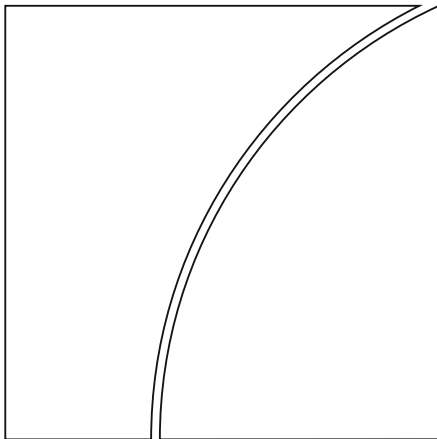




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Global real rates: a secular approach

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Foreword

The 17th BIS Annual Conference took place in Zurich, Switzerland, on 22 June 2018. The event brought together a distinguished group of central bank Governors, leading academics and former public officials to exchange views on the topic “Ten years after the Great Financial Crisis: what has changed?”. The papers presented at the conference and the discussants’ comments are released as *BIS Working Papers No 790, 791, 792 and 793*.

BIS Papers No 103 contains Panel remarks by Mervyn King (former Governor, Bank of England) and Anne Le Lorier (former First Deputy Governor, Bank of France) and a resulting Panel discussion between Agustín Carstens and them.

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Global Real Rates: A Secular Approach*

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Abstract

The current environment is characterized by low real rates and by policy rates close to or at their effective lower bound in all major financial areas. We analyze these unusual economic conditions from a secular perspective using data on aggregate consumption, wealth and asset returns. Our present-value approach decomposes fluctuations in the global consumption-to-wealth ratio over long periods of time and show that this ratio anticipates future movements of the global real risk-free rate. Our analysis identifies two historical episodes where the consumption-to-wealth ratio declined rapidly below its historical average: in the roaring 1920s and again in the exuberant 2000s. Each episode was followed by a severe global financial crisis and depressed real rates for an extended period of time. Our empirical estimates suggest that the world real rate of interest is likely to remain low or negative for an extended period of time.

*Nick Sander, Todd Messer and Jianlin Wang provided outstanding research assistance. We thank Ricardo Caballero, Emmanuel Farhi, Ralph Koijen, Martin Lettau Linda Tesar and Gabriel Zucman for comments. All errors remain our own. Special thanks to Oscar Jordà, Moritz Schularick and Alan Taylor for sharing their data with us. Contact email: pog@berkeley.edu, hrey@london.edu. Rey thanks the ERC for financial support.

1 Introduction

The current macroeconomic environment remains a serious source of worry for policymakers and of puzzlement for academic economists. Global real rates, which have been trending down since the 1980s, are at historical lows across advanced economies, both at the short and long end of the term structure. Policy rates are close to or at their Effective Lower Bound in all major financial areas. Figures 1 and 2 report the nominal policy rates and long yields for the U.S., the Eurozone, the U.K. and Japan since 1980. Large amounts of wealth are invested at zero or negative yields.¹

Despite the aggressive global monetary policy treatment administered in advanced economies, levels of economic activity have only recently normalized, suggesting a decline in the *natural interest rate*, i.e. the real interest rate at which the global economy would reach its potential output.

Understanding whether natural rates are indeed low, for how much longer, and the source of their decline has become a first-order macroeconomic question. More generally, understanding what drives movements in real rates in the long run is one of the most intriguing questions in macroeconomics.

In a celebrated speech given at the International Monetary Fund in 2013, five years after the onset of the Global Financial Crisis, Summers (2015) ventured that we may have entered an age of ‘*secular stagnation*’, i.e. an era where output remains chronically below its potential, or equivalently real rates remain above their natural rate. Not coincidentally, the secular stagnation hypothesis was first voiced by Hansen (1939), ten years after the onset of the Great Depression. Whether we are indeed in a period of ‘*secular stagnation*’, and why, remains to be elucidated. Several hypotheses have been put forward for a secular decline in real rates: a global savings glut (Bernanke (2005)), i.e. a rise in desired savings due to the fast growth of emerging market economies with relatively underdeveloped financial sectors; a decline in investment rates due to a lack of investment opportunities, potentially because of a technological slowdown (Gordon (2012)); a decline in the relative price of investment goods such as machine and robots, which

¹According to FitchRatings (2017), the total amount of fixed-rate sovereign debt trading at negative yields was \$9.7 trillion as of December 2017, slightly below its peak of \$11.7 trillion in June 2016.

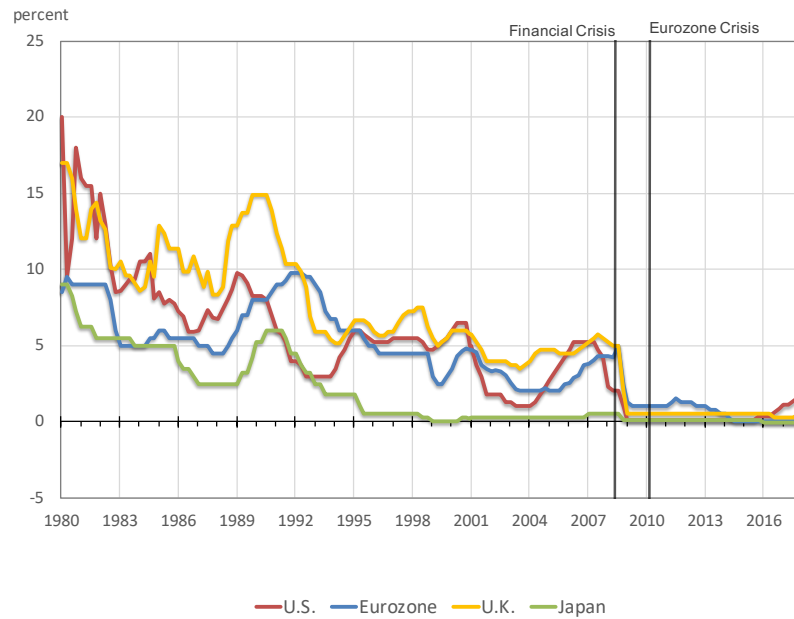


Figure 1: **Policy Rates, 1980-2017.** Sources: U.S.: Federal Funds Official Target Rate; Eurozone: until Dec. 1998, Germany's Lombard Rate. After 1998, ECB Marginal Rate of Refinancing Operations; U.K.: Bank of England Base Lending Rate; Japan: Bank of Japan Target Call Rate. Data from Global Financial Database.

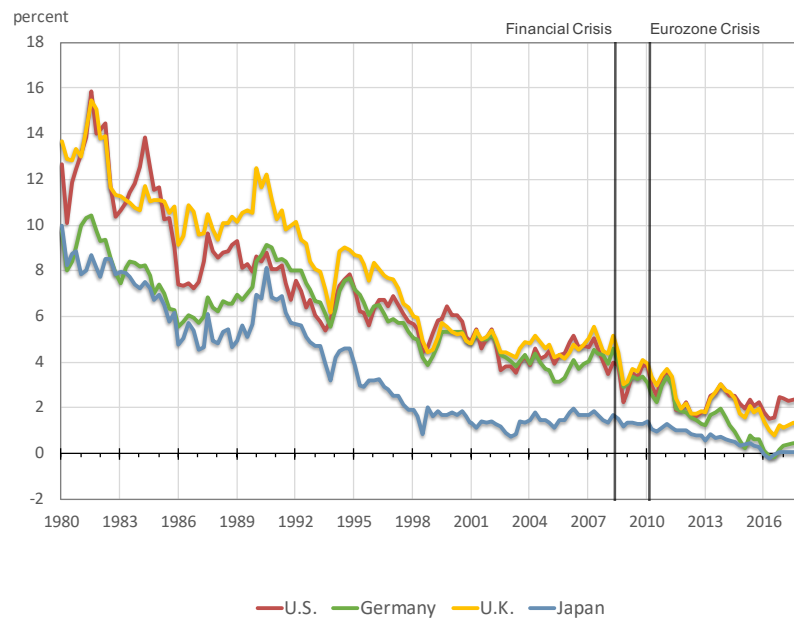


Figure 2: **Yields on Long Term Debt, 1980-2017.** Sources: U.S.: 10-year bond constant maturity rate; Germany: 10-year benchmark bond; U.K.: 10-year government bond yield; Japan: 10-year government bond yield. Data from Global Financial Database

depresses the level of investment; a decline in the rate of population growth; an increase in the demand for safe assets (Caballero et al. (2015)); or the long shadow cast by a major financial crisis and the slow process of deleveraging associated with it (Lo and Rogoff (2015)).

This paper is an empirical contribution to this debate. We take a ‘secular view,’ building from recent contributions in macroeconomic history from Jordà et al. (2016) or Piketty and Zucman (2014a) that have made a number long macroeconomic time-series available to researchers. Our focus is to analyze movements in real rates since 1870 in the U.S., and since 1920 for a group of four advanced economies: the U.S., the U.K., Germany and France.

A long historical perspective is important. As noted by others before us (e.g. Hamilton et al. (2016) for a sample of 17 countries or Vlieghe (2017) for the U.K.), real rates have historically fluctuated a lot, and the current low real rates are not unprecedented when seen from an historical perspective. Figure 3 reports estimates of the annualized ex-post 3-months real interest rates for the United States, the United Kingdom, Germany and France. The figure illustrates that real short rates were high and declining from 1870 to WW1, reached low and volatile levels in the interwar period, remained low in the post WWII period, until the early 1980s when they rose sharply before gradually declining again.

To understand the evolution of global real rates over such long periods of time, we propose an approach based on standard present-value decompositions often used in the modern finance literature (Campbell and Shiller (1988), Lettau and Ludvigson (2001) and more recently Binsbergen et al. (2010)). We apply this long run decomposition to more than a century of historical data. Under very modest assumptions, this decomposition establishes that the global consumption-to-wealth ratio encodes information about future risk-free rates, future risk-premia and/or future consumption growth. The intuition is quite straightforward: times when consumption is high relative to wealth must be followed either by lower consumption growth (the numerator), or higher returns on wealth (the denominator). Higher returns on wealth can result either from higher real risk-free rates, or from higher risk-premia. Because consumption growth and risk-premia are difficult to predict, we expect the consumption-to-wealth ratio to contain information mostly about current and future real rates. This intuition turns out to be correct: empirically, the consumption-

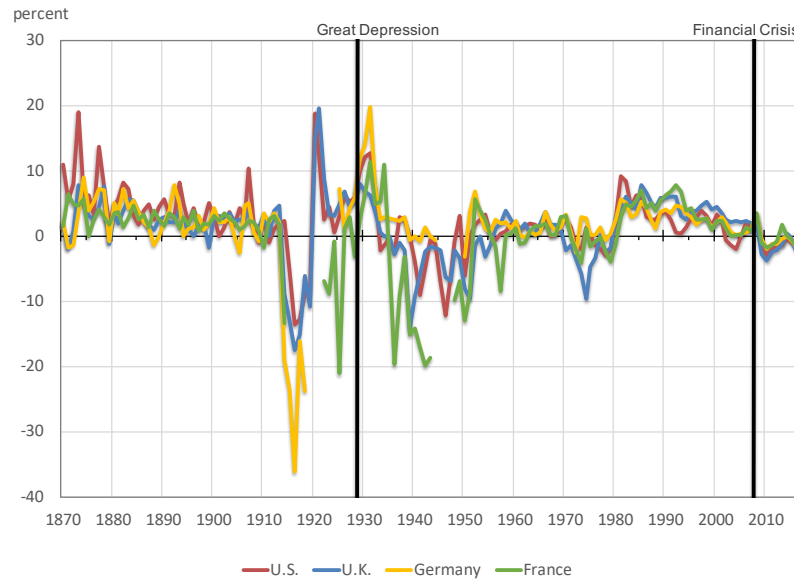


Figure 3: **3-Months Ex-Post Real Rates (p.a.), 1870-2015.** Sources: [Jordà et al. \(2016\)](#). Ex-post real rates are constructed as the nominal interest rate on 3-months Treasuries minus realized CPI inflation.

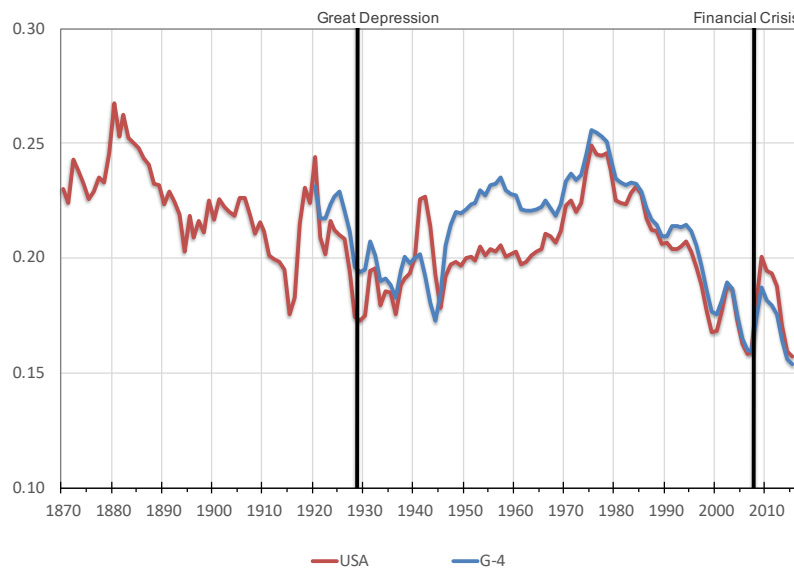


Figure 4: **Consumption-to-Private Wealth Ratio, 1870-2015, United States and G-4 (U.S., U.K., Germany and France).** Sources: [Jordà et al. \(2016\)](#), [Piketty et al. \(2017\)](#) and WDI.

to-wealth ratio, reported on Figure 4, is an excellent predictor of the low-frequency movements in global real rates. In particular, our estimation suggests that real rates will remain low for an extended period of time: our baseline empirical estimates predict an average real short-term rate of -2.35% between 2015 and 2025 for the U.S., and of -3.1% for the U.S., U.K., Germany and France combined.

Establishing the importance of the global consumption-to-wealth ratio for predictive purposes is an important result. But this brings an immediate question: why does the consumption-to-wealth ratio fluctuate over time? Returns, consumption and wealth are all endogenous. This is an identification question and as such it is much harder to answer. Yet, our decomposition does provide some useful hints. While consumption growth and risk premia are difficult to forecast, their present value still contributes to movements in the consumption-to-wealth ratio, alongside the present value of risk-free rates. However, different fundamental shocks will imply different patterns of co-movements between the different components. Consider for instance, the impact of productivity shocks. A decline in productivity growth is often claimed as a reason behind the recent decline in real rates. Standard Euler equation reasoning suggests that lower productivity growth should be associated with lower real rates, with the strength of that effect controlled by the intertemporal elasticity of substitution. Since lower productivity growth also means lower consumption growth, it follows that productivity shocks will have two opposite effects on the consumption-to-wealth ratio: lower real rates will tend to decrease the ratio; lower future consumption growth to increase it. By looking at the empirical pattern of co-movements between the different components of the ratio, we can hope to recover some information about the key drivers. We consider four such shocks: productivity growth, demographics (specifically population growth), deleveraging shocks and risk appetite. We can think of the first two as ‘*macro*’ shocks. The latter two are ‘*finance*’ shocks that have been the object of much recent focus in the literature.

Our results indicate that both macro and financial forces play a role. For the former, we do find evidence that demographic and productivity shocks play a small but significant role, especially at lower frequencies. On the financial side, we find that two historical episodes stand out, during

which the consumption-to-wealth ratio was abnormally low: in the 1930s and since 2000. In both cases the decline in the consumption-to-wealth ratio was largely driven by a rapid increase in wealth during the financial boom that preceded a major financial crisis: the Great Depression in 1929 and the Great Financial Crisis in 2008. Our decomposition suggests that low real rates in the aftermath of these crisis was driven in part by a protracted and still on-going deleveraging process associated with the financial cycle.

Review of the Literature. This is a placeholder for the literature review. It will include:

- A discussion of papers that estimate the natural rate: [Laubach and Williams \(2016, 2003\)](#); [Holston et al. \(2017\)](#); [Hamilton et al. \(2016\)](#); [Pescatori and Turunen \(2015\)](#); [Del Negro et al. \(2017\)](#); [Farooqui \(2016\)](#); [Barro and Sala-i Martin \(1990\)](#)
- A discussion of paper papers on the decline in global real rates due to heterogenous risk aversions: [Gourinchas et al. \(2010\)](#); [Barro et al. \(2014\)](#); [Hall \(2016\)](#).
- A discussion of papers on ‘secular stagnation’: [Eggertsson et al. \(2015\)](#); [Eggertsson and Mehrotra \(2014\)](#); [Caballero et al. \(2015\)](#); [Summers \(2015\)](#); [Hansen \(1939\)](#); [Sajedi and Thwaites \(2016\)](#)
- A discussion of papers on financial crises and deleveraging: [Lo and Rogoff \(2015\)](#); [Jordà et al. \(2016\)](#); [Schularick and Taylor \(2012\)](#)
- A discussion of papers on the present value approach: [Campbell and Shiller \(1991\)](#); [Lettau and Ludvigson \(2001\)](#); [Gourinchas and Rey \(2007\)](#); [Binsbergen et al. \(2010\)](#); [Lustig et al. \(2013\)](#)
- A discussion of papers on integrated macroeconomic accounts and historical data, [Piketty et al. \(2017\)](#), [Piketty and Zucman \(2014a\)](#), [Jordà et al. \(2016\)](#).

2 The Dynamics of the Consumption-to-Wealth Ratio and Natural Rates

We are interested in understanding the drivers behind the low-frequency movements in the global natural rate of interest. Our key methodological contribution consists in connecting expected current and future global risk-free rates to fluctuations in the consumption-to-wealth ratio, using a simple Present Value model (PV). This Present Value model can be derived under a minimal set of assumptions, which we make explicit, and builds from the generic implications of the global resource constraint.

2.1 The Global Resource Constraint: A Present Value Relation

Since we are interested in understanding global returns, the relevant unit of analysis is the global (i.e. world) resource constraint. Let \bar{W}_t denote the beginning-of-period global *total* private wealth, composed of the sum of global private wealth W_t and global human wealth H_t . Private wealth W_t consists of financial assets, including private holdings of government assets, and non-financial assets such as land and real estate. Human wealth H_t consists of the present value of current and future non-financial income.² Total private wealth evolves over time according to:

$$\bar{W}_{t+1} = \bar{R}_{t+1}(\bar{W}_t - C_t). \quad (1)$$

In equation (1), C_t denotes global private consumption expenditures and \bar{R}_{t+1} the gross return on total private wealth between periods t and $t + 1$. All variables are expressed in real terms. Equation (1) is simply an accounting identity that holds period-by-period.

We add some structure to this identity by observing that, in almost any sensible income-fluctuation and portfolio-choice model, optimizing households aim to smooth consumption. This tends to stabilize the consumption-to-wealth ratio, i.e. the average propensity to consume. For instance, if consumption decisions are taken by an infinitely lived representative-household max-

²See appendix A for a detailed data description. We focus on *private* wealth and consumption, and not *national* wealth or consumption, which includes government's net wealth and consumption.

imizing welfare defined as the expected present value of a logarithmic period utility $u(C) = \ln C$, then the consumption-to-wealth ratio is constant and equal to the discount rate of the representative agent.

