Topic 4
Topic 2
Topic 3
Topic 1
Topic 5

Topic3
p20, p38

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Optimal Bank Leverage and Recapitalization in Crowded Markets*

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Abstract

We study optimal bank leverage and recapitalization in general equilibrium when the supply of specialized investment capital is imperfectly elastic. Assuming incomplete insurance against capital shortfalls and segmented financial markets, ex-ante leverage is inefficiently high, leading to excessive insolvencies during systemic capital shortfall events. Recapitalizations by equity issuance are individually and socially optimal. Additional frictions can turn asset sales individually but not necessarily socially optimal. Our results hold for different bankruptcy protocols and we offer testable predictions for banks' capital structure management. Our model provides a rationale for macroprudential capital regulation that does not require moral hazard or informational asymmetries.

Keywords: Bank capital, recapitalization, macroprudential regulation, incomplete markets, financial market segmentation, constrained inefficiency

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1 Introduction

The need for specialized investment knowledge restricts the free flow of capital and leads to segmented financial markets. The resulting shortages of specialized investment capital have received considerable attention in the literature, but primarily in conjunction with asset fire sales (Shleifer and Vishny, 1992). Yet, evidence of large aggregate stock issuances raising banks' cost of capital (Lambertini and Mukherjee, 2016) suggest broader implications for equity offerings and capital structure management.[1] To study these implications, we develop a general equilibrium model of bank capital with segmented financial markets and imperfectly elastic supply of investment capital. Our model encompasses recapitalizations by equity issuances and asset sales, as well as an ex-ante leverage choice.[2]

Market segmentation is impermanent, and arises primarily in situations in which banks need to recapitalize quickly and simultaneously. Our focus is therefore on the implications of such periods for optimal ex-ante leverage and on the optimality of different recapitalization strategies once they arise. Absent additional frictions, we find equity issuances to be individually and socially preferable to asset sales. The reason is that asset sales reduce portfolio returns–including in capital-constrained states–and thus exacerbate bank-specific and aggregate capital shortages. Additional frictions, such as the loss of control benefits through equity offerings, may cause banks to change recapitalization strategies and drive a wedge between individual and social optimality.

Independent of their recapitalization strategy, banks in our model fail to incorporate the general equilibrium effect of their leverage on others' ability to recapitalize. Under the plausible assumption that systemic stress events with system-wide capital shortfalls are rare, the laissez-faire equilibrium features inefficient over-leveraging and excessive bank failures. This happens because of a *pecuniary externality* in conjunction with incomplete financial markets and contracts.

Systemic capital shortfall events of the type we have in mind, where many banks need to swiftly recapitalize, include the 2008 financial crisis and the current COVID-19 crisis.

[1] Evidence of limits to arbitrage in equity markets goes back to Asquith and Mullins (1986) and Pontiff (1996). More recently, Mitchell et al. (2007), Mitchell and Pulvino (2012), and Duffie (2010) in his presidential address to the American Finance Association, argue that capital in the market for convertible debt is slow-moving. Cornett and Tehranian (1994) show for bank equity markets that banks' share price tends to drop in response to *involuntary* share issuance, i.e. for reasons unrelated to adverse selection à la Myers and Majluf (1984).

[2] For evidence that banks' capital structure management relies on equity offerings, see (De Jonghe and Öztekin, 2015; Dinger and Vallascas, 2016). Additional evidence is available for German (Memmel and Raupach 2010), Swiss (Rime 2001), British (Ediz et al. 1998), European (Kok and Schepens 2013) and Middle Eastern banks (Alkadamani 2015).

In May 2009 the Federal Reserve assessed the 19 largest bank holding companies (BHCs) in its *Supervisory Capital Assessment Program* (SCAP). It identified 10 BHCs with significant capital shortfalls and mandated them to raise equity within 6 month to avoid a permanent government recapitalization.[3] In response US banks raised a record $45bn in new common equity within a few weeks.[4] Consistent with our model, Lambertini and Mukherjee (2016) show that these issuance volumes were associated with an increasing cost of capital for banks failing SCAP.[5,6]

Similarly, in the early stages of the COVID-19 crisis, financial regulators appealed to banks worldwide to reinforce their capital buffers by halting dividend payments and buybacks (Georgieva, 2020). The President of the Federal Reserve Bank of Minneapolis, for instance, recommended that large US BHCs quickly raise $200bn through equity offerings (Kashkari, 2020). For Europe, Schularick et al. (2020) estimated a Eurosystem-wide capital shortfall between €142 and €600 bn in different COVID-19 crisis scenarios. They too called for a rapid recapitalization of the European banking system through what we will call "liability side measures", i.e. primarily stock issuances.

We analyze the implications of such events in a two period general equilibrium model of financial intermediation, in which banks manage the maturity mismatch between long-term investments and short-term deposits. In our model it is natural to interpret the intermediaries as banks rather than firms, since we allow for asset sales (which are easier for banks) and consider an interbank market extension. Banks initially borrow, short-term and uninsured, from households and invest in a risky investment technology to which they have exclusive access.[7] At the intermediate date news about the actual risk of banks' portfolios arrives and may impair their ability to issue safe claims and roll over their debt. Bad news thus necessitate recapitalizations to protect depositors' claims, and to avoid a bank run and bankruptcy. Due to correlated bank portfolios individual recapitalization needs can in the aggregate generate a systemic shortfall with elevated recapital-

[3]The results of the Supervisory Capital Assessment Program, as well as the details on its design and implementation are available online: http://www.federalreserve.gov/newsevents/press/bcreg/20090507a.htm.

[4]See Hanson et al. (2011) and an US equity market issuance summary by Reuters: http://www.lse.co.uk/ukIpoNews.asp?ArticleCode=4a39ycmc7drz9zm.

[5]As mentioned earlier, elevated issuing costs may also arise due to an adverse selection problem (Myers and Majluf, 1984). Hanson et al. (2011), however, argue that the strong regulatory involvement in the SCAP likely muted the adverse selection problem associated with equity issuance in this case.

[6]Systemic capital shortfalls have also occurred in response to the U.S. subprime mortgage crisis, or when Italian banks were forced collectively to write down large proportions of the non-performing loans on their balance sheets.

[7]We acknowledge the relevance of deposit insurance and guarantee schemes in practice. However, we want to stress that our mechanism does not hinge on the assumption that deposits are uninsured or only partially insured. If mandatory recapitalization is triggered by a regulator (e.g. when a regulatory constraint is hit as the result of a stress test) and not in response to the risk of bank-runs, our qualitative results also hold with full deposit insurance.

ization costs if specialized investment capital is in imperfectly elastic supply. Ex-ante, banks then trade off the benefit of higher leverage in good times with the cost of recapitalizations and the likelihood of bankruptcy. Whether a bank can recapitalize however depends on individual portfolio risk *and* on the sector-wide capital shortfall, which is shaped by the pecuniary externality.[8]

Central to our mechanism is a *cost of contingencies*. Financial market segmentation separates households into *investors* and *depositors*, capturing administrative charges or informational costs, e.g., due to financial literacy (Guiso and Sodini, 2013).[9] Only investors can purchase equity claims or bank assets with risky payoffs. Becoming an investor is not equally difficult for everyone and thus entails an idiosyncratic utility cost. These costs generate the imperfectly elastic supply of specialized investment capital (Holmstrom and Tirole, 1997) and an endogenous premium. The required compensation of the marginal investor increases with the system-wide recapitalization need, which we consider to be a short-term property of the relevant financial markets.[10,11]

In the event of a systemic capital shortfall some banks' portfolios can be too risky for recapitalization. The upside these banks can offer to new investors is insufficient even if the initial shareholders' claims (the bank manager's expected profits) are fully diluted. Ex-ante leverage then affects the capital shortfall at the bank level *(intensive margin)*, as well as bank's ability to recapitalize *(extensive margin)* via what we call the *recapitalization constraint*. This constraint captures the threshold level of portfolio risk for which recapitalizations remain feasible. Since both margins are functions of endogenous market conditions, an externality emerges.

Besides analyzing banks' recapitalization choice and efficiency, we examine how bank failures depend on the degree to which bankruptcy procedures involve the market-based liquidation of intermediary assets as in Allen and Gale (1998, 2004). We also study extensions of our benchmark model with a corporate governance friction, asymmetric information about asset quality, and with an interbank market. We further revisit the merits of risk-sensitive capital regulation.

[8]The optimal leverage ratio in our model is thus determinate. That bankruptcy costs generate a determinate capital structure is known from Bradley et al. (1984) and Myers (1984), as well as from the literature on firms' optimal capital structure following Modigliani and Miller (1958) and Modigliani and Miller (1963).

[9]This micro-foundation for financial market segmentation is motivated by empirical evidence on the low direct and indirect participation in stock markets. See Vissing-Jorgensen (2003) on fixed costs of participation, Barberis et al. (2006) on loss aversion and Guiso et al. (2008) on heterogeneous beliefs or trust.

[10]Similar setups have been used by De Nicolò and Lucchetta (2013), Allen et al. (2016), and Carletti et al. (2020).

[11]Due to positive financial market participation costs, equity is *always* more costly than debt in our model. The magnitude of the cost differential, however, depends on endogenous market conditions. Our assumption is w.l.o.g., as long as *some* of the equity that banks issue is more costly than debt. Notice also, that the reason for the elevated cost of equity is different from traditional reasons related to adverse selection problems à la Myers and Majluf (1984).

Our framework resonates with the definition by Brownlees and Engle (2017), according to which *systemically risky banks* are prone to under-capitalization when the system-wide capital shortfall is particularly severe. We demonstrate that excessive exposure to systemic capital shortfalls–resulting from inefficient over-leveraging–can be individually optimal; even in the absence of deposit insurance or *Too-Big-To-Fail* guarantees. Our model therefore provides a complementary rationale for macroprudential capital regulation that does not require moral hazard or informational asymmetries. Moreover, it allows for a positive analysis of banks' capital structure management. We generate a set of novel and testable predictions about different recapitalization policies, their respective stability implications, and the link between ex-ante leverage and future costs of capital. Finally, our analysis speaks to the design and communication of supervisory stress tests, since they can create the kind of aggregate capital shortages that we have in mind.

We intentionally focus on market-based bank recapitalizations, but acknowledge the importance of government interventions in the form of financial support. Our insights remain relevant in the context of such interventions as well though, suggesting for example the need for sufficiently potent interventions. An important difference, however, which is absent in our model is the potential moral hazard associated with anticipated government support.

