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by

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Abstract

The paper presents a one-factor affine model of the term structure of Libor rates with autocorrelated measurement errors. It can be viewed as a central tendency model, with the theoretical arbitrage-free rates serving as stochastic means to which the observed rates revert. Two estimation techniques are compared, one based on a no-measurement-error assumption, the other on Kalman filtering. The estimates are then used in standard yield spread regressions with a view to accounting for the departure of future short rates from what the expectations hypothesis would predict.

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Contents

1. Introduction

Understanding the dynamics of Libor is important for the management of risk. Managers rely heavily on over-the-counter markets and organised exchanges to hedge or take positions in derivative contracts whose reference rates are linked to Libor, and this requires a consistent theory of how the market forms expectations of Eurodeposit rates.

From a monetary policy perspective, considering Libor rates instead of those derived from domestic government securities can also provide valuable insights into the market's expectations of future short rates. The Libor curve may be regarded as an approximation to the true cost of funds for prime banks participating in the London market. There are no reserve requirements or deposit insurance premiums imposed on Eurocurrency deposits, no withholding taxes levied on interests paid to non-resident depositors and, in terms of capital adequacy, no specific capital requirements for foreign banks.[1] An additional feature of the Libor market is that it provides quotes every day for a wide range of short-run maturities. By contrast, it is less easy to define the near end of the term structure with government securities. Short-term instruments used to complement bonds with a horizon of less than one year are not available for every maturity, and the determination of their rates is often affected by specific institutional features. Finally, the short-term nature of the market, together with its concentration in highly rated banks subject to strict credit monitoring, implies that the credit premium included in Libor fixings is probably small. This is not to say that the credit standing of any prime institution cannot deteriorate rapidly and eventually be reflected in higher rates. The British Banker's Association, however, polls a sample of top-rated banks, trims the highest and lowest rates, rounds the remaining yields and finally averages them. As a result, the influence of idiosyncratic credit risk in official quotes is likely to be significantly dampened.

This paper estimates an interbank term structure from a panel of weekly dollar Libor fixings for six maturities, ranging from one to 12 months. There are two key features. First, the estimation is based on an equilibrium model of the term structure. Market participants and central banks tend to "read" the term structure as if it revealed the market's expectations of future short-term rates. Forward spreads, however, are driven by changes in the price of risk as well as changes in expectations about interest rates. Rather than treating excess returns on long bonds as residuals, the equilibrium approach seeks to elicit term premia endogenously from the absence of arbitrage. Second, it is assumed that expectations

[1] Europlacements enjoy the same favored risk rating (20%) as the paper issued by federal agencies and supranational institutions, but the situation might change in the wake of the draft proposals for the Basel apital Accord.

about future rates can be captured by a single factor. The justification for this choice is to limit the number of parameters needed to reproduce the Libor curve and avoid computationally intensive formulations. Instead of increasing the number of factors to describe the entire yield curve, the paper keeps a single latent variable to fit a smaller part of the yield curve. To some extent, it tries to find a middle ground between the complicated dynamic general equilibrium models of the term structure developed in the financial literature and the simpler static curve-fitting techniques that many (central) banks use to approximate implied forward rates or their distributions from observed financial prices.

The continuous-time, square root model of Cox et al. (1985) has a few remarkable properties. First, it is tractable, and thus has the potential to be more widely used in central banks as an indicator of interest rate risk. The factor summarises the current shape of the yield curve and the way it is expected to fluctuate over time. Second, it allows for a variable term premium, and thus provides a theoretical decomposition of forward rates into expectations and risk premiums directly. One subject of obvious interest in this respect is its bearing on the rejection of the expectations hypothesis. Since the largest discrepancies arise for postwar US data at maturities under two years, it is particularly interesting to address this issue with Eurodollar data. Finally, the continuous-time formulation avoids the restrictive assumption of log-normal interest rate innovations at discrete time intervals, which is at odds with the evidence of a substantial excess kurtosis. In particular, it allows more flexibility in that it can easily generate a hump-shaped curve for the impact of shocks on the term structure.

The econometric estimation of the model is carried out using two alternative techniques. Following Chen and Scott (1993) and many others, the first one is based on the true conditional density of the underlying factor, but assumes that there is no measurement error on the six-month Libor. Because the non-central chi-square density of the factor involves a modified Bessel function, we call it the "Bessel" method. The second uses a Kalman filter to let the data determine the measurement errors, but does not use the true conditional density of the underlying factor. We call this the "Kalman" method. In either case the measurement errors are assumed to follow simple autoregressive processes, with innovations that are independent of the factors, normally distributed and possibly cross-correlated. One motivation for this assumption is that the cross-section averaging and rounding-off of Libor quotes induce serial correlation in the data. More plausibly, it deals with the risk of misspecification in the model, which can have a lasting effect on measurement errors. Although we will continue to refer to them as measurement errors, they can also be interpreted as specification errors.

Our results are as follows. First, we find that the one-factor model yields plausible parameter estimates with a reasonably small measurement error. Interestingly, it can be viewed as a central tendency model, with the theoretical arbitrage-free rates serving as stochastic means to which the observed rates revert.

The bottom line is that the factor is much more persistent than the error term, and this is sufficient to explain the main features of the interest rate data. The model has in fact two incarnations, depending on the econometric method used to estimate it. Both reproduce the various unconditional moments but the Bessel variant is better at capturing the time-varying shapes of the yield curve and the term structure of volatility. Second, we show how both econometric methods induce systematic biases, which lead to Bessel overpredicting and Kalman underpredicting future changes in rates. To this extent, the specification biases appear to arise less from the model than from the method used to estimate it. Finally, we show on the basis of standard yield spread regressions that the one-factor model accounts well for the departure of future short rates from what the expectations hypothesis would predict at the one-year maturity, although significant tensions remain between the model and the data at the three-month and six-month maturities.

Recent option pricing literature has placed greater emphasis on interbank interest rates. Jegadeesh and Pennacchi (1996) estimate a two-factor model with central tendency calibrated on the three-month Eurodollar futures contracts traded on the Chicago Mercantile Exchange. They show that more than one factor is required to fit the interest rate dynamics, but their evidence is based on the Vasicek model, which cannot account for time-varying term premia. Moreno and Peña (1996) explore a single-factor model with jumps for the Spanish overnight interest rate. They show that the existence of jumps can explain the systematic underpricing of some interest rate derivatives, but they do not use panel data. Jamshidian (1997) studies the existence of multifactor arbitrage-free models with a view to pricing Libor and swap derivatives jointly. The no-arbitrage paradigm can be questioned if, as argued by Duffie and Singleton (1997), Libor yields have distinctive features that are not shared by the longer end of the swap yield curve, for example as a result of heterogeneity in credit quality.

In monetary economics, interbank interest rates have also been studied for their informational or predictive content. Gerlach and Smets (1997) use Euromarket data for 17 currencies and show that rejection of the expectations hypothesis is for many countries less cogent than the empirical literature based on US data would suggest. Malz (1998) fits zero coupon curves on Eurodeposits, FRAs and swap rates at different points in time using the popular Nelson-Siegel-Svensson methodology, and argues that interbank rates are a valuable source of information on market expectations and the stance of monetary policy. Konstantinov (1998) derives an equilibrium model of the interbank term structure by assuming that the federal funds target rate is subject to discrete jumps and is able to replicate the predictability pattern of interest rates for part of the sample.

To better identify the place of this paper in the literature of affine-yield models characterised by Duffie and Kan (1996), it may be useful to discuss the relative merits of one-factor and multifactor models.

Much criticism has been levelled against the one-factor formulation. One main objection is that it assumes that all information is captured by a single expectation process. Fleming and Remolona (1999), for example, show that different types of announcements lead to fundamentally different reactions of the yield curve according to the way market expectations are revised. As an amalgam, a single factor is also more difficult to interpret in terms of outside factors, such as money growth rates, inflation or other factors connected to monetary policy. The empirical weaknesses of one-factor models have also been clearly documented. The main problems concern the shape of the mean yield curve, the changing patterns of autocorrelations and volatility with maturity, and the fact that interest rate innovations have substantial excess kurtosis. In addition to these shortcomings, Backus et al. (1998) show that the one-factor model is incapable of accounting for the departure from the expectations hypothesis and still maintaining an upward-sloping forward curve. Not surprisingly, all these difficulties have pointed toward a larger number of factors, and it seems that nothing can stop authors in their quest for more complicated models.

At the same time, introducing more factors is no panacea. Ideally, one would like to increase their number until the discrepancy between observed and modeled rates could be ascribed to a pure measurement error. In practice, however, the error comes both from the underlying data and the model. Statistical tests of the overidentifying restrictions, in the rare cases where they fail to reject, can reinforce confidence in the model, but it is doubtful that any equilibrium model, however refined, will decisively identify the deep structural parameters that underlie the absence of arbitrage, if there are any. The pitfall is that by imposing a theoretical straitjacket on the data one may end up modelling the error term. Of course, adding more factors results in a better fit of the yield curve at any point in time, but this does not necessarily lead to an improvement in the reliability of the model's predictions.

This paper is organised as follows. Section 2 summarises some results from the one-factor affine-yield model and its implications for the properties of rates. Section 3 outlines the statistical properties of Libor and reports the results of our estimations according to the two econometric methods referred to above. Section 4 provides a brief account of the specification biases generated by those methods. Section 5 presents estimates of standard yield spread regressions, involving ex post returns on rolling over a one-month investment over different periods, and compares the results with the same regressions obtained when the dependent variables are generated by the model. Section 6 concludes.

2. Some theory

In finance theory, the pricing kernel approach highlights the interaction between probabilities and risk. The paper follows that approach by focusing on the one-factor continuous-time affine formulation.

Writing the model in continuous time avoids the assumption of log-normal conditional distributions, but requires that the implied density recorded at discrete intervals be evaluated exactly. Since the derivations are now standard, this section briefly recalls the basic formulas and shows how they can be embedded in an econometric model as the central tendencies to which the observable interest rates revert.

The principle of arbitrage by dynamic trading states that the current price $B_{t,T}$ of a zero-coupon bond maturing at time T is determined by $M_t B_{t,T} = \mathrm{E}\left[M_T \,|\, \mathcal{F}_t\right]$, where M_t is the state price process (or pricing kernel). The process M_t is defined as

$$M_t = \exp\left\{-\int_0^t r_s\, ds\right\} \eta_t \tag{1}$$

and captures both the effect of pure discounting, where r_t is the instantaneous interest rate, and the market's valuation of risk, encapsulated in the risk-neutral density η_t. One way to arrive at a tractable specification of movements in the yield curve is to characterise state prices in terms of a few latent state variables. The paper uses a single-factor representation obeying the square root process

$$dz_t = \kappa(\theta - z_t)\, dt + \sqrt{z_t}\, dW_t \tag{2}$$

where W is a standard Brownian motion, κ the mean reversion parameter and θ the steady-state mean.[2] The z process models the arrival of information, possibly embodying expectational factors related to monetary policy or other economic news in some "reduced form" fashion.