Assumption 1 *The (log) consumption-to-wealth ratio is stationary and we denote $\overline{\ln(C/W)} < 0$ its unconditional mean.*

If the (log) average propensity to consume out of wealth is stationary, equation (1) can be log-linearized around its steady state value. Denote $0 < \rho_w \equiv 1 - \exp(\overline{\ln(C/W)}) < 1$, Δ the difference operator so that $\Delta x_{t+1} \equiv x_{t+1} - x_t$, and $\bar{r}_{t+1} \equiv \ln \bar{R}_{t+1}$, the continuously compounded real return on wealth.³ Following the same steps as [Campbell and Mankiw \(1989\)](#) or [Lettau and Ludvigson \(2001\)](#), we obtain the following log-linearized expression (ignoring an unimportant constant term):⁴

$$\ln C_t - \ln \bar{W}_t \simeq \rho_w (\ln C_{t+1} - \ln \bar{W}_{t+1} + \bar{r}_{t+1} - \Delta \ln C_{t+1}). \quad (2)$$

Equation (2) indicates that if today's consumption-to-wealth ratio is high, then either (a) tomorrow's consumption-to-wealth ratio will be high, or (b) the return on wealth between today and tomorrow \bar{r}_{t+1} will be high, or (c) aggregate consumption growth $\Delta \ln C_{t+1}$ will be low.

Since $\rho_w < 1$, Equation (2) can be iterated forward under the usual transversality condition, $\lim_{j \rightarrow \infty} \rho_w^j (\ln C_{t+j} - \ln \bar{W}_{t+j}) = 0$. Denoting $\mathbb{E}_t[\cdot]$ the conditional expectations at time t , we obtain the following ex-ante Present Value (PV) relation:

$$\ln C_t - \ln \bar{W}_t \simeq \mathbb{E}_t \sum_{s=1}^{\infty} \rho_w^s (\bar{r}_{t+s} - \Delta \ln C_{t+s}). \quad (3)$$

To understand equation (3), suppose that the (log) consumption-to-wealth ratio is currently higher than its unconditional mean, $\overline{\ln(C/W)}$. Since $\ln(C/W)$ is stationary, this ratio must be

³In steady state, it follows from (1) that $\Gamma/\bar{R} = 1 - \exp(\overline{\ln(C/W)}) \equiv \rho_w$, where Γ denotes the steady state growth rate of total private wealth and \bar{R} the steady state gross return on total private wealth. The requirement that $\rho_w < 1$ is equivalent to $\bar{R} > \Gamma$, i.e. that the real interest rate exceeds the gross rate of the economy.

⁴See appendix B for a full derivation.

expected to decline in the future. Equation (3) states that this decline can occur in one of two ways. First, expected future return on total private wealth \bar{r}_{t+s} could be high. This would increase future wealth, i.e. the denominator of C/\bar{W} . Alternatively expected future aggregate consumption growth could be low. This would reduce the numerator of C/\bar{W} .

At this stage, it is important to emphasize that the assumptions needed to derive equation (3) are *minimal*: we start from a global budget constraint, equation (1), which is an accounting identity. We then perform a log-linearization under very mild stationarity conditions, and impose a transversality condition that rules out paths where wealth grows without bounds in relation to consumption. Equation (3)'s main economic message is that today's average propensity to consume out of wealth *encodes* relevant information about future consumption growth and/or future returns to wealth.

2.2 From the Present Value Relation to Empirics

Before we can exploit this expression empirically, we need to make two important adjustments. First, as mentioned above, *total* private wealth is the sum of *private* wealth W_t and *human* wealth H_t . The former is -partly- observable, from existing wealth surveys and historical integrated macroeconomic accounts such as [Piketty and Zucman \(2014a\)](#) or [Jordà et al. \(2016\)](#). The latter is not, and often needs to be estimated with the help of auxiliary assumptions on the stochastic process of the discount factor and/or future labor income. For instance, [Lettau and Ludvigson \(2001\)](#) approximate human wealth with current aggregate labor income and construct a proxy for the left hand side of (3) by estimating a co-integration relation between consumption, financial wealth and labor income. [Lustig et al. \(2013\)](#) follow a different approach. Using data on bond yields and stock returns, they estimate an affine Stochastic Discount Factor (SDF) consistent with no-arbitrage. They then solve for total wealth \bar{W} as the market value of a claim to current and future aggregate consumption expenditures, evaluated at the estimated SDF. An advantage of their method is that it does not require any wealth data. A disadvantage is that one needs to put a lot of faith on the particular SDF that is estimated.

We follow a different route. Specifically, denote $\omega_t = W_t/\bar{W}_t$ the aggregate share of private

wealth in total private wealth. If ω_t is stationary around a mean ω , we can approximate (log) total wealth as $\ln \bar{W}_t = \omega \ln W_t + (1 - \omega) \ln H_t$, and the log return on total wealth as $\bar{r}_t = \omega r_t^w + (1 - \omega) r_t^h$ where r_t^w (resp. r_t^h) denotes the log return on private wealth (resp. human wealth).⁵ Substituting these expressions into equation (3) and re-arranging we obtain:

$$0 \simeq \omega \left(\ln C_t - \ln W_t - \mathbb{E}_t \sum_{s=1}^{\infty} \rho_w^s (r_{t+s}^w - \Delta \ln C_{t+s}) \right) + (1 - \omega) \left(\ln C_t - \ln H_t - \mathbb{E}_t \sum_{s=1}^{\infty} \rho_w^s (r_{t+s}^h - \Delta \ln C_{t+s}) \right). \quad (4)$$

This equation makes clear that if the Present Value relation holds for private wealth (the first term of the equation), then it holds for human wealth (the second term of the equation), and vice versa. More generally, we can re-arrange this expression into:

$$\ln C_t - \ln W_t \simeq \mathbb{E}_t \sum_{s=1}^{\infty} \rho_w^s (r_{t+s}^w - \Delta \ln C_{t+s}) + \varepsilon_t. \quad (5)$$

where ε_t represents an error term induced by ignoring human wealth that can be expressed as:

$$\varepsilon_t = (1 - \omega) \mathbb{E}_t \sum_{s=1}^{\infty} \rho_w^s (r_{t+s}^h - r_{t+s}^w) + (1 - \omega) (\ln H_t - \ln W_t)$$

This error term is small when expected returns on human and private wealth are similar, and when the ratio of private to human wealth is stationary (since we are ignoring constants). Equation (5) states that the consumption to private wealth ratio may be high if either (a) future returns on private wealth are high; (b) future consumption growth is low; (c) the error term is high, which can occur either if the returns on human wealth r^h are high relative to the returns on private wealth r^w or when human wealth is high relative to private financial wealth. Because human wealth and the return on human wealth are difficult to observe, we will simply assume that the error term is negligible and ignore it

Assumption 2 *The Present Value Relation (3) holds for total private wealth. Equivalently: $\varepsilon_t \approx 0$.*

⁵The gross return on human wealth may be defined as $R_{t+1}^h = \exp(r_{t+1}^h) = H_{t+1}/(H_t - WL_t)$ where WL_t denotes aggregate non-financial income in period t . See [Campbell \(1996\)](#).

The recent evidence on the decline in the labor share (see e.g. [Karabarbounis and Neiman \(2014\)](#)) and on the increase in income inequality (see e.g. [Piketty and Saez \(2003\)](#)) could invalidate these assumptions: in recent years, the return on private wealth r^w may have exceeded the return on human wealth r^h . Similarly, it is possible growing wealth inequality imply that human wealth H declined relative to private wealth W . This could translate into downward trends in the consumption-to-private wealth ratio C/W , even if consumption-to-total wealth C/\bar{W} remained stationary. However, our focus on long run data should mitigate these concerns. For instance, as documented by [Piketty and Saez \(2003\)](#), the dynamics of income inequality over the last century is characterized by large and persistent fluctuations, but no historical trend: income and wealth inequality in the U.S. are today close to what they were at the beginning of the XXth century.

The second adjustment is to realize that the return on private wealth r_{t+1}^w can always be decomposed into the sum of a real risk-free rate r_t^f (known at time t) and an excess return er_{t+1}^w according to: $r_{t+1}^w = r_t^f + er_{t+1}^w$. While we can construct reasonably accurate estimates of the real risk free rate r_t^f , it is harder to measure the excess return on private wealth er_{t+1}^w , or equivalently, the return to private wealth r_{t+1}^w . This is so since private wealth includes a variety of traded financial assets such as portfolio holdings, whose return could reasonably be approximated, but also non-financial or non-traded assets such as real estate, agricultural land and equipments whose returns are more difficult to measure. Our approach consists in proxying the excess return on private wealth with a vector of N excess returns on existing assets $\tilde{\mathbf{e}}\mathbf{r}_{t+1}$, such as equity or bond returns, as follows:

$$r_{t+1}^w = r_t^f + \nu' \tilde{\mathbf{e}}\mathbf{r}_{t+1}, \quad (6)$$

where ν is an $N \times 1$ vector that will be estimated.

Substituting (6) into the present value relation (5), we obtain our fundamental representation:

$$\begin{aligned} \ln C_t - \ln W_t &\simeq \mathbb{E}_t \sum_{s=1}^{\infty} \rho_w^s r_{t+s-1}^f + \nu' \mathbb{E}_t \sum_{s=1}^{\infty} \rho_w^s \mathbf{r}\mathbf{p}_{t+s-1} - \mathbb{E}_t \sum_{s=1}^{\infty} \rho_w^s \Delta \ln C_{t+s} + \varepsilon_t. \\ &\equiv cw_t^f + cw_t^{rp} + cw_t^c + \varepsilon_t, \end{aligned} \quad (7)$$

where $\mathbf{rp}_t = \mathbb{E}_t[\tilde{\mathbf{e}}\mathbf{r}_{t+1}]$ is the $N \times 1$ vector of one period-ahead risk premia. This equation states that the consumption-to-private wealth ratio C/W should contain information either about (a) future safe rates r_t^f , (b) future risk premia, \mathbf{rp}_t , or (c) future aggregate consumption growth, $\Delta \ln C_t$. The terms cw_t^f , cw_t^{rp} and cw_t^c summarize the relative contributions of the risk free rate, the risk premia and consumption growth, respectively.

Inspecting (7), we make two final observations. First, under a particular data generating process, it is relatively straightforward to estimate the present value terms cw_t^f and cw_t^c . However, since the vector of loadings of private wealth excess returns on market returns ν is unknown, we cannot infer the contribution of risk-premia cw_t^{rp} without additional assumptions. We will estimate ν so as to minimize the residuals in equation (7). This way of proceeding opens up the possibility that our estimate of the risk-premium component may be contaminated by the human capital component error term ε_t . For instance, if we proxy excess returns on private wealth with equity excess returns only, $N = 1$ and the OLS estimate of ν satisfies $\hat{\nu} = \nu + \text{cov}(\varepsilon, cw^{rp})/\text{var}(cw^{rp})$ where $cw^{rp} = \mathbb{E}_t \sum_{s=1}^{\infty} \rho_w^s \tilde{e}r_{t+s}$ is the estimated present value of future excess equity returns. The possible bias on $\hat{\nu}$ attributes to the risk-premium component the part of the variation in $\ln C - \ln W$ coming from fluctuations in human wealth that co-moves with the equity risk premium.

Second, and importantly for us, it is well-known that aggregate consumption expenditures is close to a random walk, while the risk premium is volatile and difficult to predict. Therefore, we expect equation (7) to connect the aggregate consumption-to-wealth ratio to the expected path of future real risk-free rates r_{t+s}^f via cw_t^f . The last step of the argument is to realize that, under the generally admitted assumption that monetary policy aims to target the risk-free rate to the natural rate denoted r_t^* , $\mathbb{E}_t r_{t+s}^f = \mathbb{E}_t r_{t+s}^*$, and the risk free component can be expressed as:

$$cw_t^f = \mathbb{E}_t \sum_{s=1}^{\infty} \rho^w r_{t+s-1}^* = cw_t^*$$

In other words, we expect to recover from the behavior of the global consumption-to-wealth

ratio information about the discounted path of future natural rates.⁶

3 Consumption-to-Wealth Ratio: Some Elements of Theory

Before we lay out our empirical strategy in more details, we discuss how different fundamental shocks can affect returns, consumption and the consumption/wealth ratio. We then show how, under more restrictive assumptions, a full characterization of the consumption-to-wealth ratio can be obtained.

3.1 Present Value Relation and Structural Shocks

Our fundamental representation (7) does not provide a *causal* decomposition: the risk-free, risk-premium and consumption growth components cw^i are endogenous and interdependent. Different fundamental shocks will imply different patterns of co-movements between risk-free rates, risk premia and consumption growth. We begin by fleshing out the implications for our fundamental representation (7) by considering productivity shocks, demographic shocks, deleveraging shocks and changes in risk appetite.

3.1.1 Productivity shocks.

To focus on the purest implications of productivity shocks, consider a closed endowment economy with no government, so consumption C is equal to the endowment Y . Equation (7) takes the form:

$$\ln C_t - \ln W_t \simeq \mathbb{E}_t \sum_{s=1}^{\infty} \rho_w^s (r_{t+s}^w - \Delta \ln Y_{t+s}). \quad (8)$$

Suppose that total output growth is expected to decline in the future, $\Delta \ln Y_{t+s} < 0$, holding output growth unchanged at other periods. For a given path of expected future returns, this should exert *upward* pressure on the current consumption-to-wealth ratio. However, and this is

⁶The assumption that $\mathbb{E}_t r_{t+s}^f = \mathbb{E}_t r_{t+s}^*$ could be violated if the economy is stuck at the Effective Lower Bound (See discussion below). In that case, $\mathbb{E}_t r_{t+s}^f \geq \mathbb{E}_t r_{t+s}^*$ and $cw_t^f \geq cw_t^*$ would provide an upper bound on the discounted path of future natural rates.

the key insight, expected future returns will not remain constant. Faced with a future slowdown in output growth, households may want to save more today. This will depress expected returns, up to the point where consumption remains equal to output. The decline in expected returns will exert *downward* pressure on the consumption-to-wealth ratio. Which of these two effects will dominate? The answer depends on whether the Intertemporal Elasticity of Substitution (IES) is above or below 1.

To see this mechanism explicitly, assume that the representative household has additively separable preferences over consumption, with a constant intertemporal elasticity of substitution (IES) $1/\gamma$ and discount rate ρ : $U_t = \mathbb{E}_t \sum_{s=0}^{\infty} e^{-\rho s} c_{t+s}^{1-\gamma} / (1 - \gamma)$. The usual log-linearized Euler equation takes the following form (up to the second order):

$$\gamma \mathbb{E}_t \Delta \ln C_{t+1} = \mathbb{E}_t r_{t+1}^w - \rho + \frac{1}{2} \sigma_{z,t}^2,$$

where $\sigma_{z,t}^2$ denotes the conditional variance of $z_{t+1} = r_{t+1}^w - \gamma \Delta \ln Y_{t+1}$ at time t .

Denote $g_t = \Delta \ln Y_t$ the (exogenous) aggregate endowment growth, which coincides here with productivity, and $\sigma_{g,t}^2$ its conditional variance. The Euler equation can be solved for the expected return on wealth:

$$\mathbb{E}_t r_{t+1}^w = \rho + \gamma \mathbb{E}_t g_{t+1} - \frac{1}{2} \sigma_{z,t}^2. \quad (9)$$

This expression encodes precisely the extent to which the expected return on wealth needs to respond to changes in expected output growth so as to clear the goods market: if output growth is expected to increase by 1%, the expected return on private wealth must increase by $\gamma\%$. Substituting the Euler equation (9) into equation (8) and ignoring constants, one obtains:

$$\ln C_t - \ln W_t \simeq \mathbb{E}_t \sum_{s=1}^{\infty} \rho_w^s \left((\gamma - 1) g_{t+s} - \frac{1}{2} \sigma_{z,t+s-1}^2 \right). \quad (10)$$

It is immediate from equation (10) that whether the consumption-to-wealth ratio increases or decreases with output growth depends on the sign of $\gamma - 1$, i.e. on the relative strength of the substitution and income effects. If $\gamma > 1$, the IES is low and expected returns need to decline

a lot in order to stimulate consumption growth. The impact of productivity changes on returns dominates and C/W co-moves *positively* with expected future productivity growth. If instead $\gamma < 1$, the IES is high and a modest decline in expected returns is sufficient to push consumption growth down. The direct impact of productivity growth dominates and C/W co-moves *negatively* with expected future productivity growth.⁷

Following similar steps, one can compute the various components cw_t^i as (up to some unimportant constants):

$$\begin{aligned} cw_t^f &= \gamma \mathbb{E}_t \sum_{s=1}^{\infty} \rho_w^s \left(g_{t+s} - \frac{\gamma}{2} \sigma_{g,t+s-1}^2 \right) \\ cw_t^{rp} &= \mathbb{E}_t \sum_{s=1}^{\infty} \rho_w^s \left(\gamma \text{cov}_t(r_{t+s}^w, g_{t+s}) - \frac{1}{2} \sigma_{r,t+s-1}^2 \right) \\ cw_t^c &= -\mathbb{E}_t \sum_{s=1}^{\infty} \rho_w^s g_{t+s}, \end{aligned}$$

where $\sigma_{r,t}^2$ is the conditional variance of the return on private wealth.

These expressions make clear that expected changes in future productivity have direct opposite effects on the risk free and consumption components, scaled by the inverse of the IES, while the risk premium component only depends on the present value of co-movements between the return on wealth and output growth. In the limit where there is no time-variation in second moments, the risk premium component is constant while the risk free and consumption components are perfectly negatively correlated, and $\text{var}(cw^f)/\text{var}(cw^c) = \gamma$.⁸

3.1.2 Demography.

Consider now the effect of demographic forces on the consumption-to-wealth ratio. To do so, decompose total consumption growth $\Delta \ln C_{t+1}$ into per capita consumption growth $\Delta \ln c_{t+1}$,

⁷In the special case where $\gamma = 1$, the consumption-to-wealth ratio is constant and independent from $\mathbb{E}_t g_{t+s}$.

⁸Of course, risk premia may not be constant. For most models of interest, however, the correlation between excess returns on wealth and consumption growth is relatively small, indicating a small role for the macroeconomic risk premium that we measure here.

and population growth n_{t+1} : $\Delta \ln C_{t+1} = \Delta \ln c_{t+1} + n_{t+1}$. Substituting into (7) we obtain:

$$\begin{aligned} \ln c_t - \ln w_t &\simeq \mathbb{E}_t \sum_{s=1}^{\infty} \rho_w^s \left(r_{t+s}^w - \Delta \ln c_{t+s} - n_{t+s} \right), \\ &\simeq cw_t^f + cw_t^{cp} + cw_t^n \end{aligned}$$

where w_t denotes real private wealth per capita. cw_t^{cp} and cw_t^n represent respectively the contribution of future growth in consumption per capita and future population growth. It is obvious from this expression that an expected decline in population growth ($\mathbb{E}_t n_{t+s} < 0$) has a direct and positive effect on $c - w$, given a path of returns and consumption per capita. The effect of a decline in population growth on equilibrium returns, and therefore the indirect effect on the consumption-to-wealth ratio, is more complex. As population growth slows down, capital per worker increases, pushing down the marginal product of capital and r^w . At the same time, a decline in population growth increases the dependency ratio, i.e. the ratio of retirees to working-age population. Since retirees save less than workers, aggregate savings may decline, pushing interest rates up. Finally, increases in life expectancy, which have been a major driver of demographic developments in the last century, lead to increased saving and therefore a decline in interest rates.

