BANK FOR INTERNATIONAL SETTLEMENTS

BIS Working Papers

No 101

Can liquidity risk be subsumed in credit risk? A case study from Brady bond prices

by Henri Pagès

July 2001

BANK FOR INTERNATIONAL SETTLEMENTS

BIS Working Papers

No 101

Can liquidity risk be subsumed in credit risk? A case study from Brady bond prices.

by Henri Pagès

July 2001

Abstract

The paper applies a reduced-form model to uncover from secondary market's Brady bond prices, together with Libor interest rates, how the risk of sovereign default is perceived to depend upon time. The methodology is implemented on a particular issue, a discount bond issued by Brazil and maturing in April 2024. It is shown that subsuming liquidity risk in default risk may result in a misspecified model that, while generating the desired negative correlation between credit spreads and default-free interest rates, also generates negative probabilities of default at long horizons.

Copies of publications are available from:

Bank for International Settlements
Information, Press & Library Services
CH-4002 Basel, Switzerland

E-mail: publications@bis.org

Fax: (+41 61) 280 9100 and (+41 61) 280 8100

This publication is available on the BIS website (www.bis.org).

ISSN 1020-0959

Table of Contents

1. Introduction[1]

In this paper I strive to explain the price movements of a specific Brady bond issue. The idea is to fit the joint Libor term structure and discount Brady bond prices to a reduced-form debt valuation model. A measure of sovereign risk can then be defined as the default risk process which is consistent with the data given the model. Unlike the traditional contingent claims approach, which links default to the variability of debtor-specific variables, the model used here is based on the recent hazard rate approach, which views default as the first jump in an exogenous Poisson process.

Extending the reduced-form model of Duffie and Singleton (1999) to allow for security-specific variation in the returns of bonds is a natural way to learn how credit and liquidity risks are actually priced in markets.[2] This alternative approach lacks a structural interpretation. However, it seems difficult to model realistically the boundary conditions which trigger default, especially in the case of sovereign debtors. Many economic and political factors affect a country's ability or even willingness to make debt payments. In this respect, the advantage of a reduced-form approach is its ability to concentrate on a country's sovereign risk, without having to specify the conditions under which default actually occurs and, in the event of default, how an agreement is reached between debtors and creditors.

The reason to focus on Brady bonds rather than other instruments is their liquidity. Brady bonds are now surpassed by Eurobonds in terms of outstanding volume, but they remain the most actively traded issues in secondary markets for developing country debt.[3] The instrument under review is the most liquid of discount bonds, Brazil Brady bond due 15 April 2024.[4] The choice of Brazil is made in light the January 1999 crisis, but the methodology could be applied to other Brady issues as well. Unfortunately, the need to deal in a consistent fashion with Brady bonds' various credit enhancements has prevented me from implementing it on a larger scale.[5]

There are two goals to this exercise. One is to determine whether sovereign risk is correlated with factors driving the default-free term structure. To this end, I follow a suggestion made by Duffie and Singleton (1997) by allowing sovereign risk to depend on the riskless interest rate. More specifically, the sovereign interest rate is decomposed into the sum $R = Br + h^*$, where r is the Libor interest rate and h^* an autonomous component which can be interpreted as a country-specific default risk factor or an "emerging markets factor" common to developing economies. The other is to investigate the implications of the estimated hazard rate process for the term structure of expected default frequencies. Indeed, the possibility to rank countries by risk according to the horizon of interest is one of the obvious advantages of the method.

The main results are the following. The effect of higher money-centre rates is to rise the sovereign interest rate less than proportionately, ie $B < 1$. The induced negative relationship between credit spreads and default-free interest rates, which may reflect the incidence of liquidity, is in line with the empirical literature. This implies that he short spread $R - r$ can become negative when r is sufficiently high and h^* sufficiently low.[6] Such a problem is common in term structure modelling, where the possibility of negative interest rates is often regarded as a small price to pay as long as this probability

1 I thank Joe Bisignano, Claudio Borio, Craig Furfine, Srichander Ramaswami, Eli Remolona and seminar participants at a BIS workshop in July 2000 for helpful comments and suggestions. This research was carried out while the author was at the Bank for International Settlements. The views stated herein are those of the author and not those of the Bank for International Settlements, the Banque de France or the European System of Central Banks.

2 Recent examples include Duffee (1999), Duffie and Singleton (1997), Duffie, Pederson and Singleton (2000).

3 The situation is changing progressively as more sovereigns retire par and discount Brady bonds, either through public exchanges or through informal buy-backs on the secondary market.

4 This issue ranked fifth among Euroclear's Most Actively Traded Issues on emerging markets in August 1999, seventh in May 2000.

5 A classical reference for the derivation of default probabilities implicit in the price of Brady bonds is Claessens and Pennacchi (1996), who construct an indicator of Mexico's repayment capacity taking explicitly into account the value of interest and principle guarantees. For other reduced-form approaches which do not assume a fully specified hazard rate process, see Bhanot (1998) and Izvorski (1998).

6 Duffie, Pedersen and Singleton (2000) link negative short spreads in the market for MinFins with the possibility of international investors "over pricing" Russian securities.

is low and the model accurate. However, the paper argues that he desired negative correlation may generate negative probabilities of default at longer horizons, even when estimated yield spreads remain positive at all horizons. This troubling result is a feature of the extended affine model with negative correlation, which does not necessarily reflect shortcomings in the market's assessment of risk.

The paper is organised as follows. Section 2 describes the model. Econometric results are presented in Section 3. The ability of the model to describe the term structure of expected default frequencies is discussed, and the section closes with a comparison between the implied hazard rate and other indicators of sovereign repayment capacity. Section 4 concludes the paper.

2. Pricing discount Brady bonds

In the reduced form approach, a defaultable security is defined as a promised stream of payment and a random time at which the issuer defaults. When default is unpredictable, agents can form expectations about its likelihood and the resulting hazard rate varies stochastically through time. Duffie and Singleton (1997) show how to parameterise the joint behaviour of the default-free short-term interest rate and the hazard rate. In the implementation to follow neither process is directly observable. One is the instantaneous rate implied by the Libor term structure. The other is the instantaneous probability of a country missing a debt payment, given that it has not defaulted before, as implied by the price of a Brady bond.

Duffie and Singleton (1997, 1999) actually distinguish the hazard rate from the fractional loss given default. In this case there are actually three risks: the interest rate, the event of default and the loss given default. The assumption of no-recovery on default is frequently maintained in the valuation of defaultable bonds In order to avoid the difficult task of modeling an additional source of uncertainty, I am content here with assuming that there is no recovery upon default, thus identifying the "mean loss rate" with the hazard rate. This arbitrary identification assumption must be kept in mind when interpreting the computed hazard rate as a likelihood of default.

2.1 Building blocks

Suppressing for convenience the dependence on time, I write the interest rate and hazard rate processed r and h as "affine" functions of two factors z_1 and z_2, representing the interest rate risk and the "emerging markets risk", respectively:

$$r = \alpha_L + \beta_L z_1 \tag{1}$$

$$h = \Delta\alpha + \Delta\beta_{z1} + \beta_h z_2 \tag{2}$$

where the Greek letters are parameters to be estimated. Pagès (2000) provides some discussion of the relevant of one-factor models in the context of estimating the Libor term structure. The risk factors are themselves assumed to follow the CIR (1985) "square-root" specification

$$dz_i = K_i(\theta_i - z_i)dt + z_i^{1/2}\, dw_i \tag{3}$$

where the w_i are two independent standard Brownian motions, the θ_i steady-state means and the K_i mean reversion parameters. The z_i processes are used as metaphors for the arrival of information, possibly embodying news or expectational facts related to the country's, as well as its creditors', position in some "reduced form" fashion. One advantage of this specification, besides yielding transition densities in closed-form at any frequency, is that it allows for a time-varying term premium. It thus provides a theoretical decomposition of forward rates into expectations and risk premiums directly.

The formulation (1-2) points to some interest rate dependence in the risk of default, but not the other way around. This assumption could be justified if debt payment crises in emerging countries were found to have little adverse impact on the Eurodollar market. If, on the other hand, events in the crisis countries had systemic effects in Western countries, for example by affecting the funding cost of a significant number of exposed banks, such an assumption would be clearly inappropriate. Kho, Lee and Stulz (2000) show in this respect that the market may distinguish well between exposed and non-exposed banks when a debt event occurs. They find no evidence of a significant effect of crisis events on non-exposed banks, implying that their impact on the prime Libor rate might be subdued.