With only one factor, it remains to determine how z impacts the price system M and the short-term interest rate r. It seems reasonable to assume that, in a risk-averse world, positive shocks to the short-term interest rate (higher r) should be associated with greater anxiety about future rates (higher state price M), thus inducing a positive correlation between r and M. Suppose, for example, that shocks to the risk-neutral density η_t are proportional to the factor's innovations:

$$\frac{d\eta_t}{\eta_t} = -\lambda\sqrt{z_t}\, dW_t$$

where λ parameterises the price of risk. With lower-case letters standing for logarithms, Ito's formula applied to (1) implies that state prices are ruled by the stochastic differential equation

$$-dm_t = r_t\, dt + (\lambda^2/2) z_t\, dt + \lambda\sqrt{z_t}\, dW_t.$$

This formulation makes clear that $-\lambda$ governs the covariance between shocks to the price system (via m_t) and shocks to the interest rate (via z_t). One would then expect λ to be negative to induce a positive correlation between r and M.

Given this setup, a simple Markovian structure obtains when the instantaneous interest rate depends on the current state. As shown by Duffie and Kan (1996), the choice $r(z) = \alpha + \beta_0 z$ leads to an affine term

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[2] When $2\kappa\theta > 1$, the process z_t never reaches 0. This condition is always met for the parameter values estimated in the sequel.

structure. A succinct derivation can be found in Appendix A. The parameter α controls for the lower bound on the short rate, while β_0 scales its conditional volatility. Letting $\tau = T - t$ denote the current maturity, the bond price $B_{t,T}$ becomes the discount function

(3) $$\psi(\tau, z) = \exp\{-(\alpha\tau + \kappa\theta\gamma(\tau) + \beta(\tau)z)\}$$

for the functions $\beta(\cdot)$ and $\gamma(\cdot)$ given in Appendix A. We call $\beta(\cdot)$ the impact curve. The corresponding spot rates are

(4) $$r(\tau, z) = \alpha + \kappa\theta\gamma(\tau)/\tau + z\beta(\tau)/\tau$$

where $\beta(\tau)/\tau$, or factor loading, expresses the sensitivity of the interest rate to information arrival. The one-factor affine model differs from the standard CIR formulation in that $r_t = r(0, z_t)$ satisfies

$$dr_t = \kappa(r_0^* - r_t)\, dt + \sqrt{\beta_0(r - \alpha)}\, dW_t$$

where $r_0^* = r(\theta) = \alpha + \beta_0\theta$. That is, α may be different from zero, and the entire yield curve is bounded below by α. (A negative α would imply that rates can become negative with positive probability.)

The cross-sectional and time series behavior of yields under the one-factor model is entirely subsumed in the five-dimensional parameter vector $(\kappa, \lambda, \beta_0, \theta, \alpha)$. This has strong implications. First, all rates are linear functions of the same factor and, in the absence of measurement errors, are perfectly correlated. In particular, they are stationary and revert to their respective long-term means at the same rate κ. Second, the yield curve tends to a limit which is independent of time as the maturity lengthens. This is illustrated in Figure 1, which displays two sets of forward curves for different values of the state variable. Of course, it is not possible to ascertain the long-term interest rate limit with Libor maturities ranging from one to 12 months. Third, the same graph shows that the average yield curve can be upward-sloping or hump-shaped. A high volatility parameter β_0 relative to the mean reversion parameter κ is sufficient to generate a hump, and the curve steepness at the short end depends only on the market price of risk λ and on interest rate volatility, as measured by $r_0^* - \alpha = \beta_0\theta$. Finally, the term structure of volatility or variability of forward rates in the maturity dimension is time-varying, with a shape that is entirely determined by the slope of the impact curve $\dot{\beta}(\tau)$. The volatility curves in Figure 2 can move up and down following changes in the factor, but their shapes are not allowed to change. The curve is downward-sloping when $\kappa + \lambda \geq 0$, hump-shaped otherwise, and displays exponential decay at long horizons, implying that medium-term rates can be more variable than short rates, but that their volatility will eventually decline with maturity.

Casual empiricism would reject the one-factor formulation because of its implication that any pair of rates would satisfy a deterministic constraint exactly. In addition, there must be only a modicum of truth in the one-factor affine representation. For both reasons, statistical noise must be added to the model. Let $(\tau_1, \ldots, \tau_n)$ be an n-tuple of selected maturities. A vector of measurement errors u_t can be defined

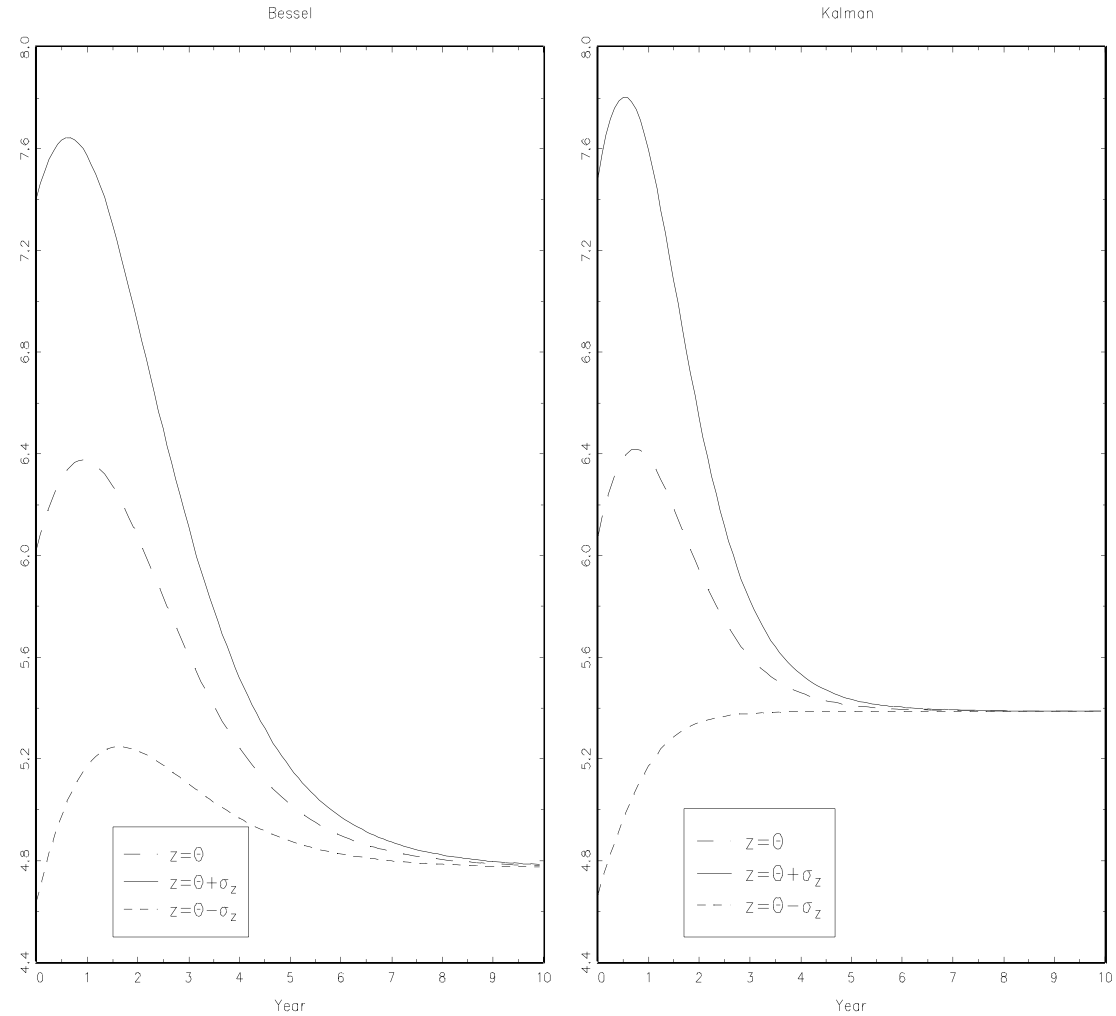

Figure 1: Forward curve

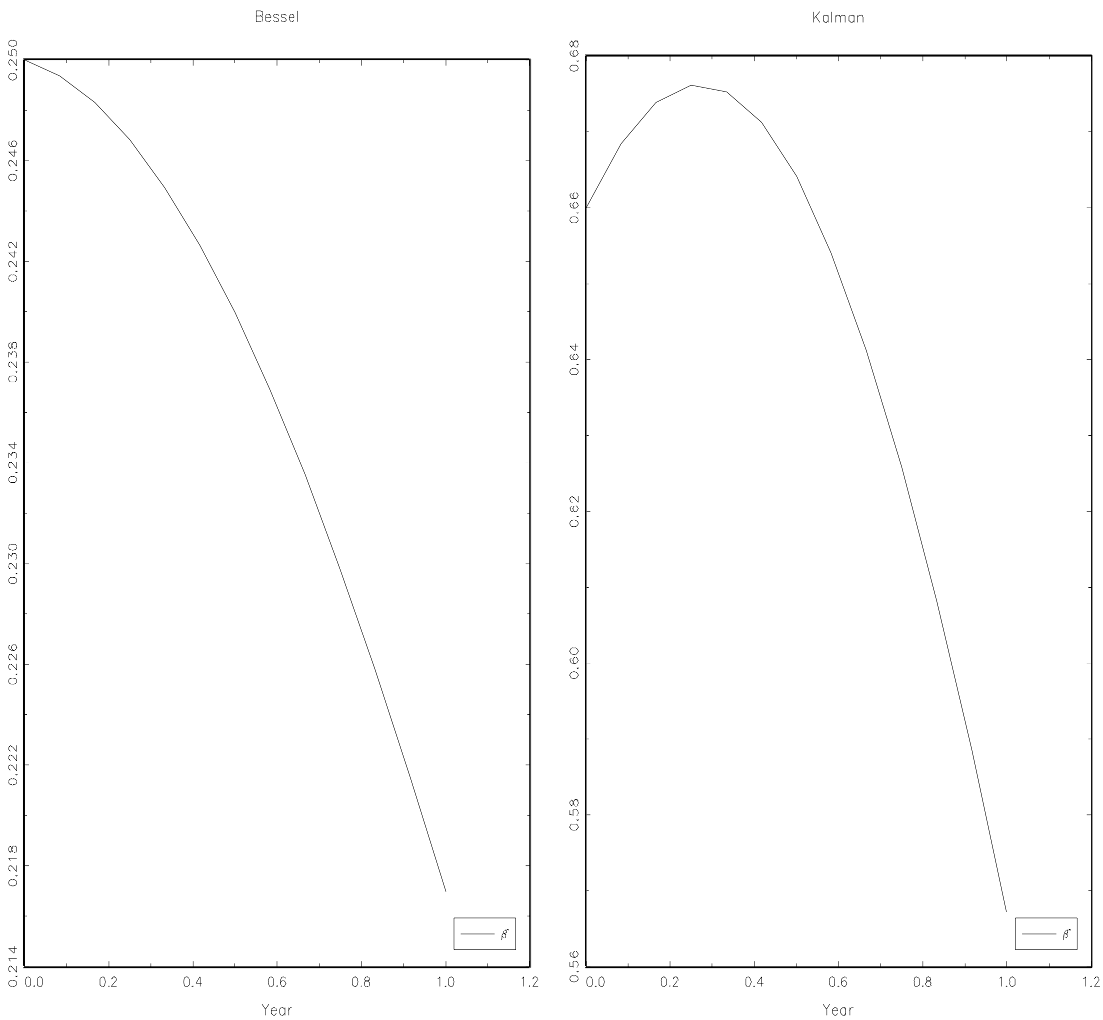

Figure 2: Volatility curve $\dot{\beta}(\tau)$

from

$$r_t^k = r(z_t, \tau_k) + u_t^k, \qquad k = 1, \ldots, n, \tag{5}$$
$$\text{where} \quad r(z_t, \tau_k) = g_k + \beta_k z_t.$$