Our work is closely related to the fire sales literature (Shleifer and Vishny, 1992, 2010). Reminiscent of the precautionary and speculative motives, which are a characteristic of this literature (Allen and Gale, 1994, 2004, 2007), we identify an insufficient precautionary motive for ex-ante capitalization due to a pecuniary externality and incomplete markets for ex-ante risk-sharing. Similarly, Lorenzoni (2008) and Dávila and Korinek (2018) analyze the efficiency properties of economies with exogenous financial constraints. Dávila and Korinek (2018) show that the equilibrium may be constrained inefficient despite a complete set of contracts, if exogenous financial constraints depend on market prices and give rise to collateral externalities. In a related paper, Biais et al. (2020) study risk-sharing in a model with complete contracts and endogenous market incompleteness, due to a moral hazard problem of protection sellers. Different from these papers, we consider an environment with exogenously incomplete markets that give rise to inefficiencies, if paired with financial market segmentation. That is, we consider a variant of Dávila and Korinek (2018), in which market incompleteness stems from the degree of financial market segmentation, which in our model is endogenous (unlike in Gromb and Vayanos (2002)). In fact, borrowers in our model do not face a collateral or moral hazard constraint, but can freely obtain funding against future income. Akin to the *borrowing constraint* in Lorenzoni (2008), which depends on asset prices and affects leverage, our *recapitalization constraint* depends on the endogenous crowdedness of the capital market. The externality in our model can thus be characterized as a *solvency externality*, which shares similarities with the *collateral externality* in Dávila and Korinek (2018). The resulting inefficiencies are similar in nature, but have distinct origins and properties.[12,13]

Other related papers, specifically on banks' capital structure and regulation, include Gorton and Pennacchi (1990), Admati et al. (2011), DeAngelo and Stulz (2015), Allen et al. (2016), Gale and Gottardi (2020) and Carletti et al. (2020). Prominently, Admati et al. (2018) build on an agency conflict between shareholders and creditors. The authors predict undercapitalization due to a "leverage ratchet effect" and study implications for recapitalizations. For different types of agency problems see Kashyap et al. (2008) and Philippon and Schnabl (2013).

While moral hazard problems play an important role in the literature on macroprudential capital regulation (Farhi and Tirole, 2012; Begenau, 2020), our paper is more closely related to papers motivating the need for regulation based on externalities (see, e.g., Gersbach and Rochet, 2012; De Nicolò et al., 2012; Klimenko et al., 2016; Malherbe, 2020 and De Nicolò et al., 2012 for a survey) and emphasizing the buffer function of equity (Repullo and Suarez, 2013). Similar to the dynamic model of Klimenko et al. (2016), banks in our model also fail to internalize the effect of their individual decisions on the loss-absorbing capacity of the banking sector. However, our work is complementary in that we do not study implications for lending but focus on the efficiency implications of different forms of private bank recapitalizations. This focus also separates us from papers studying the joint regulation of capital and liquidity (Calomiris et al., 2015; Eichenberger and Summer, 2005; Boissay and Collard, 2016; Hugonnier and Morellec, 2017).

The remainder of the paper is organized as follows. Section 2 presents the baseline economy and analyses the recapitalization choice. Section 3 solves for the laissez-faire equilibrium. Section

[12] Korinek (2012) characterizes financial amplification, building on a pecuniary externality that links asset fire sales to falling prices and tightening borrowing constraints. In his model, financially constrained firms inefficiently underinsure (due to the under-valuation of liquidity during crises). Walther (2016) develops a model with a price externality in asset markets where banks under-invest in liquidity and build up excessive leverage. While in our model the endogenous cost of equity issuance relates to the frequency of insolvency, Walther's inefficiency is not related to solvency and arises because banks do not internalize the possibility of a socially costly transfer of assets to investors.

[13] Gale and Gottardi (2015) study firms that choose leverage and investment, to balance the tax advantages of debt with default risk. In their dynamic general equilibrium model with asset fire sales by insolvent firms, inefficient underinvestment occurs because firms do not internalize how aggregate debt reduces the tax burden.

4 analyses efficiency with an appropriate second-best benchmark. Thereafter, Section 5 discusses policy, as well as various extensions and testable implications. Finally, Section 6 concludes.

2 Model

Agents, preferences and technology. Time is discrete and we consider three dates ($t = 0, 1, 2$). There are two types of agents: a continuum of mass one of banks (superscript B) and a continuum of mass one of households (superscript H). All agents are risk-neutral and we normalize the discount rate to one for simplicity. The period-utility functions are $u\left(c_t^B\right) = c_t^B, \forall c_t^B \in \mathbb{R}_0^+$ for banks and $u\left(c_t^H\right) = c_t^H, \forall c_t^H \in \mathbb{R}_0^+$ for households. Both types of agents have access to a risk-less, short-term storage technology at $t = 0, 1$. Moreover, banks have the unique opportunity to also invest in a risky long-term technology (e.g. a portfolio of risky loans) at $t = 0$.

Banks. At the beginning of $t = 0$ banks are identical. For simplicity, banks have no endowment of the consumption good at $t = 0, 1$.[14] We assume that banks are protected by limited liability and that their external financing at $t = 0$ consists entirely of short-term debt, which they can invest in a productive, but risky long-term technology and in risk-less storage.[15]

The risky long-term technology is modeled as follows: at $t = 0$ banks first transform $h\left(k_1^B\right)$ units of the consumption good into k_1^B units of capital, where h is convex with $h(0) = 0$, $h'(0) \in (1, \underline{h})$, and $\lim_{k_1^B \to +\infty} h'\left(k_1^B\right) = +\infty$. The resulting capital stock is then employed in a technology that depreciates fully and generates a stochastic return at $t = 2$. $\underline{h} \in (1, R)$ is defined together with technological parameters that we introduce next in order to ensure a positive level of investment. The returns depend on two layers of exogenous risk: First, the aggregate state $\psi \in \{\psi_1, \psi_2\}$ is realized and becomes known at the beginning of $t = 1$. It determines whether banks' portfolios are safe or risky. Banks are safe in state ψ_1, which occurs with probability $0 < \Pr\{\psi_1\} < 1$, and risky in state ψ_2, which occurs with probability $\Pr\{\psi_2\} = 1 - \Pr\{\psi_1\}$. Second, and independent of ψ, there are two equally likely states $\omega \in \{\omega_1, \omega_2\}$, which are realized and become known at

[14] The key insights go through with positive endowments, provided banks have to rely on external financing.

[15] Bank debt is short-term and rolled over at $t = 1$. This feature is motivated by banks' pervasive use of short-term or demandable debt contracts and could be endogenized, for example, as a tool against renegotiation (Diamond and Rajan, 2001). Alternative rationales for demandable debt contracts rely on liquidity risk (Diamond and Dybvig, 1983) or the disciplining role of short-term debt (Calomiris and Kahn, 1991; Grossman and Hart, 1982). Allowing banks to issue equity at $t = 0$ instead would add a layer of complication without providing additional insights.

$t = 2$. From the $t = 1$ viewpoint and conditional on the aggregate state ψ, the portfolio return of bank i, with capital, k_1^B, is characterized by:

$$F^{\psi,\omega_1}\left(k_1^B;\Delta_i\right) = \begin{cases} F^{\psi,\omega_1}\left(k_1^B;\Delta_i\right) = \left(R + \epsilon\Delta_i\right) k_1^B & \text{w.p. } \Pr\{\omega_1\} = 1/2 \\ F^{\psi,\omega_2}\left(k_1^B;\Delta_i\right) = \left(R - \Delta_i\right) k_1^B & \text{w.p. } \Pr\{\omega_2\} = 1/2, \end{cases}$$

where $R > 1 \geq \epsilon \geq 0$ and $\Delta_i \sim G^\psi$. G^ψ is the state dependent CDF with support $[\underline{\Delta}^\psi, \overline{\Delta}^\psi]$. We assume that $\underline{\Delta}^{\psi_1} = \overline{\Delta}^{\psi_1} = 0$ and $\underline{\Delta}^{\psi_2} \geq R - 1 \geq 0$ with $\overline{\Delta}^{\psi_2} \in \left(\underline{\Delta}^{\psi_2}, R\right]$. This implies that there is no risk in state ψ_1, while banks experience return volatility in state ψ_2. It will become clear that the first inequality ensures that the principal of debt claims issued by the bank is at risk. For $\epsilon = 1$, the distributions of individual banks' returns across ω are mean preserving spreads of each other and the expected return at $t = 1$ is independent of ψ: $\sum_\omega \Pr\{\omega\}\left[F^{\psi,\omega}\left(k_1^B;\Delta_i\right)\right] = Rk_1^B, \forall i, \psi$. For $\epsilon < 1$, instead, banks with a higher Δ_i not only face a more severe downside risk, but also a lower expected return in state ψ_2. With this notation, we define $\underline{h} \equiv \Pr\{\psi_1\} R$.

The general safety of banks' portfolios (i.e. the state ψ) and individual banks' types (Δ_i), become known at $t = 1$. Since banks' ability to produce safe claims is limited by their portfolio's downside risk, they may not be able to fully roll over their existing short-term debt at $t = 1$ and thus need to recapitalize. There is first a *recapitalization stage* and thereafter a *rollover stage*. At the rollover stage banks can roll over the debt that remains after the recapitalization stage. At the recapitalization stage banks can refinance some (or all) of their funding by issuing state-contingent "equity" claims (we call this a "liability side operation") or by selling some of their assets (we call this an "asset side operation"). The returns on bank assets are observable at $t = 2$, but not verifiable and thus not contractible. Hence, banks can either issue equity claims against their balance sheet, or transfer ownership of their assets in return for capital.

Bankruptcy. Bankruptcy occurs when a bank cannot refinance at $t = 1$. Because the banks' managers/original owners are protected by limited liability they receive nothing. Instead, the available resources are distributed pro-rata to the bank's claimants after accounting for potential losses during liquidation and subtraction of an exogenous bankruptcy cost, $\gamma \geq 0$. Our formulation encompasses a variety of market- and non market-based bankruptcy procedures and allows us to characterize the liquidation value of an insolvent bank i in state ψ_2 as follows:

$$\mathcal{L}_i \equiv \max\left\{0, \left(f\, q^{\psi_2} + (1-f)\,\tau\right) \sum_{\omega} \Pr\{\omega\} \left[F^{\psi,\omega}\left(k_1^B; \Delta_i\right)\right] + x_{1i}^B - \gamma\right\}. \tag{1}$$

In equation (1), $x_{1i}^B \geq 0$ denotes the investment in storage and $f \in [0,1]$ is the fraction of the loan portfolio that is sold on the specialized investment capital market at the endogenous price q^{ψ_2}; $(1-f)$ is the fraction that is "physically" liquidated at a discount, i.e. the investments are terminated and converted to consumption goods at the exogenous rate $\tau \in \left[0, q^{\psi_2}\right)$.