From (1-2), a default-adjusted discount rate can be defined as $R = r + h$, yielding

$$R = \alpha_B + \beta_B z_1 + \beta_h z_2 \tag{4}$$

where $\alpha_B = \alpha_L + \Delta\alpha$ and $\beta_B = \beta_L + \Delta\beta$. It should be emphasised that, in the present setup, *h* represents both the empirical and the risk-neutral probabilities of instantaneous default.[7] Intuitively, the jump carries no risk premium as long as it is not a defining part of the information driving bond prices. If, for example, market participants were suddenly more apprehensive about risk as default would take place, pulling back from their positions across the sovereign bond market, the risk-neutral density would depend on the unpredictable time of default. It would no longer be adapted to the filtration generated by the factors and the empirical and risk-neutral instantaneous probabilities of default would differ. Details can be found in Jarrow and Yu (1999).

Correlation between interest and default risk is controlled by the coefficient $\Delta\beta$ in (2). When $\Delta\beta = 0$, the default risk decouples from the bond pricing. In this special case, the market value of a cash flow decomposes into the product of the price a default-free zero-coupon and a probability of survival at the horizon of the cash flow. Market participants are then justified in deriving a direct measure of the debtors' underlying default probability from the spread between the defaultable and the default-free term structures.

When $\Delta\beta \neq 0$, it is no longer possible to "read" spreads as if they revealed the market's perception of the likelihood of default, because the hazard rate process *h* depends on the default-free interest rate *r*. The formulas for the price default-free and defaultable zero-coupon bonds are given in Appendix A.

2.2 Rolling guarantee

Most Brady bonds possess collateral accounts in the event interest or principal payments are missed. For dollar-denominated issues, the collateral posted for the principal generally consists in zero-coupons issued by the US Treasury for a comparable maturity (usually 30 years after issuance). These securities are often special non-marketable assets issued with the IMF's seal of approval. An additional collateral, taking the form of high-grade assets, secures interest for either 12, 14 or 18 months and is "rolled" over the next period if the payment is made.

The valuation of the interest collateral receives different treatments in practice because there is uncertainty as to when the guarantee is actually used. The 1995 Merrill Lynch Guide to Brady Bonds states:

> One method assumes *the limiting case*, which ascribes the minimal value to the collateral's current market value, effectively treating is as if there was no life to the "roll" (i.e., default occurs today and collateral is utilised and exhausted to pay the nearest 2 or 3 semi-annual interest coupons). Other methods rely on the use of probability models or option pricing formulas to assign probabilities to interest payment default, which are then incorporated in the valuation model.

Appendix B shows how to discount collateralised interest payments in the context of the present affine yield model. The formulas are then applied to the specific case of the Brazilian discount bond due 15 April 2024. Its principal is fully collateralised by US Treasury zero coupon obligations. It pays

---

7 To see this, let σ be the unpredictable time of default. With the notation used in Appendix A, the process $M_t = 1_{\{\sigma \le t\}} - \int_0^{t\wedge\sigma} h_s ds$ is a G_t - martingale under *P*; see for example Jeanblanc and Rutkowski (1999), Lemma 3.2. With a change in measure characterised by the market's valuation of risk η. Girsanov's formula says that $M_t - \int_0^t d(M,\eta)_s / \eta_s$ is a martingale under *Q*. Since *M* and η do not have common jumps (η cannot jump at σ), the last term vanishes and *M* is a martingale under both *P* and *Q*. Thus, *h*, is invariant to the change in probability from *P* to *Q*.

semiannually a floating interest rate equal to Libor plus 13/16 and is enhanced by a rolling guarantee that covers two semesters worth of interest payments. The bond is callable, but there is no further credit enhancements such as "value recovery rights" which would allow creditors to recapture part of the country's revenues, should its situation improve. If callability is neglected, the gross price of the bond can be derived as usual as the sum of the discounted prices of its individual cash flows. The clean price is then obtained by subtracting from the gross price the accrued interest.

3. Fitting the Brazilian discount bond

I follow Chen and Scott (1993) by assuming that one of the rates - six-month Libor - and the Brady bond are observed without error. This actually sidesteps the identification problem by making both factors observable variables. Their value is chosen so that the market quotes and their theoretical counterparts coincide. The remaining Libor rates are assumed to be imperfectly observed, with AR(1) measurement errors. Innovations to the measurement errors are themselves specified as independent of the factors, normally distributed and possibly cross-correlated. The estimation then proceeds under maximum likelihood based on the conditional transition of the factors and the joint distribution of the measurement errors.

3.1 Raw data

Libor rates consist of end-of-week official fixing data denominated in US dollars and obtained from Reuters from 24 October 1986 through 2 June 2000 (711 cases). The selected maturities are one, two, three, six, nine and 12 months. The British Bankers Association actually polls a sample of top-rated banks, trims the highest and lowest rates, rounds the remaining yields and averages them. This should dampen the influence of possible credit risk on official quotes.

There is no street convention for the quotation of Brady bond yields, because two of its components are calculated differently from firm to firm: the valuation of the interest collateral and the swap curve used for the determination of floaters. This is why the securities are traded on a price basis rather than on a yield basis. The paper uses end-of-week bid prices as reported by Bloomberg from 6 January 1995 through 2 June 2000 (283 cases).

The data are sampled weekly in order to reduce serial correlation resulting from infrequent trading or from the averaging of Libor quotes. Trading in both markets is not synchronised. The difference between the coupon rates reported by Bloomberg for the various settlement dates and Reuters' Libor rates is not strictly equal to the spread of 81.25 basis points. This is because the coupon is reset two trading sessions before the settlement date, as is common practice with floating rate notes. Even when the difference in timing is taken into account, discrepancies arise as Brady bond issuers seem to have referred to Libor rates that are quoted from a group of local banks rather than the group selected by the British Bankers Association.

Table 1 reports summary statistics of the Eurodollar yields, in levels, as well as prices and weekly rates of return of the bond, in log differences. The average yield curve is upward sloping, with a shape which is initially concave and then convex. Libor volatility is decreasing, with a pattern that reflects the declining influence of the factor loading as the maturity increases. Libor rates also display high persistence, as measured by the first-order autocorrelation coefficients. They are about 0.996 for rates in level and less than 0.05 for weekly changes in levels, suggesting that yields are non-stationary or borderline stationary.

The large standard deviation of the Brady bond price reflects Brazil's chequered history as a sovereign borrower. While Brady bonds are usually liquid instruments, traded in Euroclear and Cedel, the three abrupt falls occurring from October 1997 to early 1999 underscore the thin nature of the market during crisis episodes. The weekly rates of return on the discount Brady bond are quite volatile. Over the period, they fluctuate in a range between –14% to 10%. Finally, the autocorrelation coefficients are 0.984 in level and 0.07 in weekly changes. Although the relation between bond prices and factors is nonlinear, the emerging markets factor appears less persistent than money-centre rates.

Figures 1 and 2 display the six-month Libor and the Brazilian bond over their respective time intervals The evidence of the interplay between the reference rate and the Brady price is mixed.

Table 1
Summary statistics

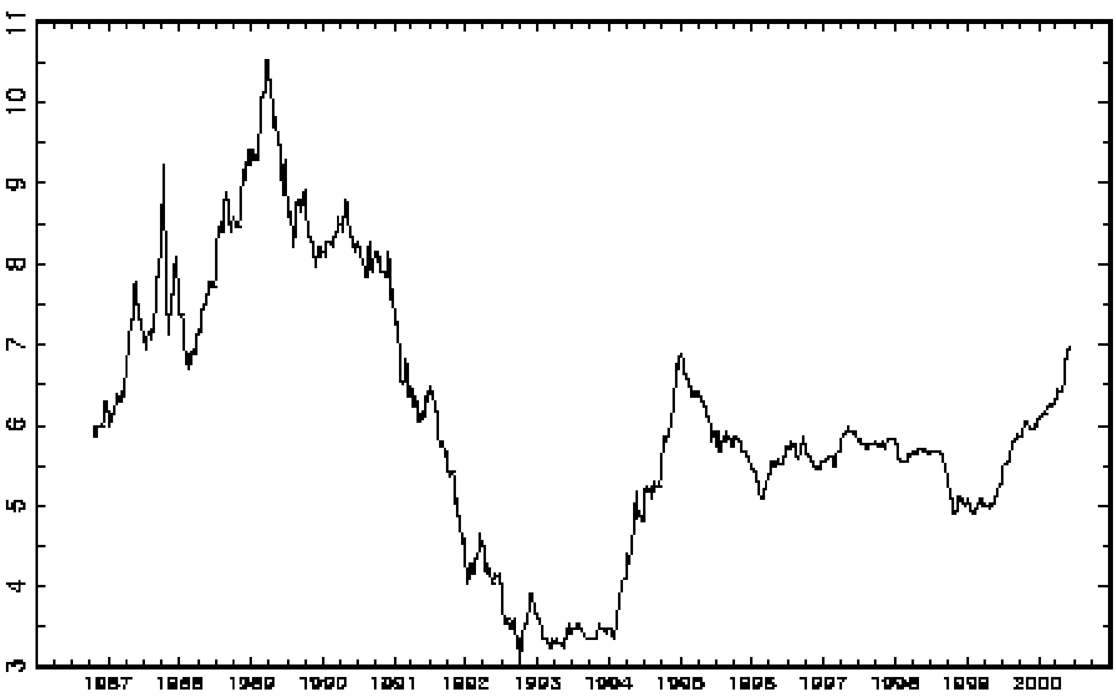
	Libor rates						Brady bond	
	1 mth	2 mths	3 mths	6 mths	9 mths	12 mths	Price	Weekly return
Mean	5.96	6.00	6.02	6.07	6.13	6.19	69.4	0.09
Std dev	1.70	1.68	1.67	1.63	1.60	1.57	10.4	2.90
Autocorr	0.995	0.997	0.997	0.996	0.997	0.996	0.984	0.07

Mean is the sample mean, Std dev is the standard deviation, Autocorr the first-order autocorrelation coefficient. There are 711 observations for the Libor rates and 283 observations for the Brady prices.