From (4), the coefficients are $g_k = \alpha + \kappa\theta\gamma(\tau_k)/\tau_k$ and $\beta_k = \beta(\tau_k)/\tau_k$. The continuous-time dynamic (2) gives rise to the difference equation

$$z_t = (1-b)\,\theta + b z_{t-1} + w_t \tag{6}$$

where $b = e^{-\kappa/52}$ is the autocorrelation of the factor (alternatively, $1-b$ is the weekly rate of mean reversion). The innovation to the factor, w_t, has a conditional distribution which is determined by the transition function of the state variable z sampled every week. One can show that this conditional distribution is a non-central chi-square with $2\kappa\theta - 1$ degrees of freedom, the analytic form of which is given in Appendix B. As a result, the unconditional distribution of w_t is non-normal[3] with variance $\sigma_w^2 = (1-b^2)\,\theta/2\kappa$. Measurement errors are assumed to be AR(1) processes,

$$u_t^k = \rho_k u_{t-1}^k + \epsilon_t^k; \qquad \mathrm{E}\,\epsilon\epsilon_t' = \Omega \tag{7}$$

with innovations that are independent of the factors, normally distributed and possibly cross-correlated. In general, z_t is not observable, and (5) can be viewed as the measurement equation of a filtering problem with transition equation (6) and error covariance structure (7).

With the general AR(1) formulation provided by (6) and (7), the signal extraction problem in (5) might seem unidentified at first sight. Indeed, if the first component were itself a generic AR(1) process, it would be difficult to disentangle the factor from the error term, and the factor loadings β_k would not be identified. However $r(z_t, \tau_k)$ is a vector process whose mean, covariance and autocorrelation are determined by an arbitrage model. Consider the following thought experiment. First, ρ-difference the data suitably at all maturities until the error terms become serially uncorrelated. This determines the ρ_ks. Second, use the covariance matrix of the observed rates to estimate the covariance of the innovation error Ω. With six maturities, there remain six first-moment conditions to estimate the twelve coefficients g_k, β_k and the parameters b, θ and σ_w^2 in (6). But all these coefficients are determined by a model based on the five-dimensional parameter $(\kappa, \lambda, \beta_0, \theta, \alpha)$. Hence, the model is actually slightly overidentified.

Finally, to draw the link between (5–7) and a model where one factor reverts to a time-varying mean, we select any maturity k and write (5) in first difference using (6) and (7) to obtain

$$\Delta r_t^k = (1-b)\left(r_k^* - r_{t-1}^k\right) + (b - \rho_k)\left(r^k(z_{t-1}) - r_{t-1}^k\right) + \epsilon_t^k + \beta_k w_t \tag{8}$$

where we have put $r^k(z_t) = r(\tau_k, z_t)$ and $r_k^* = r^k(\theta)$ to simplify notation. In addition to the fixed long-run mean r_k^*, to which the interest rate r_t^k reverts at rate $1-b$, we recover the standard central

[3] It is actually a gamma distribution (see Appendix B1).

tendency formulation where the time-varying rate $r^k(z_t)$ plays the role of the target driving the future path of the interest rate, with rate of mean reversion $b - \rho_k$. Conversely, the central tendency model

$$\begin{aligned}\Delta x_t &= (1-\rho)(\theta_{t-1} - x_{t-1}) + \epsilon_t \\ \Delta \theta_t &= (1-b)(\theta^* - \theta_{t-1}) + \eta_t\end{aligned}$$

where the factor x_t reverts to a time-varying factor θ_t can be reinterpreted in terms of the econometric model

$$\begin{aligned}x_t &= \theta_t + u_t \\ u_t &= \rho u_{t-1} + \epsilon'_t\end{aligned}$$

with $\epsilon'_t = \epsilon_t - \eta_t - (1-b)(\theta^* - \theta_{t-1})$. When b is close to one, the error term ϵ' is hardly distinguishable from a pure error innovation, and it may be that the first factor is simply the central tendency itself up to some AR(1) process. To this extent, an investigator requiring that all rates be linear functions of both x and θ may in fact be unduly imposing arbitrage constraints on the autocorrelated error term u_t. By contrast, in (5) all arbitrage constraints are captured in $r^k(z_t)$, not in the error term.

The introduction states that the one-factor formulation can easily generate hump-shaped responses to the arrival of new information. In the central tendency model, the hump comes from the interaction between two factors. When surprise information is revealed to the central tendency, the first factor starts adjusting towards its new target, but its movements are limited by the size of $1 - \rho$. During the adjustment period, the central tendency, which is itself mean-reverting, gradually declines toward its steady-state value, lowering the first factor's initial deviation from target. The impact of the innovation is therefore felt more sharply for intermediate maturities than for short and long ones. By contrast, multifactor Vasicek (1977) models with homoskedastic volatility, such as in Longstaff and Schwartz (1992) or Chen and Scott (1993), imply constant term premia and cannot generate a hump. However, time-varying conditional variances can also accommodate a hump-shape in the one-factor model.[4] A necessary and sufficient condition for the factor loading curve to be upward-sloping at inception is that the price of risk associated with holding debt λ be larger in absolute value than the mean reversion parameter κ. This implies that the initial rise in the term premium more than offsets the speed at which the short rate is expected to return to an equilibrium. The relationship between risk and expectational factors is further examined in Section 5.1.

[4] This is shown in Appendix A, where our model is taken as the limit of a sequence of heteroskedastic discrete-time models.

3. Econometric results

Our first approach follows Chen and Scott (1993). We assume that one of the rates – the often referenced six-month Libor – is observed without error. This actually sidesteps the identification problem by making z_t an observable variable. The estimation then proceeds using maximum likelihood under the conditional factor transition and the joint distribution of measurement errors. Because the true density involves a modified Bessel function, it is referred to in the sequel as the "Bessel" estimation. The second approach is based on the Kalman filter. This quasi-maximum likelihood procedure exploits the first two moments of the conditional density of observed yields. We depart from the standard application of the Kalman filter in taking the model's arbitrage conditions as well as the serial correlation of measurement errors into account. Details of the two estimations are given in Appendix B.

While each approach is relatively easy to implement, they both lead to inconsistent estimates. The no-measurement-error assumption allows direct observation of the underlying factor, but the resulting distribution of yields cannot be taken as a correct description of the data generating process. In particular, the variance of the underlying factor is equated to that of the six-month Libor, even though the latter should be larger in the presence of uncorrelated measurement errors. We thus predict that the unconditional variance of the factor, $\theta/2\kappa$, will be biased upwards. On the other hand, the Kalman filter lets the data determine the measurement errors for all yields, but uses normality assumptions that are not met by a non-Gaussian model. The estimated factor can still be interpreted as an optimal predictor in the mean squared error sense, rather than in the conditional mean sense, but the parameter estimates do not minimise the conditional density of the data generating process. These biases are examined in Section 4.

3.1 Data and summary statistics

We obtain end-of-week Libor official fixing data from Reuters as reported in the DRI database for 24 October 1986 to 20 February 1998 for maturities ranging from one to 12 months (592 observations). The selected maturities are one, two, three, six, nine and 12 months. Each bank in the panel is asked to contribute the rate at which it could borrow funds, were it to do so by asking for and then accepting interbank offers in reasonable market size, just prior to 11.00 a.m. Eurodeposit rates are volatile but, due to the trimming, rounding-off and averaging operations, official quotes are not. Table 1 shows the occurrence of identical consecutive entries in Libor fixing data for various maturities. Repetitions are indeed quite common from one week to the next, especially for one-month rates.

Some properties of Eurodollar yields are summarised in Table 2. One feature is the shape of the average yield curve, which is displayed in Figure 3. Some more perspective has been provided to the average

Table 1

Consecutive identical entries

No. of entries *	1 month	6 months	12 months
2	56	73	53
3	21	14	9
4	16	8	6

* Number of identical consecutive entries found in the data at the selected maturities. There are 592 observations.