Households. Households are endowed with $\varepsilon_t^H > 0$ units of the consumption good at $t = 0, 1$, and we assume $\epsilon_1^H \geq \epsilon_0^H \geq h\left(\check{K}_1^B\right)$, where $\check{K}_1^B$ solves $h'\left(\check{K}_1^B\right) = 1$. This ensures that households' collective endowments always exceed banks' aggregate financing needs, which allows us to focus on the problems arising from financial market segmentation. At $t = 0$, households are identical and can invest their endowment in short-term bank debt or risk-less storage. Potential losses on uninsured bank debt are anticipated and correctly priced at $t = 0$. Throughout the paper we are interested in studying an economy where system-wide financial stress events are rare, i.e. where $\Pr\{\psi_2\}$ is small. Thus there is little risk associated with $t = 0$ household deposits.

At $t = 1$, households simultaneously decide after observing the aggregate state ψ whether they remain "depositors" or become "investors." Depositors continue to be constrained to low-risk investments–for simplicity they only accept safe debt or risk-less storage. Investors, instead, can participate in financial markets, which enables them to invest in state-contingent bank equity claims *and* to buy risky bank assets. Households who become investors have to pay an idiosyncratic utility cost $\rho_j \geq 0$ that is drawn at $t = 1$ from a continuous distribution with PDF ϕ and support $\left[\underline{\rho}, \overline{\rho}\right]$, with $0 \leq \underline{\rho} \leq \overline{\rho} < +\infty$. This generates segmented financial markets.[16]

In section 3.2 we provide conditions such that there exists a marginal household, with threshold participation costs $\hat{\rho}^\psi \geq 0$, who is indifferent between remaining a depositor and becoming an investor. It follows that a fraction $\Phi\left(\hat{\rho}^\psi\right) = \int_0^{\hat{\rho}^\psi} \phi\left(\rho_j\right) d\rho_j \geq 0$ of households with sufficiently low participation costs becomes investor, while the remaining fraction, with mass $1 - \Phi\left(\hat{\rho}^\psi\right)$, is better off as depositors. There are no restrictions on debt holders of different banks to exchange

[16]It does not matter whether participation costs are observed or not, as long as the distribution is common knowledge.

contracts at $t = 1$, as long as all parties are willing to participate.

Timing. Figure 1 summarizes the game.

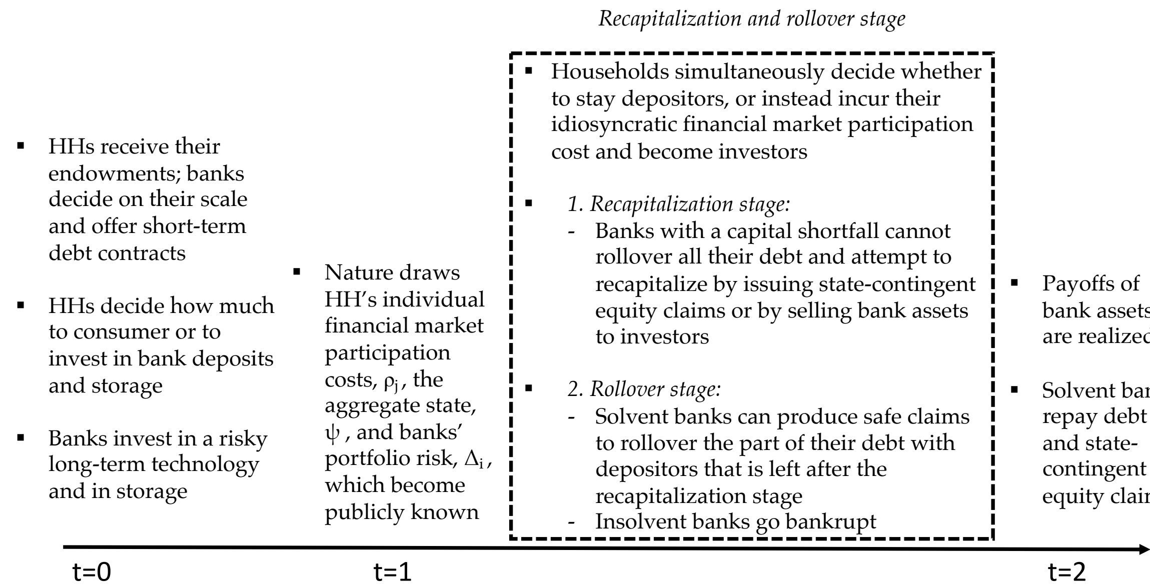

Figure 1: Timeline.

Frictions. Since portfolio risk becomes known at the beginning of $t = 1$ there is no asymmetric information. The crucial frictions, consistent with the aforementioned evidence, are that banks rely on short-term debt (i.e. contracts are incomplete at $t = 0$), and that the supply of investment capital is imperfectly elastic at $t = 1$ (due to segmented financial markets). In addition, we assume that individual portfolio returns, depending on Δ_i, are observable but not verifiable, which limits the contract space and breaks the equivalence of asset and liability side recapitalizations at $t = 1$.[17]

2.1 Household and Bank Problems

The model is solved backwards. Sections 2.1.1 and 2.1.2 discuss the problems of households and banks. Following the notation in Dávila and Korinek (2018), we first introduce the net worth of households ($n_1^{H,\psi}$) and banks ($n_1^{B,\psi}$) as state variables at $t = 1$. We further denote investments in

[17] The assumption of observable, but not verifiable returns is common in the literature (e.g., Hart and Moore (1990)) and can be motivated by the banker's ability to divert funds.

storage by $x_1^B, x_1^H \geq 0$, and bank debt by $d_1^B, d_1^{H,\psi} \geq 0, \forall\psi$, so that the state-dependent household net worth equals $n_1^{H,\psi} = \varepsilon_1^H + d_1^{H,\psi} + x_1^H$. If banks successfully refinance, their net worth is independent of the state and equal to $n_1^B = -d_1^B + x_1^B$; if they default, the net worth equals zero. The state of the economy at $t = 1$ is then characterized by households' and banks' net worth, the capital stock k_1^B, and the corresponding vector of aggregate state variables $S^\psi \equiv \left(K_1^B, N_1^{B,\psi}, N_1^{H,\psi}\right)$.

To simplify the notation, we further introduce $\vartheta\left(S^\psi\right) \in [0,1]$ as the fraction of each household's deposit holdings that gets repaid at $t = 1$ in the aggregate state ψ. If $\psi = \psi_1$ bank portfolios are safe, bankruptcies are absent and $\vartheta\left(S^{\psi_1}\right) = 1$, meaning that the true value of debt is equal to its face value $d_1^{H,\psi_1} = d_1^H$. If $\psi = \psi_2$ bank portfolios are risky, bankruptcies are possible and $\vartheta\left(S^{\psi_2}\right) < 1$ implies $d_1^{H,\psi_2} = \vartheta\left(S^{\psi_2}\right) d_1^H < d_1^H$. The probability of bankruptcies is endogenous and debt holders can recover a fraction $\mathcal{L}_i/d_1^B \in [0,1)$.

2.1.1 Household problems

At $t = 1$ households are heterogeneous in the idiosyncratic utility cost of financial market participation, $\rho_j \geq 0$. An endogenous fraction becomes "investors" (superscript HI), while all others remain "depositors" (superscript HD). We will see that households only pay the financial market participation cost if banks are willing to pay a premium for specialized investment capital.

Depositors at $t = 1$ in state ψ. The problem of an individual household j who does not participate in the financial market at $t = 1$ and takes all prices as given reads:

$$V_{1j}^{HD,\psi}\left(n_{1j}^{H,\psi}; S^\psi\right) = \max_{\left\{c_{1j}^{HD,\psi}, c_{2j}^{HD,\psi}, d_{2j}^{HD,\psi}, x_{2j}^{HD,\psi}\right\}} u^H\left(c_{1j}^{HD,\psi}\right) + \sum_\omega \Pr\{\omega\}\left[u^H\left(c_{2j}^{HD,\psi,\omega}\right)\right] \tag{2}$$

$$\begin{aligned}
s.t.\quad & \left[\lambda_{1j}^{HD,\psi}\right]: \quad c_{1j}^{HD,\psi} + p_{d,2}^\psi d_{2j}^{HD,\psi} \leq n_{1j}^{H,\psi} - x_{2j}^{HD,\psi} \\
& \left[\lambda_{2j}^{HD,\psi}\right]: \quad c_{2j}^{HD,\psi} = c_{2j}^{HD,\psi,\omega} \leq d_{2j}^{HD,\psi} + x_{2j}^{HD,\psi}, \ \forall\omega \\
& \left[\eta_{tj}^{HD,\psi}\right]: \quad c_{1j}^{HD,\psi} \geq 0, \ \forall t \qquad \left[\xi_{dj}^{HD,\psi}\right]: \ d_{2j}^{HD,\psi} \geq 0 \qquad \left[\xi_{xj}^{HD,\psi}\right]: \ x_{2j}^{HD,\psi} \geq 0,
\end{aligned}$$

where we introduce the Lagrange multipliers for each inequality constraint in brackets. Unlike debt claims at $t = 0$, debt claims at $t = 1$ are risk-free since they can only be issued by sufficiently well-capitalized banks. The first-order necessary conditions yield the Euler equations:

$$p_{d,2}^{\psi} = \frac{\lambda_{2j}^{HD,\psi} + \xi_{dj}^{HD,\psi}}{\lambda_{1j}^{HD,\psi}} \qquad \text{and} \qquad 1 = \frac{\lambda_{2j}^{HD,\psi} + \xi_{xj}^{HD,\psi}}{\lambda_{1j}^{HD,\psi}}, \tag{3}$$

with $\lambda_{1j}^{HD,\psi} = u^{h\prime}\left(c_{1j}^{HD,\psi}\right) + \eta_{1j}^{HD,\psi}$ and $\lambda_{2j}^{HD,\psi} = u^{h\prime}\left(c_{2j}^{HD,\psi}\right) + \eta_{2j}^{HD,\psi}$.

Provided an interior solution with $d_{2j}^{HD} > 0$ exists, it follows from the Euler equations in (3) that $p_{d,2}^{\psi} < 1$ requires $\xi_{xj}^{HD,\psi} > 0$ and thus $x_{2j}^{HD,\psi} = 0$, while households are indifferent between storage and deposits for $p_{d,2}^{\psi} = 1$. Due to the assumption that households' collective endowments exceed the financing needs of banks, the relevant case is $x_{2j}^{HD,\psi} > 0$, $d_{2j}^{HD,\psi} > 0$, and $p_{d,2}^{\psi} = 1$. This further implies that $\lambda_{1j}^{HD,\psi} = \lambda_{2j}^{HD,\psi}$, since depositors have positive consumption in at least one of the periods and $u^{h\prime}\left(c_{1j}^{HD,\psi}\right) = u^{h\prime}\left(c_{2j}^{HD,\psi}\right) = 1$, $\eta_{1j}^{HD,\psi} = \eta_{2j}^{HD,\psi} = 0$.