In 1995, as Brazil overcomes the "Tequila effect", the sharp increase in the price of the discount Brady bond is supported by a strong decline in US dollar interest rates. On the other hand, substantial gains have been made since the float of the real in January 1999, despite a continued surge in interest rates. It may be argued that market conditions are markedly different over the two periods, implying that the diffusion parameters could have changed accordingly. In the model of sovereign risk considered here, changes in the hazard rate are impounded entirely in the factors z_1 and z_2. The identifying assumption is that changes in those factors, not in the parameters, account for variations in sovereign spreads.

Figure 1: Six-month Libor rate

Libor 6m



3.2 Estimation

The primary difficulty encountered in the estimation is to pin down the Brady shift parameter α_B with any reliability. Although consistently negative, the computed estimates depend on their start values. For this reason, I follow Pearson and Sun (1994) and Duffee (1999) by setting the parameter to an arbitrary level. I choose $\alpha_B = -1\%$, as lower values do not lead to a noticeable improvement in fit. Table 2 reports the maximum likelihood estimates of the other parameters.

Figure 2: Brazilian discount Brady bond price

Brady bond

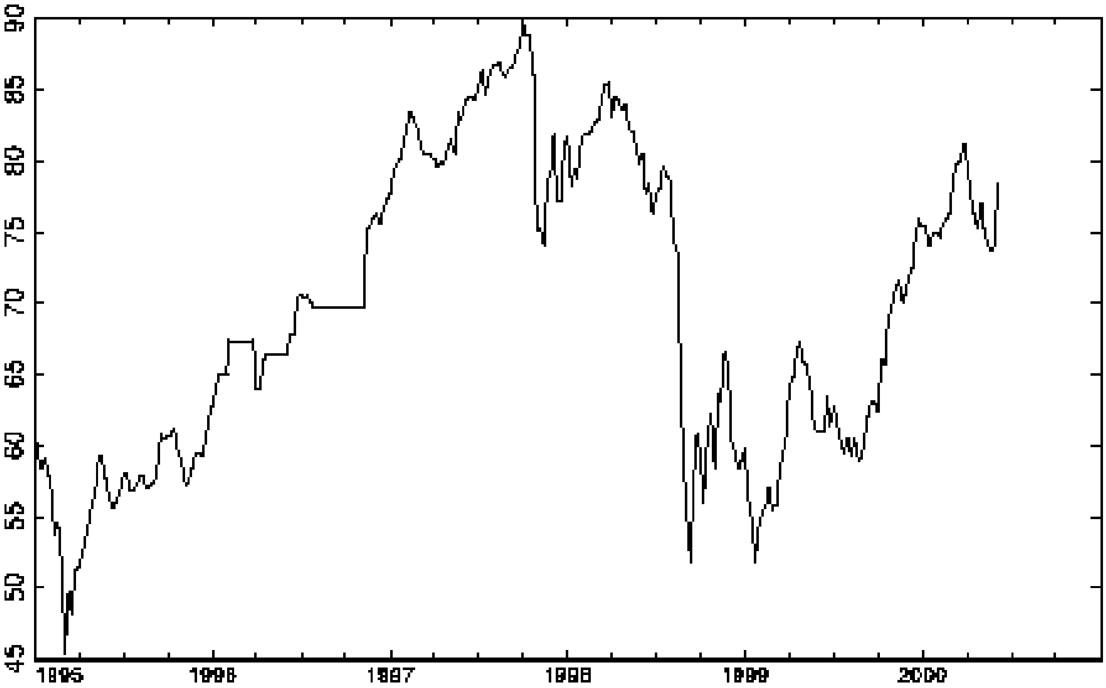


Table 2

Affine-yield model estimates

Parameter	**Estimate**	**Standard error**
κ_1	0.35	0.10
λ_1	– 0.26	0.09
θ_1	16.00	3.81
α_L	1.83	0.14
κ_2	1.14	0.09
λ_2	– 4.51	0.39
θ_2	0.44	0.03
β_L	0.31	0.01
β_B	0.11	0.19
β_h	1.01	0.89
ρ_{1m}	0.84	0.01
ρ_{2m}	0.87	0.01
ρ_{3m}	0.87	0.01
ρ_{9m}	0.89	0.01
ρ_{12m}	0.92	0.01
Mean log-likelihood	13.40	

Standard errors are computed as the outer product of the score vector. As is commonly the case in small samples, they differ from the usual Hessian matrix of the log likelihood function. They are not reported in the table because a few of them appear improbably small.

The first four estimates relate to the Libor yield curve. They are significant and consistent with the values I reported earlier with the same data but a somewhat shorter time interval; cf Pagès (2000). The mean reversion parameter κ_1 of 0.35 corresponds to a half-life of two years. The implied autocorrelation coefficient is 0.993, slightly less than the sample autocorrelations of yields at various maturities. Thus, the factor absorbs all the persistence in yields, leaving measurement errors with substantially less autocorrelation. The autocorrelation coefficients of the measurement errors, indicated by the various $\rho_{im}s$ at the bottom of the table, range between 0.84 and 0.92. This can be interpreted as follows. The theoretical spot rates are affine functions of the time-varying factor and represent the long-run means to which the observed rates revert. With a single factor, changes in the slope of the yield curve cannot be modeled separately. Instead, the autocorrelated measurement errors play the role of the additional independent factors, with faster rates of mean-reversion. The main difference with a model in which one factor would revert to a time-varying mean is that, here, arbitrage constraints are not imposed on the deviations of observed rates from their central tendencies.

The parameter λ_1 corresponds to the price of risk associated with holding Libor debt. It seems reasonable to believe that, in a risk-averse world, higher interest rates ($dr > 0$) should be associated with greater anxiety about future interest rates ($d\eta > 0$). As can be seen from (1) and (6), the correlation between shocks to the short-term Libor rate r and the market's valuation of risk η is governed by $-\lambda_1$. Table 2 confirms that λ_1 is indeed negative. The risk parameter has also a bearing on the variability of interest rates across maturity. In the single factor affine model, the term structure of volatility can move up and down following variations in the factor, but its shape is not allowed to change. It is decreasing when $\kappa+\lambda \geq 0$ and hump-shaped otherwise. It also displays exponential decay at long horizon, implying that medium-term rates may be more variable than short rates, but that their volatility eventually declines with maturity. The estimate of $\kappa_1+\lambda_1$ found here is positive and thus consistent with the decreasing pattern of standard deviation of yields reported in Table 1.

The estimated theoretical long-run mean θ_1 of 16.0 is above the population mean $\overline{z_1}$ = 12.45. It is often difficult to pin down the shift parameter α_L in the context of an affine model with two independent factors. As reported by Duffee (1999), the reason is that it is necessary to raise the "slope" risk factor to accommodate flat term structures. By contrast, the shift parameter α_L in Table 2 is positive and significant. This, at least, is a small benefit of the single factor model.

The following three parameters κ_2, λ_2 and θ_2 pertain to the emerging markets factor z_2. The mean reversion parameter κ_2 is higher than its Libor counterpart. With a half-life of 0.6 years, the second factor is significantly less persistent than the interest rate risk factor. The estimate of λ_2 is significant and negative. Given (2) and (6), the large absolute value of λ_2 means that, holding interest rates constant, higher hazard rates $(dz_2 > 0)$ impart greater anxiety in terms of the market's underlying valuation of risk $(d_\eta > 0)$. In fact, the mean reversion parameter under the risk neutral measure, $\kappa_2+\lambda_2$, is negative. As mentioned before, this implies that the volatility curve associated with the second factor is hump-shaped. A hump arises because the increase in the risk premium required to compensate investors for variations in the hazard rate more than offsets the rate at which the hazard rate itself is expected to decline.