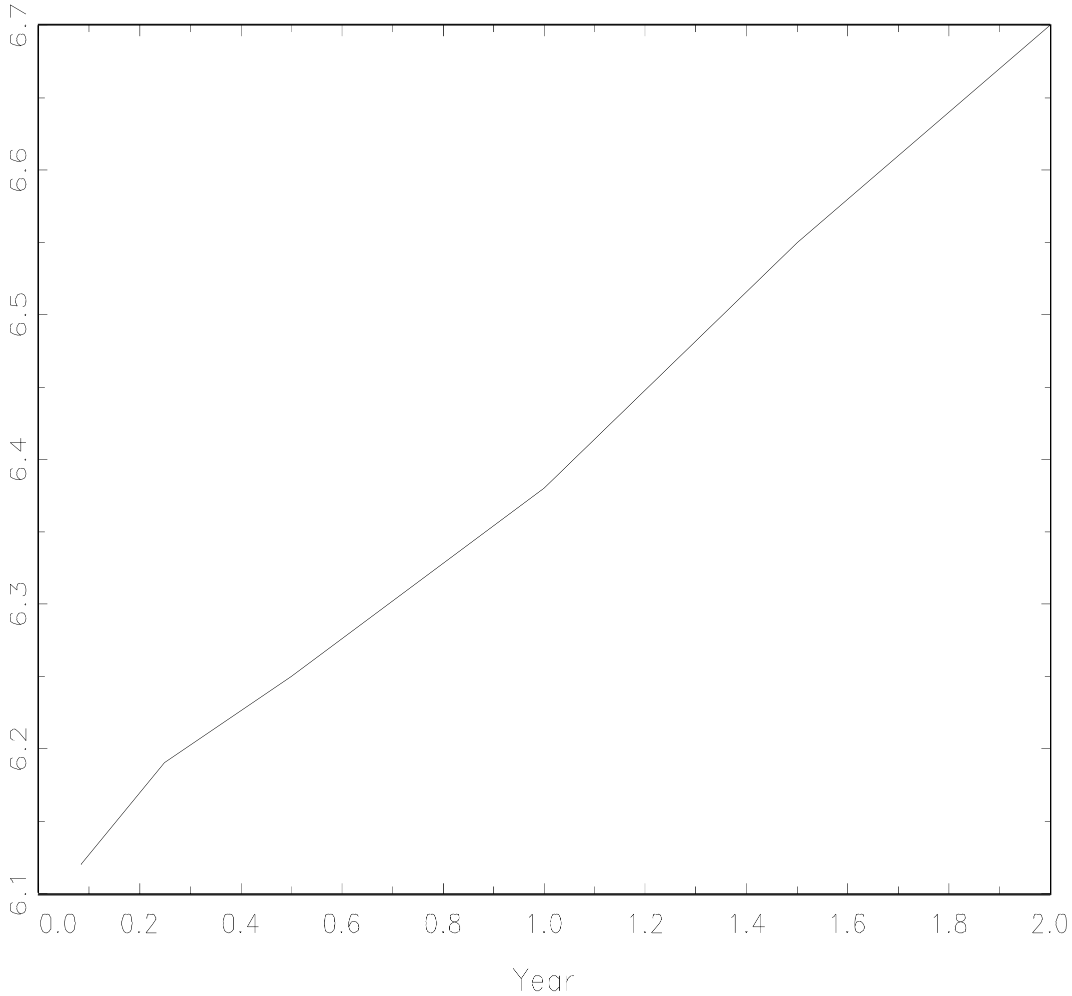

Figure 3: LIBOR and FRA average yield curve

Table 2

Descriptive statistics of observed and estimated Libor

Average of Libor yields/spreads							
	$\tau_k =$	*1 month*	*2 months*	*3 months*	*6 months*	*9 months*	*12 months*
Sample	$r^1/r^k - r^1$	$6.132/-$	$6.170/0.038$	$6.194/0.062$	$6.247/0.114$	$6.307/0.175$	$6.372/0.239$
Bessel	$\hat{r}^1/\hat{r}^k - \hat{r}^1$	$6.130/-$	$6.168/0.038$	$6.192/0.062$	$6.248/0.118$	$6.307/0.177$	$6.369/0.239$
Kalman	$\hat{r}^1/\hat{r}^k - \hat{r}^1$	$6.112/-$	$6.146/0.034$	$6.178/0.066$	$6.254/0.142$	$6.304/0.192$	$6.329/0.216$
Standard deviation of Libor							
	r^k	1.860	1.841	1.828	1.781	1.739	1.697
Sample	$r^k - r^1$	—	0.130	0.177	0.287	0.385	0.464
	Δr^k	0.174	0.133	0.141	0.148	0.162	0.159
	$\hat{r}^k$	1.840	1.827	1.816	1.774	1.735	1.692
Bessel	$\hat{r}^k - \hat{r}^1$	—	0.103	0.141	0.234	0.309	0.393
	$\Delta\hat{r}^k$	0.161	0.132	0.141	0.147	0.155	0.155
	$\hat{r}^k$	1.766	1.775	1.781	1.783	1.760	1.715
Kalman	$\hat{r}^k - \hat{r}^1$	—	0.008	0.015	0.017	0.006	0.050
	$\Delta\hat{r}^k$	0.143	0.143	0.144	0.144	0.142	0.139
First autocorrelation of Libor							
	r^k	0.996	0.997	0.997	0.996	0.996	0.995
Sample	$r^k - r^1$	—	0.65	0.78	0.89	0.92	0.94
	Δr^k	- 0.02	- 0.04	0.04	0.05	- 0.01	0.03
	$\widehat{r}^k$	0.996	0.997	0.997	0.996	0.996	0.996
Bessel	$\widehat{r}^k - \widehat{r}^1$	—	0.63	0.78	0.89	0.92	0.94
	$\Delta\widehat{r}^k$	- 0.01	- 0.02	0.05	0.05	0.02	0.04
	$\widehat{r}^k$	0.997	0.998	0.998	0.998	0.997	0.996
Kalman	$\widehat{r}^k - \widehat{r}^1$	—	0.63	0.77	0.89	0.92	0.94
	$\Delta\widehat{r}^k$	0.10	0.13	0.19	0.37	0.24	0.14
Skewness/kurtosis of weekly changes in Libor							
Sample	Δr^k	- 0.07/17.7	- 1.21/12.3	- 1.92/25.0	- 2.00/25.1	- 1.50/22.2	- 1.55/19.2
Bessel	$\Delta\widehat{r}^k$	- 0.40/15.4	- 1.37/13.4	- 1.98/25.6	- 2.00/25.1	- 1.71/22.5	- 1.65/20.2
Kalman	$\Delta\widehat{r}^k$	- 0.22/16.9	- 1.20/11.8	- 1.80/23.9	- 1.53/19.8	- 1.21/19.1	- 1.48/18.4

The data are end-of-week 11 a.m. fixing rates as calculated by the British Bankers' Association. The sample period is from 24 October 1986 to 20 February 1998 (592 observations). r^k is the continuously compounded annual yield, $r^k - r^1$ the spread relative to the one-month rate and Δr^k is the weekly change in yields. Estimated rates are predicated on equation (8). Bessel means estimation with no measurement error on the six-month LIBOR, Kalman means estimation using a Kalman filter. The parameter values are $\kappa = 0.24$, $\lambda = -0.22$, $\beta_0 = 0.25$, $\theta = 14.68$, $\alpha = 2.35$ for the former and $\kappa = 0.31$, $\lambda = -0.49$, $\beta_0 = 0.66$, $\theta = 2.83$, $\alpha = 4.21$ for the latter.

Libor curve by adding spot rates implicitly derived from FRA 12×18 and 12×24 rates. The overall shape must be treated with caution, since Libor and FRA quotes are not synchronous and come from different banks. Average yields rise with maturity, with a rate of increase that initially falls and then rebounds between six and 18 months. The shape of the term structure is thus neither concave nor convex. This peculiar feature militates against pasting the Libor and FRA rates as if they were adjoining parts of the same curve.

Another feature is persistence, measured by the first-order autocorrelation coefficients. They are about 0.996 for all Libor rates and less than 0.05 for weekly changes, suggesting that yields are non-stationary or borderline stationary. Spreads over the one-month rate exhibit substantially less autocorrelation than levels, with a rising pattern from the short to the long end. To account for this pattern we note that, for neighbouring maturities, the factor must exert a similar influence on both components of the spread. With a negligible contribution of the factor, the autocorrelation of the spread resembles that of the measurement errors. By contrast, for more distant maturities the contribution of the common factor to changes in the spread becomes dominant. With a highly persistent underlying factor, the autocorrelation of the spread is all the higher the more distant the maturities.

A third feature concerns volatility. Libor volatility as measured by the standard deviation of yields does not appear to have a hump shape but instead decreases with maturity. When considered in terms of their weekly changes, the standard deviations increase slighltly after an initial sharp drop at the one-month maturity. This significant difference could not be explained if the single factor were the only source of uncertainty, because both levels and first differences would evince the same pattern of volatility. On data observed in levels, the factor, which is the most persistent, induces more variability than the measurement errors. Hence, the pattern of volatility mirrors that of the factor, and the downward-sloping volatility curve suggests that the factor loading curve decreases with maturity. The situation is different for weekly changes in yields. The contribution of the factor, which has a near-unit root, is sharply reduced. By contrast, the measurement errors, which are less than perfectly correlated, induce a perceptible variance of yield changes over two consecutive periods. In this case, the maturity pattern is contaminated by variations in the volatility of measurement errors.

The last feature relates to skewness and kurtosis. The excess kurtosis of weekly changes is huge, which clearly shows that weekly innovations to interest rates cannot be modelled as Gaussian distributions.

3.2 Estimation

Table 3 reports parameter estimates for the Bessel and Kalman variants. The primary difficulty was encountered in the Kalman filter, as the estimated variance-covariance Ω in (7) did not converge to a

Table 3

Affine-yield model estimates

	Bessel			**Kalman**		
Parameter	*Estimate*	*Std error*	*Student t*	*Estimate*	*Std error*	*Student t*
κ	0.24	0.07	3.4	0.31	0.08	3.8
λ	- 0.22	0.06	- 3.5	- 0.49	0.13	- 3.8
β_0	0.25	0.03	10.0	0.66	0.16	4.2
θ	16.2	4.7	3.4	2.4	1.2	2.0
α	1.8	0.3	6.1	5.5	0.9	6.4
ρ_1	0.81	0.02	43	0.82	0.02	46
ρ_2	0.85	0.02	55	0.86	0.01	59
ρ_3	0.85	0.02	54	0.86	0.01	58
ρ_4	—	—	—	0.72	0.06	11
ρ_5	0.74	0.02	28	0.75	0.03	28
ρ_6	0.87	0.02	54	0.88	0.01	59
Mean log likelihood		13.9			13.8	

The parameters κ, λ, β_0, θ and α refer to the mean reversion rate, price of risk, short rate volatility, long-run mean of the factor and lower bound on yields, respectively. The parameters ρ_k are the autocorrelation coefficients of measurement errors at maturities of one, two, three, six, nine and 12 months. The concentrated covariance matrix of the Bessel log-likelihood function (no measurement error) is used in the Kalman filter. The standard errors and t-statistics of parameter estimates are based on the usual Hessian matrix.

positive-definite matrix. The Bessel estimation does not have that problem since the variance-covariance matrix of residuals is actually concentrated out of the likelihood function. For simplicity, we have equated the Kalman variance of residuals with the Bessel one, interpolating the six-month variance from those of the neighbouring maturities and setting all remaining cross-variances to zero in the estimation. This restriction is admittedly arbitrary, but reflects our prior that, if the biases implied by both methods are not too severe, the covariance structure of the innovations to fitting errors should be relatively close. The Kalman parameter estimates are not sensitive to the particular value chosen for the unknown variance at the six-month maturity within its prescribed range.

The estimates are of the correct sign and magnitude. They are statistically significant, although the small-sample properties of our Libor data do not vindicate the use of asymptotic theory. They differ significantly across the two approaches, but remain broadly consistent with the features of the data highlighted in the previous subsection. The mean reversion parameter κ is lower for the Bessel than for the Kalman estimation. The estimated values are 0.24 and 0.32 respectively, which corresponds to half-lives of 2.9 and 2.2 years. As argued before, the no-measurement-error assumption makes the factor look more volatile than it actually is. This higher variance is supported by a lower κ, since a low mean reversion leads to a more volatile factor. The two mean reversion estimates are associated with autocorrelation coefficients of 0.995 and 0.994 (weekly), slightly less than the sample autocorrelations of yields at various maturities, which are about 0.996. Thus, the common factor absorbs most of the high persistence in yields, leaving measurement errors with substantially less autocorrelation. This difference in persistence between the factor and the measurement errors was a key feature needed to explain the contrasted patterns of volatility and persistence observed in Libor data. The estimated autocorrelation coefficients of the errors, between 0.7 and 0.9, are quite comparable across the two estimations, but this may be a consequence of the common covariance assumption mentioned above.

The identifying restriction used to disentangle the factor and the measurement errors is the absence of arbitrage, which places cross-section restrictions on the coefficients of the measurement equation (5). If a second factor has been left out of the analysis, these restrictions will be violated, since the single factor will not capture all the restrictions implied by arbitrage. A test of the overidentifying restrictions should thus bear out the model on the number of factors. We estimate the model, under both approaches, with no restriction placed on (5) and carry out standard likelihood ratio tests, keeping the autocorrelation parameters and the covariance of measurement errors constant. The likelihood ratio statistic (twice the change in sample likelihood), which is χ^2 with 12 degrees of freedom, is 27.0 for Bessel and 13.8 for Kalman. The corresponding significance levels are 1% and 30%. The test thus favors the latter approach while rejecting the former, but the result must be treated with care since, among other caveats, it should be noted that the Kalman likelihood is not the true density of the underlying model.