The empirical evidence as well as calibrated overlapping generation models such as see [Carvalho et al. \(2016\)](#) generally indicate that slowdowns in population growth are associated with increased savings.⁹ This should push down expected returns and the consumption-to-wealth ratio, with the strength of that effect, again, controlled by the IES $1/\gamma$. In this case, as in the case of productivity shocks, the impact of demographic shocks will have opposite effects on the risk-free and population growth components: $\text{corr}(cw_t^f, cw_t^n) < 0$. We can measure the direct effect of demographic shocks on the consumption-to-wealth ratio by constructing an empirical counterpart to $cw_t^n = -\mathbb{E}_t \sum_{s=1}^{\infty} \rho_w^s n_{t+s}$.

⁹With open economies, the same phenomenon manifests itself in the form of current account surpluses for countries, such as Japan, Germany and China, with more rapid slowdown in population growth and aging.

3.1.3 Deleveraging shock.

Consider next what happens if there is an expected shift in individuals' desire to save. At an abstract level, one can model this shift as a decrease in ρ , the discount rate of households. Such deleveraging shocks have been studied by [Eggertsson and Krugman \(2012\)](#), as well as [Guerrieri and Lorenzoni \(2011\)](#). To understand how these shocks may affect the consumption-to-wealth ratio, we need to consider two cases, depending on whether the economy is above or at the Effective Lower Bound on nominal interest rates (ELB). In the presence of nominal rigidities, the ELB may constrain the equilibrium real interest rate in the economy at a level that is excessively high, pushing the economy into a recession.

Consider first the case where the economy is above the ELB. For simplicity, assume that (potential) output is constant. With the economy outside the ELB, it is possible for the real interest rate to adjust so that consumption equals output. The Euler equation takes the form:

$$\mathbb{E}_t r_{t+1}^w = \rho_t - \frac{1}{2} \sigma_{r,t}^2, \quad (11)$$

where ρ_t is the now time-varying discount rate of the representative household between periods t and $t+1$, known at time t . A decline in ρ_t pushes down the equilibrium expected return on wealth. Under the assumption that the economy remains permanently above the ELB, the present-value equation (8) becomes:

$$\ln C_t - \ln W_t = \mathbb{E}_t \sum_{s=1}^{\infty} \rho_w^s (\rho_{t+s-1} - \frac{1}{2} \sigma_{r,t+s-1}^2).$$

We can express the different components as:

$$\begin{aligned} cw_t^f &= \mathbb{E}_t \sum_{s=1}^{\infty} \rho_w^s \rho_{t+s-1} \\ cw_t^{rp} &= -\frac{1}{2} \mathbb{E}_t \sum_{s=1}^{\infty} \rho_w^s \sigma_{r,t+s-1}^2 \\ cw_t^c &= 0. \end{aligned}$$

An expected deleveraging shock, i.e. a decline in $\mathbb{E}_t \rho_{t+s}$, has a direct negative effect on the consumption-to-wealth ratio because it lowers the real risk-free rate one for one, but it has no effect on the consumption or risk-premia components.

Consider now what happens at the ELB. If prices are nominally rigid and real interest rates cannot decrease further to satisfy (11), the economy will experience a recession, as in Eggertsson and Krugman (2012) or Caballero and Farhi (2015). For simplicity, suppose that the effective lower bound is zero and that prices are permanently fixed so that $r^f = 0$ while the economy remains at the ELB (i.e. while the natural rate ρ_t remains negative). The Euler equation for the risk-free rate requires that:

$$\gamma \mathbb{E}_t \Delta \ln C_{t+1} = -\rho_t + \frac{\gamma^2}{2} \text{var}_t (\Delta \ln C_{t+1}).$$

Consumption is expected to increase at a rate that reflects the (positive) gap between the real interest rate (0) and the natural real interest rate ($\rho_t < 0$). Since potential output is constant this expression makes clear that the economy must experience a recession today (i.e. consumption and output need to be below potential). The expected return on wealth (equal to the expected excess return) now satisfies:

$$\mathbb{E}_t r_{t+1}^w = \gamma \text{cov}_t(r_{t+1}^w, \Delta \ln C_{t+1}) - \frac{1}{2} \sigma_{r,t}^2,$$

and may increase as the economy hits the ELB, as emphasized by Caballero et al. (2016). If the economy is expected to remain permanently at the ELB, the different components of the consumption-to-wealth ratio can be expressed as:

$$\begin{aligned} cw_t^f &= 0 \\ cw_t^{rp} &= \mathbb{E}_t \sum_{s=1}^{\infty} \rho_w^s \left(\gamma \text{cov}_{t+s-1}(r_{t+s}^w, \ln C_{t+s}) - \frac{1}{2} \sigma_{r,t+s}^2 \right) \\ cw_t^c &= \mathbb{E}_t \sum_{s=1}^{\infty} \rho_w^s \left(\frac{1}{\gamma} \rho_{t+s-1} - \frac{\gamma}{2} \text{var}_{t+s-1} (\Delta \ln C_{t+s}) \right). \end{aligned}$$

This expression makes clear that at the ELB, the adjustment in the consumption-to-wealth ra-

tio occurs through the consumption component. Expected future consumption growth requires that the consumption-to-wealth ratio be low today. In the general case where the economy does not remain stuck at the ELB permanently, the adjustment will occur both via a decline in future real risk free rates -when the economy is expected to leave the ELB and via an increase in consumption growth while the economy is at the ELB. Both terms depress the consumption-to-wealth ratio, so cw^f and cw^c will be *positively* correlated.

3.1.4 Risk Appetite.

A deleveraging shock increases the demand for savings and therefore depresses the returns on all assets, leaving risk-premia largely unchanged outside the ELB. Let's now consider a shock to risk appetite, i.e. a shift in the demand for *safe* versus *risky* assets. The safe asset scarcity, arising from instance from an increase in desired holdings of safe assets, has been one of the leading explanations for the secular decline in real risk-free rates (Hall (2016), Caballero et al. (2015)).

An easy way to capture such a shift would be via an increase in risk aversion. However, with CES preferences, it is well known that the coefficient of risk aversion is also the inverse of the intertemporal elasticity of substitution $1/\gamma$. In order to isolate the effect of a shift in risk appetite from that of a change in the IES, assume that the representative household has Epstein-Zin recursive preferences:

$$U_t = \left\{ (1 - e^{-\rho}) C_t^{1-\sigma} + e^{-\rho} (\mathbb{E}_t U_{t+1}^{1-\gamma_t})^{\frac{1-\sigma}{1-\gamma_t}} \right\}^{\frac{1}{1-\sigma}},$$

where γ_t is the now time-varying coefficient of relative risk aversion. The IES is assumed constant and equal to $1/\sigma$. Given these preferences, we can solve the Euler equation for the risk-free rate:

$$r_t^f = \rho + \sigma \mathbb{E}_t \Delta \ln C_{t+1} + \frac{\theta_t - 1}{2} \sigma_{r,t}^2 - \frac{\theta_t \sigma^2}{2} \sigma_{g,t}^2$$

where $\theta_t \equiv (1 - \gamma_t)/(1 - \sigma)$. When $\theta_t = 1$, this formula collapses to the Euler equation (9) for the risk-free return. By contrast, when $\theta_t \neq 1$, the risk free rate depends on the variance of the market return $\sigma_{r,t}^2$. Standard derivations provide the following expression for the expected risk

premium:

$$\mathbb{E}_t r_{t+1}^w - r_t^f = \theta_t \sigma \text{cov}_t(r_{t+1}^w, g_{t+1}) + (1 - \theta_t) \sigma_{r,t}^2.$$

To highlight the role of fluctuations in risk appetite, consider an environment where output is constant, so $\sigma_{g,t}^2 = 0$. It follows that the consumption-to-wealth ratio can be expressed as (up to some constant):

$$\ln C_t - \ln W_t = \frac{1}{2} \mathbb{E}_t \sum_{s=1}^{\infty} \rho_w^s (1 - \theta_{t+s-1}) \sigma_{r,t+s-1}^2.$$

An increase in risk aversion γ_t raises $1 - \theta_t = (\gamma_t - \sigma)/(1 - \sigma)$ and leads to an increase in the consumption-to-wealth ratio. This is intuitive: while current consumption is unchanged (by assumption), the decline in risk appetite lowers the present value of future income, hence the current value of wealth. The decomposition (7) yields :

$$\begin{aligned} cw_t^f &= -\frac{1}{2} \mathbb{E}_t \sum_{s=1}^{\infty} \rho_w^s (1 - \theta_{t+s-1}) \sigma_{r,t+s-1}^2 \\ cw_t^{rp} &= \mathbb{E}_t \sum_{s=1}^{\infty} \rho_w^s (1 - \theta_{t+s-1}) \sigma_{r,t+s-1}^2 \\ cw_t^c &= 0. \end{aligned}$$

An expected increase in future risk aversion increases the risk premium component cw^{rp} and decreases the risk free rate component cw^f . The consumption component remains unchanged. This is also intuitive: the decline in risk appetite requires an increase in risk premia. This increase in risk-premia is achieved via an increase in the expected return on risky assets and a decline in the risk-free rate. It follows that $\text{corr}(cw_t^f, cw_t^{rp}) = -1$. Overall, the increase in risk premia dominates, driving up the consumption-to-wealth ratio so $\text{corr}(cw_t, cw_t^f) = -1$.

Summary. The preceding discussion highlights that, while the decomposition (7) does not provide a causal interpretation of the different components, the co-movements of the different components offers a natural signature about the various economic forces at play: If the consumption and risk free rate components are negatively correlated, we would conclude that productivity and/or demographic shocks play an important role. If instead the consumption and risk free rate

Shock	$\ln C/W$	cw^f	cw^{rp}	cw^c
Productivity	sign of $\gamma - 1$	+	\sim	-
Population Growth	sign of $\gamma - 1$	+	\sim	-
Deleveraging (outside ELB)	-	-	\sim	0
Deleveraging (at the ELB)	-	0	\sim	-
Risk Appetite	-	+	-	0

Table 1: Summary of Sign Restrictions. The table reports the sign of $\ln C/W$ and its components cw^i in response to various expected future structural shocks. For instance, in response to an expected future positive productivity shock, cw^c decreases, cw^f increases, cw^{rp} is mostly unchanged and the sign of $\ln C/W$ depends on $\gamma - 1$.

components are either poorly correlated or positively correlated and the consumption-to-wealth ratio is positively correlated to the risk-free component, then we would conclude that deleveraging shocks are likely to be more relevant. Finally, if we find that the risk free component is both negatively correlated with the risk premium components and the consumption-to-wealth ratio, we would infer that shocks to risk appetite are an important part of the story. Table 1 summarizes the different co-movements implied by the theory.

3.2 Orders of Magnitude

Our empirical approach is flexible: it does not require imposing a particular stochastic discount factor, and allows for a flexible parametrization of the data generating process. Under additional restrictions, it is possible to express the consumption-to-wealth ratio in closed form as a function of the underlying fundamental parameters. For instance, following [Martin \(2013\)](#) and [Vlieghe \(2017\)](#), assume that there is a representative agent with separable constant elasticity preferences, so that the real Stochastic Discount Factor takes the form: $\mathcal{M}_{t,t+1} = e^{-\rho - \gamma \Delta \ln C_{t+1}}$. Assume further that consumption growth is i.i.d. with Cumulant Generating Function $\mathcal{C}(\theta) =$

$\ln \mathbb{E}[\exp(\theta \Delta \ln C_{t+1})]$. Then, the consumption-to-total wealth ratio is constant and satisfies:¹⁰

$$\ln C_t - \ln \bar{W}_t = \ln \left(1 - e^{-\rho + \mathcal{C}(1-\gamma)} \right).$$

The risk-free return is also constant,

$$r_t^f = \ln R_t^f = -\ln \mathbb{E}_t [\mathcal{M}_{t,t+1}] = \rho - \mathcal{C}(-\gamma)$$

while the return on total wealth follows

$$\bar{r}_{t+1} = \ln \bar{R}_{t+1} = \ln \left(\frac{\bar{W}_{t+1}}{\bar{W}_t - C_t} \right) = \ln \left(\frac{1}{1 - C/\bar{W}} \frac{C_{t+1}}{C_t} \right) = \rho - \mathcal{C}(1 - \gamma) + \Delta \ln C_{t+1},$$

and the expected risk premium satisfies:

$$ERP \equiv \ln \mathbb{E}_t \bar{R}_{t+1} - \ln R_t^f = \mathcal{C}(1) + \mathcal{C}(-\gamma) - \mathcal{C}(1 - \gamma)$$

This representation is obviously too restrictive, since it implies a constant consumption-to-wealth ratio and a constant risk-free rate, but it nevertheless allows us to consider some relevant orders of magnitudes. For instance, if we follow [Martin \(2013\)](#) and postulate that log consumption growth follows a jump-diffusion process $\Delta \ln C_{t+1} = g + \sigma_g^2 \varepsilon_{t+1} + v_{t+1}$ where ε_{t+1} is a standard normal and v_{t+1} is a Poisson ‘disaster’ process with arrival rate p per unit of time and, where the disaster size is distributed $\mathcal{N}(-b, s^2)$, then the cumulant generating function satisfies:

$$\mathcal{C}(\theta) = g\theta + \frac{1}{2}\sigma_g^2\theta^2 + p \left(e^{-\theta b + \frac{1}{2}\theta^2 s^2} - 1 \right).$$

Substituting the parameters from [Barro \(2006\)](#), $\rho = 0.03$, $\gamma = 4$, $g = 0.025$, $\sigma_g = 0.02$,

¹⁰To obtain this result, observe that we can substitute the return \bar{R}_{t+1} into the fundamental asset pricing equation $\mathbb{E}_t[\mathcal{M}_{t,t+1}R_{t+1}] = 1$ and iterate forward to obtain

$$\bar{W}_t = C_t \sum_{s=0}^{\infty} \mathbb{E}_t \left[e^{-\rho s} \left(\frac{C_{t+s}}{C_t} \right)^{1-\gamma} \right] = C_t \sum_{s=0}^{\infty} e^{-\rho s} (\mathbb{E} [\exp \Delta \ln C_{t+1}])^s = \frac{C_t}{1 - e^{-\rho + \mathcal{C}(1-\gamma)}}.$$

$p = 0.017$, $b = 0.39$ and $s = 0.25$, we obtain $C/\bar{W} = 0.0465$, with a real risk-free rate $r^f = 1.04\%$ and an expected risk premium $ERP = 5.73\%$.

As we will see in the empirical section, the observed consumption-to-private wealth ratio for the U.S. between 1870 and 2015 has a mean of 0.209, which implies that the ratio of private wealth to total wealth is equal to $0.0465/0.209 = 22.25\%$. According to this crude calculation, human wealth represents the bulk of total wealth (77.75%), a figure that is roughly in line with -albeit smaller than- the calculations of [Lustig et al. \(2013\)](#) who estimate that human wealth represents 92% of total wealth. Similar calculations for the U.S., the U.K., Germany and France between 1920 and 2015 yield a consumption-to-private wealth ratio of 0.210, which implies a very similar estimate of the ratio of private wealth to total wealth (22.14%).

4 Estimating the Present Value Relation

We implement our empirical strategy in three steps. First, we construct estimates of the consumption-to-wealth ratio over long periods of time. Next, we evaluate the empirical validity of equation (7) by constructing the empirical counterparts of the right hand side of that equation, and testing whether they capture movements in the consumption-to-wealth ratio. Lastly, we investigate the role of various drivers of the consumption-to-wealth ratio.

4.1 Data description and Long-run Covariability

We use historical data on private wealth, population and private consumption for the period 1870-2015 for the United States, the United Kingdom, Germany and France from [Piketty and Zucman \(2014a\)](#), [Piketty et al. \(2017\)](#), the World Inequality Database, as well as [Jordà et al. \(2016\)](#) to construct measures of real per capita consumption and (beginning of period) private wealth, expressed in constant 2010 US dollars. Private wealth is defined as the sum of non-financial assets, including housing and other tangible assets such as software, equipment and agricultural land, and net financial assets, including equity, pensions, life insurance and bonds. Private wealth does

not include government assets, but includes private holdings of government issued liabilities as an asset.

Figure 5 reports real per capita private wealth and consumption for the United States between 1870 and 2015. As expected, historical time series on consumption and private wealth show a long term positive trend. U.S. real per capita consumption increased from \$2,829 in 2010 dollars in 1870 to \$35,771 in 2015, while real per capita private wealth increased over the same period from \$12,304 to \$227,283. The resulting consumption-to-wealth ratio, already reported on Figure 4 appears relatively stable over this long period of time, with a mean of 20.94 percent, decreasing from roughly 23 percent in 1870 to about 16 percent in the latter part of the sample. As noted above, we observe two periods during which the consumption-to-wealth ratio was significantly depressed: the first one spans the 1930s, starting shortly before the Great Depression and ending at the beginning of the 1940s. Interestingly, in 1939 Professor Alvin Hansen writes his celebrated piece about ‘secular stagnation’ (Hansen (1939)). The second episode of low consumption-to-wealth ratio starts around 1995 with a pronounced downward peak in 2008. The consumption-to-wealth ratio temporarily rebounds after 2008 largely as a result of the decline in private wealth. Perhaps not coincidentally, in the Fall 2013 at a conference at the International Monetary Fund, Larry Summers resuscitates the idea of secular stagnation, an idea which is still haunting us in 2018 (Summers (2015)).

Figure 6 reports real consumption and wealth per capita for an aggregate of the U.S., the U.K., Germany and France since 1920. We label this aggregate the ‘G-4’. Over the period considered, these four countries represent a sizable share of the world’s financial wealth and consumption. London, New-York, and to a lesser extent Frankfurt and Paris, represent major financial centers. As for the U.S., real consumption and wealth per capita for the G-4 show a long term positive trend with a few major declines during the two World Wars and the Great Depression.¹¹ Real

¹¹The effect of the wars on the financial wealth and consumption of Germany and France is most dramatic during WWI. The U.S. consumption-to-wealth ratio is somewhat insulated and does not show swings of the amplitude of the U.K., French and German consumption wealth ratios at the time during that period. This, and concerns about data quality prior to 1920 are the two reasons we only begin the ‘G4’ aggregate after 1920. In particular, as discussed in the appendix, wealth data is not available annually before 1954 for France, 1950 for Germany, 1920 for the U.K. and 1916 for the U.S. and is imputed based on savings data and estimates of the rate of capital gains on wealth for each country.