The last three estimates correspond to the sensitivity parameters in the default-free and defaultable interest rate equations (1) and (4). As regards the first one, the equivalent CIR formulation

$$dr_t = \kappa_1\left(r^* - r_t\right)dt + \beta_L^{1/2}\sqrt{r_t - \alpha_L dw_t} \ ,$$

with $r^* = \alpha_L + \beta_L\theta_1$, shows that the first parameter, β_L , is a scale parameter for both the conditional variance and the steady-state mean of the Libor instantaneous interest rate. The estimate is statistically significant. On the other hand, the model is unable to determine with any precision the coefficients β_B and β_h, which govern the relation between default intensities, the Libor term structure and the emerging markets factor. Table 1 documents a lower β_B, the sensitivity of the risk-adjusted interest rate to z_1, than β_L, the corresponding sensitivity of the default-free interest rate. As can be seen from (2), this implies that the hazard rate is negatively related to variations in the Libor term structure. Holding the emerging markets factor constant, a one percentage point increase in the instantaneous Libor rate lowers the hazard rate by about 60 basis points.

Supporting empirical evidence of a negative correlation between default intensities and default-free interest rates is also found in Duffee (1999). Eichengreen and Mody (1998) and Kamin and Kleist (1997). It can be interpreted in terms of supply (higher US interest rates discourage emerging countries from issuing new debt, producing excess demand for sovereign bonds) or as a flight to safety phenomenon (higher probability of default triggers the retrenchment of capital flows, pushing US interest rates down). A more structural explanation is provided by Longstaff and Schwartz (1995), who assume that changes in interest rates influence the borrower's liability side, but leave unaffected the assets side of the balance sheet. A conjecture put forth by Izvorsky (1998) is that spreads in emerging countries are compressed when US rates rise because investors submit them to tighter scrutiny. On the other hand, the absence of a significant effect suggests that counterbalancing effects could prevail over other periods, such as the search for high yields (investors shift into emerging markets when money centre rates decline, lowering the hazard rate).

3.3 Possible sources of misspecification

The failure of the model to accurately estimate the last two parameters indicates that the available data do not reflect well the features of the defaultable yield curve which it predicts. There are several possible explanations. First and foremost, the observation of a single bond on the term structure is not enough to approach a reasonable level of precision in the parameter estimates. The method leans heavily on theory so as to require less data but, given our ignorance of the market's pricing behaviour, a highly non-linear model based solely on the absence of arbitrage opportunities is unlikely to be successful. Second, the affine-yield model may have implausible implications for the cross-sectional and time series behaviour of yields. For example, the impact functions $\beta_B(T)$ and $\beta_h(T)$ tend to finite limits as the maturity lengthens. This implies that the factor loadings $\beta_B(T)/T$ and $\beta_h(T)/T$ eventually vanish as T increases, forcing the spot rates at the longer end towards time-invariant constants. While this is not a concern when modelling the small stretch spanned by the Libor curve, the situation is different when pricing sovereign bonds with maturities of up to 30 years. Third, the demands of tractability require strong assumptions which inevitably entail some risk of misspecification. For example, in (1), the Libor interest rates are unaffected by changes in the emerging markets factor z_2. Conversely, the time of default is constructed on an independent space and thus unaffected by the path followed by the Libor interest rates. Perhaps more importantly, in (6), the "pricing kernel" η is specified as a function of the fundamentals and does not adjust upon default. It would be more realistic to assume that earlier private misjudgements could lead to jumps in the market's valuation of risk, as investors recover far less that they had bargained for during previous expansions in capital inflows.

Further insights into the risk of misspecification come from examination of the determinants of default. In (2), the hazard rate is correlated with the factor driving the Libor term structure. More generally, there are factors other then default risk that could play a role in the determination of sovereign spreads, such as the relative liquidities of the international and US bond markets. For example, working with spreads for maturities of two through 10 years, Grinblatt (1995) argues that the variation in spreads is driven by a liquidity yield to Treasuries rather than default risk. In the hazard rate approach, however, these alternative explanations are always subsumed into a risk- and liquidity-adjusted short-rate process called the "hazard rate". By allowing factors to influence pricing indirectly through the hazard rate, the approach implicitly presumes that the term structure of credit spreads originates solely from the likelihood of default. Under this pure credit risk hypothesis, there is no room from an impact of factors on pricing other than through the hazard rate.

To understand how this relates to the consistency of the model, consider again (2) which, given the estimates in Table 2, becomes

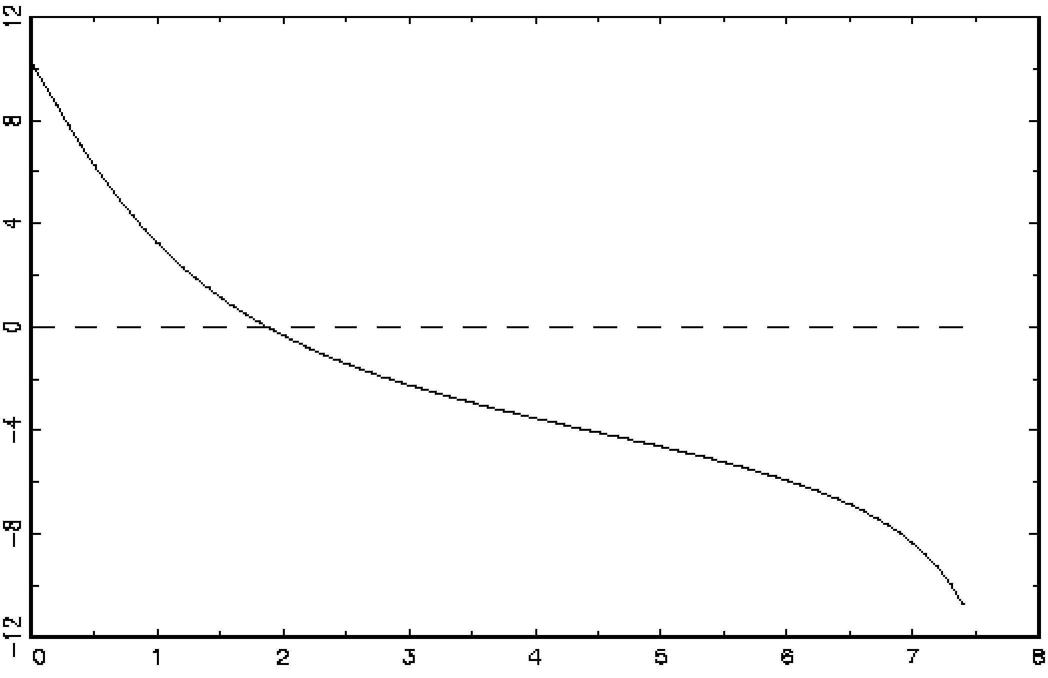
$$h = -2.8 - 0.20 z_1 + 1.01 z_2 \tag{5}$$

With $\overline{z_1} = 11.2$ (from 1995 to 2000) and $\overline{z_2} = 15.3$, the mean fitted hazard rate for Brazil is about 10.3% per annum. Provided z_1 is sufficiently high or z_2 sufficiently low, equation (5) allows for a negative hazard rate through the negative constant term and the negative correlation between interest rates and default. This problem is common in term structure modelling, where the possibility of negative interest rates is regarded as a small price to pay as long as this probability is low and the model accurate.

The argument is not foolproof, however. Even though the hazard rates turn out to be always positive over the sample period, the expected default frequencies computed at any point in time fail to exist for long maturities. Appendix C shows that expected default frequencies run out to minus infinity over a fixed horizon, whether in actual or in risk-neutral terms, provided the impact coefficient from interest rates to hazard rates is sufficiently low and negative. Hence, it is not always possible to derive a term structure of expected default frequencies from the underlying default risk process.

The term structure of the actual expected default frequencies implied by Table 2 can be seen from Figure 3. Initially positive at 10.3%, the estimates become negative after two years and explode after eight years. The level and the slope of the term structure may vary following fluctuations in the factors, but the critical maturity does not change. Although hazard rates are invariant to the change from the historical to the risk-neutral probability, expected default frequencies at longer horizons are not. Under risk neutrality, the critical maturity slightly exceeds five years. This estimate is even lower, because the dynamics of the hazard rate is less mean-reverting when risk aversion is subsumed into the probability. In any event, the divergent path of expected default frequencies is driven by the negative correlation between interest and default risk. When discounted over successive future periods, the negative impact of interest rates produces divergent expectations over relatively large horizons. This casts doubts on the ability of the "negative affine-yield models" to price bonds accurately, to the extent that the critical maturities have the same order of magnitude as the duration of the bonds.