It is interesting to contrast the performance of the two methods, since their parameter estimates imply very different shapes for the theoretical forward and volatility curves. Figure 4 compares the mean theoretical curves $r(\tau, \widehat{\theta})$ with the actual mean curve. Average Libor increases by about two basis points each month beyond the two-month maturity, and this quasi-linear relation makes it difficult to fit a model which generates a hump-shaped term structure. Statistical information about the fitting errors $u_t^k = r_t^k - r(\tau_k, z_t)$ is reported in Table 4. The model curves approximate the average curve to within a few basis points.[5] Except for the one-month maturity, the standard deviations of the Bessel innovations are about half the bid/ask spread of 12.5 basis points during the sample period. By contrast, the estimated standard deviations of the Kalman innovations are about three times as large. In this respect, the former method fares better than the latter. However, the autocorrelations of fitting errors, ρ, are in both cases different from the estimates $\widehat{\rho}$ taken from Table 3. It turns out that the Bessel (Kalman) method systematically underestimates (overestimates) the autocorrelations of fitting errors, suggesting

[5] Because the slow rates of mean reversion imply that the long-run mean θ is not estimated with precision, the population mean $\bar{z}$ of the factor is different from the estimated value $\widehat{\theta}$. One finds $\bar{z} = 16.9$ for Bessel and $\bar{z} = 0.88$ for Kalman, while the estimates of the long-run means are $\widehat{\theta} = 16.2$ and $\widehat{\theta} = 2.4$. The unconditional variance of $\bar{z}$ can be approximated as $\theta/\kappa^2 T$, where T is the length of the period (in years). The corresponding standard errors are 4.9% and 1.4%. We calibrate θ and the shift parameter α to minimize the discrepancy between the theoretical and the mean yield curves, leaving unchanged the three parameters κ, λ and β_0, which determine the shape of the forward and volatility curves.

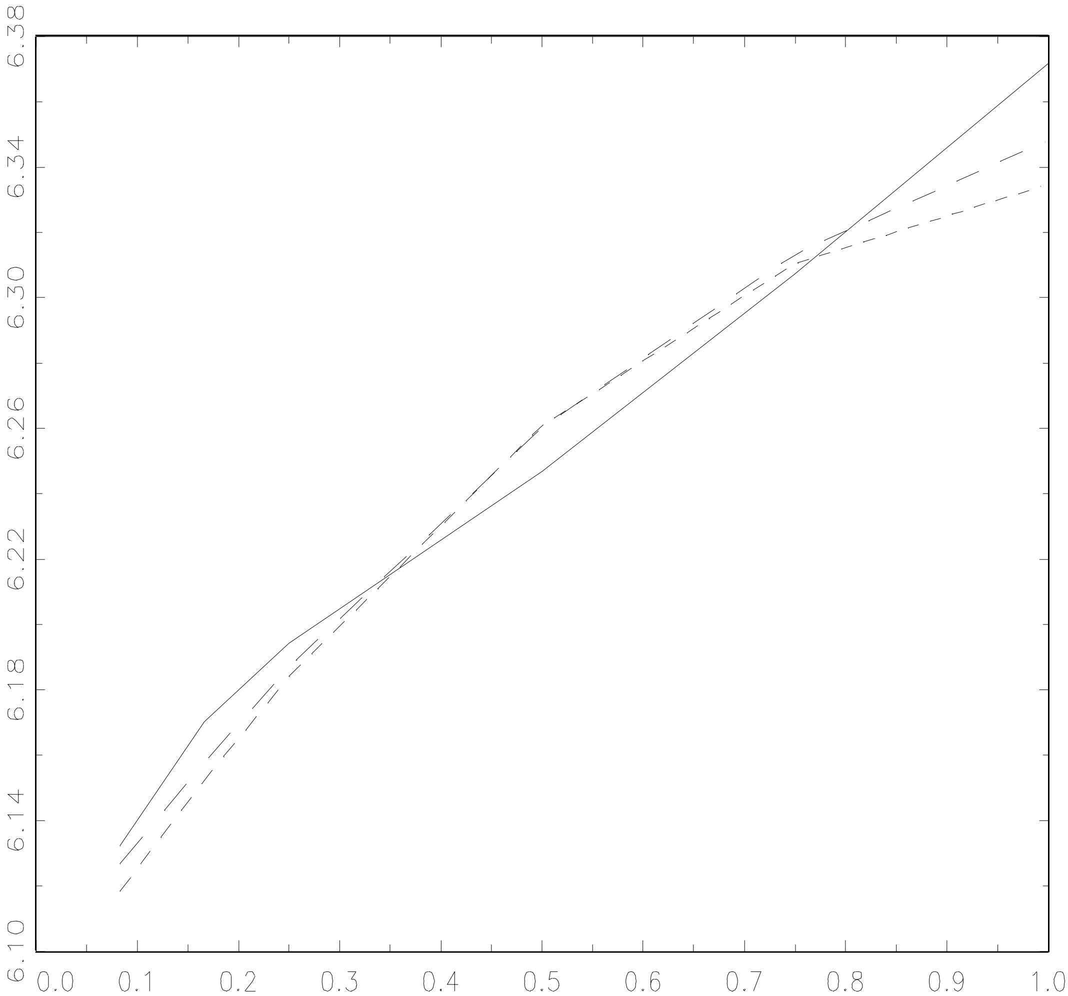

Figure 4: **Observed and estimated average yield curves**

Table 4

Fitting errors

	$\tau_k =$	1 month	2 months	3 months	6 months	9 months	12 months
	$\overline{u}$	0.02	0.03	0.02	0	0.01	0.04
	σ_u	0.28	0.21	0.14	—	0.11	0.18
Bessel	σ_ϵ	0.13	0.08	0.05	—	0.07	0.06
	ρ	0.88	0.93	0.93	—	0.82	0.94
	$\widehat{\rho}$	0.81	0.84	0.84	—	0.74	0.88
	$\overline{u}$	0.02	0.02	0.02	- 0.01	0.00	0.04
	σ_u	0.34	0.26	0.22	0.15	0.17	0.21
Kalman	σ_ϵ	0.21	0.18	0.18	0.18	0.18	0.19
	ρ	0.78	0.71	0.58	0.06	0.25	0.55
	$\widehat{\rho}$	0.82	0.86	0.86	0.72	0.76	0.88

The first entry is the sample average of the fitting errors. Entries σ_u and σ_ϵ are the standard deviations of fitting errors and their innovations. The last two variables ρ and $\widehat{\rho}$ are the empirical and estimated first-order correlation coefficients.

that the orthogonality conditions are violated in both cases. We will show in the next section that these biases can be ascribed to the specific assumptions on which the two methods rely.

To gauge the unconditional properties of the one-factor model, we construct estimates $\widehat{r}^k_{t|t-1}$ of the different yields on the basis of equation (8) by

$$\widehat{r}^k_{t|t-1} = r^k_{t-1} + (1-b)\left(r^*_k - r^k_{t-1}\right) + (b - \rho_k)\left(r^k(z_{t-1}) - r^k_{t-1}\right)$$

where $\widehat{r}^k_{t|t-1}$ has the nature of a one-step-ahead predictor. The expression relates the expected change in rate $\widehat{r}^k_{t|t-1} - r^k_{t-1}$ to the sum of two terms. The first accounts for the slow mean-reversion of yields toward their steady-state $r^*_k = r^k(\theta)$ and the second for their deviations from target: a high z_{t-1} implies that r^k_{t-1} is below its theoretical value $r^k(z_{t-1})$ and is expected to rise. The interpretation is slightly different for the Bessel and the Kalman approaches. In the first case, the six-month Libor at time $t-1$ serves to identify z_{t-1} directly, from which the whole term structure at time t is derived. In the second, the term structure at time t is used in the "correction step" to refine the last estimate of z_{t-1} obtained at time $t-1$, and this new estimate is used in (8) to uncover a "filtered" term structure at date t.

Table 2 compares the unconditional moments of these estimates with their actual sample counterparts. The Bessel estimates match the average yields and the standard deviations of yield changes to the third decimal place. They reproduce reasonably well the persistence and volatility of yields, yield spreads and yield changes, as well as the skewness and kurtosis of yield changes. In comparison, the Kalman estimates appear less sensible. In particular, the term structure of volatility is hump-shaped, a feature which is not supported by the data. One concludes that the left panel of Figure 2 gives the correct shape, while the right panel is invalidated by the data.

Table 5 reports the standard deviations of the weekly changes in yields $r_t^k - r_{t-1}^k$, along with those of the explained component $\widehat{r}_{t|t-1}^k - r_{t-1}^k$ and of the error component $r_t^k - \widehat{r}_{t|t-1}^k$. The last is simply the term $\epsilon_t^k + \beta_k\, w_t$ in (8). The volatilities of prediction errors and weekly changes in yields are very close. The Bessel method fails to improve on the “naive” prediction based on the rule $\widehat{r}_{t|t-1}^k = r_{t-1}$, while the improvement for the Kalman method is only marginal. More specifically, expected yield changes and prediction errors are negatively correlated in the former case and positively correlated in the latter. The Bessel method will thus tend to overpredict real changes in yields, as the predicted changes in yields will have to increase by more than necessary to counteract the negative contribution of the error term. This excess variability is severe enough to make (8) no better than the naive prediction rule. The next section attempts to explain the source of this anomaly.

Table 5

Standard deviations of prediction errors and expected changes in yields

	$\tau_k =$	*1 month*	*2 months*	*3 months*	*6 months*	*9 months*	*12 months*
	Δr_e^k	0.05	0.03	0.03	0.01	0.03	0.02
Bessel	$r^k - \widehat{r}^k$	0.17	0.13	0.14	0.15	0.16	0.16
	Δr^k	0.17	0.13	0.14	0.15	0.16	0.16
	Δr_e^k	0.06	0.04	0.03	0.04	0.04	0.03
Kalman	$r^k - \widehat{r}^k$	0.15	0.12	0.12	0.11	0.13	0.15
	Δr^k	0.17	0.13	0.14	0.15	0.16	0.16

Entries are standard deviations of expected changes $\Delta r_e^k = \widehat{r}^k - r_{-1}^k$, of prediction errors $r^k - \widehat{r}^k$ and of yield changes $\Delta r^k = r^k - r^{k-1}$.

4. Analysis of specification biases

Table 4 has shown that the estimates $\widehat{\rho}$ of the autocorrelation of measurement errors were biased in specific (and opposite) directions. Moreover, we have just seen that each method consistently misrepresents the direction of weekly changes in yields. We argue in this section that the origin of the misspecification is more likely to be found in the econometric approaches that were implemented than in the model itself. To see this, we consider again (8), written more compactly as

$$\Delta r_t = -(1-b) r_{t-1} - (b-\rho)\, u_{t-1} + \epsilon_t + \beta\, w_t \tag{9}$$

where the maturity superscript and the constant terms have been omitted for simplicity. The very assumptions on which the econometric methods are predicated prevent ϵ_t from being the true innovation to the fitting errors u_t. It is the violation of this orthogonality condition which leads to the systematic biases documented above.