Finally, the envelope condition is given by $\left(V_{1j}^{HD,\psi}\right)_{n_{1j}^{H}} = \lambda_{1j}^{HD,\psi}$.

Investors at $t = 1$ in the aggregate state ψ. Households who participate in financial markets at $t = 1$ take all prices as given and maximize their expected utility from consumption at dates $t = 1, 2$ by choosing their investments in state-contingent equity claims, bank assets, and risk-less storage. In principle, investors can also invest in deposits. Since we have just shown that households are indifferent between storage and deposits at $t = 1$, however, we simplify the problem and abstract from deposits without loss of generality:

$$V_{1j}^{HI,\psi}\left(n_{1j}^{H,\psi}; S^{\psi}\right) = \max_{\left\{c_{1j}^{HI,\psi}, c_{2j}^{HI,\psi,\omega}, e_{j}^{HI,\psi,\omega}, a_{j}^{HI,\psi}, x_{2j}^{HI,\psi}\right\}} u^{H}\left(c_{1j}^{HI,\psi}\right) + \sum_{\omega} \Pr\{\omega\}\, u^{H}\left(c_{2j}^{HI,\psi,\omega}\right) \tag{4}$$

$$\begin{array}{lll}
s.t. & \left[\lambda_{1j}^{HI,\psi}\right]: & c_{1j}^{HI,\psi} + \sum_{\omega} \Pr\{\omega\}\left[p_{e,2}^{\psi,\omega} e_{j}^{HI,\psi,\omega}\right] + q^{\psi} a_{j}^{HI,\psi} \leq n_{1j}^{H,\psi} - x_{2j}^{HI,\psi} \\
& \left[\lambda_{2j}^{HI,\psi,\omega}\right]: & c_{2j}^{HI,\psi,\omega} \leq e_{j}^{HI,\psi,\omega} + a_{j}^{HI,\psi}\Theta^{\psi,\omega} + x_{2j}^{HI,\psi}, \forall\omega \\
& \left[\eta_{tj}^{HI,\psi}\right]: & c_{1j}^{HI,\psi} \geq 0, \forall t = 1, 2 \\
& \left[\xi_{ej}^{HI,\psi,\omega}\right]: & e_{j}^{HI,\psi,\omega} \geq 0, \forall\omega \qquad \left[\xi_{aj}^{HI,\psi}\right]: \; a_{j}^{HI,\psi} \geq 0 \qquad \left[\xi_{xj}^{HI,\psi}\right]: \; x_{2j}^{HI,\psi} \geq 0.
\end{array}$$

To ease the exposition we assume that investors can fully diversify idiosyncratic–but not aggregate–bank risk. Purchases of state-contingent equity claims are conditional on the state ψ and denoted by $e_{j}^{HI,\psi,\omega}$; each unit guarantees the right to one consumption good unit in state ω. We think of these claims as equity, because they provide non-negative state-contingent payoffs. Purchases of bank assets, instead, are denoted by $a_{j}^{HI,\psi} \geq 0$. We assume that investors cannot individually operate bank assets (e.g., because they do not have the necessary monitoring capacity/skills) and that they therefore buy a share in a composite asset comprising all assets divested by the banking sector. Buying $a_j^{HI,\psi}$ units of the composite assets yields a return of $\Theta^{\psi,\omega}$, which is normalized such that $\sum_\omega \Pr\{\omega\}\,\Theta^{\psi,\omega} = 1, \forall\psi$. Finally, investments in risk-less storage are denoted by $x_{2j}^{HI,\psi} \geq 0$.

The corresponding first-order optimality conditions yield the following Euler equations:

$$p_{e,2}^{\psi,\omega} = \frac{\lambda_{2j}^{HI,\psi,\omega} + \xi_{ej}^{HI,\psi,\omega}}{\lambda_{1j}^{HI,\psi}}, \forall\omega \tag{5}$$

$$q^{\psi} = \frac{\sum_\omega \Pr\{\omega\}\left[\lambda_{2j}^{HI,\psi,\omega}\Theta^{\psi,\omega}\right] + \xi_{aj}^{HI,\psi}}{\lambda_{1j}^{HI,\psi}} \tag{6}$$

$$1 = \frac{\sum_\omega \Pr\{\omega\}\left[\lambda_{2j}^{HI,\psi,\omega}\right] + \xi_{xj}^{HI,\psi}}{\lambda_{1j}^{HI,\psi}} \tag{7}$$

where $\lambda_{1j}^{HI,\psi} = u^{h\prime}\left(c_{1j}^{HI}\right) + \eta_{1j}^{HI,\psi}$ and $\lambda_{2j}^{HI,\psi,\omega} = u^{h\prime}\left(c_{2j}^{HI,\psi,\omega}\right) + \eta_{2j}^{HI,\psi,\omega}, \forall\omega$.

If investors invest in state-contingent equity claims but not in storage, we have $\xi_{ej}^{HI,\psi,\omega} = 0, \forall\omega$ and $\xi_{xj}^{HI,\psi} > 0$. Equations (5) and (7) then imply $\sum_\omega \Pr\{\omega\}\, p_{e,2}^{\psi,\omega} < 1$. That is, the expected return on equity has to exceed the return from storage. Since $u^{h\prime}\left(c_{1j}^{HI}\right) = u^{h\prime}\left(c_{2j}^{HI,\psi,\omega}\right) = 1$, this further implies $\eta_{1j}^{HI,\psi} > 0$ and thus $c_{1j}^{HI} = 0$; investors store and consume zero at $t = 1$. This result is intuitive as investors want to take advantage of the premium on specialized investment capital. If investors also purchase the composite bank asset, then positive consumption is guaranteed for states ω_1 and ω_2, so that $\eta_{2j}^{HI,\psi,\omega} = 0, \forall\omega$, and thus $\lambda_{2j}^{HI,\psi,\omega} = 1, \forall\omega$. Equations (5) and (6) thus imply the following indifference condition: $q^{\psi} = p_{e,2}^{\psi,\omega},\ \forall\omega$.

The envelope condition is given by $\left(V_{1j}^{HI,\psi}\right)_{n_{1j}^H} = \lambda_{1j}^{HI,\psi}$.

Segmentation of households. At the beginning of $t = 1$ households draw their financial market participation cost $\rho_j \sim \phi(\rho_j)$ and decide whether or not to become investors. The condition for which household j is indifferent is determined by the endogenous participation constraint:[18]

$$\hat{\rho}_j^{\psi} = u^H\left(c_{1j}^{HI,\psi}\right) + \sum_\omega \Pr\{\omega\}\left[u^H\left(c_{2j}^{HI,\psi,\omega}\right)\right] - u^H\left(c_{1j}^{HD,\psi}\right) - u^H\left(c_{2j}^{HD,\psi}\right) \geq 0. \tag{8}$$

[18]In Proposition 4 we show that households take symmetric choices at $t = 0$ so that equation (8) defines the participations threshold for the marginal household, $\hat{\rho}^{\psi}$, who is just willing to become investor.

Since there is no portfolio risk in state ψ_1, and thus no demand for specialized investment capital, we have $\hat{\rho}_j^{\psi_1} = 0, \forall j$, because no household is willing to incur the participation cost if there are no securities to invest in. Instead, banks may need to recapitalize in state ψ_2 and also divest assets in the market if they become insolvent and $f > 0$. When discussing the bank problem at $t = 1$, we will see that both factors lead to a strictly positive demand for specialized investment capital and hence to investment opportunities for household investors. In these cases, $\hat{\rho}_j^{\psi_1} > 0$ and the required compensation for investors implies an endogenous premium for specialized investment capital, so that $\sum_\omega \Pr\{\omega\} \left[p_{e,2}^{\psi_2,\omega}\right] < p_{d,2}^{\psi_2}$. This is a key feature of our model and generates an imperfectly elastic supply of specialized investment capital at $t = 1$.

Household problem at $t = 0$. Next, we consider the household problem at $t = 0$:

$$V_{0j}^H \equiv \max_{\left\{c_{0j}^H, d_{1j}^H, x_{1j}^H\right\}} u^H\left(c_{0j}^H\right) + \sum_\psi \Pr\{\psi\} \left[\begin{array}{c} \Phi\left(\hat{\rho}_j^\psi\right) V^{HI,\psi}\left(n_{1j}^{H,\psi}; S^\psi\right) - \int_0^{\hat{\rho}_j^\psi} \rho\, \phi(\rho)\, d\rho \\ + \left(1 - \Phi\left(\hat{\rho}_j^\psi\right)\right) V^{HD,\psi}\left(n_{1j}^{H,\psi}; S^\psi\right) \end{array} \right] \tag{9}$$

$$\begin{aligned} s.t. \quad \left[\lambda_{0j}^H\right]: \quad & c_{0j}^H + p_{d,1} \textstyle\sum_\psi \Pr\{\psi\} \vartheta\left(S^\psi\right) d_{1j}^H \leq \epsilon_0^H - x_{1j}^H \\ \left[\eta_{0j}^H\right]: \quad & c_{0j}^H \geq 0 \qquad \left[\xi_{dj}^H\right]: \ d_{1j}^H \geq 0 \qquad \left[\xi_{xj}^H\right]: \ x_{1j}^H \geq 0 \\ & n_{1j}^{H,\psi} = \epsilon_1^H + \vartheta\left(S^\psi\right) d_{1j}^H + x_{1j}^H \\ & \hat{\rho}_j^\psi = V_j^{HI,\psi}\left(n_{1j}^{H,\psi}; S^\psi\right) - V^{HD,\psi}\left(n_{1j}^{H,\psi}; S^\psi\right), \end{aligned}$$

where prices and the expected repayment on each nominal unit of debt, $\sum_\psi \Pr\{\psi\} \vartheta\left(S^\psi\right)$, are taken as given. Unlike debt claims at $t = 1$, which are issued by sufficiently well capitalized banks, debt claims at $t = 0$ carry some risk since the debt is uninsured and undercapitalized banks may go bankrupt at $t = 1$ in state ψ_2.