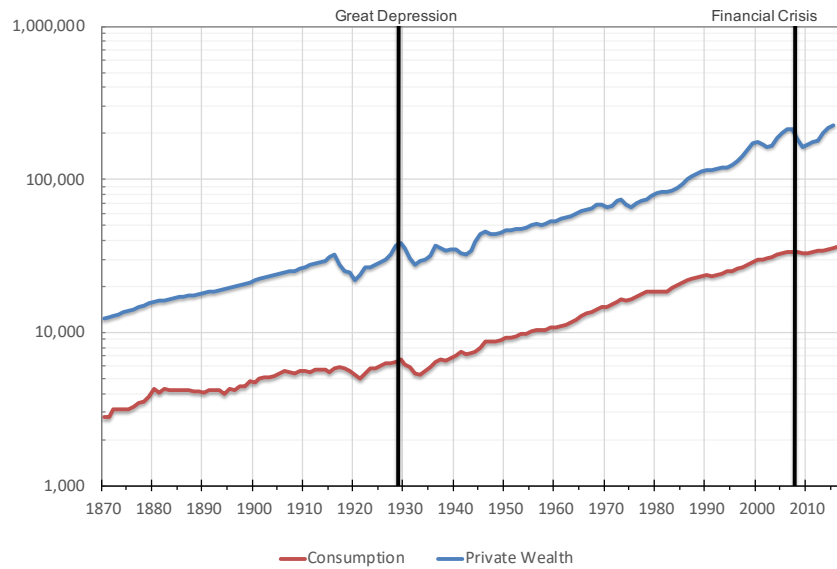


Figure 5: **Real Consumption and Private Wealth per capita, 2010 USD, United States, 1870-2015.**

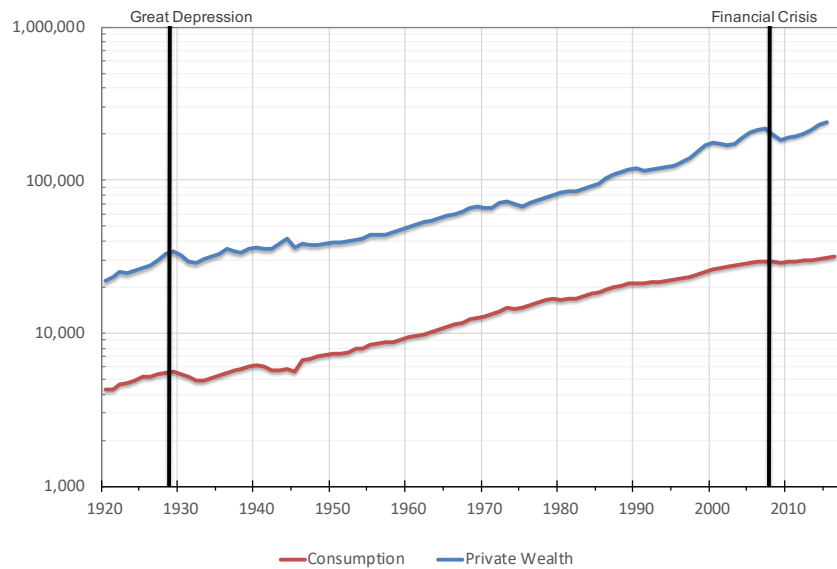


Figure 6: **Real Consumption and Private Wealth per capita, 2010 USD, United States, United Kingdom, Germany and France, 1920-2015.**

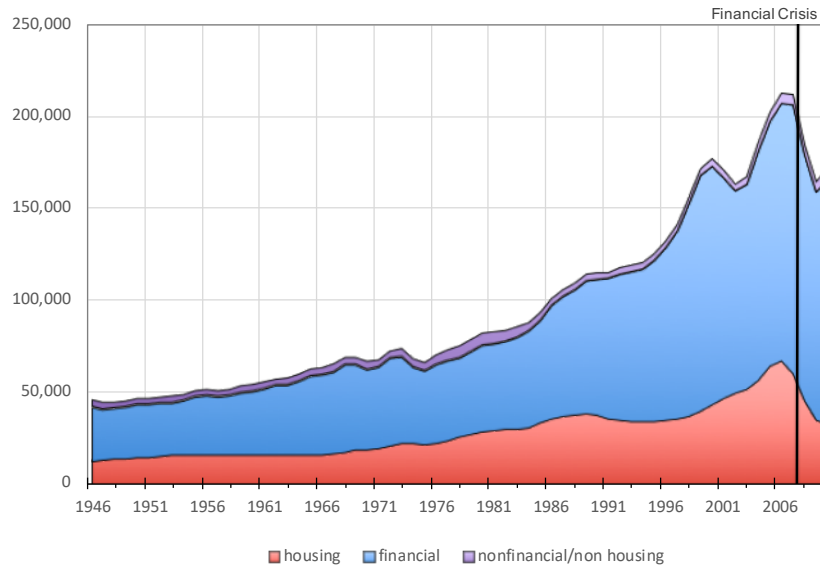


Figure 7: **Housing, Financial and Private Wealth per capita, 2010 USD, United States, 1946-2010.** Source: [Piketty and Zucman \(2014a\)](#).

per capita consumption increased from \$4,282 in 1920 to \$31,198 in 2015 in 2010 constant dollars while real per capita private wealth increased from \$21,818 to \$238,535 over the same period. The consumption-to-wealth ratio exhibits the same pattern as that of the U.S., with a mean of 20.97 percent. While both consumption and wealth per capita look quite smooth over long periods of time, the ratio C/W exhibits substantial fluctuations, as seen in Figure 4.

Looking at Figures 4 and 5-6, it is clear that the decline in the consumption-to-wealth ratio observed in the 1930s and in the 2000s was associated in both cases with faster growth in private wealth, rather than slower growth in consumption. The growth rate of U.S. real private wealth per capita reached 4.88% p.a. between 1920 and 1930 and 4.35% between 1997 and 2007. Over the same periods, the growth rate of real consumption per capita was 1.56% and 2.4% respectively.¹²

Figure 7 uses the [Piketty and Zucman \(2014a\)](#) data to decompose U.S. real private wealth per capita into housing, financial and a non-housing/non-financial residual components between 1946 and 2010.¹³ The figure illustrates that housing wealth declined as a fraction of private wealth

¹²Over the 1870-2015 period, the average growth rate of U.S. real private wealth per capita was 2.01%, that of real consumption per capita was 1.75%.

¹³For the U.S., the non housing/non financial component includes software, equipment and agricultural land. This represent a very small share of private wealth.

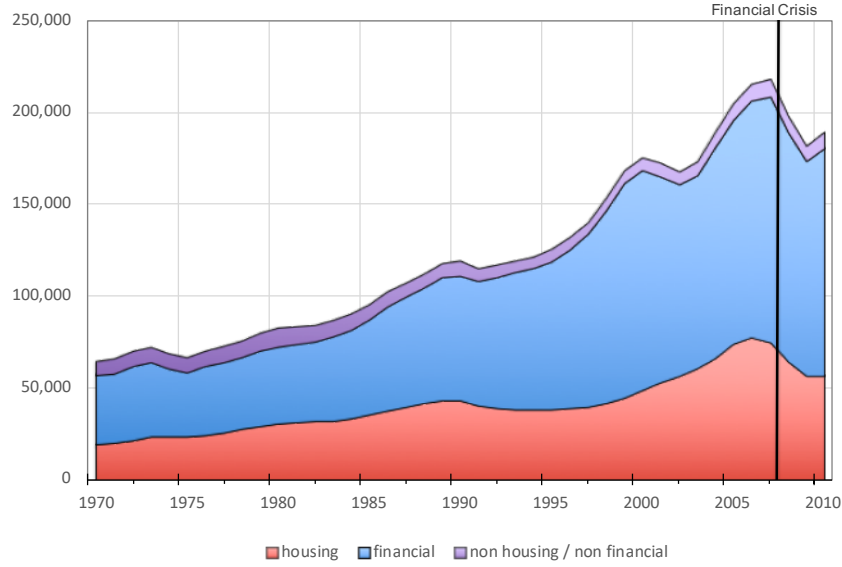


Figure 8: **Housing, Financial and Private Wealth per capita, 2010 USD, United States, United Kingdom, Germany and France, 1970-2010.** Source: [Piketty and Zucman \(2014a\)](#).

during that period, from 28-30% in 1946 to 20% by 2010. The figure also illustrates that the first decline in C/W in 2000 was associated with an increase in financial wealth (the growth rate of real financial wealth per capita between 1990 and 2000 was 5.66%, at the time of the dotcom boom), while the second decline in 2007 was associated with rapid growth in housing wealth (5.2% p.a. between 1997 and 2007 during the U.S. housing boom). Figure 8 reports a similar decomposition for our G-4 group, but on the shorter period 1970-2010 and shows a similar pattern, with rapid growth in housing wealth, but also financial wealth in the 2000s, when the ratio C/W was rapidly decreasing.¹⁴

For each country or group of country, we measure the real ex-post interest rate as the 3-month nominal yield minus realized CPI inflation.¹⁵ Lastly, we use excess returns on equities r^e , long term bonds r^l and the rate of growth of house prices r^h to instrument for the risk premium on private wealth.¹⁶

¹⁴Our consumption measure includes input rent for homeowners. Hence, increases in housing prices could be reflected in higher inputted rents.

¹⁵For the G4 aggregate, we use the average of the U.S. and U.K. real interest rates, weighted by relative wealth. Appendix C provides the details of the aggregation procedure. We do not include the real rate for Germany and France, since episodes of monetary instability in the 1920s and during WWII in both countries generate very volatile measures of the ex-post real interest rate.

¹⁶As for the risk free rate, we use a wealth-weighted average of the equity excess returns, term premium and

Table 2 reports some summary statistics for the different variables.

	C/W	$\Delta \ln c$	$\Delta \ln w$	n	r^f	$r^e - r^f$	$r^l - r^f$	$r^h - r^f$
Panel A: United States. Sample: 1870-2015								
Mean	20.94	1.750	2.011	1.431	1.984	4.456	0.473	-0.493
Median	20.86	1.643	2.197	1.343	2.060	6.158	0.819	-1.461
Standard deviation	2.295	3.422	4.897	0.522	4.932	18.460	1.608	7.825
Maximum	26.77	10.798	14.537	2.545	19.029	41.305	3.316	31.783
Minimum	15.74	10.000	-15.181	0.489	-13.482	-59.453	-7.979	-38.161
Panel B: G-4. Sample: 1920-2015								
Mean	20.96	2.021	2.356	0.774	1.975	5.361	0.888	-0.992
Median	21.47	2.127	3.129	0.754	1.949	8.830	0.980	-1.890
Standard deviation	2.41	2.394	4.061	0.372	4.315	18.149	1.298	5.440
Maximum	25.60	10.953	9.273	1.412	18.710	38.164	3.176	18.597
Minimum	15.39	-5.784	-11.024	-1.491	-10.421	-49.705	-3.311	-11.758

Table 2: Summary Statistics. The table shows summary statistics for the consumption-to-wealth ratio C/W , the growth rate of real consumption per capita $\Delta \ln c$, the growth rate of real private wealth per capita $\Delta \ln w$, population growth n , the risk-free real rate r^f , the equity excess return $r^e - r^f$, the term premium $r^l - r^f$, housing capital gain minus the risk free rate $r^h - r^f$. r^h is available starting in 1891 for the U.S.

The table exhibits a number of interesting findings. First, the consumption-wealth ratio declines slightly over time since the rate of growth of consumption is small than that of private wealth by about 0.3% p.a. Second, wealth growth is more volatile than consumption growth. Hence, as discussed above, fluctuations in the consumption-to-wealth ratio will likely be driven by endogenous changes in. Third, the realized excess return on equities is sizable, around 4.5% for the U.S. and 5.3% for the G-4, numbers that are consistent with historical estimates of the equity premium. Third, the capital gain on housing is slightly lower than the risk free rate, and this excess return is highly volatile. As discussed above, r^h does not represent the full return on housing since it does not include rental income. Nevertheless, this suggests that the long run return to housing is largely driven by rental income and not capital gains.

Table 3 reports estimates of long-run covariability between pairs of variables. Long run co-variability estimates developed by Müller and Watson (2018) are designed to allow long run inference on the co-movements between two variables that is robust to the degree of long-run

housing returns for the global excess return. Note that the rate of growth of house prices differs from the true return on housing by the rent/price ratio which was not available to us. Since the rent/price ratio is positive, the rate of growth of housing price underestimates the actual return on housing.

Variable \ Variable	C/W	$\Delta \ln c$	$\Delta \ln w$	n	r^f	$r^e - r^f$	$r^l - r^f$
$\Delta \ln c$	0.05 [-0.20,0.60]						
$\Delta \ln w$	-0.12 [-0.44,0.13]	0.64 [0.40,0.78]					
n	0.46 [-0.10,0.85]	-0.12 [-0.35,0.50]	-0.13 [-0.25,0.20]				
r^f	0.18 [-0.30,0.80]	0.00 [-0.35,0.38]	0.10 [-0.15,0.43]	0.27 [-0.07,0.75]			
$r^e - r^f$	-0.15 [-0.70,0.12]	0.34 [-0.08,0.59]	0.38 [0.03,0.61]	-0.05 [-0.50,0.39]	-0.22 [-0.65,0.19]		
$r^l - r^f$	-0.52 [-0.85,-0.07]	0.04 [-0.40,0.40]	0.10 [-0.10,0.55]	-0.50 [-0.80,-0.03]	-0.53 [-0.85,-0.17]	0.27 [-0.08,0.75]	
$r^h - r^f$	-0.03 [-0.55,0.32]	0.10 [-0.13,0.47]	0.04 [-0.20,0.41]	0.03 [-0.40,0.55]	-0.70 [-0.85,-0.44]	0.04 [-0.25,0.50]	0.16 [-0.26,0.65]
	[-0.65,0.45]	[-0.23,0.53]	[-0.31,0.46]	[-0.50,0.60]	[-0.89,-0.38]	[-0.33,0.65]	[-0.30,0.65]

Table 3: Long-Run Co-Variability — US, 1870-1915. The table reports the long-run correlation between any two variables, estimated as in [Müller and Watson \(2018\)](#). 67% and 90% CI reported in brackets. Variables: C/W : consumption-to-wealth ratio; $\Delta \ln c$: growth rate of real consumption per capita; $\Delta \ln w$: growth rate of real private wealth per capita; n : population growth; r^f : real ex-post risk free rate; $r^e - r^f$: realized excess equity return; $r^l - r^f$: term premium (10-year minus 3-months); $r^h - r^f$: excess of housing capital gains over risk-free rate.

persistence in the data. For a pair of variables x_t and y_t , Müller & Watson estimates the long-run correlation as the correlation between low frequency transformations of the variables, using low-pass filters. The table also present 67% and 90% confidence intervals, estimated using Müller and Watson ABcde model. Not surprisingly, the results indicate that consumption and wealth co-move positively in the long run, with a long run correlation of the growth rates of 0.64. Beyond this finding, the table illustrates the absence of strong long-run co-movements between real risk-free rates and the usual suspects: real interest rates do not systematically co-move with real consumption growth or population growth. Real risk-free rates do appear to covary negatively with the term premium, i.e. the difference between the yield on 10-year government bonds and the 3-months rate. This is consistent with the expectation hypothesis, with long term rates encoding future short term real rates and the later mean reverting slowly over time. Similarly, while the consumption-to-wealth ratio does not seem to covary strongly with the level of the risk free

rate, it is strongly and statistically negatively correlated with the term premium (-0.52).¹⁷

4.2 Vector-Auto-Regression Results

We construct an empirical estimate of the right hand side of equation (7) using a Vector Auto Regression (VAR). We form the vector $\mathbf{z}_t = \left(\ln C_t - \ln W_t, r_t^f, \mathbf{er}_t', \Delta \ln C_t \right)'$ and estimate a Vector Auto Regression (VAR) of order p . Using this VAR, we then construct the forecasts $\mathbb{E}_t \mathbf{z}_{t+k}$ to construct:¹⁸

$$\begin{aligned}\hat{cw}_t^f &\simeq \mathbb{E}_t \sum_{s=1}^{\infty} \rho_w^s r_{t+s-1}^f \\ \hat{cw}_t^{rp} &\simeq \hat{\nu} \mathbb{E}_t \sum_{s=1}^{\infty} \rho_w^s \mathbf{er}_{t+s} \\ \hat{cw}_t^c &\simeq -\mathbb{E}_t \sum_{s=1}^{\infty} \rho_w^s \Delta \ln C_{t+s}.\end{aligned}$$

Each of these components has a natural interpretation as the contribution of the risk free rate, the risk premium and the consumption growth components to the consumption-to-wealth ratio. We assume an annual discount rate $\rho_w = 1 - 0.0465$. Recall that according to our derivations $\rho_w = 1 - C/\bar{W}$ and that we calibrated $C/\bar{W} = 0.0465$ in section 3.2. Importantly, observe that we do not need to identify structural shocks to form the forecasts \hat{cw}_t^i .

Our approach requires an estimate of ν . As indicated earlier, we estimate this parameter by regressing $\ln C_t - \ln W_t - \hat{cw}_t^f - \hat{cw}_t^c$ on $\mathbb{E}_t \sum_{s=1}^{\infty} \rho_w^s \mathbf{er}_{t+s}$. Recall that we do not observe the return on private wealth, so this method gives the highest chance to the model to match the observed consumption-to-wealth ratio. This calls for two observations. First, as noted above, this method leaves cw^f and cw^c unchanged so the correlation between the consumption growth component and the risk free rate component is unaffected by $\hat{\nu}$. Second, as we noted, while this method is appropriate if there is measurement error in the return to private wealth, it may induce some spurious movements if the residual in Eq. (7) due to fluctuations in human wealth relative to

¹⁷Results are similar for the G-4. We omit that table in the interest of space.

¹⁸See the details of the empirical VAR methodology in Appendix D.

private wealth, is correlated with the excess return on equities and bonds. In that case, $c\hat{w}_t^{tp}$ is best interpreted as capturing both the risk premium as well as the component of the excess return on human wealth that is correlated with it. We start by using the equity excess return $r^e - r^f$ to forecast the risk premium component and discuss later how our results change as we include the term premium and a proxy for housing returns.

Figure 9 shows the consumption wealth ratio as well as the components of the right hand side of equation (7) for the US. The results are striking. First, we note that the fit of the VAR is excellent.¹⁹ The grey line reports the predicted consumption-to-wealth ratio, i.e. the sum of the three components $cw_t^f + cw_t^{tp} + cw_t^c$.²⁰ Our empirical model is able to reproduce quite accurately the annual fluctuations in the consumption-to-wealth ratio over more than a century of data. This is all the more striking since the right hand side of equation (7) is constructed entirely from the reduced form forecasts implied by the VAR estimation.

Second, most of the movements in the consumption-to-wealth ratio reflect expected movements in the future risk-free rate, i.e. the cw_t^f component. The estimated risk-premium component cw^{tp} (in black) is never very significant economically. We do observe, however, a negative co-movement between the consumption cw^c and both $\ln C/W$ and the risk-free component cw^f . This is consistent with productivity and/or demographic shocks driving part of the movements in $\ln C/W$ as discussed in section 3.1. It follows that the consumption-to-wealth ratio contains significant information on current and future real short term rates, as encoded in equation (7). As discussed above, the two historical episodes of low consumption-to-wealth ratios occurred during periods of rapid asset price and wealth increases followed each time by a severe financial crisis. Our empirical results indicate that in the aftermath of these crises real short term rates remain low (or negative) for an extended period of time.

Table 4 decomposes the variance of $\ln C - \ln W$ into components reflecting news about future real risk-free rates, future risk premia, and future consumption growth. The decomposition accounts for 93 percent of the variance in the average propensity to consume, with the risk free rate

¹⁹The lags of the VAR are selected by standard criteria.

²⁰The overall fit is excellent, with an $R^2 = 0.92$. However, this result is obtained with a some attenuation of the equity excess return since we estimate $\hat{\nu} = 0.64$.