Consider first the Bessel approach. Here we assume that there is no error on the six-month Libor, mistakenly identifying the factor as $\beta\widehat{z} = \beta z + v$, where v is the measurement error on the six-month rate. Substituting $\widehat{z}$ for z in (9), we find that the measurement innovation becomes $\widehat{\epsilon}_t = \epsilon_t - (v_t - \rho v_{t-1})$. Thus, $\text{cov}\,(\widehat{\epsilon}_t, u_{t-1}) = (\rho - \rho_v)\,\text{cov}\,(u, v)$, where ρ_v is the autocorrelation of the omitted six-month error. In general, the sign of the correlation will depend on the relative persistence and covariances of measurement errors. However, the pattern of autocorrelations in Table 3 strongly suggests that a minimum is reached at the six-month maturity. Assuming positive comovements between measurement errors, this implies $\text{cov}\,(\widehat{\epsilon}_t, u_{t-1}) > 0$. Table 6 confirms that all correlations are indeed positive and significant, pointing to a clear violation of the orthogonality conditions. This has two implications. First, noting that the model defines measurement errors as $\widehat{\epsilon}_t = u_t - \widehat{\rho} u_{t-1}$, one has

$$\begin{aligned}\text{cov}\,(\widehat{\epsilon}_t, u_{t-1}) &= \text{cov}\,(u_t - \widehat{\rho} u_{t-1}, u_{t-1}) \\ &= (\rho - \widehat{\rho})\,\text{var}\,(u)\end{aligned}$$

so that $\widehat{\rho} < \rho$, as Table 6 reveals. Second, the prediction error and predicted yield changes in (9) tend to move in opposite directions, as yields are expected to decline when u is high. The Bessel forecast must overestimate the direction of change.

Table 6
Orthogonality conditions
Cross-correlation of measurement errors and lagged error term

	$\tau =$	*1 month*	*2 months*	*3 months*	*6 months*	*9 months*	*12 months*
Bessel		0.16	0.23	0.24	—	0.13	0.19
Kalman		-0.07	-0.19	-0.31	-0.58	-0.46	-0.37

Consider now the Kalman approach. Here, the error term $u_{t-1} = r_{t-1} - r(z_{t-1})$ is no longer exogenous and equation (9) can be rewritten as

(10) $$\Delta r_t = -(1-\rho) r_{t-1} + \beta(b-\rho)\, z_{t-1} + \epsilon_t + \beta w_t, \qquad w_t = z_t - b z_{t-1}.$$

The Kalman filter reestimates both z_{t-1} and z_t on the basis of new information at time t. A large interest rate innovation may be due to a large innovation to the factor $-$ in which case w_t will be increased $-$ or to the fact that the state at time $t-1$ was underestimated $-$ in which case z_{t-1} will be increased. If the weighting is applied properly, both z_t and z_{t-1} will be revised in a manner that leaves the resulting measurement error ϵ_t uncorrelated with z_{t-1}. Unfortunately, the method defines z_t as the mean squared error forecast of a variable that has a large kurtosis. As a result, it gives more weight to w_t than necessary, squeezing both ϵ_t and z_{t-1} in the process. The excessive factor variability thus generates positive comovements between ϵ_t and z_{t-1}. Table 6 indicates that the correlation between ϵ_t and $u_{t-1} = r_{t-1} - \beta\, z_{t-1}$ is indeed negative. This naturally has the reverse implications for the estimated autocorrelations of measurement errors ($\widehat{\rho} > \rho$) and the direction of change (underprediction).

5. Implications for the expectations hypothesis

The one-factor model is the most parsimonious way to generate a risk premium endogenously. It thus has implications for the predictability of interest rates. In this section, we use the estimations above to see if the one-factor model is capable of accounting for the evidence against the expectations hypothesis. The predictability "smile", i.e. the fact that yield spreads help forecast future short rates at short and long horizons but less so at horizons of about a year, is a major stumbling block of the expectations hypothesis.

To understand intuitively how the one-factor model behaves with respect to the expectations hypothesis, we assume away measurement errors and start with a simple forward rate regression involving the short rate. We then use a standard form of yield spread regression involving changes in one-month Libor to examine whether the one-factor model conforms with the predictability pattern of interest rates observed in the data.

5.1 A simple forward rate regression

The expectations hypothesis has several forms. A typical statement is that forward rates are the expectations of future short rates, up to a constant term premium

$$f_t^\tau - r_t = (\mathrm{E}_t\, r_{t+\tau} - r_t) + p^\tau \tag{11}$$

where $f_t^\tau - r_t$ is the forward spread and $\mathrm{E}_t\, r_{t+\tau} - r_t$ is the expected change in the instantaneous rate. The expectations hypothesis can be tested by estimating the regression

$$r_{t+\tau} - r_t = a + b(f_t^\tau - r_t) + v_t \tag{12}$$

where $v_t = r_{t+\tau} - \mathrm{E}_t\, r_{t+\tau}$ is the forecast error. If the hypothesis is true, then $b = 1$ and $a = -p^\tau$. Many explanations for the rejection of the expectations hypothesis have focused on a time-varying term premium. As is well known, if the residual v_t is contaminated by a risk premium, b will deviate from one due to a standard omitted variable problem.

According to the one-factor model, the theoretical expressions for the expected change in yields and the risk premium are, respectively,

$$\begin{aligned} \mathrm{E}_t\, r_{t+\tau} - r_t &= \beta_0(1 - e^{-\kappa\tau})\,(\theta - z_t) \\ p_t^\tau \equiv f_t^\tau - \mathrm{E}_t\, r_{t+\tau} &= \theta\,\left(\kappa\beta(\tau) - \beta_0(1 - e^{-\kappa\tau})\right) + \left(\dot{\beta}\,(\tau) - \beta_0 e^{-\kappa\tau}\right)\, z_t. \end{aligned}$$

To understand intuitively these expressions, consider a positive shock to the expectations process z_t, which shifts the term structure upwards. Expected yields are revised downwards, since mean reversion implies that the short rate will gradually return to its equilibrium level. On the other hand, the shape of the yield curve is also influenced by variations in the term premium.[6] When interest rates are expected to fall, borrowers prefer to pay high short rates until long rates eventually fall. Similarly, investors attempt to invest in long-term bonds in the hope of locking in high yields. The result of both responses is to raise the risk premium at the shorter end of the maturity spectrum and reduce it at the longer end, causing the term premium to be first negatively then positively correlated with expected yield changes.

To ascertain b in the context of the one-factor model without measurement error, we consider (11) as a signal extraction problem, where the term premium varies through time. The implied regression slope

$$b(\tau) = \beta_0(1 - e^{-\kappa\tau})/(\beta_0 - \dot{\beta}(\tau)) \tag{13}$$

is displayed in Figure 5. The affine model endogenously creates a predictability smile, but it generates a regression coefficient that is greater than one for maturities under two years. To this extent, it worsens the situation by moving the regression coefficient in the wrong direction. The reason is clear. Over short-term horizons, a positive shock to the expectations process z_t moves the expected change in the short rate and the term premium in opposite directions. Thus, a falling forward spread indicates that the expected component has declined even further. In this case, expected changes in the short rate tend to overpredict the magnitude of forward spreads, making b greater than one. The inability of affine models to generate both a rising mean forward rate curve and a regression slope between zero and one is well known, see Backus et al. (1998). This is also true in the present continuous-time framework. The average forward curve is upward-sloping if and only if $\lambda + \beta(\tau) < 0$. This implies that the volatility curve $\dot{\beta}(\tau)$ cannot fall at a faster rate than κ. With $\dot{\beta}(0) = \beta_0$, this yields $\dot{\beta}(\tau) > \beta_0 e^{-\kappa\tau}$, and the numerator in (13) is larger than the denominator.

The regression slope equation (13) assumes that all shocks to interest rates are captured by a single factor. As a result, the expected yield change and the term premium are perfectly correlated. A large negative correlation raises the regression slope above one. In practice, however, one would expect a less than perfect correlation. With systematic measurement errors, the one-factor model provides at best a rough approximation to the true expected yield changes and risk premia. Indeed, part of the interest rate dynamics has been left unexplained and shifted instead to measurement errors. Thus, even though in theory the one-factor model cannot account for regression slopes of less than one in a simple regression such as (12), in practice the presence of systematic measurement errors will bias the slope estimates downwards. It would be unfortunate to discard the affine model just because we require that

[6] The effect on the risk premium depends on $\dot{\beta}(\tau) - \beta_0 e^{-\kappa\tau}$. Since $\ddot{\beta}(\tau)/\dot{\beta}(\tau) = -(\kappa + \lambda + \beta(\tau))$, whether the rate of change of the impact curve first derivative, $\dot{\beta}(\tau)$, is above or below $-\kappa$ depends on the sign of $\lambda + \beta(\tau)$. For short maturities $\beta(\tau)$ is close to zero so $\lambda + \beta(\tau)$ is negative and $\dot{\beta}(\tau)$ must lie above $\beta_0 e^{-\kappa\tau}$. For long maturities, $\lambda + \beta(\tau)$ becomes eventually positive and $\dot{\beta}(\tau)$ decreases faster than $\beta_0 e^{-\kappa\tau}$. The term premium turns negative as $\dot{\beta}(\tau)$ cuts below $\beta_0 e^{-\kappa\tau}$.

