x)^{2} = 1$$
$$(\sec x)^{2} - (\tan x)^{2} = 1$$
$$(\csc x)^{2} - (\cot x)^{2} = 1$$

# Vinklar

$$\begin{array}{|c|c|c|c|c|c|c|c|}\hline & 0 & \frac{\pi}{12} & \frac{\pi}{6} & \frac{\pi}{4} & \frac{\pi}{3} & \frac{5\pi}{12} & \frac{\pi}{2}\\ \hline \sin & 0 & \frac{\sqrt{6}-\sqrt{2}}{4} & \frac{1}{2} & \frac{\sqrt{2}}{2} & \frac{\sqrt{3}}{2} & \frac{\sqrt{6}+\sqrt{2}}{4} & 1\\ \hline \cos & 1 & \frac{\sqrt{6}+\sqrt{2}}{4} & \frac{\sqrt{3}}{2} & \frac{\sqrt{2}}{2} & \frac{1}{2} & \frac{\sqrt{6}-\sqrt{2}}{4} & 0\\ \hline \tan & 0 & 2-\sqrt{3} & \frac{\sqrt{3}}{3} & 1 & \sqrt{3} & 2+\sqrt{3} & \infty\\ \hline \cot & \infty & 2+\sqrt{3} & \sqrt{3} & 1 & \frac{\sqrt{3}}{3} & 2-\sqrt{3} & 0\\ \hline \sec & 1 & \sqrt{6}-\sqrt{2} & \frac{2\sqrt{3}}{3} & \sqrt{2} & 2 & \sqrt{6}+\sqrt{2} & \infty\\ \hline \csc & \infty & \sqrt{6}+\sqrt{2} & 2 & \sqrt{2} & \frac{2\sqrt{3}}{3} & \sqrt{6}-\sqrt{2} & 1\\ \hline \end{array}$$

### Vinkeltransformationer

Addition

$$\sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta$$
$$\cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta$$
$$\tan(\alpha \pm \beta) = \frac{\tan \alpha \pm \tan \beta}{1 \mp \tan \alpha \tan \beta}$$

Halva vinkeln

$$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}} \qquad \cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$$
$$\tan\frac{\theta}{2} = \csc\theta - \cot\theta = \frac{\sin\theta}{1+\cos\theta} = \frac{1-\cos\theta}{\sin\theta}$$

Dubbla vinkeln

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$\cos 2\theta = (\cos\theta)^2 - (\sin\theta)^2$$

$$\tan 2\theta = \frac{2\tan\theta}{1 - (\tan\theta)^2}$$

 $Trippla\ vinkeln$ 

$$\sin 3\theta = 3(\cos \theta)^2 \sin \theta - (\sin \theta)^2$$
$$= 3\sin \theta - 4(\sin \theta)^3$$
$$\cos 3\theta = (\cos \theta)^3 - 3(\sin \theta)^2 \cos \theta$$
$$= 4(\cos \theta)^3 + -3\cos \theta$$

# Arcusfunktionerna

$$\sin^{-1} x + \cos^{-1} x = \pi/2$$

$$\tan^{-1} x + \cot^{-1} x = \pi/2$$

$$\tan^{-1} x + \tan^{-1} 1/x = \begin{cases} \pi/2 & \text{if } x > 0\\ -\pi/2 & \text{if } x < 0 \end{cases}$$

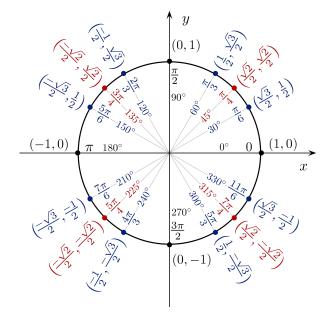
$$\sin[\arccos(x)] = \sqrt{1 - x^2} \qquad \tan[\arcsin(x)] = \frac{x}{\sqrt{1 - x^2}}$$

$$\sin[\arctan(x)] = \frac{x}{\sqrt{1 + x^2}} \qquad \tan[\arccos(x)] = \frac{\sqrt{1 - x^2}}{x}$$

$$\cos[\arctan(x)] = \frac{1}{\sqrt{1 + x^2}} \qquad \cot[\arcsin(x)] = \frac{\sqrt{1 - x^2}}{x}$$

$$\cos[\arcsin(x)] = \sqrt{1 - x^2} \qquad \cot[\arccos(x)] = \frac{x}{\sqrt{1 - x^2}}$$

# Enhetscirkeln med standardvinklar



Trigonometri Referens, trigref.pdf v. 0.1 Viktor Qvarfordt – September 19, 2011