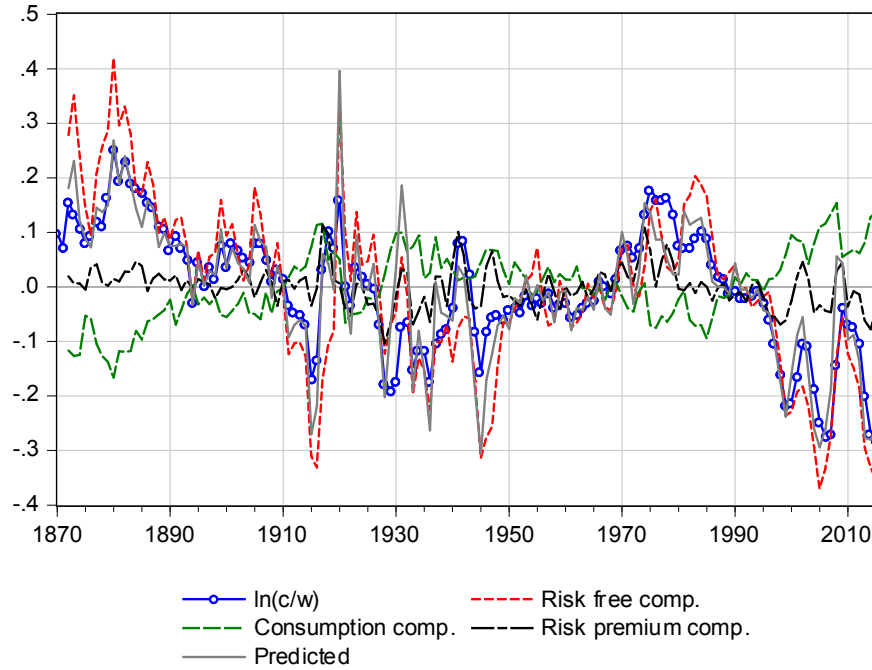


Figure 9: **Consumption Wealth, Risk-free, Equity Premium and Consumption Growth Components. United States, 1870-2015.** Note: The graph reports the (log, demeaned) private consumption-wealth ratio together with the risk-free, risk premium and consumption growth components. Estimates a VAR(2) with $\hat{\nu} = 0.64$. Source: Private wealth from WID. Consumption and short term interest rates from [Jordà et al. \(2016\)](#). Equity return from Global Financial Database.

representing 118 percent of the variation and the consumption growth component -45 percent.

Figure 10 reports a similar decomposition for the ‘G-4’ aggregate between 1920 and 2015. The results are very similar. First, the overall fit of the VAR remains excellent.²¹ As before, we find that the risk-free component explains most of the fluctuations in the consumption-to-wealth ratio. The adjusted risk premium and consumption growth components remain smaller and the risk free component remains strongly negatively correlated with the consumption growth component. Finally, the variance decomposition, presented in Table 4 confirms again the importance of the risk free component. Overall, these results are consistent with the main drivers of being deleveraging shocks as well as productivity/demographic shocks.

To explore further the distinction between productivity and demographic shocks, Figure 11 reports an alternate decomposition where we separate total consumption growth into growth in

²¹The attenuation of the equity risk premium is stronger, however since we estimate $\hat{\nu} = 0.33$.

#	percent	U.S.		G4	
1	β_{rf}	1.178	1.446	1.448	1.361
2	β_{rp}	0.197	0.191	0.255	0.266
3	β_c	-0.446	-0.420	-0.604	-0.548
	of which:				
3	β_{cp}		-0.096		-0.315
4	β_n		-0.324		-0.232
5	Total	0.929	1.217	1.099	1.079
	(lines 1+2+3)				

Table 4: **Unconditional Variance Decomposition of $\ln C - \ln W$**

Note: β_{rf} (resp. β_{rp} , and β_c) represents the share of the unconditional variance of $\ln C - \ln W$ explained by future risk free returns (resp. future risk premia and future total consumption growth); β_{cp} (β_n) represents the share of the unconditional variance of $\ln C - \ln W$ explained by per capita consumption growth (population growth). The sum of coefficients $\beta_{cp} + \beta_n$ is not exactly equal to β_c due to numerical rounding in the VAR estimation. Sample: U.S: 1870-2015; G4: 1920:2015

consumption per capita and population growth: $\Delta \ln C = \Delta \ln c + n$. The results are largely unchanged. Table 4 provides the unconditional variance decomposition. This suggests that productivity shocks and demographic shocks play similar role in the dynamics of C/W . Both are negatively correlated with the risk free component

The fact that equity risk premia account for almost none of the movements in C/W is perhaps surprising in light of [Lettau and Ludvigson \(2001\)](#)'s findings that a cointegration relation between aggregate consumption, wealth and labor income predicts reasonably well U.S. equity risk premia. A number of factors may account for this result. First and foremost, we assume that $\ln C/W$ is stationary over the long run, and thus do not estimate a cointegrating vector with labor income. Second, we consider a longer sample period, going back to 1870 for the U.S and 1920 for the other countries. Thirdly, as argued above, our sample is dominated by two large financial crises and their aftermath. Lastly, we view our analysis as picking up low frequency determinants of real risk-free rates while [Lettau and Ludvigson \(2001\)](#) seem to capture business cycle frequencies.

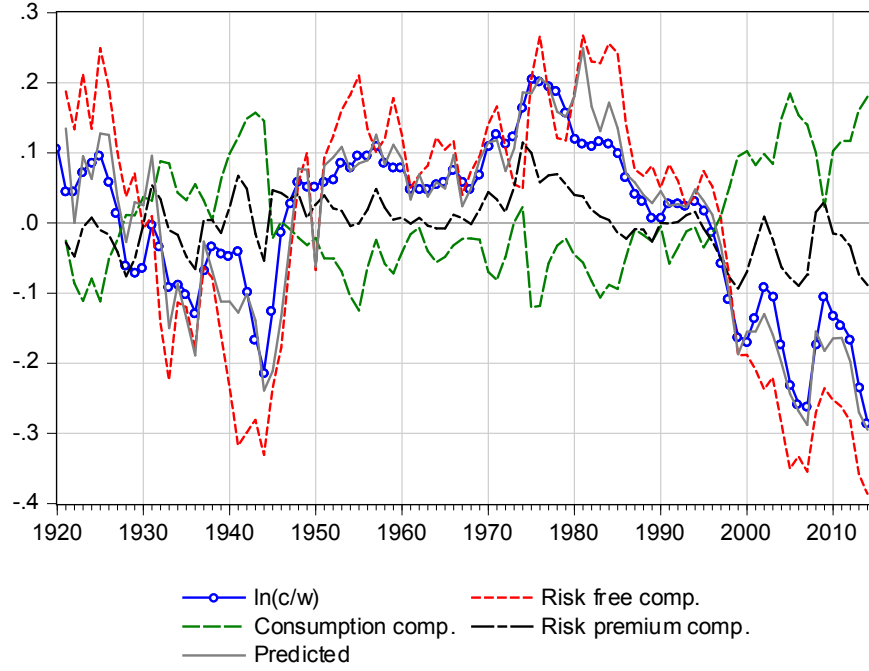


Figure 10: **Consumption Wealth: Risk-free, Equity Premium and Consumption Growth Components. United States, United Kingdom, Germany and France, 1920-2015.** Note: The graph reports the (log, demeaned) private consumption-wealth ratio for the U.S. U.K., Germany and France, together with the risk-free, risk premium and consumption growth components. Estimates a VAR(2) with $\hat{\nu} = 0.329$. Source: Private wealth from [Piketty and Zucman \(2014a\)](#). Consumption and short term interest rates from [Jordà et al. \(2016\)](#). Equity return from Global Financial Database.

5 Predictive regressions

The third step consists in directly evaluating the forecasting performance of the consumption-wealth variable for future risk-free interest rates, risk premia and aggregate consumption growth.

Our decomposition exercise indicates that the consumption-wealth ratio contains information on future risk-free rates. We can evaluate directly the predictive power of $\ln C_t/W_t$ by running regressions of the form:

$$y_{t+k} = \alpha + \beta \ln (C_t/W_t) + \epsilon_{t+k} \quad (12)$$

where y_{t+k} denotes the variable we are trying to forecast at horizon k . We consider the following candidates for y : the average real risk free rate between t and $t + k$; the average one-year excess return between t and $t + k$; the average annual real per capita consumption growth between t

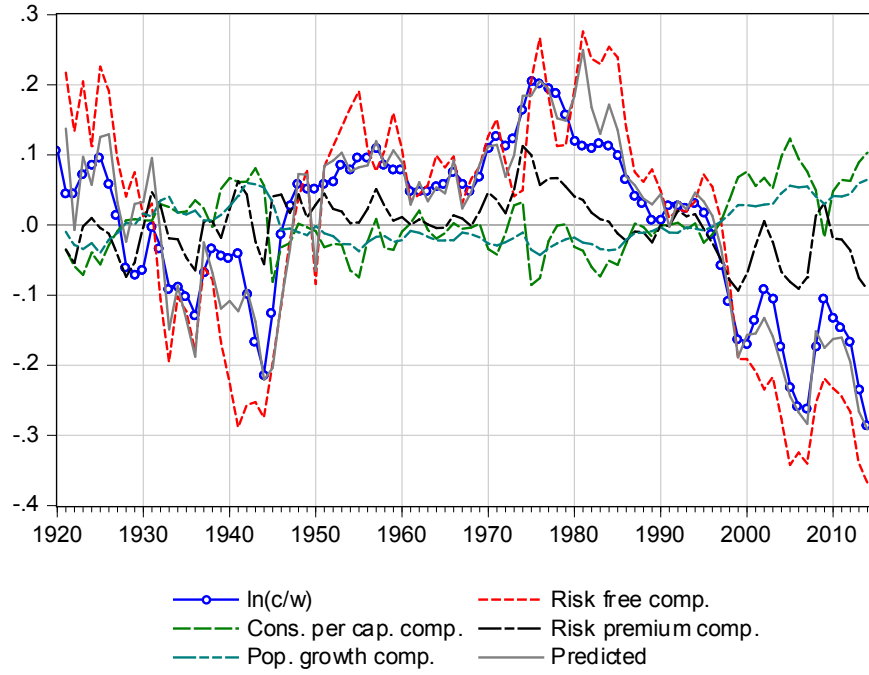


Figure 11: **Consumption Wealth: Risk-free, Equity Premium, Consumption per capita and Population Growth Components. United States, United Kingdom, Germany and France, 1920-2015.** Note: The graph reports the (log, demeaned) private consumption-wealth ratio together with the riskfree, risk premium, consumption per capita and population growth components. Estimates a VAR(2) with $\hat{\nu} = 0.19$. Source: Private wealth from [Piketty and Zucman \(2014a\)](#). Consumption, population and short rates from [Jordà et al. \(2016\)](#). Equity return from Global Financial Database.

and $t + k$; the average annual population growth between t and $t + k$; the average term premium between t and $t + k$; the average growth of real credit to the non-financial sector per capita between t and $t + k$.

Tables 5 presents the results for the US and the G4 aggregate. We find that the consumption-to-wealth ratio always contains substantial information about future short term risk free rates (panel A). The coefficients are increasing with the horizon and become strongly significant. They also have the correct sign, according to our decomposition: a low $\ln C/W$ strongly predicts a period of below average real risk-free rates up to 10 years out. By contrast, the consumption-to-wealth ratio has almost no predictive power for the equity excess returns, and more limited predictive power for per-capita consumption growth. The regressions indicate some predictive power for population growth for the U.S.: a low $\ln C/W$ predicts a low future population growth

United States					U.S., U.K., France and Germany				
Forecast Horizon (Years)					Forecast Horizon (Years)				
	1	2	5	10		1	2	5	10
A. Short term interest rate					A. Short term interest rate				
$\ln C_t/W_t$.13	.14	.14	.15	$\ln C_t/W_t$.06	.07	.11	.15
	(.04)	(.05)	(.04)	(.03)		(.03)	(.04)	(.05)	(.05)
R^2	[.08]	[.10]	[.18]	[.29]	R^2	[.03]	[.03]	[.13]	[.27]
B. Consumption growth (per-capita)					B. Consumption growth (per-capita)				
$\ln C_t/W_t$	-.03	-.006	-.001	-.04	$\ln C_t/W_t$.04	.04	.04	.04
	(.02)	(.03)	(.02)	(.08)		(.02)	(.02)	(.02)	(.02)
R^2	[0.01]	[0.0]	[0.0]	[.00]	R^2	[.02]	[.05]	[.10]	[.10]
C. Equity Premium					C. Equity Premium				
$\ln C_t/W_t$.15	.13	-.02	-.07	$\ln C_t/W_t$.24	.26	.21	.14
	(.16)	(.15)	(.09)	(.08)		(.19)	(.15)	(.10)	(.10)
R^2	[0.0]	[0.0]	[0.0]	[.02]	R^2	[.01]	[.04]	[.08]	[.05]
D. Population Growth					D. Population Growth				
$\ln C_t/W_t$.03	.03	.03	.02	$\ln C_t/W_t$.01	.02	.01	.01
	(.01)	(.01)	(.01)	(.01)		(.01)	(.01)	(.01)	(.01)
R^2	[.30]	[.30]	[.33]	[.29]	R^2	[.06]	[.06]	[.06]	[.03]
E. Term Premium					E. Term Premium				
$\ln C_t/W_t$	-.05	-.05	-.05	-.04	$\ln C_t/W_t$	-.03	-.04	-.5	-.04
	(.01)	(.01)	(.01)	(.01)		(.01)	(.01)	(.01)	(.01)
R^2	[.10]	[.14]	[.26]	[.25]	R^2	[.7]	[.11]	[.36]	[.35]
D. Credit Growth					D. Credit Growth				
$\ln C_t/W_t$.06	.09	.13	.09	$\ln C_t/W_t$.15	.15	.14	.06
	(.05)	(.05)	(.07)	(.08)		(.04)	(.05)	(.07)	(.06)
R^2	[0.0]	[.02]	[.08]	[.06]	R^2	[0.08]	[.12]	[.14]	[.02]

Table 5: **Long Horizon Regressions.** Note: The table reports the point estimates, Newey-West corrected standard errors and the R^2 of the forecasting regression.

which suggests that the indirect effect (via changes in real risk-free rates) dominates the direct effect. Finally, there is significant predictive power for the term premium, i.e. the difference between the yield on 10-year Treasuries and short term rates. According to the estimates, a decrease in C/W is associated with a significant increase in term premia. This result is consistent with our long-run co-variability estimates.

Figures 12-18 report our forecast of the risk free rate, equity premium, population growth, cumulated consumption growth per capita, term premium and the growth rate of credit to the nonfinancial sector, using the G-4 consumption-to-wealth ratio at 1, 2, 5 and 10 year horizon. For each year t , the graph reports $y_{t,k}^f = \frac{1}{k} \sum_{s=0}^{k-1} y_{t+s}^f$, the average of the variable z to forecast

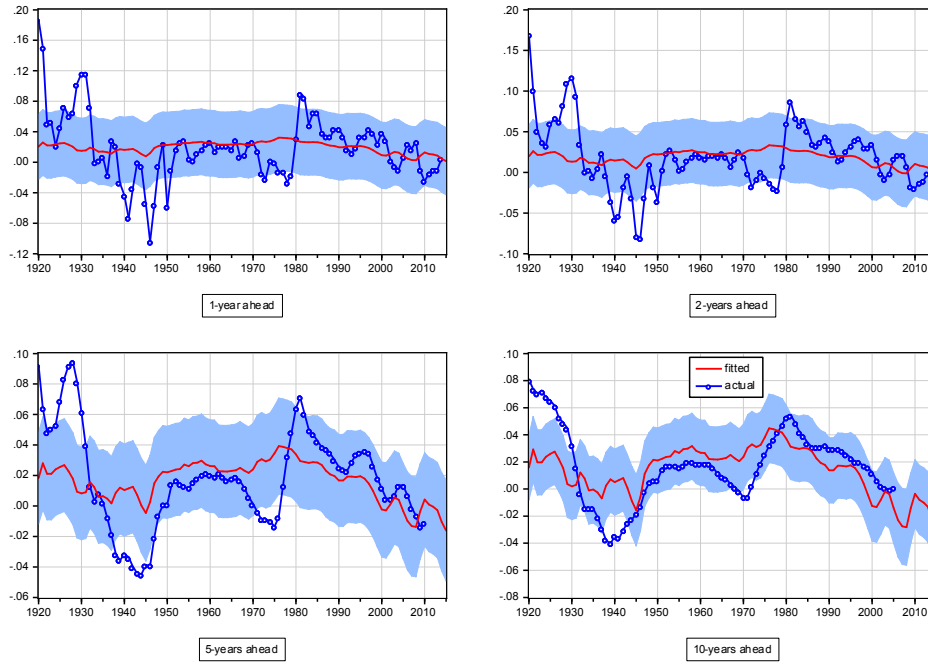


Figure 12: **Predictive Regressions: Risk Free Rate, 1920-2015.** Note: The graph reports forecasts at 1, 2, 5 and 10 years of the annualized global real risk free rate from a regression on past $\ln(C/W)$.

one-year real risk-free rate between t and $t + k$, where k is the forecasting horizon. The graph also reports the predicted value $\hat{y}_{t,k}^f$ based on predictive regression (12) together with a 2-standard error confidence band, computed using Newey-West robust standard errors. For two variables, the average future global short rate and the average future global term premium, the fit of the regression improves markedly with the horizon. The last forecasting point is 2015, indicating a forecast of -3.1 percent for the global short real interest rate until 2025 (bottom right graph). The corresponding figure using U.S. data is -2.35 percent.

Finally, figure ?? reports, for the U.S., the forecast of the average risk-free rate at 10 years, together with a Kalman-Filter estimate constructed using the Present Value representation, as in Ventura (2001). The Kalman-Filter estimate tracks the realized 10-year average riskfree rate extremely well. The estimated risk free rate for 2015-2025 is slightly higher, at -1.37 percent, but still remarkably low compared to historical averages.

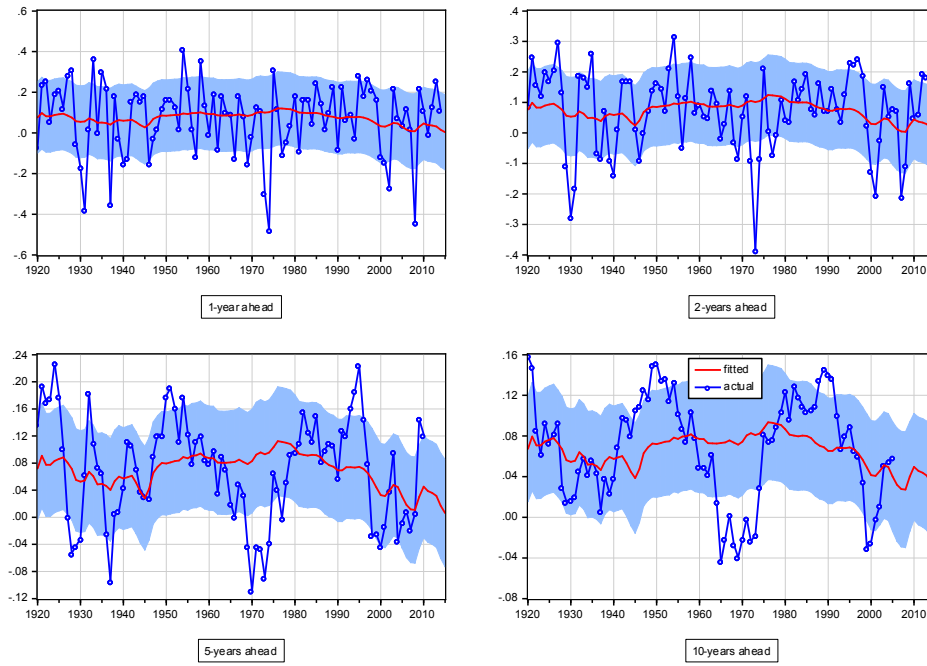


Figure 13: **Predictive Regressions: Equity Premium, 1920-2015.** Note: The graph reports forecasts at 1, 2, 5 and 10 years of the annualized equity premium from a regression on past $\ln(C/W)$.