Using the previously derived envelope conditions, we derive the following Euler equations:

$$\lambda_{0j}^H p_{d,1} \sum_\psi \Pr\{\psi\} \vartheta\left(S^\psi\right) - \xi_{dj}^H = \sum_\psi \Pr\{\psi\} \vartheta\left(S^\psi\right) \Omega\left(S^\psi\right) \tag{10}$$

$$\lambda_{0j}^H - \xi_{xj}^H = \sum_\psi \Pr\{\psi\} \Omega\left(S^\psi\right), \tag{11}$$

where $\lambda_{0j}^H = u^{H\prime}\left(c_{0j}^H\right) + \eta_{0j}^H$. $\Omega\left(S^\psi\right) \equiv \Phi\left(\hat{\rho}^\psi\right) \lambda_{1j}^{HI,\psi} + \left(1 - \Phi\left(\hat{\rho}^\psi\right)\right) \lambda_{1j}^{HD,\psi}$ is the expected return from having one more unit of net worth when entering $t = 1$ in state ψ. Suppose an interior solution exists, then it follows from equations (10) and (11) that $p_{d,1} < 1$. The fact that the expected return on debt exceeds one compensates households for a low debt repayment in state ψ_2 due to $\vartheta\left(S^{\psi_2}\right) < 1$; which is the state where they could as investors benefit most from having more net worth at $t = 1$ in order to take advantage of the premium on specialized investment capital.

Finally, the envelope condition is given by $\left(V_{0j}^H\right)_{\epsilon_0^H} = \lambda_{0j}^H$.

2.1.2 Bank problems

We allow for heterogeneous investment levels, k_{1i}^B, and net worth, n_{1i}^B, but show in Proposition 4 that banks optimally take symmetric $t = 0$ choices. Hence, we use $k_{1i}^B = k_1^B$ and $n_{1i}^B = n_1^B$ to simplify notation and to highlight bank heterogeneity at $t = 1$, which is governed by the realization of the portfolio level of risk Δ_i characterizing the lowest possible return that each bank can realize, $F^{\psi,\omega_2}\left(k_1^B; \Delta_i\right) = \left(R - \Delta_i\right) k_1^B$. Since bank types are publicly observable at $t = 1$, their ability to issue safe debt claims in order to roll over pre-existing liabilities is entirely determined by this risk. Provided the premium for investment capital is positive, banks prefer to issue debt. If a full debt rollover is impossible so that $d_{2i}^{B,\psi} < d_1^B$, we say that banks face a positive *capital shortfall*.[19]

The two-stage game depicted in Figure 1 features similarities with the debt renegotiation game in Gale and Gottardi (2015). Differently, we consider segmented financial markets with household depositors who demand safe claims and specialized household investors who participate in bank recapitalizations that can be interpreted as debt renegotiations. In the second stage, the *rollover stage*, depositors decide simultaneously whether or not to rollover debt with a face value of d_1^B. They are willing to do so whenever the repayment of d_1^B at $t = 2$ is guaranteed, i.e. if the bank can produce sufficient safe claims at $t = 1$. This is the case if a bank does not face a capital shortfall or if it is able to recapitalize in the first stage. In the first stage, the *recapitalization stage*, banks make take-it-or-leave-it offers for either state-contingent equity claims or bank assets to specialized investors who simultaneously decide whether or not to accept these offers.

[19]It will turn out that this is the natural case to consider in state ψ_2, given the assumptions that portfolio returns are positively correlated and $\underline{\Delta}^{\psi_2} > R - 1$. In an extension in Section 5.3 we study how the analysis is affected when some banks have additional risk-bearing capacity, which gives rise to an interbank market.

Bank problem at $t = 1$. Taking all prices as given, the problem of bank i at $t = 1$ is:

$$V_{1i}^{B,\psi}\left(n_1^B, k_1^B; S^\psi\right) = \max_{\left\{c_{1i}^{B,\psi}, c_{2i}^{B,\psi,\omega}, x_{2i}^{B,\psi}, e_i^{B,\psi,\omega}, d_{2i}^{B,\psi}, a_i^{B,\psi}\right\}} u^B\left(c_{1i}^{B,\psi}\right) + \sum_\omega \Pr\{\omega\}\left[u^B\left(c_{2i}^{B,\psi,\omega}\right)\right] \quad (12)$$

$$\begin{aligned}
s.t. \quad & \left[\lambda_{1i}^{B,\psi}\right]: && c_{1i}^{B,\psi} + x_{2i}^{B,\psi} \leq n_1^B + \textstyle\sum_\omega \Pr\{\omega\}\left[p_{e,2}^{\psi,\omega} e_i^{B,\psi,\omega}\right] + p_{d,2}^{\psi} d_{2i}^{B,\psi} + q^\psi a_i^{B,\psi} \\
& \left[\lambda_{2i}^{B,\psi,\omega}\right]: && c_{2i}^{B,\psi,\omega} \leq x_{2i}^{B,\psi} + \left(1 - \frac{a_i^{B,\psi}}{\sum_\omega \Pr\{\omega\} F^{\psi,\omega}\left(k_1^B; \Delta_i\right)}\right) F^{\psi,\omega}\left(k_1^B; \Delta_i\right) - e_i^{B,\psi,\omega} - d_{2i}^{B,\psi}, \forall\omega \\
& \left[\eta_{1i}^{B,\psi}\right]: && c_{1i}^{B,\psi} \geq 0 \qquad \left[\eta_{2i}^{B,\psi,\omega}\right]: \ c_{2i}^{B,\psi,\omega} \geq 0, \forall\omega \\
& \left[\xi_{x,i}^{B,\psi}\right]: && x_{2i}^{B,\psi} \geq 0 \qquad \left[\xi_{e,i}^{B,\psi,\omega}\right]: \ e_i^{B,\psi,\omega} \geq 0, \forall\omega \\
& \left[\xi_{a,1i}^{B,\psi}, \xi_{a,2i}^{B,\psi}\right]: && 0 \leq a_i^{B,\psi} \leq \textstyle\sum_\omega \Pr\{\omega\} F^{\psi,\omega}\left(k_1^B; \Delta_i\right) \\
& \left[\xi_{d,1i}^{B,\psi}, \xi_{d,2i}^{B,\psi}\right]: && 0 \leq d_{2i}^{B,\psi} \leq \Gamma_i^{B,\psi}\left(x_{2i}^{B,\psi}, a_i^{B,\psi}, k_1^B; \Delta_i\right).
\end{aligned}$$

The first and second inequalities are the resource constraints at $t = 1, 2$. At $t = 1$ banks consume, $c_{1i}^{B,\psi}$, and invest in storage, $x_{2i}^{B,\psi}$. The expenditures are met by the bank's net wealth and the income from issuing state-contingent equity claims, $e_i^{B,\psi,\omega}$, non-contingent debt claims, $d_{2i}^{B,\psi}$, and/or from divesting assets, $a_i^{B,\psi}$. The total value of bank i's assets at the beginning of $t = 1$ in state ψ comprises two parts: *(1)* the sum of the expected return on its portfolio (which can be partially or fully liquidated at the price q^ψ) and *(2)* the return on storage. At $t = 2$ consumption, $c_{2i}^{B,\psi,\omega}$, depends on the realization of ω. It equals the proceeds from storage and from retained bank assets, net of non-contingent payments to creditors and state-contingent payments to investors.

From the rollover stage, $\Gamma_i^{B,\psi}$ defines the maximum amount of safe claims bank i can issue:

$$\Gamma_i^{B,\psi} = \Gamma^{B,\psi}\left(x_{2i}^{B,\psi}, a_i^{B,\psi}, k_1^B; \Delta_i\right) \equiv x_{2i}^{B,\psi} + \left(1 - \frac{a_i^{B,\psi}}{\sum_\omega \Pr\{\omega\} F^{\psi,\omega}\left(k_1^B; \Delta_i\right)}\right) \overbrace{F^{\psi,\omega_2}\left(k_1^B; \Delta_i\right)}^{=(R-\Delta_i)k_1^B}, \quad (13)$$

depending on the downside risk. Its capital shortfall is: $\mathcal{C}_i \equiv \max\left\{0, -n_1^B + x_{2i}^{B,\psi} - \Gamma_i^{B,\psi} - q^{\psi_2} a_i^{B,\psi_2}\right\}$.

The debt issuance constraint, $d_{2i}^{B,\psi} \leq \Gamma_i^{B,\psi}$, and the solvency constraints, $c_{1i}^{B,\psi}, c_{2i}^{B,\psi,\omega} \geq 0, \forall\omega$, ensure that debt claims issued at $t = 1$ are safe and that the bank can offer a sufficiently high upside to investors while maintaining weakly positive consumption levels. Observe that solvent banks are indifferent about whether to raise additional debt and invest it in the risk-less storage technology, given that $p_{d,2}^\psi = 1$ from the household problem at $t = 1$. In state ψ_1 bank portfolios are safe and the capital shortfall is zero, since $d_1^B > Rk_1^B$ is inconsistent with bank optimality at $t = 1$, as we will show below. Conversely, the capital shortfall may be positive in state ψ_2 when bank portfolios are risky. If there is no feasible combination of choice variables such that the solvency constraints hold, a solution to the inner maximization problem does not exist and the bank fails. In this case, the continuation value of the bank is zero and a fraction f of its assets are divested.

Given that $p_{d,2}^{\psi} = 1$ from Section 2.1, the problem in (12) can be simplified since the debt issuance constraint holds with equality.[20] Assuming a solution to the inner maximization problem exists (i.e. $c_{1i}^{B,\psi} \geq 0$ and $c_{2i}^{B,\psi,\omega} \geq 0, \forall\omega$), the corresponding Euler equations are:

$$\lambda_{1i}^{B,\psi} \Pr\{\omega\} p_{e,2}^{\psi,\omega} - \Pr\{\omega\} \lambda_2^{B,\psi,\omega} + \xi_{e,i}^{B,\psi,\omega} = 0, \forall\omega \tag{14}$$

$$\lambda_{1i}^{B,\psi} q^{\psi} + \frac{\Pr\{\omega_1\} \lambda_{2i}^{B,\psi,\omega_1} \left[F_i^{\psi,\omega_2} - F_i^{\psi,\omega_1}\right] - \lambda_{1i}^{B,\psi} F_i^{\psi,\omega_2}}{\sum_{\omega} \Pr\{\omega\} F_i^{\psi,\omega}} + \xi_{a,1i}^{B,\psi} - \xi_{a,2i}^{B,\psi} = 0, \tag{15}$$

where $F_i^{\psi,\omega} = F^{\psi,\omega}\left(k_1^B; \Delta_i\right)$, $\lambda_{1i}^{B,\psi} = u^{B\prime}\left(c_{1i}^{B,\psi}\right) + \eta_{1i}^{B,\psi}$ and $\lambda_{2i}^{B,\psi,\omega} = u^{B\prime}\left(c_{2i}^{B,\psi,\omega}\right) + \eta_{2i}^{B,\psi,\omega} / \Pr\{\omega\}, \forall\omega$.