Figure 3: Term structure of EDF



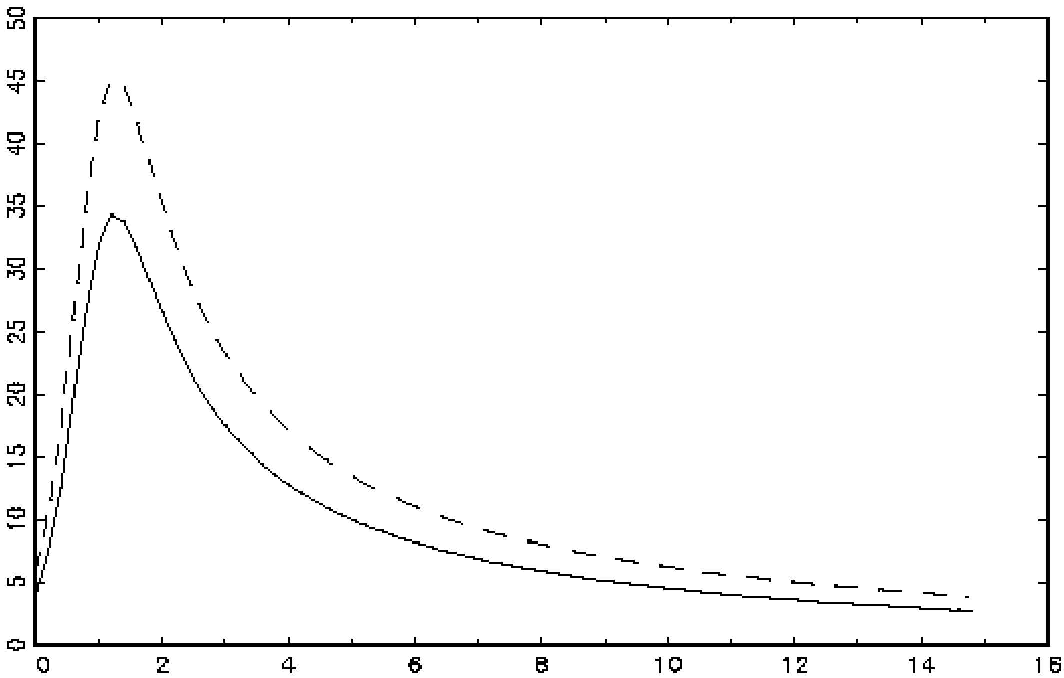
3.4 Term structure of credit spreads

While it has long been known that credit risk has a term structure, empirical analysis has tended to focus more on yield spreads. The method can generate a country-specific curve synthetically, as shown in Figure 4. The solid line represents the spread curve at the beginning of October 1997, just before Brazil is thrown into the Asian crisis contagion, and the dotted line represents the same curve as of March 2000, when the Brady bond reaches its peak of the first semester. Unlike the stylised term structure of investment grade bonds, the spread curve is hump-shaped.[8] It shows that investors price sovereign bonds as if they expected the hazard rate to rise over a horizon of two years.

One possible interpretation is that the hump reflects the institutional features of Brady bonds. The short-lived effect of the interest collateral, together with the absence of a supranational bankruptcy court, means that sovereign lenders run large risks beyond the one year horizon. Over larger horizons, the vulnerabilities of emerging markets economies may be more readily assuaged. For example, longer-dated hedging instruments may become available; creditors' bargaining power may improve as a result of the penalties they impose; the existence of a fully collateralised principal also implies that, as the residual maturity shortens, the bond becomes less risky. Hence, the credit curve pattern is not entirely unreasonable.

Finally, examination of Figure 4 gives little indication that a crisis was imminent in October 1997, as the margin between the two spread curves signals that the situation at that time was not recognised as more disturbing than in March 2000. Although it might seem that sovereign spreads would reflect advanced market information about the deteriorating conditions in Brazil in late 1997, they did not.

Figure 4: Term structure of credit spreads



3.5 Comparison with other benchmarks

The last goal is to compare the time-series behaviour of the model-implied measure of sovereign risk with other market-based indicators. Gauging the relevance of the new measure is important in light of the large imprecision in the estimates of the sovereign interest rate equation. Formal tests of parameter instability would have limited power because the parameters of interest are not statistically significant. Besides, the estimation methodology assumes that there is no measurement error on the six-month Libor and on the Brady bond price in order to identify the factors. This precludes the possibility of devising specification tests based on the orthogonality of prediction errors. Instead, I fall

[8] The empirical analysis of sovereign term structures has been hindered by sparse data. Empirical evidence about sub-investment-grade spread curves is examined by Helwege and Turner (1997) and Sarig and Warga (1989).

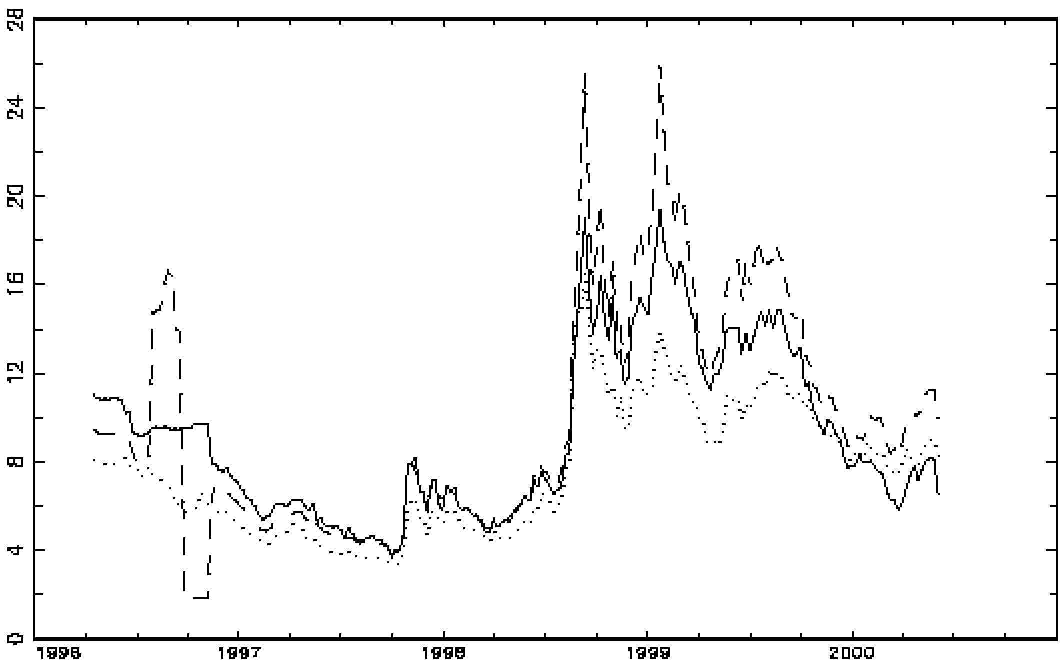
back on graphical evidence to compare the Brazilian hazard rate estimated from equation (5) with two benchmarks.

The first is the Brazilian sovereign "stripped" spread reported by Bloomberg, obtained by subtracting the value of the collateralised obligations embedded in the Brady bond price. Stripped spreads are not the same as hazard rates. They are calculated as a constant spread over a zero coupon default free curve. Viewed from a given date, the default probabilities over all future dates are taken as constant, even if they are allowed to change from day to day to match the sovereign bond price. Moreover, when the coupons are variable, all expected cash flows must be estimated from the forward rates implied by some default-free term structure, such as the US Treasury or the dollar swap curves.

The second is a broader indicator, the sovereign stripped spread associated with JP Morgan's Emerging Market Bond Index (EMBI) The index tracks total returns for traded external debt instruments from a panel of emerging countries, reflecting in part the size and liquidity of three major Latin American economies.[9]

Figure 5 plots these three different measures of sovereign risk. The continuous line is the hazard rate, the dashed line corresponds to Brazil's Brady 2024 stripped spread,[10] and the dotted line represents JP Morgan's EMBI spread. The comovements between the three time series are pronounced, as peaks and troughs are synchronised with the price reversals in emerging markets bonds. From the end of 1996 through 1999, the hazard rate and the Brady 2024 spread series are closely knit. After 1999, the rising trend in Libor rates exerts a dampening influence on the hazard rate, which lies below the sovereign stripped spread. In early 1996, the opposite situation prevails. The Brady 2024 spread has a tendency to overshoot both the hazard rate and the EMBI spread, at least during the period from the Russian default to the free float of the real. This of course reflects the influence of other emerging markets - like Poland and, to a lesser extent, the Philippines - which performed comparatively well. The EMBI spread is a lower bound, lying about 2% below on average.