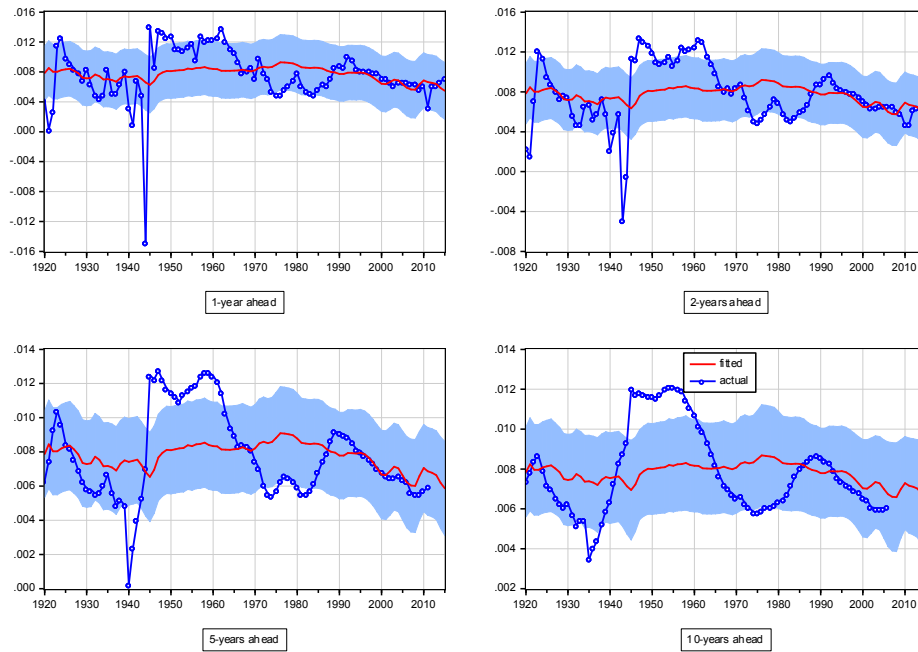


Figure 14: **Predictive Regressions: Population Growth, 1920-2015.** Note: The graph reports forecasts at 1, 2, 5 and 10 years of the annualized global population growth rate from a regression on past $\ln(C/W)$.

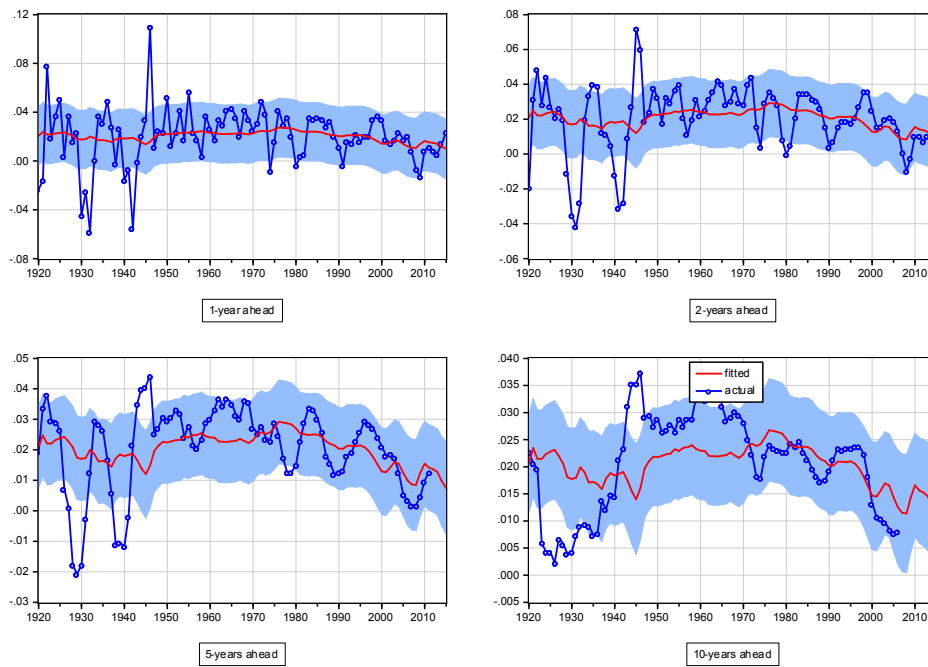


Figure 15: **Predictive Regressions: Consumption growth per capita, 1920-2015.** Note: The graph reports forecasts at 1, 2, 5 and 10 years of the annualized global per capita real consumption growth from a regression on past $\ln(C/W)$.

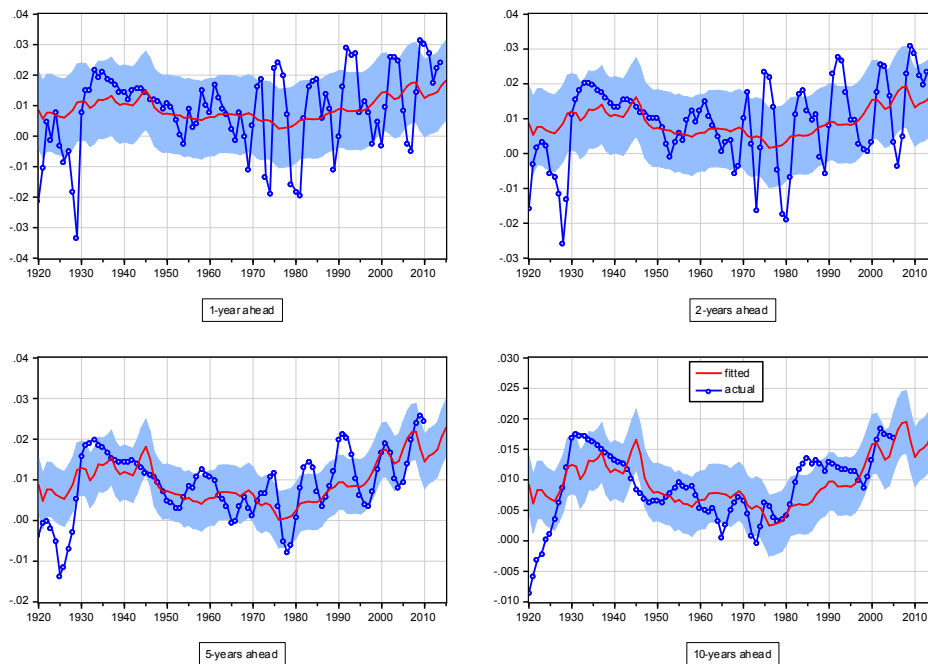


Figure 16: **Predictive Regressions: Term premium, 1920-2015.** Note: The graph reports forecasts at 1, 2, 5 and 10 years of the annualized global term premium from a regression on past $\ln(C/W)$.

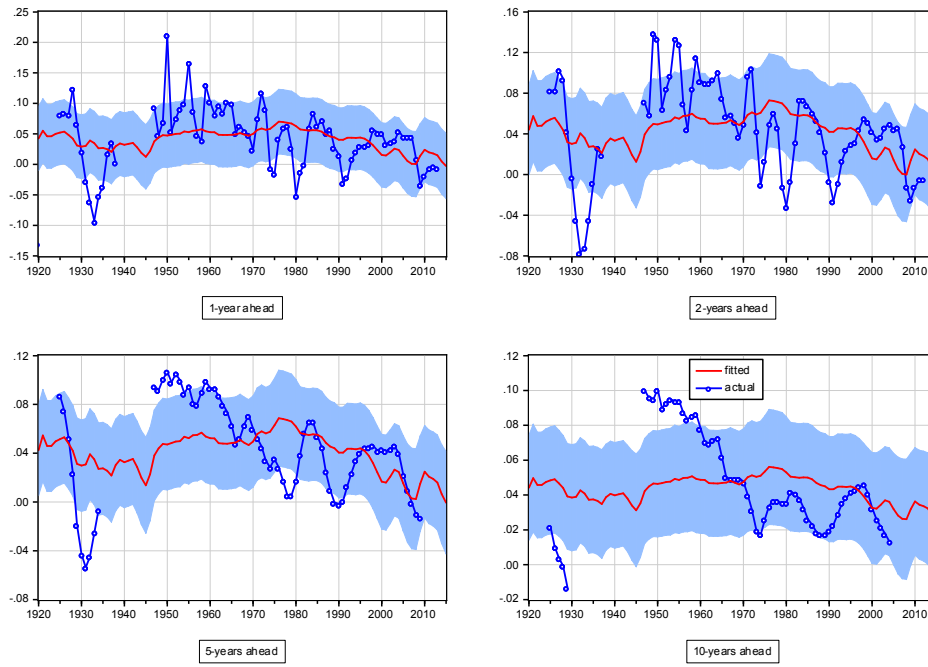


Figure 17: **Predictive Regressions: Growth rate of credit to non-financial sector, 1920-2015.** Note: The graph reports forecasts at 1, 2, 5 and 10 years of the annualized global term premium from a regression on past $\ln(C/W)$.

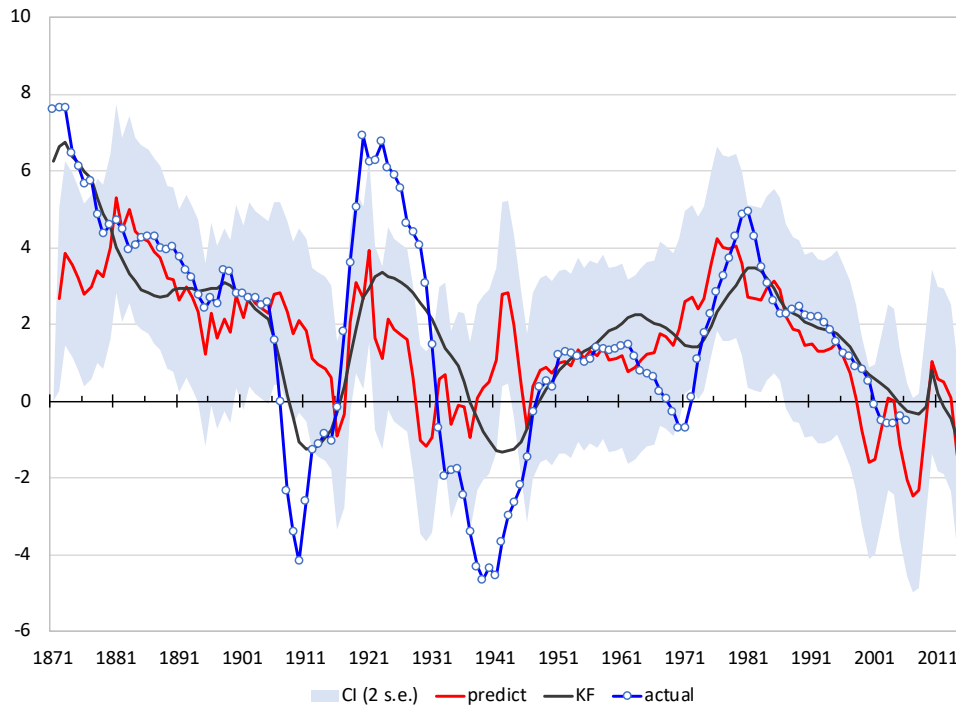


Figure 18: **Predictive Regressions: Real interest rate, 1870-2015.** Note: The graph reports forecasts at 10 years of the annualized risk-free rate: a simple forecast using $\ln(C/W)$ and a Kalman Filter estimate using [Ventura \(2001\)](#).

6 Conclusion.

Our results suggest that macro and financial shocks are both important determinants of global real rates. On the macro side, there is some evidence that productivity growth and demographic shocks affect global rates. On the financial side, the two significant declines in C/W occurred in the years preceding -and in the aftermath- of global financial crises. These boom-bust financial cycles are a strong determinant of real short term interest rates. During the boom, private wealth increases rapidly, faster than consumption, bringing down the ratio of consumption to private wealth. This increase in wealth can occur over the course of a few years, fueled by increased leverage, financial exuberance, and increased risk appetite. Two such historical episodes for the global economy are the roaring 1920s and the 2000s. In the subsequent bust, asset prices collapse, collateral constraints bind, and households, firms and governments attempt to simultaneously de-leverage, as risk appetite wanes. The combined effect is an increase in desired saving that depresses persistently safe real interest rates. An additional force may come from a weakened banking sector and financial re-regulation or repression that combine to further constrain lending activity to the real sector. Our estimates indicate that short term real risk free rates are expected to remain low or even negative for an extended period of time.

The central object of our analysis are risk free rates. In recent years, an abundant empirical literature has attempted to estimate the natural rate of interest, r^* , defined as the real interest rate that would obtain in an equivalent economy without nominal frictions. Many estimates indicate that this natural rate may well have become significantly negative. Our analysis speaks to this debate. Outside of the effective lower bound, monetary policy geared at stabilizing prices and economic activity will set the policy rate so that the real short term rate is as close as possible to the natural rate. Therefore, to the extent that the economy is outside the ELB, our estimate of future global real rates should coincide with estimates of r^* . At the ELB, this is not necessarily the case since global real rates must, by definition of the ELB, be higher than the natural rate. Therefore, our estimates provide an upper bound on future expected natural rates. Given that our estimates are quite low (-2.35 percent on average between 2015 and 2025), we conclude that

the likelihood of the ELB binding remains quite elevated.

Our empirical results suggest that over long horizons, global real rates are driven both by standard structural forces, such as productivity or demographic forces, as well as financial forces, especially the leveraging cycle that accompanied the boom and bust in the 1930s and in the 2000s.

We view these empirical results very much in line with interpretations of recent events that emphasize the global financial cycle ([Miranda-Agrippino and Rey \(2015\)](#), [Reinhart and Rogoff \(2009\)](#))).

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Appendix

A Data description

The data used in Section 4 were obtained from the following sources:

1. Consumption:

Real per-capita consumption going back to 1870 and covering the two world wars was taken from [Jordà et al. \(2016\)](#) who in turn obtained the data from [Barro and Ursúa \(2010\)](#). As this consumption series is an index rather than a level, we convert it to a level using the consumption data from [Piketty and Zucman \(2014a\)](#). To convert to a level we could use any year we have level data for but chose to use the year 2006 (the year that the index of consumption was 100). In addition, the consumption data was adjusted so that instead of being based on a 2006 consumption basket, it was based on a 2010 consumption basket to match the wealth data.

2. Wealth:

Real per capita wealth data was taken from [Piketty and Zucman \(2014b\)](#). The wealth concept used here is private wealth. As such it does not include government assets but includes private holdings of government issued liabilities as an asset. Where possible, wealth data is measured at market value. Human wealth is not included. Private wealth is computed from the following components: “Non-financial assets” (includes housing and other tangible assets such as software, equipment and agricultural land), and net financial assets (includes equity, pensions, value of life insurance and bonds). Prior to 1954 for France, 1950 for Germany, 1920 for the UK and 1916 for the USA, wealth data is not available every year (see Piketty-Zucman’s appendix for details on when data is available for each country or refer to Table 6f in the data spreadsheets for each country). When it is available it is based on the market value of land, housing, other domestic capital assets and net foreign assets less net government assets. For the remaining years the wealth data is imputed based on savings rate data and assumptions of the rate of capital gains of wealth (see the Piketty-Zucman appendix for details of the precise assumptions on capital gains for each country. The computations can be found in Table 5a in each of the data spreadsheets for each country).

3. Short term interest rates:

These were taken from [Jordà et al. \(2016\)](#) and are the interest rate on 3-month treasuries.

4. Long term interest rates:

These were taken from [Jordà et al. \(2016\)](#) and are the interest rate on 10 year treasuries.

5. Return on Equity:

This data is the total return on equity series taken from the Global Financial Database.

6. CPI:

CPI data is used to convert all returns into real rates and is taken from [Jordà et al. \(2016\)](#).

7. Population:

These were taken from [Jordà et al. \(2016\)](#).

Figure 6 reports consumption per capita, wealth per capita, the consumption/wealth ratio as well as the short term real risk free rate for our G4 aggregate between 1920 and 2011.

B Loglinearization of the budget constraints and aggregation

For a country i the budget constraint takes the form:

$$\bar{W}_{t+1}^i = \bar{R}_{t+1}^i (\bar{W}_t^i - C_t^i) \quad (13)$$

where \bar{W}_t^i denotes total wealth at the beginning of period t , C_t^i is private consumption during period t and \bar{R}_{t+1}^i is the gross return on total wealth between periods t and $t+1$. All variables are measured in real terms. [Lettau and Ludvigson \(2001\)](#) propose a log-linear expansion around the steady state consumption-to-wealth ratio and steady state return. Define $cw_t^i = \ln C_t^i - \ln \bar{W}_t^i$. cw_t^i is stationary with mean \bar{cw}^i . Dividing both side of (13) by \bar{W}_t^i and taking logs, we obtain:

$$\begin{aligned} \ln \bar{W}_{t+1}^i / \bar{W}_t^i &= \bar{r}_{t+1}^i + \ln(1 - C_t^i / \bar{W}_t^i) \\ &= \bar{r}_{t+1}^i + \ln(1 - e^{\bar{cw}^i} \exp(cw_t^i - \bar{cw}^i)) \\ &\approx \bar{r}_{t+1}^i + \ln(1 - e^{\bar{cw}^i} - e^{\bar{cw}^i} (cw_t^i - \bar{cw}^i)) \\ &\approx \bar{r}_{t+1}^i + \ln \left((1 - e^{\bar{cw}^i}) \left(1 - \frac{e^{\bar{cw}^i}}{1 - e^{\bar{cw}^i}} (cw_t^i - \bar{cw}^i) \right) \right) \\ &\approx \bar{r}_{t+1}^i + \ln(1 - e^{\bar{cw}^i}) - \frac{e^{\bar{cw}^i}}{1 - e^{\bar{cw}^i}} (cw_t^i - \bar{cw}^i) \\ &\approx \bar{r}_{t+1}^i + k + \left(1 - \frac{1}{\rho_w} \right) cw_t^i \end{aligned}$$

where $\rho_w = 1 - e^{\bar{cw}^i}$ and k is an unimportant constant. The next step is to rewrite the left hand side as

$$\ln \bar{W}_{t+1}^i / \bar{W}_t^i = \ln(\bar{W}_{t+1}^i / C_{t+1}^i) - \ln(\bar{W}_t^i / C_t^i) + \Delta \ln C_{t+1}^i = -cw_{t+1}^i + cw_t^i + \Delta \ln C_{t+1}^i$$

to obtain (again, ignoring the constant):

$$cw_t^i = \rho_w (cw_{t+1}^i - \Delta \ln C_{t+1}^i + \bar{r}_{t+1}^i) \quad (14)$$

which can be iterated forward to obtain (under the usual transversality condition):

$$c_t^i - \bar{w}_t^i = \sum_{s=1}^{\infty} \rho_w^s (\bar{r}_{t+s}^i - \Delta \ln C_{t+s}^i)$$

C Aggregation

From Eq. (1) we can aggregate across countries:

$$\sum_i \frac{\bar{W}_{t+1}^i}{\bar{R}_{t+1}^i} = \sum_i \bar{W}_t^i - C_t^i = \bar{W}_t - C_t$$

where $\bar{W}_t = \sum_i \bar{W}_t^i$ and $C_t = \sum_i C_t^i$. From this expression we can derive

$$\bar{W}_{t+1} = \bar{R}_{t+1}(\bar{W}_t - C_t)$$

where

$$\frac{1}{\bar{R}_{t+1}} = \sum_i \frac{\bar{W}_{t+1}^i}{\bar{W}_{t+1}} \frac{1}{\bar{R}_{t+1}^i}$$

The global period return on private wealth is an harmonic weighted mean of the individual country returns.