As before, we can further determine the following envelope conditions:

$$\begin{aligned}
\left(V_{1i}^{B,\psi}\right)_{n_1^{B,\psi}} &= \lambda_{1i}^{B,\psi}, \ \forall\psi \in \{\psi_1, \psi_2\} \\
\left(V_{1i}^{B,\psi_1}\right)_{k_1^B} &= \lambda_{2i}^{B,\psi_1} R \\
\left(V_{1i}^{B,\psi_2}\right)_{k_1^B} &= \lambda_{2i}^{B,\psi_2} \sum_{\omega} \Pr\{\omega\} \frac{d\left(x_{2i}^{B,\psi_2} + \left(1 - \frac{a_i^{B,\psi_2}}{\sum_{\omega} \Pr\{\omega\} F_i^{\psi_2,\omega}}\right) F_i^{\psi_2,\omega} - e_i^{B,\psi_2,\omega} - d_{2i}^{B,\psi_2}\right)}{dk_1^B},
\end{aligned}$$

where $e_i^{B,\psi_2,\omega}$, a_i^{B,ψ_2}, d_{2i}^{B,ψ_2} and x_{2i}^{B,ψ_2} are solved for in Section 2.2.

Whether or not banks are able to recapitalize and rollover their debt depends on a critical threshold level of risk and the chosen recapitalization strategy. We derive these thresholds in turn for liability side and asset side recapitalizations.

Liability side recapitalization. Absent asset sales, the maximum amount of state-contingent equity claims that can be issued by banks with a capital shortfall, $\mathcal{C}_i > 0$, is $\overline{e}_i^{B,\psi,\omega_2} = 0$ and $\overline{e}_i^{B,\psi,\omega_1}\left(k_1^B; \Delta_i\right) = F_i^{\psi,\omega_1} - F_i^{\psi,\omega_2} = (\epsilon + 1)\Delta_i k_1^B$. Plugging into the first inequality constraint of the problem in (12), we can use the limited liability assumption and show that solvency requires

[20] In state ψ_2, with a positive capital shortfall, the debt issuance constraint, $d_{2i}^{B,\psi} \leq \Gamma_i^{B,\psi}$, is binding. Plugging $d_{2i}^{B,\psi_1} = \Gamma_i^{B,\psi_1}$ in state ψ_1 implicitly assumes that banks' consumption is shifted to $t = 1$. This simplification allows us to focus on the key choice variables, without changing the nature of the problem since all banks are risk-less in state ψ_1 and there is no discounting of time.

$p_{e,2}^{\psi,\omega_1}\overline{e}_i^{B,\psi,\omega_1}/2 \geq -n_1^B - p_{d,2}^{\psi}F_i^{\psi,\omega_2}$. Intuitively, the ability to recapitalize and successfully meet the solvency requirement improves with a higher upside, i.e. a higher $\overline{e}_{2i}^{B,\psi,\omega_1}$, and with a higher net worth, i.e. a higher n_1^B, while it deteriorates with a lower downside, i.e. a lower F_i^{ψ,ω_2}. This allows us to define the downside level of risk, $\widetilde{\Delta}_E^{\psi}$, for which banks with a capital shortfall are just able to conduct a liability side recapitalization:[21]

$$\widetilde{\Delta}_E^{\psi}\left(n_1^B, k_1^B; p_{d,2}^{\psi}, p_{e,2}^{\psi,\omega_1}\right) \equiv \min\left\{\max\left\{\underline{\Delta}^{\psi}, \frac{n_1^B + p_{d,2}^{\psi}Rk_1^B}{k_1^B\left(p_{d,2}^{\psi} - p_{e,2}^{\psi,\omega_1}\frac{\epsilon+1}{2}\right)}\right\}, \overline{\Delta}^{\psi}\right\}. \tag{16}$$

In state ψ_1 banks have no capital shortfall and, hence, $\widetilde{\Delta}_E^{\psi_1} = \underline{\Delta}^{\psi_1} = \overline{\Delta}^{\psi_1} = 0$. In state ψ_2, instead, banks have a capital shortfall and equation (16) defines the level of downside risk, $\widetilde{\Delta}_E^{\psi_2}$, for which banks are just able to conduct a liability side recapitalization. In particular, all banks of type $\Delta_i > \widetilde{\Delta}_E^{\psi_2}$ fail. Below, we will show that the relevant case is when $0 < p_{d,2}^{\psi_2}R + n_1^B/k_1^B < p_{d,2}^{\psi_2}R$ and $\sum_{\omega}\Pr\{\omega\}\,p_{e,2}^{\psi_2,\omega} < p_{d,2}^{\psi_2}$, i.e. both the nominator and denominator in (16) are positive. Notably, more banks are able to recapitalize when ϵ increases, i.e. when banks can promise a higher upside.

Asset side recapitalization. Absent liability side recapitalizations, we can similarly derive the downside level of risk for which an asset side recapitalization is just feasible by manipulating $n_1^B + q^{\psi}\sum_{\omega}\Pr\{\omega\}\,F_i^{\psi_2,\omega} \geq 0$:

$$\widetilde{\Delta}_A^{\psi}\left(n_1^B, k_1^B; q^{\psi}\right) \equiv \min\left\{\max\left\{\underline{\Delta}^{\psi}, \frac{n_1^B + q^{\psi}Rk_1^B}{q^{\psi_2}\frac{1-\epsilon}{2}k_1^B}\right\}, \overline{\Delta}^{\psi}\right\}. \tag{17}$$

As before, $\widetilde{\Delta}_A^{\psi_1} = \underline{\Delta}^{\psi_1} = \overline{\Delta}^{\psi_1} = 0$ in state ψ_1. Instead, the threshold is non-zero in state ψ_2 and all banks of type $\Delta_i > \widetilde{\Delta}_A^{\psi_2}$ are unable to recapitalize. Like $\widetilde{\Delta}_E^{\psi_2}$, $\widetilde{\Delta}_A^{\psi_2}$ is increasing in ϵ, since more banks are able to recapitalize when they can promise a higher upside. For $\epsilon = 1$ the ability to recapitalize by selling assets depends exclusively on the expected return of the investment project and on the market price, meaning that bank specific portfolio risk is irrelevant. In this case, banks are able to conduct asset side recapitalizations if and only if $n_1^B + q^{\psi_2}Rk_1^B \geq 0$.

This concludes the analysis of the bank problem at $t = 1$.

[21]Note that $x_{2i}^{B,\psi} = 0$ for the marginally solvent bank.

Bank problem at $t=0$. Taking all prices as given, the problem at $t=0$ can be written as:

$$V_0^B \equiv \max_{\{c_0^B,k_1^B,d_1^B,x_1^B\}} u^B\left(c_0^B\right) + \sum_{\psi} \Pr\{\psi\} \int_{\underline{\Delta}^{\psi}}^{\widetilde{\Delta}^{\psi}} V_{1i}^{B,\psi}\left(n_1^B, k_1^B; S^{\psi}\right) g^{\psi}\left(\Delta_i\right) di \tag{18}$$

$$\begin{aligned} s.t. \quad & \left[\lambda_0^B\right]: \quad c_0^B + h\left(k_1^B\right) \leq p_{d,1} \textstyle\sum_{\psi} \Pr\{\psi\}\,\vartheta\left(\cdot; S^{\psi}\right) d_1^B - x_1^B \\ & \left[\eta_0^B\right]: \quad c_0^B \geq 0 \qquad \left[\xi_{d,0}^{B,\psi}\right]: \; d_1^B \geq 0 \qquad \left[\xi_{x,0}^{B,\psi}\right]: \; x_0^B \geq 0 \\ & \qquad\quad n_1^B = -d_1^B + x_1^B \qquad \vartheta\left(\cdot; S^{\psi}\right) = \textstyle\int_{\underline{\Delta}^{\psi}}^{\widetilde{\Delta}^{\psi}} g^{\psi}\left(\Delta_i\right) di + \int_{\widetilde{\Delta}^{\psi}}^{\overline{\Delta}^{\psi}} \frac{\mathcal{L}\left(\cdot;\Delta_i,S^{\psi}\right)}{d_1^B} g^{\psi}\left(\Delta_i\right) di, \end{aligned}$$

where $\widetilde{\Delta}^{\psi} = \widetilde{\Delta}_E^{\psi}\left(\cdot; S^{\psi}\right)$ if banks favor liability side recapitalizations at $t=1$ and $\widetilde{\Delta}^{\psi} = \widetilde{\Delta}_A^{\psi}\left(\cdot; S^{\psi}\right)$ if banks favor asset side recapitalizations. The threshold for the downside risk level depends on the vector of aggregate state variables via the prices and is defined in equations (16) and (17). Banks are indexed by their exposures to downside risk, Δ_i, and $g\left(\Delta_i\right)$ denotes the probability density function. Note that $V_0^{B,\psi} \geq 0, \; \forall \psi$ due to limited liability. From the $t=0$ budget constraint we can see that the size of the investment, k_1^B, is positively associated with bank leverage, i.e. with a higher debt issuance at $t=0$. Importantly, the value of the debt claim is corrected by anticipated bankruptcies, with $\vartheta\left(\cdot; S^{\psi}\right) \leq 1$ being the expected repayment rate. The problem can be simplified drastically since no bank fails in state ψ_1, i.e. $\widetilde{\Delta}^{\psi_1} > \underline{\Delta}^{\psi_1} = \overline{\Delta}^{\psi_1} = 0$ and $\vartheta\left(\cdot; S^{\psi_2}\right) = 1$. Instead, with a positive mass of insolvent banks in state ψ_2, $\underline{\Delta}^{\psi_2} \leq \widetilde{\Delta}^{\psi_2} < \overline{\Delta}^{\psi_2}$, we take the bank-specific expected recovery rates into account, which are a linear in the expected value of banks' assets.

