Homework Rakhmatullin

#1 $S = 2\pi r^2 + 2\pi r \cdot h$; $V = \pi r^2 h \Rightarrow h = \frac{V}{2\pi r^2}$

$S = 2\pi r^2 + \frac{2V}{r}$; $\frac{dS}{dr} = 4\pi r - \frac{2V}{r^2}$;

$S'(r) = 0$; $4\pi r^3 - 2V = 0 \Rightarrow r = \sqrt[3]{\frac{V}{2\pi}}$;

if $r < \sqrt[3]{\frac{V}{2\pi}}$ then $S'(r) < 0$ else $S'(r) > 0$

$r = \sqrt[3]{\frac{V}{2\pi}}$ is the minimum arg

Answer: $r = \sqrt[3]{\frac{V}{2\pi}}$; $h = \frac{V}{\pi \sqrt[3]{\left(\frac{V}{2\pi}\right)^2}}$

#2 $f(x_1, x_2) = \frac{3}{2}(x_1^2 + x_2^2) + (1+a) x_1 x_2 - (x_1 + x_2) + b$

$f'_{x_1} = 3x_1 + (1+a) x_2 - 1$; $f''_{x_1 x_1} = 3$

$f'_{x_2} = 3x_2 + (1+a) x_1 - 1$; $f''_{x_2 x_2} = 3$

$f''_{x_1 x_2} = 1 + a$

Necessary condition $\begin{cases} f'_{x_1} = 0 \\ f'_{x_2} = 0 \end{cases}$ $\begin{pmatrix} 3 & 1+a \\ 1+a & 3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$|A| = 9 - (1+a)^2$, $\Delta_1 = 2 - a$ $\quad x_1 = \frac{2-a}{9 - (1+a)^2}$; $x_2 = \frac{2-a}{9-(1+a)^2}$

$\Delta_2 = 2 - a$

Sufficient condition:

$\begin{vmatrix} 3 & 1+a \\ 1+a & 3 \end{vmatrix} = 9 - (1+a)^2 = -(a-2)(a+4) > 0$

$(a-2)(a+4) < 0$

$+ \quad -4 \quad - \quad 2 \quad +$

Answer: $a \in (-4, 2)$ $b \in (-\infty, \infty)$

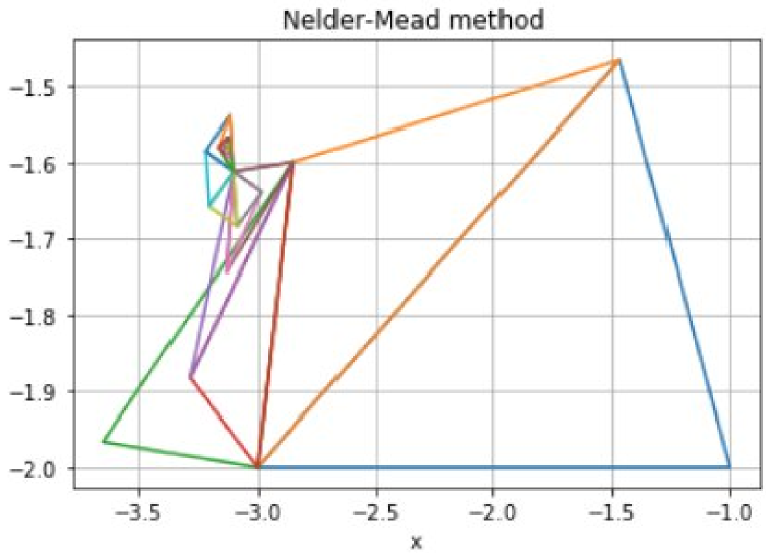
Task #3

Given:

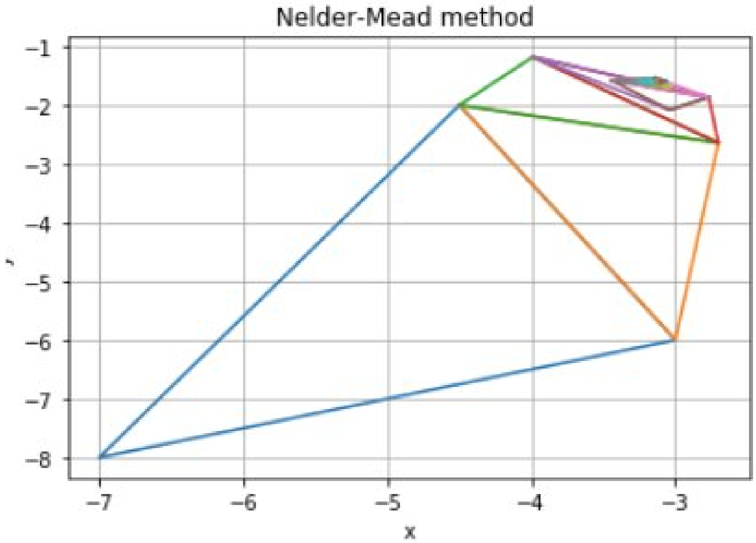
eps = 1e-6
refl, expan, contr = 1, 0.5, 2
simplex = list([(-1.0,-1.0),(-1.0,-2.0), (-3.0,-2.0)])