D VAR methodology

Consider the present value relation in Eq. 4. We form $\mathbf{z}_t = (\ln C_t - \ln W_t, r_t, \mathbf{ep}_t', \Delta \ln C_t)'$ and estimate Vector AutoRegression of order p, VAR(p), which can be expressed in companion form as:

$$\bar{\mathbf{z}}_t = \bar{\mathbf{A}}\bar{\mathbf{z}}_{t-1} + \bar{\varepsilon}_t$$

where $\bar{\mathbf{z}}_t' = (\mathbf{z}_t', \mathbf{z}_{t-1}', \dots, \mathbf{z}_{t-p}')$. Using the estimated VAR matrix $\bar{\mathbf{A}}$, conditional forecasts of $\bar{\mathbf{z}}_t$ can be directly obtained as:

$$\mathbb{E}_t \bar{\mathbf{z}}_{t+k} = \bar{\mathbf{A}}^k \bar{\mathbf{z}}_t$$

from which we recover:

$$\mathbb{E}_t \sum_{s=1}^{\infty} \rho_w^s \bar{\mathbf{z}}_{t+s} = \sum_{s=1}^{\infty} \rho_w^s \bar{\mathbf{A}}^s \bar{\mathbf{z}}_t = \rho_w \bar{\mathbf{A}} (\mathbf{I} - \rho_w \bar{\mathbf{A}})^{-1} \bar{\mathbf{z}}_t.$$

Denote \mathbf{e}_x the vector that ‘extracts’ variable x from $\bar{\mathbf{z}}$, in the sense that $\mathbf{e}_x' \bar{\mathbf{z}} = x$. It follows that

$$\mathbb{E}_t \sum_{s=1}^{\infty} \rho_w^s \mathbf{x}_{t+s} = \rho_w \mathbf{e}_x' \bar{\mathbf{A}} (\mathbf{I} - \rho_w \bar{\mathbf{A}})^{-1} \bar{\mathbf{z}}_t$$

From this we can construct the various components as:

$$\begin{aligned} cw_t^f &= \rho_w \mathbf{e}_r' \bar{\mathbf{A}} (\mathbf{I} - \rho_w \bar{\mathbf{A}})^{-1} \bar{\mathbf{z}}_t \\ cw_t^c &= -\rho_w \mathbf{e}_{\Delta \ln C}' \bar{\mathbf{A}} (\mathbf{I} - \rho_w \bar{\mathbf{A}})^{-1} \bar{\mathbf{z}}_t \\ cw_t^{rp} &= \nu \rho_w \mathbf{e}_{erp}' \bar{\mathbf{A}} (\mathbf{I} - \rho_w \bar{\mathbf{A}})^{-1} \bar{\mathbf{z}}_t \\ cw_t^{\Delta \ln c} &= -\rho_w \mathbf{e}_{\Delta \ln c}' \bar{\mathbf{A}} (\mathbf{I} - \rho_w \bar{\mathbf{A}})^{-1} \bar{\mathbf{z}}_t \\ cw_t^n &= -\rho_w \mathbf{e}_n' \bar{\mathbf{A}} (\mathbf{I} - \rho_w \bar{\mathbf{A}})^{-1} \bar{\mathbf{z}}_t \end{aligned}$$

Global Real Rates: A Secular Approach by Pierre-Olivier

Gourinchas and Helene Rey

Discussion by Sebnem Kalemli-Ozcan

This paper asks two very important and related questions: Why do real risk free rates decline and how long they will stay low? Differently than the existing literature on the topic, the authors adopt a present value approach to decompose fluctuations in global consumption-to-wealth ratio over long periods of time. This decomposition involves three components: risk free rate, risk premia, and consumption growth. The authors write down a model to analyze role of shocks on each of these components to discover the underlying deep causes of fluctuations in consumption-to-wealth ratio. They undertake such an exercise since consumption-to-wealth ratio predicts risk free rates. Their main results are as follows:

1. Consumption-to-wealth ratio is a strong predictor of risk-free rates, term premium, and population growth.
2. Macro shocks and financial shocks both have a role in explaining real risk free rates.
3. Risk free rates will stay low for an extended period of time.
4. Overall the results suggest a decline in *natural* interest rate.

This is an excellent paper with thought provoking results. In my comments, I will try to clarify certain issues to make the interpretation of the results sharper. The first issue is on measurement of real risk free rates and relation to natural rate. Real rate is the sum of real risk free rate and risk premium. And natural rate is only equal to real rate when real rate is the

one that equates output to potential output. Put it differently, real rate only equals to natural rate under monetary policy neutrality. Hence, a decline in real risk free rate may or may not suggest a decline in natural rate since this will also depend on risk premium and monetary policy. In this juncture, it is important how to measure risk free real rates. The authors use a measure of nominal rates on 3 month treasury bills after subtracting CPI inflation. Maybe a better measure is nominal rates minus inflation expectations and/or yields on inflation-indexed bonds.

The next issue is on the key cause of decline in real rates. The existing literature takes two opposing views. The “investment view” or the “savings view.” The investment view, associated with Larry Summers, argue that the decline in real rates is due to a decline in investment due to lack of good investment opportunities given the lower relative price of investment. The savings view can have two different components. The first one, associated with Ben Bernanke, is about savings glut in the rest of the world due to demographic changes and those savings are invested in US risk free assets, lowering their yield. The second component of savings view is about debt accumulation and associated deleveraging that can take a long time, as argued by Carmen Reinhart and Ken Rogoff. There can also be a twist to this story in terms of preferences where investors prefer risk free assets to risky ones so savings are channelled to risk free assets, that leads to a decline in risk free rate and an increase in risk premium. The early proponents of this view are Ricardo Caballero, Emmanuel Farhi and Pierre-Olivier Gourinchas.

The current paper is different than all the other papers given their long-run historical approach. However, it is similar to the papers that support the savings view since the current paper’s long-run approach makes it clear that savings especially after the financial crises have a big role in explaining declining real risk free rates. The problem is that, it is not fully clear if increases in savings after big financial crises is the only force behind the declining real risk free rates or there are also other factors at play? For example, savings might be higher due to demographics changes and not due to deleveraging effect of financial crises.

The authors realize this and run predictive regressions to sort this out. Their predictive regressions regress several outcome variables, namely, risk free rates, consumption growth,

equity premium, population growth, term premium, and credit growth, on consumption-to-wealth ratio and find that this ratio can predict risk free rates, term premium, and population growth at long horizons. Revisiting these results suggest that a *stationary* consumption-to-wealth ratio, as assumed by their decomposition approach, can only predict risk free rates and term premium and not population growth. Checking for stationarity of the consumption-to-wealth ratio and adding a trend to the predictive regressions deliver these results as shown in Tables 1 and 2. A horse race predicting regression in Table 3, shows that, it is not only consumption-to-wealth ratio but also term premium can predict risk free rates.

The final issue is on identification. The decomposition of consumption-to-wealth ratio does not have a causal interpretation. But, we want to know what causes the fluctuations in consumption-to-wealth ratio. Again, realizing this fully, the authors run several different exercises, where each delivers a different result. For example, the VAR analysis says risk premium is not important for consumption-to-wealth ratio. But, the OLS says risk/term premium is very important for consumption-to-wealth ratio. The VAR says productivity shocks and demographic shocks seem to be more important than deleveraging shock but simple plots of data seems to suggest a bigger role for deleveraging and risk appetite shocks. The key reason for this dilemma is the fact that the model based VAR forces the Euler equation to hold and hence there is a negative association between the risk free rate component and consumption component, whereas a deleveraging shock implies a positive association between the risk free rate and the consumption growth component.

I propose that the authors can further delve into this issue by using their model to identify the effect of shocks on consumption-to-wealth ratio and risk free rates. They have four different shocks, that are a productivity shock, a demographic shock, a deleveraging shock, and a risk appetite shock. They investigate each separately using a reduced form VAR but if they evaluate all together instead, pushing their structural model further then the most important determinant of consumption-to-wealth ratio can survive. This can be done by adding all the shocks to the model and calculating the share of variation explained by each shock. Undertaking this exercise shows that deleveraging shock explains consumption-to-wealth ratio and deleveraging shock together with the risk appetite shock explain risk free rate. Table 4 shows

the fit of such a model is good. Table 5 shows that deleveraging shock and risk appetite shock explain about 31 and 63 percent of risk free rate movements, respectively. Productivity shock only explains 6 percent of risk free rate fluctuations and the effects of demographic shock are negligible. On the other hand, consumption to wealth ratio is mainly explained by deleveraging shock (92 percent). It implies that deleveraging shock primarily increases the correlation between risk free rate and consumption to wealth ratio. Appendix provides details on the model with all the shocks.

In closing, I want to re-emphasize that this is an important contribution showing effects of savings increases as a result of debt super cycle and deleveraging on real risk free rate decline. There are also important policy implications: First, the result that term premium can predict short-term risk free rates, suggest an important role of expectations, which is essential for monetary policy making. The second important policy implication is how persistent the effects of debt driven financial crises on risk free rates can be and the final one is on the effectiveness of monetary policy since under persistent low interest rates, this will be in doubt.

Table 1: Testing for Stationarity

Null hypothesis: The variable has a unit root				
Variable	Sample	Specification [†]	t-statistic	p-value
U.S. $\ln(C/W)$	1870 - 2015	No intercept and trend	0.534	0.830
		Intercept only	-2.592	0.097
		Intercept and trend	-3.430	0.052
	1920 - 2015	No intercept and trend	0.629	0.851
		Intercept only	-1.173	0.683
		Intercept and trend	-1.303	0.881
G-4 $\ln(C/W)$	1920 - 2015	No intercept and trend	0.876	0.897
		Intercept only	-0.862	0.796
		Intercept and trend	-1.123	0.919

The equation for the augmented Dickey-Fuller test is specified as $\Delta y_t = \gamma y_{t-1} + \sum_{s=1}^k \delta_s \Delta y_{t-s} + c + \beta t + \epsilon_t$. Reject unit root and establish stationarity only for 1870-2015 for US.

Table 2: Predictive Regressions With Trend

United States (1870 - 2015)								
Forecast Horizon	1	2	5	10	1	2	5	10
	(1) No Trend				(2) With Trend			
A. Short term interest rate								
$\ln(C/W)_t$	0.13** (0.06) [0.09]	0.14** (0.06) [0.11]	0.14*** (0.04) [0.19]	0.15*** (0.03) [0.29]	0.09 (0.08) [0.10]	0.10 (0.08) [0.13]	0.12** (0.05) [0.21]	0.13*** (0.03) [0.30]
R^2								
B. Consumption Growth (per capita)								
$\ln(C/W)_t$	-0.03 (0.03) [0]	0 (0.03) [0]	0.01 (0.03) [0]	-0.01 (0.02) [0]	-0.03 (0.04) [0]	0.01 (0.04) [0]	0.03 (0.03) [0.03]	0 (0.02) [0.07]
R^2								
C. Equity Premium								
$\ln(C/W)_t$	0.13 (0.15) [0]	0.12 (0.15) [0]	0.01 (0.09) [0]	-0.04 (0.07) [0]	0.29 (0.19) [0.01]	0.26 (0.19) [0.03]	0.09 (0.10) [0.02]	-0.01 (0.07) [0.02]
R^2								
D. Population Growth								
$\ln(C/W)_t$	0.03*** (0.01) [0.30]	0.03*** (0.01) [0.32]	0.03*** (0.01) [0.34]	0.02*** (0.01) [0.31]	0.01 (0.01) [0.62]	0.01 (0.01) [0.64]	0.01* (0.01) [0.67]	0.01* (0.01) [0.68]
R^2								
E. Term Premium								
$\ln(C/W)_t$	-0.05*** (0.01) [0.11]	-0.05*** (0.01) [0.15]	-0.05*** (0.01) [0.27]	-0.04*** (0.01) [0.27]	-0.03 (0.02) [0.17]	-0.03* (0.01) [0.23]	-0.03*** (0.01) [0.40]	-0.02** (0.01) [0.52]
R^2								
U.S., U.K., France and Germany (1920 - 2015)								
A. Short term interest rate								
$\ln(C/W)_t$	0.07 (0.05) [0.03]	0.08 (0.05) [0.05]	0.12*** (0.04) [0.18]	0.17*** (0.04) [0.35]	0.07 (0.06) [0.02]	0.08 (0.06) [0.04]	0.13*** (0.05) [0.17]	0.17*** (0.04) [0.35]
R^2								
E. Term Premium								
$\ln(C/W)_t$	-0.03** (0.02) [0.07]	-0.04** (0.01) [0.12]	-0.05*** (0.01) [0.36]	-0.04*** (0.01) [0.38]	-0.03 (0.02) [0.09]	-0.03** (0.02) [0.14]	-0.04*** (0.01) [0.40]	-0.04*** (0.01) [0.44]
R^2								

Table 3: Horse-Race Predictive Regressions

United States (1870 - 2015)								
Horizon	1	2	5	10	1	2	5	10
	No C/W and other variables				All variables			
$\ln(C/W)_t$					0.06 (0.06)	0.08 (0.07)	0.09** (0.05)	0.11*** (0.03)
C. growth _t	-0.02 (0.13)	-0.05 (0.10)	0.03 (0.06)	0.00 (0.06)	-0.04 (0.13)	-0.07 (0.09)	0.01 (0.05)	-0.02 (0.04)
EP _t	-0.01 (0.02)	0.02 (0.02)	0.03* (0.01)	0.01 (0.01)	0.00 (0.02)	0.03 (0.02)	0.03* (0.02)	0.02* (0.01)
Pop. growth _t	1.34 (1.13)	1.25 (1.13)	1.69* (0.93)	1.83** (0.83)	0.77 (1.36)	0.50 (1.39)	0.81 (1.12)	0.94 (0.81)
TP _t	-1.22*** (0.36)	-1.20*** (0.39)	-0.81*** (0.27)	-0.58*** (0.20)	-1.17*** (0.36)	-1.13*** (0.40)	-0.73*** (0.27)	-0.47*** (0.19)
R ²	[0.21]	[0.22]	[0.24]	[0.27]	[0.21]	[0.25]	[0.30]	[0.38]

U.S., U.K., France and Germany (1920 - 2015)								
Horizon	1	2	5	10	1	2	5	10
	No C/W and other variables				All variables			
$\ln(C/W)_t$					0.02 (0.04)	0.04 (0.04)	0.10** (0.04)	0.14*** (0.03)
C. growth _t	-0.05 (0.16)	0.01 (0.15)	0.16 (0.14)	0.17* (0.09)	-0.07 (0.15)	-0.03 (0.13)	0.06 (0.13)	0.05 (0.09)
EP _t	-0.01 (0.02)	0.01 (0.02)	0.01 (0.02)	-0.01 (0.02)	-0.01 (0.02)	0.01 (0.02)	0.01 (0.02)	0.00 (0.02)
Pop. growth _t	1.01 (1.00)	1.18 (1.16)	1.66* (0.93)	1.32* (0.74)	0.91 (1.15)	0.80 (1.20)	0.72 (0.93)	0.15 (0.60)
TP _t	-1.42*** (0.34)	-1.49*** (0.36)	-1.00** (0.39)	-0.80*** (0.29)	-1.40*** (0.34)	-1.45*** (0.37)	-0.93** (0.37)	-0.65*** (0.24)
R ²	[0.24]	[0.27]	[0.20]	[0.19]	[0.24]	[0.28]	[0.28]	[0.41]

Table 4: Model Fit

Data Risk Free Rate Predicting (U.S., 1870 - 2015)					Model Risk Free Rate Predicting				
	Forecast Horizon (Years)					Forecast Horizon (Years)			
	1	2	5	10		1	2	5	10
$\ln(C/W)$	0.13**	0.14**	0.14***	0.15***	$\ln(C/W)$	0.11***	0.11***	0.10***	0.08***
R^2	[0.09]	[0.11]	[0.19]	[0.29]	R^2	[0.20]	[0.23]	[0.25]	[0.22]

Table 5: Using Model for Identification

U.S. (1870 - 2015), Contribution of each shock (percent)				
	Productivity (g)	Demographics (n)	Deleveraging (ρ)	Risk App. (θ)
$\ln(C/W)$	2.18	1.34	92.01	4.47
Risk free rate	5.80	0.24	30.58	63.38

The table reports the share of unconditional variance of log consumption to wealth (C/W) and risk free rate explained by each shock. The share of productivity and population growth shocks includes both first and second moment shock.

1 Appendix: Model

Adopt the model in the section 3.1.4 of Gourinchas and Rey (2018). The model assumes Epstein-Zin recursive preference of representative household. Add productivity (g), demographics (n), deleveraging (ρ), and risk appetite (θ) shock together in the model. Notations are the same as in Gourinchas and Rey (2018), unless stated otherwise. Using the fact that consumption growth $\Delta \ln C_{t+1}$ equals the sum of consumption per capita growth (g_{t+1}) and population growth (n_{t+1}), one can obtain the Euler equation for the risk free rate as follows:

$$r_t^f = \rho_t + \sigma E_t[g_{t+1} + n_{t+1}] + \frac{\theta_t - 1}{2} \sigma_{r,t}^2 - \frac{\theta_t \sigma^2}{2} (\sigma_{g,t}^2 + \sigma_{n,t}^2 + 2cov_t(g_{t+1}, n_{t+1})) \quad (1.1)$$

where $cov_t(g_{t+1}, n_{t+1})$ is assumed to be small and constant for simplicity.

The budget constraint is:

$$\ln C_t - \ln W_t = \rho_w (\ln C_{t+1} - \ln W_{t+1} + r_{t+1}^w - g_{t+1} - n_{t+1}) + \kappa \quad (1.2)$$

where κ is an unimportant constant in the log-linearized budget constraint. For simplicity, only consider private wealth W_t .

Assume consumption per capita growth (g) and population growth (n) follows a AR(1) process with time-varying volatility $\sigma_{g,t}^2$ and $\sigma_{n,t}^2$. Consumption per capita growth (g) and population growth (n) fluctuate along with first moment shocks (ϵ_g, ϵ_n) and second moment shocks (u_g, u_n), where these shocks exhibit normal distribution with zero mean.

$$g_{t+1} = (1 - \rho_g)\mu_g + \rho_g g_t + \epsilon_g, \quad \epsilon_g \sim N(0, (1 - \rho_g^2)\sigma_{g,t}^2) \quad (1.3)$$

$$\sigma_{g,t+1}^2 = \alpha_g + \beta_g g_t^2 + \gamma_g \sigma_{g,t}^2 + u_g, \quad u_g \sim N(0, \sigma_{u_g}^2) \quad (1.4)$$

$$n_{t+1} = (1 - \rho_n)\mu_n + \rho_n n_t + \epsilon_n, \quad \epsilon_n \sim N(0, (1 - \rho_n^2)\sigma_{n,t}^2) \quad (1.5)$$

$$\sigma_{n,t+1}^2 = \alpha_n + \beta_n n_t^2 + \gamma_n \sigma_{n,t}^2 + u_n, \quad u_n \sim N(0, \sigma_{u_n}^2) \quad (1.6)$$

Deleveraging (ρ) and risk appetite (θ) shock are assumed to follow a AR(1) process with constant variance and innovations to level $\epsilon_\rho, \epsilon_\theta$.

$$\rho_{t+1} = (1 - \rho_\rho)\mu_\rho + \rho_\rho \rho_t + \epsilon_\rho, \quad \epsilon_\rho \sim N(0, \sigma_\rho^2) \quad (1.7)$$

$$\theta_{t+1} = (1 - \rho_\theta)\mu_\theta + \rho_\theta \theta_t + \epsilon_\theta, \quad \epsilon_\theta \sim N(0, \sigma_\theta^2) \quad (1.8)$$

Expected risk premium (ERP) is given by

$$ERP_t = E_t r_{t+1}^w - r_t^f = \theta_t \sigma \text{cov}_t(r_{t+1}^w, g_{t+1} + n_{t+1}) + (1 - \theta_t) \sigma_{r,t}^2 \quad (1.9)$$

where $\text{cov}_t(r_{t+1}^w, g_{t+1} + n_{t+1})$ is assumed to be small and constant.