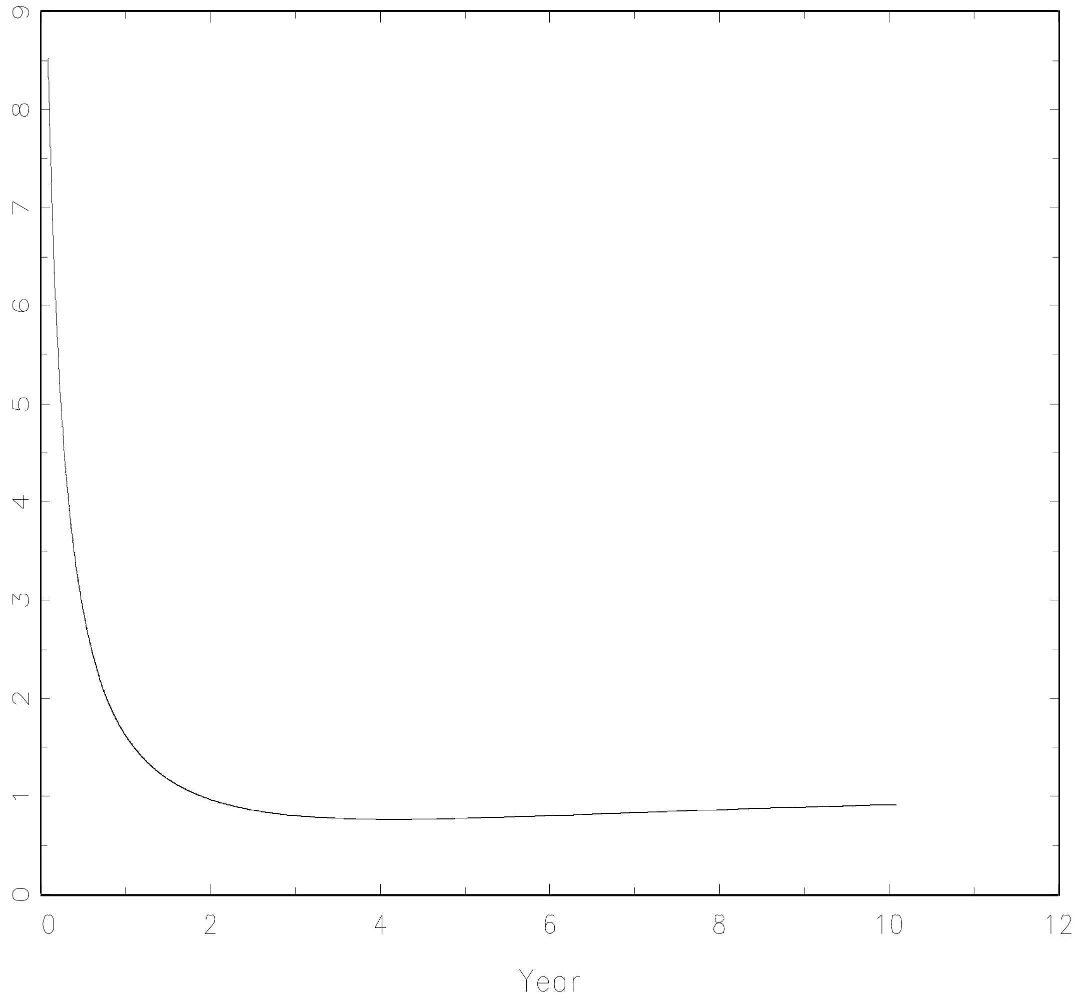

Figure 5: Predictability pattern implied by the one-factor model

it fit all rates exactly, for we would not know if a given shift in the term structure were due to a shift in expectations or risk premia.

5.2 Yield spread regressions

Because the variables in (11) are not directly observable, the expectations hypothesis is usually based on the approximate relationship

$$r_t^k = (1/k) \sum_{i=0}^{k-1} \mathrm{E}_t \, r_{t+i}^1 + p_t^k$$

which states that the expected return on holding the k-period bond should equal the return on rolling over one-period bonds, up to a term premium. A standard regression test is derived by relating the ex post changes in the short rate over the maturity of the investment to the yield spread

$$rs_t^k = a + b \left(r_t^k - r_t^1 \right) + v_t \tag{14}$$

where $rs_t^k = \sum_{i=1}^{k-1} \left(r_{t+i}^1 - r_t^1 \right) / k$ is the rollover spread. The constant a equals the mean of the risk premium and the error term v_t can be viewed as the ex post excess return relative to the one-month rate. As in the simple forward regression, $b = 1$ if the hypothesis is true, and deviates from one if v_t contains a risk premium correlated with the yield spread.

Table 7 provides estimates of the parameters of (14) and their standard errors for the three-, six- and 12-month maturities. In all cases, b is estimated to be positive and significantly different from one. As in most of the empirical literature based on data for the United States, the estimates differ substantially from the expectations hypothesis. Forward spreads contain information that can be used to forecast future one-month rates, but the regression slopes decrease monotonically, with the largest departure occurring at the one-year maturity with a coefficient of 0.66. The constant term rises concomitantly in absolute value, pointing to an increase in the average risk premium with maturity. This feature of the data conforms well with theory: as the risk premium increases, forward spreads are less informative about the future path of interest rates and the regression slope is tilted towards zero.

In order to assess the relevance of the one-factor model in this context, we run the same regressions by replacing ex post with expected rolling spreads. To construct the new series at different points in time, we compute one-month interest rate forecasts from the theoretical expression (4) evaluated at $\tau = 1/12$ and use the property that the conditional expectation of the factor is a weighted average of its current value and its long-run mean. The expected change in the one-month yield over n weeks can be written as

$$\begin{aligned} \widehat{r}_{t+n|t}^1 - r_t^1 &= \beta(1/12) \ \mathrm{E}_t(z_{t+n} - z_t) \\ &= \beta(1/12)(1 - e^{-\kappa n/52}) \ (\theta - z_t) \end{aligned}$$

Table 7

Rollover spread regressions

Yield spread regression $rs_t^k = a + b\left(r_t^k - r_t^1\right) + v_t$	Maturity	$\widehat{a}$	(s.e.)	$\widehat{b}$	(s.e.)
rs_t^k	3 months	- 0.05	(0.01)	0.77	(0.09)
	6 months	- 0.09	(0.02)	0.69	(0.07)
	12 months	- 0.20	(0.03)	0.66	(0.06)
$rs_t^{k,e}$	3 months	- 0.07	(0.00)	0.98	(0.03)
	6 months	- 0.12	(0.00)	0.87	(0.01)
	12 months	- 0.17	(0.01)	0.64	(0.01)

The regressions use the Newey and West (1987) correction for MA errors of order k. The dependent variable is either rs_t^k, the ex post rollover spread, or $rs_t^{k,e}$, the rollover spread estimated from the Bessel model.

where the factor z_t is taken from one of the two approaches described above. For space reasons we discuss only the results obtained for the Bessel estimation.

The coefficient estimates from the model are presented in Table 7 for comparison. The model is able to replicate the predictability smile, and the fitted values accord particularly well with the evidence at the one-year horizon. The estimated mean risk premia also track their sample counterparts closely. The one-month regression slope estimate, by contrast, is very close to what the expectations hypothesis would dictate. This may be evidence that the one-factor model imposes an excessive negative correlation between the rolling spread and the risk premium, or that it underestimates the variability of the risk premium. To assess whether the rollover spread identified with the model rs_t^e is a good predictor of the ex post rollover spread rs_t, we also estimate univariate n-step-ahead forecasting equations of the form

(15) $$rs_t = c + d\,rs_t^e + u_t,$$

using various yield spreads as overidentifying restrictions. The results in Table 8 present compelling evidence that the model 12-month rollover spread is a good predictor of its empirical counterpart, but the same hypothesis is soundly rejected at the three- and six-month maturities. One plausible interpretation is that the one-factor model is too parsimonious to capture the behaviour of interest rates in the short term.The low p-values associated with the tests of overidentifying restrictions indicate that current short- and long-term rates help improve forecasts, so that the predictions based on (15) are not informationally efficient. In short, the one-factor model provides a good approximation to the behaviour of interest rates at the horizon of one year and leads to better forecasts than the expectations hypothesis, but there remain significant tensions between the model and the data at short maturities.

Table 8

Forecasting equations of the rollover spread

Forecasting equation $rs_t^k = c + d\,rs_t^{k,e} + u_t$	Maturity	$\widehat{c}$	(s.e.)	$\widehat{d}$	(s.e.)	p-value
		Parameter estimates				
1 overidentifying restriction	1 month	-0.01	(0.01)	0.45	(0.06)	0.00
	3 months	-0.02	(0.01)	0.70	(0.07)	0.01
	12 months	-0.02	(0.03)	0.99	(0.07)	0.45
3 overidentifying restrictions	1 month	0.00	(0.01)	0.42	(0.05)	0.00
	3 months	-0.01	(0.01)	0.69	(0.06)	0.01
	12 months	-0.03	(0.03)	0.96	(0.07)	0.05

The forecasting equations use either one overidentifying restriction (the yield spread with the same horizon as the rollover spread) or three overidentifying restrictions (all yield spreads). The p-values are the marginal probabilities associated with the tests of the overidentifying restrictions.

6. Conclusion

This paper tries to explain movements in Libor rates with a single-factor affine-yield model. We find that this model provides a reasonable approximation of the unconditional and conditional properties of Libor rates, with the Bessel approach providing more sensible parameter estimates than the Kalman method on the basis of the implied volatility patterns and the size of error innovations. We point out that it approximates well the behaviour of future rates at the horizon of one year, although significant tensions remain between the model and the data at shorter maturities. We also argue that the evidence of misspecification is likely to originate from the implausible assumptions on which the econometric methods are predicated rather than from the model itself.

A consistent finding of the term structure literature is that, to reconcile the time-series dynamics of interest rates with their cross-sectional shapes, models with more than one factor are necessary. We feel nevertheless that the paper's effort to confine strictly the number of factors needed to explain Libor rates is justified.

First, the choice of a model should reflect the application to which it is put. If the objective is to manage interest rate risk through sophisticated trading strategies, the need to capture all sources of uncertainty may require a large number of factors. For central banks, however, the situation is different. They are more interested in extracting markets' expectations about future interest rates than in pricing interest rate risk accurately. As latent variables, factors are neither amenable to straightforward economic interpretation, nor very suggestive of the joint behavior of short- and long-term interest rates. They may also fail to provide a sensible account of the predictability pattern of interest rates.

Second, as emphasised in Gong and Remolona (1997), it is important to know whether separate models, with a reduced number of factors, can fit small stretches of the yield curve. Explaining only part of the curve holds some promise that it will become possible to fit the whole curve with an appropriately specified model. This seems especially true with an interbank term structure including FRA prices and interest rate swaps, since market players have different creditworthiness. An interest rate swap curve, for example, could be derived by discounting future Libor fixings at a risk-adjusted rate. A "driving factors" approach to the joint behaviour of interest and default rates would gather evidence on how banks have shifted their undiversifiable risks to other institutions, without having to assume that the Libor and swap markets have homogeneous credit quality. A related example can be found in the pricing of discount Brady bonds which, with different contractual arrangements, could be used to uncover sovereign credit risk. Both examples are part of the author's research agenda.

Appendix A

A1. One-factor continuous-time model

The price of a discount bond maturing at T is given by

$$B_{t,T} = \mathrm{E}\left[\exp\left\{-\int_t^T r(z_s)\,ds\right\}\frac{\eta(T)}{\eta(t)} \mid \mathcal{F}_t\right].$$

Let Q be the risk-neutral probability, defined by its density η_t in restriction to $\mathcal{F}_t$. Girsanov's theorem implies that if the risk-neutral density η_t is governed by $d\eta_t/\eta_t = -\lambda\sqrt{z_t}\,dW_t$, the process

$$\widehat{W}_t = W_t + \lambda\sqrt{z_t}\,dt$$

is a Brownian motion under Q. The square root process z follows the stochastic differential equation

$$dz_t = (\kappa\theta - (\kappa+\lambda)z_t)\,dt + z^{1/2}d\widehat{W}_t$$

under Q. Using the time-homogeneous property of the process, we see that $B_{t,T}$ can be written as

$$\psi(\tau,z) = \mathrm{E}^Q\left[\exp\left\{-\int_0^\tau r(z_s)\,ds\right\}\right]_{z(0)=z}$$

where $\tau = T - t$.