Since banks are assumed not to be borrowing constrained initially, the problem, together with the envelope conditions, implies the following Euler and optimal investment conditions:

$$\left[d_1^B\right]: \quad \lambda_0^B p_{d,1} \textstyle\sum_{\psi} \Pr\{\psi\} \left(\vartheta^{\psi} + d_1^B \frac{\partial \vartheta}{\partial d_1^B}\right) - \xi_{d,0}^{B,\psi} = \displaystyle\sum_{\psi} \Pr\{\psi\} \int_{\underline{\Delta}^{\psi}}^{\widetilde{\Delta}^{\psi}} \left(V_{1i}^{B,\psi}\right)_{n_1^B} g^{\psi}\left(\Delta_i\right) di \tag{19}$$

$$\left[k_1^B\right]: \quad \lambda_0^B \left(h'\left(k_1^B\right) - p_{d,1} d_1^B \textstyle\sum_{\psi} \Pr\{\psi\} \frac{\partial \vartheta}{\partial k_1^B}\right) = \displaystyle\sum_{\psi} \Pr\{\psi\} \int_{\underline{\Delta}^{\psi}}^{\widetilde{\Delta}^{\psi}} \left(V_{1i}^{B,\psi}\right)_{k_1^B} g^{\psi}\left(\Delta_i\right) di \tag{20}$$

$$\left[x_1^B\right]: \quad \lambda_0^B \left(1 - p_{d,1} d_1^B \textstyle\sum_{\psi} \Pr\{\psi\} \frac{\partial \vartheta}{\partial x_1^B}\right) - \xi_{x,0}^{B,\psi} = \displaystyle\sum_{\psi} \Pr\{\psi\} \int_{\underline{\Delta}^{\psi}}^{\widetilde{\Delta}^{\psi}} \left(V_{1i}^{B,\psi}\right)_{n_1^B} g^{\psi}\left(\Delta_i\right) di, \tag{21}$$

where $\lambda_0^B = u^{B\prime}\left(c_0^B\right) - \eta_0^B$ and $\vartheta^{\psi} \equiv \vartheta\left(\cdot; S^{\psi}\right)$. Moreover, we used that $V_{1i}^{B,\psi_2}\big|_{\Delta_i^{\psi}=\widetilde{\Delta}^{\psi_2}} = 0$.

From equations (19) and (21) we can show that $d_1^B > 0$ and $x_1^B = 0$, whenever there is a positive incidence of bankruptcies. Moreover, the assumption that $h'(0) \in (1, \Pr\{\psi_1\} R)$ assures that $k_1^B > 0$ if the probability of the crisis state is sufficiently low, i.e. if $\Pr\{\psi_2\}$ is small. Consequently, banks find it optimal to invest all resources in the portfolio of risky loans. Since we are exactly interested in such a scenario, this allows us to drastically simplify the problem in (18).

Finally, recall that all banks are identical ex ante. As a result, whenever the $t = 0$ choices of banks are symmetric and imply a positive mass of bankruptcies in the aggregate state ψ_2, i.e. if $\int_{\tilde{\Delta}^{\psi_2}}^{\overline{\Delta}^{\psi_2}} g^{\psi_2}(\Delta_i)\, di > 0$, the pro rata repayment of debt holders of bankrupt institutions is $d_1^{B,\psi_2} = r(\cdot; S^{\psi})\, d_1^B < d_1^B$, while $V_{1i}^{B,\psi_2} = 0$. This concludes the discussion of the bank problems.

2.2 Feasibility of bank recapitalizations

After analyzing the household and bank problems, we next study under which conditions liability and asset side recapitalizations are feasible. Comparing inequalities (16) and (17), we find $\widetilde{\Delta}_E^{\psi_2} \geq \widetilde{\Delta}_A^{\psi_2}$ with $\widetilde{\Delta}_E^{\psi_2} = \widetilde{\Delta}_A^{\psi_2}$ if an only if specialized investment capital does not come at a premium, i.e. if $q^{\psi_2} = p_{e,2}^{\psi_2,\omega} = 1$. To see this, we use the result that $p_{d,2}^{\psi_2} = 1$ from the household problem at $t = 1$. All banks in the range $\Delta_i \in \left[\widetilde{\Delta}_E^{\psi_2}, \widetilde{\Delta}_A^{\psi_2}\right)$ are able to recapitalize by selling state-contingent equity claims, while an asset side recapitalization is not possible. Lemma 1 summarizes the results.

Lemma 1. ***(Feasibility of bank recapitalizations)*** *Banks are always solvent in state ψ_1. Given a strictly positive premium for specialized investment capital, $q^{\psi_2} < 1$, all banks with $\Delta_i > \widetilde{\Delta}_E^{\psi_2}$ are insolvent in state ψ_2, while all banks with $\Delta_i \in \left[\underline{\Delta}^{\psi_2}, \widetilde{\Delta}_E^{\psi_2}\right]$ are able to recapitalize. For these banks liability side recapitalizations are always feasible, while asset side recapitalizations are only feasible for banks with $\Delta_i \in \left[\underline{\Delta}^{\psi_2}, \widetilde{\Delta}_A^{\psi_2}\right]$, where $\widetilde{\Delta}_A^{\psi_2} < \widetilde{\Delta}_E^{\psi_2}$.*

Given $q^{\psi_2} = p_{e,2}^{\psi_2,\omega} < 1$ recapitalizations are costly. For a solvent bank conducting liability side recapitalizations, i.e. if $\Delta_i \leq \widetilde{\Delta}_E^{\psi_2}$, we have $e_i^{B,\psi_2,\omega} = \mathcal{C}_i/(\Pr\{\omega_1\}\, p_{e,2}^{\psi_2,\omega_1}) = 2(-n_1^B - (R - \Delta_i)k_1^B)/p_{e,2}^{\psi_2,\omega_1}$, $a_i^{B,\psi_2} = 0$ and $d_{2i}^{B,\psi_2} = (R - \Delta_i)k_1^B$. We can set $x_{2i}^{B,\psi_2} = 0$ without loss of generality since $p_{d,2}^{\psi_2} = 1$. Moreover, for a solvent bank conducting asset side operations, i.e. if $\Delta_i \leq \widetilde{\Delta}_A^{\psi_2}$, we have $a_i^{B,\psi_2} = \left(-n_1^B - (R - \Delta_i)\, k_1^B\right) / \left(q^{\psi_2} - \frac{R-\Delta_i}{R+\frac{\epsilon-1}{2}\Delta_i}\right)$, $d_{2i}^{B,\psi_2} = -n_1^B$ and $x_{2i}^{B,\psi_2} = q^{\psi_2} a_i^{B,\psi_2}$.

The result in Lemma 1 on the feasibility of bank recapitalizations has direct implications for stability, i.e. for the incidence of bankruptcies illustrated in Figure 2 below. Before analyzing the pecking order for banks' recapitalization choices in Section 2.5 we first discuss the destabilizing role of asset sales on the bank-specific and sector-wide level in Sections 2.3 and 2.4.

2.3 The destabilizing role of asset-side recapitalizations for individual banks

An important implication of the results in Lemma 1 is that asset side recapitalizations are destabilizing at the individual bank-level. This is illustrated graphically in Figure 2 where we inspect the probability of a bank to go bankrupt in state ψ_2 in *partial equilibrium*; that is, for given leverage, d_1^B, debt remuneration, $\overline{p}_{d,1}$, and price for specialized investment capital, $q^{\psi_2} = p_{e,2}^{\psi_2,\omega_1}$.

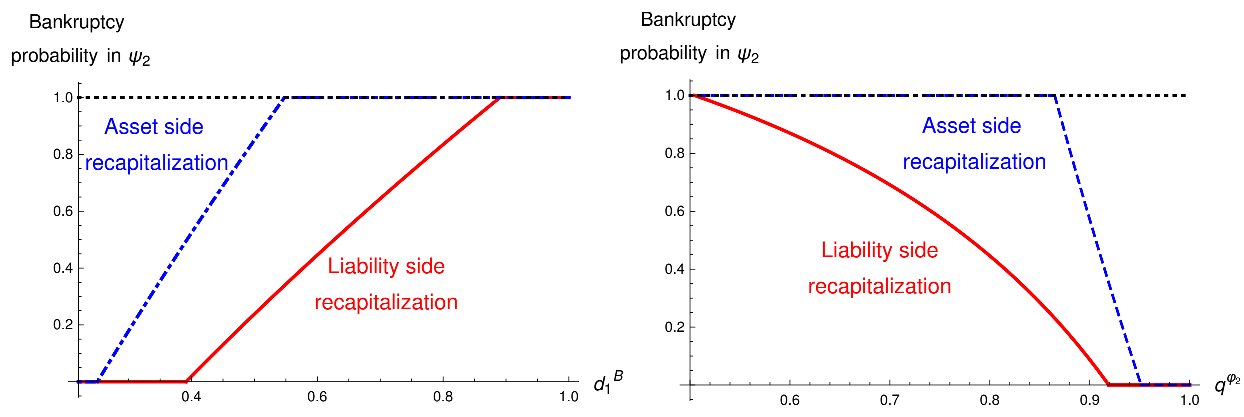

Figure 2: A comparison of liability and asset side recapitalizations. Left panel: the incidence of bankruptcies in state ψ_2 as a function of ex-ante leverage, d_1^B, for a fixed price of specialized investment capital, $q^{\psi_2} = 0.89$. Right panel: the incidence of bankruptcies as a function of the price of specialized investment capital, q^{ψ_2}, for a fixed ex-ante leverage $d_1^B = 0.45$. In both cases the remuneration of initial debt is set to one (e.g. assume that $\Pr\{\psi_2\} \to 0$), $\overline{p}_{d,1} = 1$, which implies that $d_1^B = h(k_1^B)$. All model parameters are as in the baseline numerical example $BL1$ of Section A.1.

The bankruptcy probability is given by $\int_{\widetilde{\Delta}_E^{\psi_2}}^{\overline{\Delta}^{\psi_2}} g^{\psi_2}(\Delta_i)\, di$ and $\int_{\widetilde{\Delta}_A^{\psi_2}}^{\overline{\Delta}^{\psi_2}} g^{\psi_2}(\Delta_i)\, di$ for liability and asset side recapitalizations, respectively. The left panel shows that there are more bankruptcies when ex-ante leverage, d_1^B, is higher. Intuitively, banks with higher debt face a higher capital shortfall in state ψ_2, which makes it harder to recapitalize since $d\widetilde{\Delta}_E^{\psi_2}/dd_1^B \geq 0$ and $d\widetilde{\Delta}_A^{\psi_2}/dd_1^B \geq 0$.[22] For low levels of d_1^B no bank fails and for high levels of d_1^B all banks fail in state ψ_2. For intermediate levels of d_1^B, asset and liability side recapitalizations deliver markedly different outcomes. The threshold ranking from Lemma 1, $\widetilde{\Delta}_A^{\psi_2} \leq \widetilde{\Delta}_E^{\psi_2}$, is reflected in a lower fraction of banks failing under liability side recapitalizations. For a given d_1^B, asset sales are thus *destabilizing* because they are less effective in eliminating downside risk. The right panel shows that the fraction of failing banks also increases in the investor premium, i.e. if q^{ψ_2} decreases, which makes it more difficult

[22]Both inequalities are strict whenever we have an interior solution for the recapitalization thresholds.

for banks to recapitalize; especially if they have to sell assets at a depressed price.

We next discuss the important role played by the endogenous premium for specialized investment capital in amplifying the destabilizing role of asset sales by generating a systemic feedback.