Volatility of the return on wealth $\sigma_{r,t}^2$ is assumed to be time-varying as follows:

$$\sigma_{r,t+1}^2 = \alpha_r + \beta_r r_t^2 + \gamma_r \sigma_{r,t}^2 + u_r, \quad u_r \sim N(0, \sigma_{u_r}^2) \quad (1.10)$$

1.1 Solving the Model

Solve the model using the standard perturbation method, which can be easily implemented by Dynare. Given the initial parameter values, simulate the economy with 1200 periods and drop the first 500 periods to make sure the economy starts from around the steady state value. Repeat the simulation for 100 times. After each simulation, calculate the moments of interest from the model. And then, take average of these moments and match data moments. I describe targeted moments and procedures for the parameterization in the following section.

1.2 Parameterization

First, estimate the GARCH model using real data to obtain information about the parameters for productivity and demographic process, which are mainly determined outside the model.

The estimated coefficients of the GARCH model are plugged into the parameters of equation (1.3) - (1.6). GARCH terms γ_g and γ_n are adjusted to match the observed volatility of consumption per capita growth and population growth, respectively. Next, parameters for deleveraging and risk appetite process, which include persistence (ρ_ρ , ρ_θ) and volatility (σ_ρ , σ_θ), are set to match empirical target moments. A key strategy is to utilize the relations between consumption to wealth (C/W) and risk free rate and risk premium implied by the Euler equation and budget constraint. Deleveraging shock (ϵ_ρ) affects both C/W and risk free rate but does not affect risk premium. Thus, the OLS coefficients β_{rf} and β_{rp} obtained from the regression of risk free rate and risk premium on $\ln(C/W)$ contain information in that increasing the persistence or volatility of deleveraging shock increases β_{rf} , while β_{rp} remain almost unchanged. On the other hand, risk appetite shock (ϵ_θ) affects C/W, risk free rate, and risk premium at the same time. Given the deleveraging shock process, persistence or volatility of risk appetite shock is set to match β_{rp} . I also target the volatility of risk free rate and equity premium (σ_{rf} and σ_{rp}) to pin down the volatility of deleveraging shock and risk appetite shock.

Other parameters such as steady state value of risk aversion (γ) are calibrated according to existing literatures. Table 6 compares moments from the model and data.¹ Table 7 summarizes the parameterization of the model.

Table 6: Targeted Moments

symbol	interpretation	model	data (U.S., 1870 - 2015)
σ_{rf}	volatility of risk free rate	4.93	4.93
σ_{rp}	volatility of equity premium	7.57	18.46
σ_g	volatility of consumption per capita growth	3.42	3.42
σ_n	volatility of population growth	0.52	0.52
μ_g	mean of consumption per capita growth	1.7498	1.7498
μ_n	mean of population growth	1.4326	1.4326
β_{rf}	$E_t r_{t+1}^f = \alpha + \beta_{rf} \ln(C/W)_t$	0.11	0.13
β_{rp}	$E_t rp_{t+1} = \alpha + \beta_{rp} \ln(C/W)_t$	0.04	0.15

Note: Standard deviation and mean are in percentage.

¹The model fails to match the observed volatility of equity premium and the OLS coefficient β_{rp} . If I increase the volatility of return on wealth shock σ_{u_r} , I can match the data better. σ_{u_r} is assumed to be zero, since it will affect variance decomposition results, and there is no good interpretation for σ_{u_r} in this model.

Table 7: Parameterization

symbol	interpretation	value	target / information
ρ_ρ	persistence of deleveraging shock	0.9	β_{rf}
σ_ρ	volatility of deleveraging shock	0.03	σ_{rf}
ρ_θ	persistence of risk appetite shock	0.5	β_{rp}
σ_θ	volatility of risk appetite shock	2.555	σ_{rp}
μ_g	mean of consumption per capita growth	0.017498	GARCH estimate and σ_g
ρ_g	persistence of productivity shock	0.187	
α_g	constant term of productivity volatility	0.000258	
β_g	ARCH term of productivity volatility	0.259	
γ_g	GARCH term of productivity volatility	0.714	
σ_{u_g}	standard error of productivity volatility shock	0.0344	
μ_n	mean of population growth	0.014326	GARCH estimate and σ_g
ρ_n	persistence of population shock	0.933	
α_n	constant term of population volatility	3.74E-06	
β_n	ARCH term of population volatility	-0.006	
γ_n	GARCH term of population volatility	0.915	
σ_{u_n}	standard error of population volatility shock	0.0019	
α_r	constant term of return on wealth volatility	0.001563	GARCH estimate
β_r	ARCH term of return on wealth volatility	0.0091	
γ_n	GARCH term of return on wealth volatility	0.945	
σ_{u_r}	standard error of return on wealth volatility shock	0	By assumption
κ	constant in the budget constraint	-0.1984	C/W: steady state value = average
ρ_w	constant annual discount rate	1 - 0.0465	$\rho_w = 1 - \exp(\ln(C/W)^{ss})$
$1/\sigma$	intertemporal elasticity of substitution	2	similar to standard literature
γ	steady state value of risk aversion	2	
μ_θ	$(1-\gamma)/(1-\sigma)$	-2	
μ_ρ	steady state value of stochastic discount rate	0.03	

Comments on “Global Real Rates: A Secular
Approach” by P.O. Gourinchas and H. Rey.
Gianluca Benigno (Federal Reserve Bank of New York and London
School of Economics)

1 Introduction

Pierre Olivier Gourinchas and Helene Rey’s inspiring and clever paper proposes a new approach for studying the drivers of real interest rates. Their analysis is both theoretical and empirical. From a theoretical point of view, they propose a present-value approach based on the representative consumer’s budget constraint to link fluctuations in the consumption-to-wealth ratio to future movements of the real risk-free rate. In their empirical analysis, this approach is applied over a long-time period from 1870 to 2015 aggregating France, Germany, United Kingdom and United States. The key result of the paper is to document that the global consumption-to-wealth ratio anticipates the observed global real interest rate.

This is an important paper that address a topic of highly relevance as the current economic environment is characterized by low real rates and by low policy rates mainly in advanced economies. Indeed, one key question for monetary policymakers, is: “Where are interest rates headed?”

In the long run, economists assume that nominal interest rates will tend toward some equilibrium, or “natural,” real rate of interest plus an adjustment for expected long-run inflation. Indeed, this concept serves as a benchmark to evaluate the monetary policy stance (expansionary or contractionary) by comparing the observed real rate with the estimated natural one (see for example the IMF G-20 report on Growth, October 2017).

In their leading work, Laubach and Williams (2003) defines the “natural rate of interest as the real rate consistent with real GDP equals to its potential in the absence of shocks to demand. In turn potential GDP is defined to be the level of output consistent with stable price inflation absent transitory supply shocks.”

Understanding the determinant of the real interest rate is then a challenging topic since we don’t observe it similarly to the natural rate of unemployment or potential output.

2 Summary of the paper and brief literature review

The key contribution of this paper is to apply the present-value approach as in Lettau and Ludvigson (2001). The starting point of this approach is the global period resource constraint, which is the basis for consumption and portfolio

choice models with optimizing agents. By making few assumptions (stationarity of the consumption to wealth ratio) and by forward iteration of a first order approximation of the global resource constraint, they retrieve a fundamental representation that links the consumption to wealth ratio with future risk-free rates, future risk premia and future aggregate consumption growth.

The second step in the analysis is to link the present value relation to structural shocks. To this end, the authors assume a representative world household with standard CES preferences and then discuss how different shocks (productivity, demographic, deleveraging and to risk appetite) affect the pattern of the variables in the fundamental representation.

Finally, the identification of the natural rate of interest occurs by assuming that monetary policy aims to target the risk-free rate to the natural rate so that the global consumption-to-wealth ratio would contain information on the path of future natural real interest rate.

Before entering in the details of my comments I would like to briefly refer to few selected contributions to the literature that will guide my discussion.

The seminal paper in the literature is the study by Laubach and Williams (2003). They propose a semi-structural approach to estimate the natural rate of interest. They start from a variation of the basic new Keynesian model and estimate it to retrieve the natural rate of interest as a latent variable. It is worth noting here that in this context the theoretical notion of the natural rate of interest is a function of exogenous shocks (productivity shocks, demand shocks, preference shocks).

A recent paper of interest is the work by Borio et al. (2018): their analysis emphasizes the role of monetary factors as important drivers in determining the real interest rates over long horizons. Another aspect of interest of this paper is the emphasis on a global factor as a driver of countries' real interest rate.

Finally, the speech by Vlieghe (2017) highlights how the equilibrium interest rate might depend on how risky the economy is and how this riskiness changes over time. Interestingly, in examining the behavior of the real interest rate for the UK over more than two centuries (from 1800) he also shows how breaks in monetary policy regimes along with other high-risk events (financial crises, wars) matter for determining the equilibrium real interest rate and its properties.

3 Discussion

Now in my discussion I would like to focus on some conceptual issues that arise once we start thinking about determinants of (natural) real interest rates, discuss some of the assumptions that are used in this paper and offer some final thoughts.

3.1 Conceptual issues

As I mentioned earlier, this is a challenging topic. Indeed, the first issue that arises is related to the measurement of real interest rate. The standard way in which, in practice, the real interest rate, R_t , is constructed (ex-post), is by taking the nominal interest rate, i_t , minus realized Consumer Price Index inflation, π_{t+1} , over the relevant horizon. That is:

$$R_t = i_t - \pi_{t+1} \quad (1)$$

This is what we measure and observe in the data and I will refer to it as the "market measured real interest rate". Most of the time though, the natural interest rate does not coincide with the "market" measured real interest rate. Indeed, the natural real interest rate is a notional object that is defined conditional on an underlying theory. In most structural approaches to the analysis of the determinants of the real interest rate, the underlying theory relates the real interest rate to the inverse of the price of the risk free one period bond. In standard macro model this price is derived from the Euler equation. Once we take a second-order approximation of that equilibrium condition we would obtain

$$r_t = \alpha + \alpha_1 E_t [g_{t+1}] + \alpha_2 Var_t [g_{t+1}]. \quad (2)$$

Where r_t is the log of the time t to $t + 1$ risk free rate, g_{t+1} is the log of the consumption growth rate in the next period and E_t is the expectation operator conditioning on information at time t , Var_t is the variance operator conditioning on information at time t ; α, α_1 and α_2 are function of structural parameters that depend on preference specifications.

Most structural or semi-structural approaches (as for example Laubach and Williams, 2003) assume that the law of motion of the natural real interest rate follows from a first order approximation of the Euler equation (or a variation of equation (2)) and abstract from higher moments. But more recently, Hartzmark (2016) shows empirically that the volatility term is important in explaining real interest rates over time (see also Vlieghe (2017)).

Up to now I have discussed the theoretical underpinnings of the natural real interest rate (see as from equation (2)) based on the assumption that there exists a risk-free rate for which there is no analog in the data (except TIPS and indexed bonds). As I mentioned above, the measured real interest rate is obtained as in equation (1) so that once we extend our reasoning about the importance of second moments for the determination of the real interest rate, we find that the measured real interest rate obtained in the context of the pricing of a nominal bond is affected by the volatility of inflation and the covariance between the growth rate of consumption and inflation:

$$i_t - E_t \pi_{t+1} = \alpha + \alpha_1 E_t [g_{t+1}] + \alpha_2 Var_t [g_{t+1}] + \alpha_3 Var_t [\pi_{t+1}] + \alpha_4 Cov_t [g_{t+1}, \pi_{t+1}].$$

In my opinion, the nominal side of the measuring of the real interest rate is important especially in the context of long-run analysis during which there are

different policy regimes with differences in terms of inflation behavior. With respect to this, the evidence presented in Vlieghe (2017) for the UK economy suggests that there is a possible role of inflation for understanding the determinant of the measured real interest rate. Indeed, in theory, the properties of inflation depend on the monetary regime so that the developing a framework for studying the drivers of interest rates, should consider an environment in which nominal variables are determined endogenously.

Changes in inflation volatilities across monetary policy regimes could be helpful in understanding the message that emerges from the analysis by Borio et al. who emphasize the role and the importance of monetary policy regimes.

For example the following graphs for the UK economy from 1870s to recent years show tight link between measured real interest rate and inflation (see for example during the 1970s).



Figure 1



Figure 2

Ideally a structural approach to understand the determinant of the natural real interest rate should not only be explicit in terms of nominal factors but also be successful in replicating the properties of the measured market real interest rates and its drivers.

3.2 Assumptions

After emphasizing the potential role of nominal factors, in the second part of my discussion I would like to discuss some of the underlying assumptions in the Gourinchas and Rey's approach. The virtue of their approach lies in its simplicity but it is worth examining some of the assumptions behind their framework.

The analysis conducted in the paper encompasses a long-time horizon. It runs from the 1870s to recent years. The limit of using the same framework over such long time period is that there have been many changes from a structural point of view and also in terms of policy regimes (for example Bretton Woods, the Gold Standard and the post Bretton Woods period) that could affect the estimated coefficients. To address this concern, we run a time-varying VAR to study how innovations in the consumption/wealth ratio affect the real interest rate.

In the following analysis, see Figure 3, (credit to Vassili Bazinas) we run the same VAR as in Gourinchas and Rey's paper but we restrict our focus only to the US economy using the same sample as they do in their work.

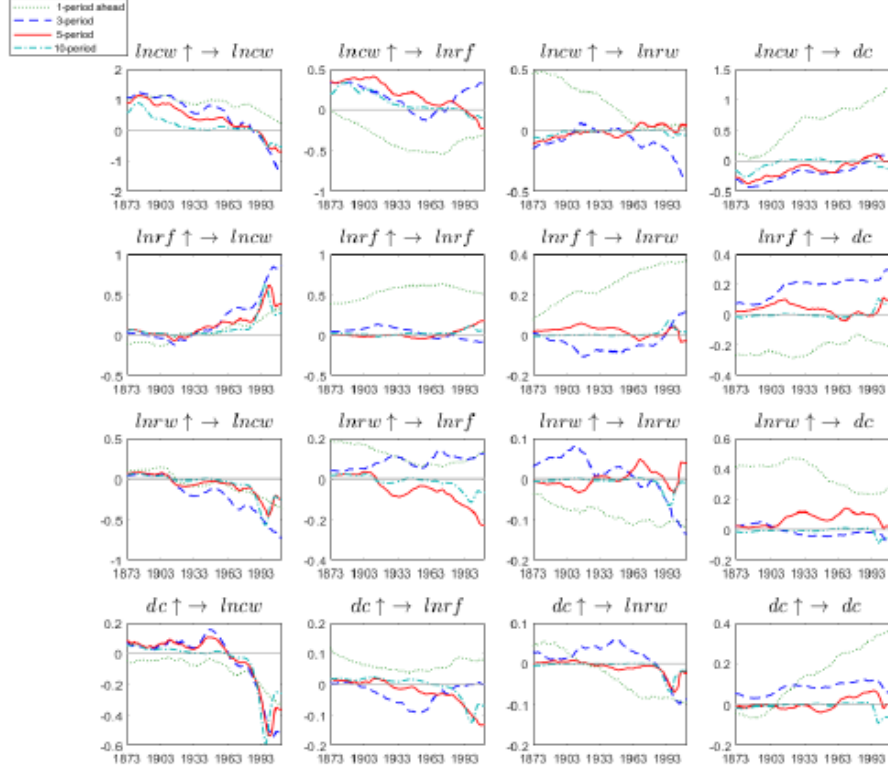


Figure 3

The innovation in consumption wealth ratio on the real interest rate is shown in the first row and second column: it is interesting to note how the response of the real interest rate varies over time and over different time horizons. The one-period ahead response tend to be negative over the sample period and becomes more stable in terms of its size from the late 1930s while as we increase the forecast horizon there is less stability in terms of sign and size. For example, the three-period ahead response is positive at the beginning of the sample, turn negative in the middle and then return to be positive and bigger toward the end of the sample.

This preliminary analysis using a VAR with time-varying coefficients highlights the possibility that the coefficient might change as the economy experiences structural breaks and that possibly this approach would be more powerful in assessing the role of the consumption to wealth ratio innovation over time horizon during which there are no big structural breaks.

Another implicit assumption of the analysis is related to the use of the global budget constraint: in the analysis, the budget constraint at the country level are aggregated by weighting them by relative wealth ratios. This aggregation is consistent with the assumption of international market completeness and as

such it captures an economic environment in which there is perfect financial integration. While the recent trend in international financial integration is consistent with this assumption, most of the Bretton Woods period is best thought as a situation in which international financial integration is limited.

Thirdly, the present value approach builds upon the hypothesis that the economy eventually returns to the steady state. The authors approximate the budget constraint up to first order before integrating it forward and decomposing it in the various terms. I think this is a reasonable assumption considering the countries on which they focus but there might be situations in which this assumption might not be ideal. The recent Japanese experience is an example of such a situation with nominal interest rate at a lower bound for more than two decades: something unprecedented that could be equally consistent with a multiple equilibria world.

Finally, as I mentioned in my summary remarks, the authors identify the natural interest rate by stating that the monetary policy rate should target the natural real interest rate then the consumption to wealth ratio would be informative in terms of predicting the path of the natural rate of interest. In my view this is a strong identification assumption since monetary policy is not always successful in achieving the natural rate even outside the special case of the zero lower bound.

Theoretically also, in a world in which there are multiple frictions the target real rate (the efficient real rate from a monetary policy point of view) might be different from the natural real interest rate (the one that would prevail under stable price level).

3.3 Remarks

In the last part of my comments I would like to reflect on some common features of all these approaches and proposes an alternative view. In the context of the structural or semi-structural approach to the estimation of the natural real interest rate the underlying framework is a New-Keynesian model in which the natural real of interest would coincide and identified with the interest rate that would prevail under flexible prices. This implies that the natural rate of interest is a function of the underlying exogenous shocks. It follows that the natural rate of interest is invariant to policy choices and exogenous to the cycle as it does not depend on any endogenous current variables.

I would like to suggest and propose an alternative framework for thinking about macroeconomic fluctuations and the determination of the natural real interest rate. This framework is what, in a joint work with Luca Fornaro (Benigno and Fornaro, 2018) we label Keynesian Growth as it provides a theory of long-run growth that depends on a theory of economic fluctuations based on Keynesian frictions. In this theory the natural real interest rate is determined by endogenous productivity development that are affected by the economic cycle so

that the natural real interest rate is endogenous and might be affected by policy variables as well. In this approach to macroeconomics monetary non-neutrality is not only a short-run phenomenon but it occurs also in the long-run.

One more aspect that is often neglected in the literature but relevant (see Hartzman (2016) and Vlieghe (2017)) is the role of higher moments in explaining the behavior of the real interest rate. This paper is an exception to most of the literature as, in their section 3, they allow for the possibility of time-varying risks once they explore the links between underlying structural shocks and the different pattern of comovements between risk-free rates, risk premia and consumption growth.

Ideally it would be interesting to develop an environment in which risks arises endogenously as for example in Adrian and Duarte (2018) or Mendoza (2010) where the presence of occasionally binding constraints creates the possibility that risks changes endogenously as the constraint binds.

4 Conclusions

This is an interesting and stimulating paper: it advances on our understanding of the determinants of real interest rates by providing an approach that is complementary to others in the literature. I predict it will be the benchmark and reference paper for a lot of future works.

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