Figure 5: Risk of sovereign default



[9] These countries are Argentina, Brazil and Mexico. The other non-Latin countries are Bulgaria, Morocco, Philippines, Poland, Russia and South Africa.

[10] The first observation, April 1996, determines the size of the sample.

This graphical evidence is generally borne out by the results of Table 3. The three estimates of default probability have a high cross correlation, with coefficients in excess of 90%. In light of the country composition of the EMBI, this points to the influence of an emerging markets factor, rather than a country-specific factor, on the Brazilian sovereign spread. The Brady 2024 sovereign stripped spread is the most variable, perhaps reflecting an excess sensitivity to shifts in mood that are subsequently reversed. In terms of policy, however, it appears that the estimated hazard rate contains relatively little information in the form of risk assessment or early warning that is not already captured by the other two indicators.

Table 3

Indicators of Brazil's sovereign risk: summary statistics

	Sample mean	Standard deviation	Correlation matrix				Auto corr
			Libor 6m	Hazard rate	B2024 spread	EMBI spread	
Libor 6m	5.65	0.40	1	– 0.57	– 0.42	– 0.33	0.989
Hazard rate	9.17	3.74	– 0.57	1	0.91	0.93	0.976
B2024 spread	9.93	5.23	– 0.42	0.91	1	0.92	0.955
EMBI spread	7.61	2.92	– 0.33	0.93	0.92	1	0.975

Auto corr is the first-order autocorrelation coefficient. "Libor, 6m" is the six-month Libor "Hazard rate" the estimated hazard rate from equation (5). "B2024 spread" Bloomberg's sovereign stripped spread derived from the Brazilian discount Brady bond due April 2024 and "EMBI spread" the sovereign stripped spread derived from traded debt instruments associated with JP Morgan's Emerging Market Bond Index.

4. Conclusion

This paper tries to explain movements in Libor interest rates and Brady bond prices with a two-factor affine-yield model. By extending the hazard rate approach to the pricing of collateralised debt, it uses market quotes of a specific Brazilian discount bond to uncover the term structure of the underlying sovereign risk. A few observed properties of interest rates and bond prices can be traced to the factors, like the declining term structure of Libor volatility, the relatively low persistence of credit spreads and the dampening effect of default-free interest rates on credit spreads. The model-implied measure of sovereign risk is also broadly consistent with conventional indicators of repayment capacity. However, the explosive nature of expected default frequencies, which can be linked to the negative interaction between the hazard rate and the interest rate, is a limitation of the affine-yield model in the present context. From a theoretical perspective, default and liquidity are two important sources of security-specific variation in the returns of bonds. Default risk is event-related, possibly attributable to unobservable characteristics of borrowers, while liquidity risk is more closely related to the institutional features of the markets. Unless there are a sufficient number of securities to hedge those risks, bond markets are likely to be incomplete. The empirical evidence documenting that bond markets are incomplete is mounting.[11] Thus, a framework for explaining how yields are determined ought to specify how those security-specific risks interact with the "fundamental" factors affecting the returns on bonds.

The affine-yield models considered so far make strong assumptions about the nature of incompleteness. First, liquidity risks are ruled out, except maybe through a convenience yield in the default-free interest rate. Even if the hazard rate is interpreted as a composite rate adjusting for liquidity differences, a short cut to sidestep the difficult task of modeling liquidity, the liquidity risk premium gets buried in the hazard rate. Second, spurts of new information generated by the onset of

[11] For a recent empirical survey, see Longstaff (2000).

default events do not induce shifts in the market participants' pricing behaviour. The market's valuation of risk depends only on the common factors proxying the fundamentals. Thus, investors are not allowed to revise their apprehensions about risk as default takes place, as if the issuer-specific jump risks were not "priced" by the market. Relaxing those assumptions are important directions for theoretical and applied research.

Appendix

A. Building blocks

I call the information set F_t generated by the risk factors the “fundamental” filtration. Here the word “fundamental” refers loosely to global economic conditions, epitomised by the instantaneous US dollar interest rate, and to country characteristics like the ratio of debt to GDP, the history of ratings recorded by various credit rating agencies, the GDP growth rate and so on. This filtration has to be enlarged to incorporate default. I assume that the information set G_t available to market participants consists both of the history of fundamentals, F_t, and of the knowledge of whether or not default has already occurred. This may create complications when pricing default-free securities, because a martingale under F_t is not necessarily a martingale under G_t. To preclude this possibility, I assume that the time of default is constructed on an independent space, thereby ensuring that the martingale property is preserved under the larger information set.[12]

The principle of arbitrage states that the price at time t of a default-free zero-coupon bond maturing at $t + T$ is given by the expectation, under a “risk-neutral” measure Q, of the cumulative discount rate from now to maturity:

$$\varphi_L(T, z_1) = E^Q\left[\exp\left\{-\int_t^{t+r} r(z_1(s))ds\right\} G_t\right]$$

$$= E^Q\left[\exp\left\{-\int_t^{t+r} r(z_1(s))ds\right\} F_t\right]$$

Given the time-invariant parameters in (3), the dependence on current conditions is only implicit through the value of the underlying state variable at time t, $z_1(t) = z_1$.

The density η of Q with respect to P, the empirical probability, represents the market’s valuation of risk. To arrive at a tractable specification, shocks to η are assumed proportional to the innovations in the factors:

$$-\frac{d\eta}{\eta} = \lambda_1\sqrt{z_1}dw_1 + \lambda_2\sqrt{z_2}dw_2, \qquad (6)$$

where λ_i parameterises the price of risk. This implies that the factors carry a risk premium in the sense that

$$dz_i = (k_i\theta_i - (k_i + \lambda_i)z_i)dt + z_i^{1/2}d\hat{w}_i$$

where $\hat{w}_i$ are now independent standard Brownian motions under Q. Under the new measure, the mean reversion parameters are $k_{i+\lambda_i}$. If $k_{i+\lambda_i} < 0$, the dynamics of z_i is explosive, because higher levels of z_i lead to higher expected drifts. This should not lead to confusion, however, since the risk-neutral drift would correspond to the true expected returns only in a world where risk would have been hedged away.

The derivation of the zero-coupon bond prices implied by (1), (2) and (3) is well known and can be found for example in Pearson and Sun (1994) or, with the notation used in this paper, in Pagès (2000). The price of a default-free bond can be explicitly computed as $\psi_L(T, z_1) = e^{-\alpha_i T}\phi_L(T, z_1)$, where

---

12 This can be done with Cox processes, the construction of which is explained in Lando (1998). The condition ensuring that the martingale property is preserved under a larger information set is tersely known as “the H hypothesis”, in the mathematical literature. In this setup, it amounts to assuming that, conditionally on present and past prices, future prices do not provide any information about the probabilities of default up to the current time. See Jeanblanc and Rutkowski (1999) for details.

$$\phi_L(T,z_1)=\exp\{-k_1\theta_1\gamma_L(T)-\beta_L(T)z_1\} \tag{7}$$

The impact functions are in turn:

$$\beta_L(T)=\beta_L\frac{e^{q_1T}-e^{q_2T}}{q_1e^{q_2T}-q_2e^{q_1T}}$$

$$\gamma_L(T)=21n\frac{q_1e^{q_2T}-q_2e^{q_1T}}{q_1-q_2},$$

where q_1 and q_2 are the positive and negative roots of $q^2+(k_1+\lambda_1)q-\beta_L/2=0$, respectively.

Note that $\beta_L(0)=0$ and $\dot{\beta}_L(0)=\beta_L$.

Now consider the price of a defaultable zero-coupon, given by:

$$\psi_B(T,z_1,z_2)=E^Q\left[\exp\left\{-\int_t^{t+r}r_s ds\right\}1_{\{\sigma>t+r\}} \mid G_t\right]$$

$$=1_{\{\sigma>t\}}E^Q\left[\exp\left\{-\int_t^{t+r}(r_s+h_s)ds\right\} \mid F_t\right]$$

I have used the property that, provided that default has not yet happened, ψ_B is also the expectation of the cumulative discount rate $R = r + h$ under the fundamental filtration. Using the same tools as before, the price of the defaultable zero-coupon can be computed as

$$\psi_B(T,z_1,z_2)=e^{-\alpha_BT}\varphi_B(T,z_1)\varphi_h(T,z_2), \tag{8}$$

where

$$\phi_B(T,z_1)=\exp\{-K_1\theta_1\gamma_B(T)-\beta_B(T)z_1\} \tag{9}$$

$$\phi_h(T,z_1,z_2)=\exp\{-K_2\theta_2\gamma_h(T)-\beta_h(T)z_2\} \tag{10}$$

The impact functions in (9) (resp. (10)) are similar, with β_B (resp. β_h) in place of β_L and the second order equation defining q_1 and q_2 being $q^2+(K_1+\lambda_1)q-\beta_B/2=0$ (resp. $q^2+(K_2+\lambda_2)q-\beta_h/2=0$).