Taking $r(z) = \alpha + \beta_0 z$, the discount function can be expressed as $\psi(\tau,z) = e^{-\alpha\tau}\varphi(\tau,z)$, where φ solves the backward Kolmogorov equation

$$-\frac{\partial\varphi}{\partial\tau} + \mathcal{L}\varphi - \beta_0 z\varphi = 0,$$

and $\mathcal{L}$ is the generator associated with z_t under Q. The following guess

$$\varphi(\tau,z) = \exp\left\{-(\kappa\theta\gamma(\tau) + \beta(\tau)z)\right\},$$

together with the boundary condition $\varphi(0,z) = 1$, leads to the ordinary difference equations

$$\begin{aligned} \dot{\beta}(\tau) + (\kappa+\lambda)\beta(\tau) + \beta^2(\tau)/2 &= \beta_0 \\ \dot{\gamma}(\tau) &= \beta(\tau) \end{aligned} \tag{16}$$

with $\beta(0) = \gamma(0) = 0$. The first equation (Ricatti) has as its solution

$$\beta(\tau) = \beta_0\frac{e^{q_1\tau} - e^{q_2\tau}}{q_1 e^{q_2\tau} - q_2 e^{q_1\tau}}$$

where q_1 and q_2 are, respectively, the positive and negative roots of $q^2 + (\kappa+\lambda)q - \beta_0/2 = 0$. Upon integrating $\beta(\cdot)$ we find

$$\gamma(\tau) = 2\ln\frac{q_1 e^{q_2\tau} - q_2 e^{q_1\tau}}{q_1 - q_2}.$$

The impact function $\beta(\tau)$ is characterised by the property that its curvature $\ddot{\beta}(\tau)/\dot{\beta}(\tau)$ is equal to $-(\kappa+\lambda+\beta(\tau))$ for all τ. When $\kappa+\lambda < 0$, the impact curve $\beta(\tau)$ is initially convex and then concave, implying that the factor loading $\beta(\tau)/\tau$ is hump-shaped.

To study the average yield curve, it is convenient to consider forward rates. Taking the derivative of the discount function yields

$$f(\tau, z) = \alpha + \kappa\theta\beta(\tau) + \dot{\beta}(\tau)\, z$$

and the mean forward rate curve has a slope given by

$$\dot{f}(\tau, \theta) = -\theta(\lambda + \beta(\tau))\dot{\beta}(\tau)\,.$$

The slope at the origin is positive and equal to $\dot{f} = -\theta\lambda\beta_0 = -\lambda\,(r^* - \alpha)$. It eventually turns negative if $\lambda + 2\beta_\infty = \lambda + 2q_1 > 0$, in which case it is hump-shaped. The last condition is met if either $\kappa^2 < 2\beta_0$ or $\kappa^2 > 2\beta_0$ and $\lambda \in \left(-\kappa - \sqrt{\kappa^2 - 2\beta_0}, -\kappa + \sqrt{\kappa^2 - 2\beta_0}\right)$.

The volatility term structure can be derived by fixing the maturity date T so that $\tau = T - t$. The change of variable $f_t^T = f(t, T - t)$ leads to

$$df_t^T = (\lambda + \beta(\tau))\dot{\beta}(\tau)\, z_t\, dt + \dot{\beta}(\tau)\, z_t^{1/2}\, dW_t$$

so that the forward volatility curve is defined by $\dot{\beta}(\tau)\, z_t^{1/2}$. Since the rate of increase in $\dot{\beta}(\tau)$ is $-(\kappa + \lambda + \beta(\tau))$, its slope is governed initially by $\kappa + \lambda$ and, for large τ, decays at a rate of $\kappa + \lambda + 2q_1$.

A2. Hump shapes in independent factor models

We show that discrete-time affine models with independent factors can accommodate hump shapes provided they account for a time-varying term premium. Consider the recursion that characterises the factor loadings in a one-dimensional Cox-Ingersoll-Ross model; see for example Backus et al. (1998), p. 9:

$$B_{n+1} = 1 + \lambda^2/2 + B_n\varphi - (\lambda + B_n\sigma)^2/2 \tag{17}$$

starting with $B_0 = 0$. We posit

$$\begin{aligned} \sigma &= \sqrt{\beta_0}\epsilon \\ \varphi &= 1 - \kappa\epsilon \end{aligned}$$

for some β_0, κ and small $\epsilon > 0$. By this token, the difference equation (17) can be seen as an element of the sequence of recursions indexed by ϵ and defined by

$$B_{n+1}^\epsilon = 1 + \lambda^2/2 + B_n^\epsilon(1 - \kappa\epsilon) - (\lambda + B_n^\epsilon\sqrt{\beta_0}\epsilon)^2/2.$$

Let $\beta_n^\epsilon = \beta_0\epsilon\, B_n^\epsilon$. We obtain

$$\frac{\beta_{n+1}^\epsilon - \beta_n^\epsilon}{\epsilon} = \beta_0 - (\kappa + \lambda')\,\beta_n^\epsilon - \beta_n^{\epsilon 2}/2$$

where $\lambda' = \lambda\sqrt{\beta_0}$. Thus, β_n^ϵ converges to the solution to (16) as $\epsilon \to 0$. A necessary and sufficient condition for a hump is that $\kappa + \lambda' < 0$, a condition equivalent to $1 - \varphi + \lambda\sigma < 0$.

Appendix B

B1. No-measurement-error assumption (Bessel method)

Let r^4 be the six-month Libor rate. From (4) and (5) with $u^4 = 0$, we obtain

$$r_t^4 = g_4 + \beta_4 \, z_t.$$

The sample loglikelihood at time t can be decomposed into two parts.

Conditional Libor transition. With the above change of variable, the first part of the log-likelihood is $\ln f(z_t|z_{t-1}) - \ln \beta_4$, where $f(z_t|z_{t-1})$ is the density of z_t conditional on z_{t-1} one week before. The function f can be written as

$$\begin{aligned}
f(z_t|z_{t-1}) &= c\left(\frac{x}{\phi}\right)^{\nu/2} \exp\left\{-(x+\phi)/2\right\} I_\nu\left(\sqrt{x\phi}\right) \\
\text{where} \quad c &= \frac{2\kappa}{1-e^{-\kappa/52}} \\
x &= 2cz_t \\
\phi &= 2ce^{-\kappa/52}\, z_{t-1} \\
\nu &= 2\kappa\theta - 1 > 0 \\
I_\nu(\cdot) &= \text{modified Bessel function of the first kind.}
\end{aligned}$$

For the Bessel function, we have used the integral representation provided by Gradshteyn and Ryzhik (1980), p. 958. Details about the numerical implementation are available from the author. For the first observation, we used the unconditional distribution of z, which is the gamma function

$$g(z) = \frac{(2\kappa)^{1+\nu}}{\Gamma(1+\nu)} z^\nu e^{-2\kappa\, z}.$$

Distribution of measurement errors. Letting $\epsilon_t^k = u_t^k - \rho_k u_{t-1}^k$, where u_t is the vector of fitting errors for the five remaining rates, the second part is

$$-\frac{1}{2}\epsilon_t' \Omega^{-1}\epsilon_t - \frac{1}{2}\ln|\Omega| - \frac{1}{2}\ln(2\pi).$$

The covariance matrix can be concentrated out of the likelihood function, using

$$\Omega = \left(\Sigma_{t=2}^T \epsilon_t \epsilon_t'\right)/(T-1).$$

B2. Linear filter (Kalman method)

We start the Kalman filter by writing the model (2–5) as

$$\begin{aligned}
z_t &= \theta(1-b) + bz_{t-1} + w_t, \quad \text{var}(w_t) = q = (1-b^2)\theta/2\kappa \\
r_t &= g + \beta z_t + u_t \\
u_t &= Ru_{t-1} + \epsilon_t, \quad \text{var}(\epsilon) = \Omega
\end{aligned}$$

where $b = e^{-\kappa\tau}$ and R is the diagonal matrix containing the autocorrelation coefficients of measurement errors. Using $\widetilde{r}_t = r_t - R\,r_{t-1}$, we can transform the model as

$$\begin{aligned} z_t &= \theta(1-b) + bz_{t-1} + w_t \\ \widetilde{r}_t &= G + H\,z_{t-1} + \epsilon_t + \beta w_t \end{aligned}$$

with $G = \theta(1-b)\beta + (I-R)g$ and $H = (bI - R)\beta.$ Note that the measurement innovation of the transformed model is no longer uncorrelated with the factor innovation, and that the state variable at date t is z_{t-1}.

In the prediction step, we use the rules

$$\begin{aligned} z_{t-1|t-1} &= \theta(1-b) + bz_{t-2|t-1} + w_{t-1|t-1} \\ w_{t|t-1} &= 0 \\ \widetilde{r}_{t|t-1} &= G + H\,z_{t-1|t-1.} \end{aligned}$$

Let $\sigma^2_{t|t-1}$ be the conditional variance of z_{t-1}. The covariance matrix of the prediction errors is

$$\begin{bmatrix} \sigma^2_{t|t-1} & 0 & \sigma^2_{t|t-1}H' \\ 0 & q & q\beta' \\ \sigma^2_{t|t-1} & q\beta & \Sigma_{t|t-1} \end{bmatrix}$$

where $\Sigma_{t|t-1} = \Omega + q\beta\beta' + \sigma^2_{t|t-1}HH'$.

In the correction step, we define $E_t = \left(\widetilde{r}_t - G - H\,z_{t-1|t-1}\right)$ and find

$$\begin{aligned} z_{t-1|t} &= z_{t-1|t-1} + \sigma^2_{t|t-1}H'\Sigma^{-1}_{t|t-1}\,E_t \\ w_{t|t} &= q\beta'\Sigma^{-1}_{t|t-1}\,E_t \end{aligned}$$

the conditional variance of which is

$$\Sigma^{z,w}_{t|t} = \begin{bmatrix} \sigma^2_{t|t-1} & 0 \\ 0 & q \end{bmatrix} - \begin{bmatrix} \sigma^2_{t|t-1}H' \\ q\beta' \end{bmatrix} \Sigma_{t|t-1} \begin{bmatrix} \sigma^2_{t|t-1}H & q\beta \end{bmatrix}.$$

Thus,

$$\sigma^2_{t+1|t} = \begin{bmatrix} b & 1 \end{bmatrix} \Sigma^{z,w}_{t|t} \begin{bmatrix} b \\ 1 \end{bmatrix} = b^2\sigma^2_{t|t-1} + q - \left(q\beta' + b\sigma^2_{t|t-1}H'\right)\Sigma^{-1}_{t|t-1}\left(q\beta + b\sigma^2_{t|t-1}H\right)$$

and the Kalman filter is

$$z_{t|t} = \theta(1-b) + bz_{t-1|t-1} + K_t\,E_t$$

with $K_t = \left(b\sigma^2_{t|t-1}H' + q\beta'\right)\Sigma^{-1}_{t|t-1}$. To start the filter, we extract $z_{1|1}$ from

$$r_1 = g + \beta z_{1|1} + u_1, \quad \text{var}(z_1) = \theta/2\kappa, \quad \text{var}(u_1) = V$$

with $V_{ij} = \Omega_{ij}/(1 - \rho_i\rho_j)$.

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