Result:

func_min = -106.76453076037151   iterations = 53
test point ([-3.13004039 -1.58210909])

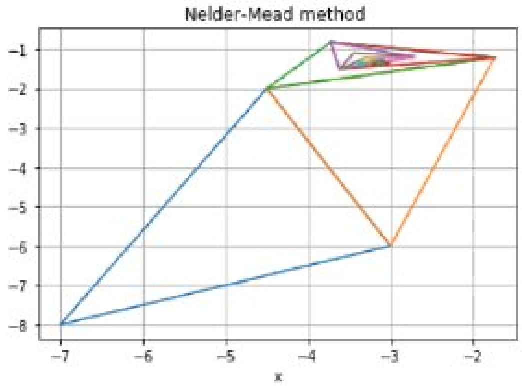


Given:
eps = 1e-6
refl, expan, contr = 1, 0.5, 2
simplex = list([(-4.5,-2.0),(-3.0,-6.0), (-7.0,-8.0)])
Result:
func_min = -106.76453674925574 iterations = 47
test point [-3.13024566 -1.58214159]

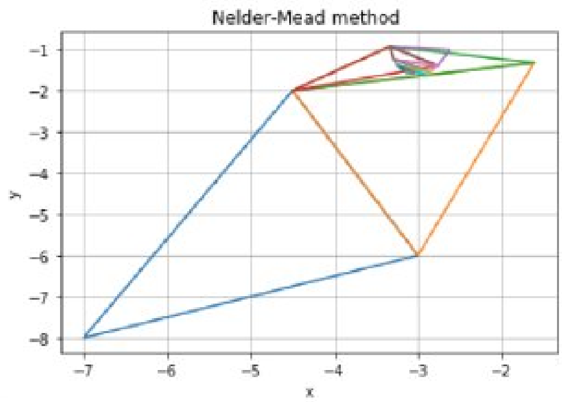


Given:
eps = 1e-6
simplex = list([(-4.5,-2.0),(-3.0,-6.0), (-7.0,-8.0)])

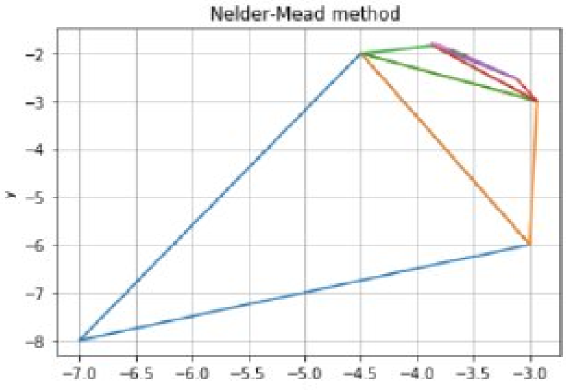
#	F min	Last Point	reflection	expansion	contraction	iterations
1	-106.764536749	-3.13024477 -1.58214304	1	0.5	2	47
2	-102.027	-3.0997595 -1.39631015	2	0.25	1.5	95
3	-106.66637667	-3.1189989 -1.55781545	0.75	0.75	1.75	69
4	-63.867308	-3.58028,-2.037996	0.25	0.75	1.2	25
5	-106.76453	-3.13024477 -1.58214304	1.5	1.75	4.2	64



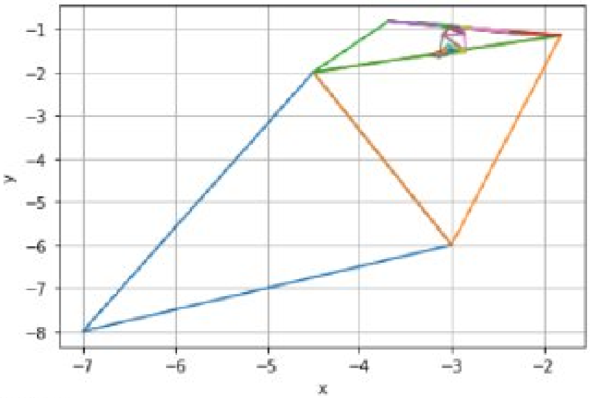
Case 2: min = -102.027



Case 3: min =-106.666



Case 4: min = `-63.867`



Case 5

Simplex from task #3: simplex =([(-4.5,-2.0),(-3.0,-6.0), (-7.0,-8.0)])
The best simplex vertex from task 3 : (-4.5,-2.0) is selected for Task#4

alpha	steps	Converged to point	Func min
0.1	109	-5.37626007, -5.61627474	1.487
0.01	100000 (stuck)	[-2.75568398 -1.91600928]	-77.875
0.001	72	[-3.13062191 -1.58221856]	-106.7645
0.002	34	[-3.13043115 -1.5821679]	-106.764531
0.005	18 (best result)	[-3.13056379 -1.5821175]	-106.7645228
0.0075	192	[-3.25394942 -1.58329201]	-104.68213
0.0001	684	[-3.13169126 -1.58250793]	-106.764232
e10-5	6026	[-3.1349531 -1.58332265]	-106.76131

Conclusion:

Nelder-Mead algorithm rarely exceed 100 iterations in the conducted experiments while coordinate descent may take more than 1000 and even stuck.
Both algorithms are sensitive to initial guesses and selected coefficients and can easily converge to local minimum.