B. Discounting Libor forward rates

Let t_n be the length of the rolling interest guarantee. Since Brady bond interest payments are semi-annual, the variable n is counted in semesters. A coupon maturing at $T+t_n$ starts being insured at $t = r_t$, and its value can be ascertained as of date T by discounting the insured pay-off at the default-free interest rate. Let $r_{0.5}^L$ be (half of) the six-month Libor rate set at time $T+t_n-0.5=T+t_{n-1}$ for $T+t_n$, ie, $r_{0.5}^L=\psi_L^{-1}(0.5,z_1(T+t_{n-1}))-1$. The present value at T of the floating coupon maturing at $T+t_n$ comprises both the six-month Libor rate and the spread over Libor δ. Discounting this quantity between 0 and T at rate R, one gets

$$E^Q\left[\exp\left\{-\int_0^T R(z_1,z_2)ds\right\}\exp\left\{-\int_T^{T+t_n}r(z)ds\right\}\left(r_{0.5}^L+\delta\right)\right]$$

$$E^Q\left[\exp\left\{-\int_0^T R(z_1,z_2)ds\right\}E_r\left[\exp\left\{-\int_T^{T+t_n}r(z)ds\right\}\left(r_{0.5}^L+\delta\right)\right]\right]$$

$$E^Q\left[\exp\left\{-\int_0^T R(z_1,z_2)ds\right\}\left(\psi_L(t_{n-1},z_1(T))-(1-\delta)\psi_L(t_n,z_1(T))\right)\right] \tag{11}$$

where the Markovian structure of the model has been used to compute the conditional expectation.

To discount the variable Libor coupon, expressions of the form

$$\pi_n(T,z_1,z_2) = E^Q\left[\exp\left\{-\int_0^T R(z_1(s), z_2(s))ds\right\}\psi_L(t_n, z_1(T))\right] \tag{12}$$

must be evaluated, where z_1 and z_2 denote the value of the factors as of the current date. The function $\pi_n(T,z_i)$ gives the market price of a promise to deliver a default-free zero-coupon with time to maturity t_n at date T, knowing that the current fundamentals are z_i. With this notation, the value in (11) of a Libor coupon maturing at $T + t_n$ with collateral length t_n and spread over Libor δ can be written as $\pi_{n-1}(T, z_1, z_2) - (1-\delta)\,\pi_n(T,z_1,z_2)$.

The computation of the expectation in (12) is somewhat involved. The following result is instrumental in expressing a discounted variable coupon in terms of the building blocks of the model.

Lemma 1 *Let* $k \in (2q_2^B, 2q_1^B)$, *where* $q_1^B > 0 > q_2^B$ *are the roots of* $q^2 + (K_1 + \lambda_1)q - \beta_B/2 = 0$. *Then*

$$E^Q\left[\exp\left\{-\beta_B \int_0^t z_1 ds\right\} e^{-kz_1(T)}\right]_{z_1(0)=z_1} = \frac{\phi_B(T+\bar{t}, z_1)}{\phi_B(\bar{t}, z_1)} e^{-k z_1},$$

where $\bar{t}$ *is implicitly defined as* $\beta_B(\bar{t}) = k$.

Proof: If $\bar{t}$ is defined by $\beta_B(\bar{t}) = k$, the left hand side of the displayed equation in the lemma can be written as

$$E^Q\left[\exp\left\{-\beta_B \int_0^T z_1 ds\right\} \exp\{-\beta_B(\bar{t}) z_1(T)\}\right]_{z_1(0)=z_1}$$

$$= \exp\{K_1\theta_1\gamma_B(\bar{t})\} E^Q\left[\exp\left\{-\beta_B \int_0^T z_1 ds\right\} \phi_B(\bar{t}, z_1(T))\right]_{z_1(0)=z_1}$$

$$= \exp\{K_1\theta_1\gamma_B(\bar{t})\}\phi_B(T+\bar{t}; z_1)$$

$$= \frac{\phi_B(T+\bar{t}, z_1)}{\phi_B(\bar{t}, z_1)} e^{-k z_1},$$

as desired.

In practice when discounting a floating coupon, either a flat term structure of interest rates or the actual term structure, such as that provided by the US Treasury yield curve, is used. The expected floating coupons can themselves be derived from swap curves extracted at various maturities and interpolated across the necessary maturities. Lemma 2 shows how the correlation between interest and credit risk affects the computations. Essentially, discounting at the default-prone rate $R = r + h$ does not involve the discount process $\psi_B(T, z_1, z_2)$ given in (8), but an arrangement of its components with a change of time from t_n to $\overline{t_n}$.

Lemma 2 *Let* π_n *be as in (12). Then*

$$\pi_n(T, z_1, z_2) = e^{-\alpha_B T}\,\phi_h(T, z_2)\frac{\phi_L(T + t_n, z_1)}{\phi_L(\overline{t_n}, z_1)} e^{-\alpha_L T}\,\phi_L(T_n, z_1)$$

where $\overline{t_n}$ *is implicitly defined by* $\beta_B(\overline{t_n}) = \beta_L(t_n)$.

To see what happens in the special case on no correlation, assume that $\Delta\beta = 0$. Then $\phi_B(T, z_1) = \phi_L(T, z_1)$ and, in particular, $\overline{t_n} = t_n$. This implies:

$$\pi_n(T, z_1, z_2) = e^{-\alpha_B T}\,\phi_h(T, z_2)\frac{\phi_L(T + t_n, z_1)}{\phi_L(\overline{t_n}, z_1)} e^{-\alpha_L T}\,\phi_L(t_n, z_1)$$

$$=e^{-\Delta\alpha T}\phi_h(T,z_2)e^{-\alpha_L(T+t_n)}\phi_L(T+t_n,z_1)$$

$$=E^Q\left[\exp\left\{-\int_t^{t+T}h_s ds\right\} \mid F_t\right]\psi_L\left(T+t_n,z_1\right),$$

so the price π_n of a promise to deliver at date T a zero-coupon with time maturity t_n decomposes into the product of the default-free discount bond $\psi_L(T+t_n,z_1)$ and the risk-neutral probability of survival $Q(\sigma\geq T \mid G_t)$.

Finally, the following result shows how the change of time from t_n *to* $\overline{t_n}$ can be solved explicitly in terms of the primitives of the model.

Lemma 3 *Let, as before,* $\bar{t}=f(t)$ *be the solution to* $\beta_B(\bar{t})=\beta_L(t)$. *Then*

$$f(t)=\frac{2\mu}{\Delta\beta(k_1-k_2)}\left[1n\frac{q_1e^{-\mu s}-q_2k_1}{q_1e^{-\mu s}-q_2k_2}\right]_{s=0}^{s=t},$$

where $q_1>0>q_2$ *are the solutions to* $q^2+(K_1+\lambda_1)q-\beta_L/2=0, \mu=q_1-q_2$ *and* $k_1>k_2$ *are the solutions to* $k^2-2\left(1+\mu^2/\Delta\beta\right)k+1=0.$

Proof: The functions $\beta_L(\cdot)$ and $\beta_B(\cdot)$ are the solutions to the following two ordinary differential equations (Ricatti):

$$\dot{\beta}_L(t)+(K_1+\lambda_1)\beta_L(t)+\beta_L^2(t)/2=\beta_L$$

$$\dot{\beta}_B(t)+(K_1+\lambda_1)\beta_B(t)+\beta_B^2(t)/2=\beta_B,$$

respectively. Differentiating $\beta_B(f(t))=\beta_L(t)$ with respect to t, one gets

$$\dot{f}(t)=\frac{\dot{\beta}_L(t)}{\beta_B-\beta_L+\dot{\beta}_L(t)}=\beta_L\frac{\mu^2}{\Delta\beta}\frac{e^{-\mu t}}{\left(q_1e^{-\mu t}-q_2a\right)^2-q_2^2\left(a^2-1\right)},$$

where q_i are the roots used in the impact functions $\beta_L(\cdot)$ and $\gamma_L(\cdot), \mu=q_1-q_2, \Delta\beta=\beta_B-\beta_L$ and $a=1+\mu^2/\Delta\beta$. When $a^2>1$, the change in variable $z(t)=-\left(q_L^1e^{-\mu t}-q_L^2a\right)/\left(q_L^2\sqrt{a^2-1}\right)$ yields

$$f(t)=\frac{\mu}{\Delta\beta\sqrt{a^2-1}}\frac{-2\dot{z}(t)}{z^2-1}.$$

Integrating from 0 to t, one has

$$f(t)=\frac{\mu}{\Delta\beta\sqrt{a^2-1}}\left[1n\frac{z+1}{z-1}\right]_0^t$$

$$\frac{\mu}{\Delta\beta\sqrt{a^2-1}}\left[1n\frac{q_L^1e^{-\mu t}-q_L^2k_1}{q_L^1e^{-\mu t}-q_L^2k_2}\right]_0^t,$$

where $k_1>k_2$ are the solutions to $k^2-2ak+1=0$. (The case $a^2<1$ gives rise to a different expression which is not used in the paper.)

