Independent Set Lab Report

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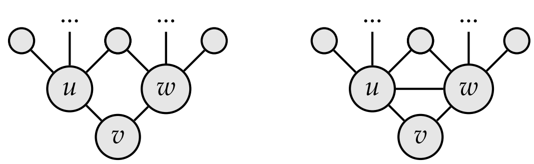
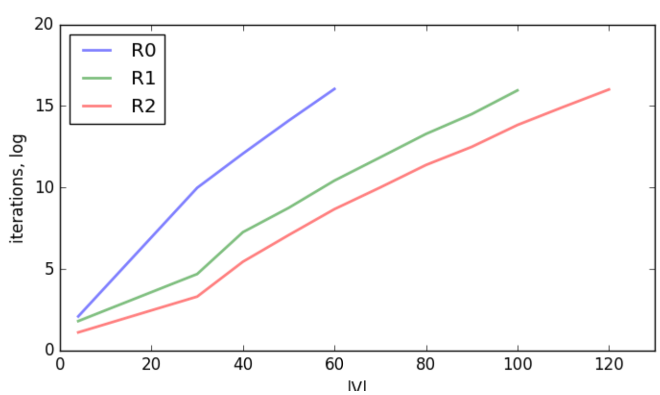
Correctness

Algorithm R1 correctly computes $\alpha(G)$ because at most one of the nodes u and v can be chosen, where the nodes from removing v, R_v, is guaranteed to be a subset of R_u, and removing v can be done without affecting the remaining graph. Hence if neither u, nor v are chosen, v can always be included in the independent set.
Algorithm R2 correctly computes $\alpha(G)$ because from u, v, and w, two nodes can be chosen if and only if nodes u and w are selected to the independent set, and there is no edge between u and w. Choosing two nodes therefore implies that the independent set cannot include any node in N[u] or N[w]. If only one node is included in the independent set, then the node can always be chosen as v. A choice to include the super node z to the independent set, results in excluding both N[u] and N[w] from the independent set.



Empirical Running time

Experiments.



The running times of algorithm R_0, R_1, and R_2 appear to be $O(1.31^n)$, $O(1.17^n)$, and $O(1.14^n)$, respectively.

Theoretical Upper Bound

Denote be $T_i(n)$ the worst runtime of algorithm Ri on *any* graph on n vertices. Note that $T_i(n)$ is a non-decreasing function of n. For R_0 we can conclude that

$$\begin{aligned} T_0(n) &\leq \max(T_0(n-1), T_0(n-1) + T_0(n-1-d_{\text{max}})) \\ &\leq T_0(n-1) + T_0(n-2) \end{aligned}$$

with d_{max} the degree of the vertex we branch on. The hard part is the one when there are no isolated vertices, in which case the vertex u we are branching on has at least one neighbor.

For R1 we have that

$$\begin{aligned} T_1(n) &\leq \max(T_1(n-1), T_1(n-1) + T_1(n-1-d_{\text{max}}), T_1(n-2)) \\ &\leq T_1(n-1) + T_1(n-2) \end{aligned}$$

For R2 we have that

$$T_2(n) = T_2(n-1) + T_2(n-2)$$

Worst Case Upper Bound The running times of algorithm R_0, R_1, and R_2 are in [...], [...], and [...], respectively.[1].

[1] Replace the [...] by a function of n on the form $O(c^n)$. Use the recursive bounds you've derived above. *Hint: A recurrence of the form* $T(n) \leq \sum_{i=1}^{k} a_i T(n-i)$ *is called a linear homogeneous recurrence relation with constant coefficients. To solve it, you can set* $T(n) \leq c^n$ *where* c *is the largest real root to the characteristic polynomial* $x^k - \sum_{i=1}^{k} a_i x^{k-i}$.