C. The term structure of expected default frequencies

Under the empirical probability, the survival probability conditional on not having defaulted at the current time is:

$$1_{\{\sigma>t\}}P\left(\sigma > t+T\right) \mid G_t = 1_{\{\sigma>t\}}\frac{P(\sigma > t+T \mid F_t)}{P(\sigma < t \mid F_t)}$$

$$= 1_{\{\sigma>t\}}\frac{E\left[\exp\left(-\int_0^{t+T} h_s ds\right) \mid F_t\right]}{\exp\left(-\int_0^t h_s ds\right)}$$

$$= 1_{\{\sigma>t\}}E\left[\exp\left(-\int_t^{t+T} h_s ds\right) \mid F_t\right].$$

From the decomposition in (2), the survival probability can be recovered directly from the above conditional expectation. The only difficulty is that, when $\Delta\beta < 0$, the expression

$$\phi_{\Delta\beta}(T,z) = E\left[\exp\left(-\Delta\beta\int_0^T z_1(s)ds\right)\right]_{z_1(0)=z}$$

may explode as T increases. More specifically, when the discriminant $\Delta = (K_1 + \lambda_1)^2 + 2\Delta\beta$ is negative, one finds

$$\phi_{\Delta\beta}(T,z) = \exp\{-K_1\theta_1\gamma(T) - \beta(T)z\}$$

with

$$\beta(T) = \Delta\beta\frac{\tan(bT)}{b - a\tan(bT)}$$

$$\gamma(T) = 2\left(aT + 1n\left(b\cos(bT) - a\sin(bT)\right)\right),$$

where $a = -(K_1 + \lambda_1)/2$ and $b = \sqrt{-\Delta/2}$. The explosion time is found by solving tan $(bT) = b/a$ for $bT \in [0,\pi]$. For example, when a < 0, the critical time is $T = (\arctan(b/a) + \pi)/b$. The expected default frequencies shown in Figure 3 are obtained as one minus the survival probabilities.

To carry out the same analysis under the risk-neutral probability, it suffices to set $\lambda_1 = 0$. This results in higher a, hence in a lower critical time.

References

Bhanot K (1998): "Recovery and implied default in Brady bonds", *The Journal of Fixed Income,* pp 47-52.

Chen, R and L Scott (1993): "Maximum-likelihood estimation for a multifactor equilibrium model of the term structure of interest rates", *Journal of Fixed Income,* 3, pp 14-31.

Claessens, S and G Pennacchi (1996): "Estimating the likelihood of Mexican default from the market prices of Brady bonds", *Journal of Financial and Quantitative Analysis,* 31, pp 109-26.

Cox, J, J Ingersoll and S Ross (1985): "A theory of the term structure of interest rates", *Econometrica*, 53, pp 385-407.

Duffee, G (1999): "Estimating the price of default risk", *The Review of Financial Studies*, 19,pp 197-226.

Duffie, D, L Pedersen and K Singleton (2000): "Modeling sovereign yield spreads: a case study of Russian debt", Graduate School of Business, Stanford University.

Duffie, D and K Singleton (1997): "An econometric model of the term structure of interest rate swap yields", *Journal of Finance,* 52, pp 1287-323.

——— (1999): "Modeling term structures of defaultable bonds", *Review of Financial Studies*, 12, pp 687-720.

Eichengreen, B and A Mody (1998): "What explains changing spreads on emerging-market debt: fundamentals or market sentiment?", *Working Paper 6408*, National Bureau of Economic Research, Cambridge, Massachussetts.

Grinblatt, M (1995): "An analytic solution for interest rate swaps", *Working Paper 9-94*, Anderson Graduate School of Management, UCLA.

Helwege, J and C M Turner (1997): "The slope of the credit yield curve for speculative-grade issuers", forthcoming in *Journal of Finance.*

Izvorski, I (1998): "Brady bonds and default probabilities", *Working Paper 98-16*, International Monetary Fund, Washington, DC.

Jarrow, R A, D Lando and S M Turnbull (1997): "A Markov model for the term structure of credit risk spreads", *Review of Financial Studies*, 10, pp 481-523.

Jarrow, R and F Yu (1999): "Diversification and default risk: an equivalence theorem for Martingale and empirical default intensities", *Working Paper*, Cornell University, Ithaca.

Jeanblanc, M and M Rutkowski (1999): "Modelling of default risk: an overview", Prépublication 115, Èquipe d'Analyse et Probabilités, Université d'Évry Val d'Éssonne, Evry.

Kamin, S and K von Kleist (1997): "The evolution and determinants of emerging market credit spreads in the 1990s", Manuscript, Federal Reserve Board and Bank for International Settlements.

Kho, B-C, L Dong and R Stulz (2000): "US banks, crises and bailouts: from Mexico to LTCM". *Working Paper*, Fisher College of Business, Ohio.

Lando, D (1998): "On Cox processes and credit risky securities", *Review of Derivatives Research*, 2, pp 99-120.

Longstaff, F (2000): "Arbitrage and the expectations hypothesis", *The Journal of Finance*, 55, pp 989-94.

Longstaff F and E Shwartz (1995): "A simple approach to valuing risky fixed and floating rate debt", *Journal of Finance*, 50, pp 789-819.

Madan, D B and H Unal (1998): "Pricing the risks of default", *Review of Derivatives Research*, pp 121-60.

Nielsen, S and E Ronn (1996): "The valuation of default risk in corporate bonds and interest rate swaps", *Working Paper*, University of Texas at Austin.

Pagès, H (2000): "Interbank interest rates and the risk premium", *The Journal of Fixed Income*, 9, pp 75-95.

Pearson, N D and T-S Sun (1994): "Exploring the conditional density in estimating the term structure: an application to the Cox, Ingersoll and Ross model", *Journal of Finance*, 49, pp 1279-304.

Sarig, O and A Warga (1989): "Some empirical estimates of the risk structure of interest rates", *Journal of Finance*, 49, pp 1351-360.

Recent BIS Working Papers

No	Title	Author
82 November 1999	Sacrifice ratios and the conduct of monetary policy in conditions of low inflation	Palle S Andersen and William L Wascher
83 January 2000	Switching from single to multiple bank lending relationships: determinants and implications	Luísa A Farinha and João A C Santos
84 January 2000	What have we learned from recent financial crises and policy responses?	William R White
85 January 2000	A defence of the expectations theory as a model of US long-term interest rates	Gregory D Sutton
86 March 2000	Information, liquidity and risk in the international interbank market: implicit guarantees and private credit market failure	Henri Bernard and Joseph Bisignano
87 May 2000	Monetary policy in an estimated optimisation-based model with sticky prices and wages	Jeffery D Amato and Thomas Laubach
88 June 2000	Evidence on the response of US banks to changes in capital requirements	Craig Furfine
89 August 2000	Forecast-based monetary policy	Jeffery D Amato and Thomas Laubach
90 September 2000	Bank capital regulation in contemporary banking theory: a review of the literature	João A C Santos
91 October 2000	Measuring potential vulnerabilities in emerging market economies	John Hawkins and Marc Klau
92 October 2000	Recent initiatives to improve the regulation and supervision of private capital flows	William R White
93 October 2000	Trading volumes, volatility and spreads in foreign exchange markets: evidence from emerging market countries	Gabriele Galati
94 November 2000	Corporate hedging: the impact of financial derivatives on the broad credit channel of monetary policy	Ingo Fender
95 November 2000	The impact of corporate risk management on monetary policy transmission: some empirical evidence	Ingo Fender
96 December 2000	The real-time predictive content of money for output	Jeffery D Amato and Norman N Swanson
97 December 2000	Information flows during the Asian crisis: evidence from closed-end funds	Benjamin H Cohen and Eli M Remolona
98 January 2001	Money and inflation in the euro area: a case for monetary indicators?	Stefan Gerlach and Lars E O Svensson
99 June 2001	The interbank market during a crisis	Craig Furfine
100 July 2001	The impact of the euro on Europe's financial markets	Gabriele Galati and Kostas Tsatsaronis