2.4 The destabilizing role of asset sales on the sector-wide level

In Section 2.3, we took $p_{e,2}^{\psi_2,\omega_1}$ and d_1^B as given. Now we go one step further in our analysis and endogenize the investor premium to study the destabilizing role of asset side recapitalizations via the capital market for given leverage. This sector-wide (or systemic) feedback can work through two channels: *(1)* the market-based divestments of bank assets by insolvent banks and *(2)* asset side recapitalizations by solvent banks, which both increase the crowdedness of the capital market. The resulting market pressure is associated with a higher premium demanded by the marginal investor, and increases the incidence of insolvencies, $d\widetilde{\Delta}_E^{\psi_2}/dp_{e,2}^{\psi_2,\omega_1} > 0$.

Propositions 1 and 2, as well as Figure 3 summarize the key insights formally and graphically.

Proposition 1. ***(Unique market-clearing price)*** *Suppose there is a positive mass of bankruptcies in state ψ_2. For a given level of leverage, d_1^B, and for a given exogenous supply schedule with a sufficiently high price elasticity (which for uniformly distributed financial market participation costs can be achieved if $\overline{\rho}$ is small), there exists a unique market-clearing price, $q^{\psi_2} = p_{e,2}^{\psi_2,\omega_1}$.*

Proof. See Appendix Section A.2.2. □

Intuitively, the supply of specialized investment capital decreases in $p_{e,2}^{\psi_2,\omega_1}$ (i.e. it increases in the premium for specialized investment capital), while demand may increase or decrease. Recall that we denote with f the fraction of assets divested on the market by insolvent banks. If $f = 0$, demand clearly increases in $p_{e,2}^{\psi_2,\omega_1}$ since more banks are solvent and seek capital to meet their shortfall. Instead, if $f > 0$ insolvent banks shed assets on the market, thereby generating additional demand. For high values of f the market pressure exerted by an insolvent bank outweighs the market pressure exerted by a solvent bank. This effect is counter-balanced by the price effect on the value of assets shed by insolvent banks, which increases $p_{e,2}^{\psi_2,\omega_1}$. In spite of that, the demand schedule may be moderately decreasing in $p_{e,2}^{\psi_2,\omega_1}$ when f is high. To assure a single-crossing even in such extreme scenarios, Proposition 1 invokes a sufficiently high price elasticity of supply. Note that for uniformly distributed financial market participation costs an arbitrarily high price elasticity can be achieved if $\overline{\rho}$ is small.

Based on the existence of a unique market-clearing price we can study its systemic role.

Proposition 2. ***(Systemic destabilization)*** *Under the conditions of Proposition 1, if the fraction of assets divested on the market by insolvent banks, f, increases, then the market-clearing price q^{ψ_2} for specialized investment capital strictly decreases leading to a higher incidence of insolvencies.*

Proof. See Appendix Section A.2.2. □

Figure 3 illustrates for a given ex-ante leverage, d_1^B, the market-clearing price for specialized investment capital (dotted brown line) and the portfolio threshold level of risk (solid red line) below which recapitalization is feasible in state ψ_2. The left panel shows the case with $f = 1$ and the right panel contrasts it with $f = 2/3$; areas colored in light red characterize banks that are too risky to recapitalize. Given the result in Proposition 3 all solvent banks conduct liability side recapitalizations. Notably, insolvent banks divest a larger quantity of assets on the market if $f = 1$, which translates into a higher investor premium (a lower $q^{\psi_2}_{f=1}$). As a result, the incidence of insolvencies is higher throughout, which is reflected in $\widetilde{\Delta}^{\psi_2}_{E,f=1} < \widetilde{\Delta}^{\psi_2}_{E,f=2/3}$, $\forall \widetilde{\Delta}^{\psi_2}_{E,f=2/3} \in \left(\underline{\Delta}, \overline{\Delta}\right)$.

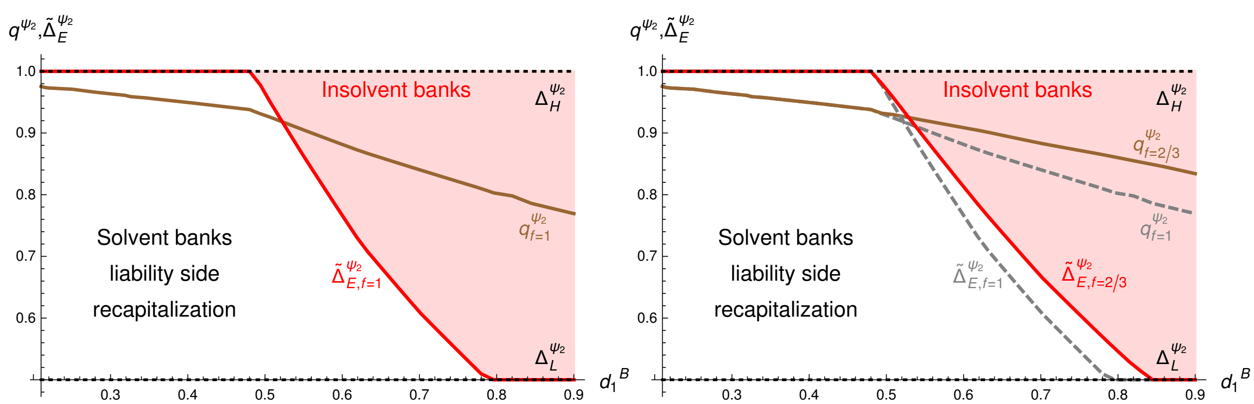

Figure 3: The market-clearing price, q^{ψ_2}, and the portfolio threshold of risk, $\widetilde{\Delta}^{\psi_2}_E$, for which a recapitalization is just feasible in the respective scenarios. Left panel: $f = 1$ with $q^{\psi_2}_{f=1}$ and $\widetilde{\Delta}^{\psi_2}_{E,f=1}$. Right panel: $f = 2/3$ with $q^{\psi_2}_{f=2/3}$ and $\widetilde{\Delta}^{\psi_2}_{E,f=2/3}$ compared to the previous case (dashed lines, gray color). Household net wealth is set to two, $n_1^{H,\psi} = 2$, and the remuneration of initial debt is set to one (e.g. assume that $\Pr\{\psi_2\} \to 0$), $\overline{p}_{d,1} = 1$, which implies that $d_1^B = h\left(k_1^B\right)$. All model parameters are as in the baseline numerical examples $BL1$ and $BL4$ of Section A.1.

2.5 Pecking order in the baseline model

Proposition 3 presents the results on banks' preferred recapitalization choice in the baseline model.

Proposition 3. ***(Pecking order)*** *Whenever banks have (1) a capital shortfall, $\mathcal{C}_i > 0$, (2) recapitalization is feasible, $\Delta_i \leq \widetilde{\Delta}_E^{\psi_2}$, and (3) the premium for specialized investment capital is strictly positive, $q^{\psi_2} < 1$, then banks prefer liability side recapitalizations, i.e. $a_i^{B,\psi_2} = 0$ and $e_{2i}^{B,\psi_2,\omega_1} > 0$. Instead, banks are indifferent if there is a zero premium, i.e. if $q^{\psi_2} = 1$.*

Proof. See Appendix Section A.2.1. □

Intuitively, divesting assets when facing a capital shortfall is less effective in eliminating the downside risk for depositors than the issuance of state-contingent equity claims against the bank's balance sheet. This is because bank assets have a strictly positive return in the downside risk state, $R - \Delta_i > 0$, that remains unused when assets are divested. The result then requires the assumption that bank asset returns are observable but not verifiable. Without this friction, securitization with tranching would allow banks to replicate the outcome under liability side operations. While the preference for equity issuance over asset sales may be inconsistent with the traditional pecking-order theory in corporate finance, evidence for the banking sector by-and-large suggests that in particular undercapitalized banks *do* see equity issuance as an important way to raise capital (e.g. De Jonghe and Öztekin (2015); Dinger and Vallascas (2016)).

We acknowledge that factors such as agency problems, asymmetric information and specialization costs may affect the result of Proposition 3. Section 5.1 discusses a relevant extension where some banks find it optimal to conduct asset side recapitalizations, which gives rise to the destabilizing effects discussed in Sections 2.3 and 2.4.

3 General equilibrium

We first define the competitive laissez-faire equilibrium.

Definition. *The allocation ($C_t^{\{H,B\}}$,$X_1^{\{H,B\}}$,$X_2^{\{H,B\}}$,K_1^B), $\forall t = 0, 1, 2$, as well as the price vector ($p_{d,1}$,$p_{d,2}^{\psi}$,$p_{e,2}^{\psi}$, q^{ψ}) constitute a competitive equilibrium if:*

(i) given ($p_{d,1}$,$p_{d,2}^{\psi}$,$p_{e,2}^{\psi}$, q^{ψ}), the vector (c_{0i}^B,k_{1i}^B,d_{1i}^B,x_{1i}^B,$c_{1i}^{B,\psi}$,$c_{2i}^{B,\psi,\omega}$,$c_{2i}^{B,\psi,\omega}$,$x_{2i}^{B,\psi}$,$d_{2i}^{B,\psi}$, $e_i^{B,\psi,\omega}$, $a_i^{B,\psi}$) solves the optimization problem for each bank i at $t = 0, 1$;

(ii) given $(p_{d,1}, p^{\psi}_{d,2}, p^{\psi}_{e,2}, q^{\psi})$, the vector $(c^H_{0j}, d^H_{1j}, x^H_{1j}, c^{HD,\psi}_{1j}, c^{HD,\psi}_{2j}, d^{HD,\psi}_{2j}, x^{HD,\psi}_{2j}, c^{HI,\psi}_{1j}, c^{HI,\psi}_{2j}, e^{HI,\psi}_{j}, a^{HI,\psi}_{j}, x^{HI,\psi}_{2j})$ solves the optimization problem for each household j at $t = 0, 1$;

(iii) given $p_{d,1}$ and $p^{\psi}_{d,2}$, deposit markets clear at $t = 0$ and for each state ψ at $t = 1$;

(iv) given $(p^{\psi}_{d,2}, p^{\psi}_{e,2}, q^{\psi})$, markets for specialized investment capital clear in each state ψ at $t = 1$.

We begin the equilibrium analysis with the derivation of the aggregate resource constraints and market clearing in Section 3.1. Thereafter, Section 3.2 discusses existence and uniqueness.

3.1 Aggregate resource constraints and market clearing

At $t = 0$. As in the equilibrium definition, we denote aggregate choice variables with capital letters. Using the respective budget constraints and market clearing on the short-term debt market ($D^B_1 = \int_i d^B_{1i} di = \int_j d^H_{1j} dj = D^H_1$), aggregation over all households and all banks at $t = 0$ gives:

$$C^B_0 + h\left(K^B_1\right) + X^H_1 + X^B_1 = \epsilon^H_0 .$$

At $t = 1$. Similarly, aggregating at $t = 1$ over budget constraints of households (i.e., depositors and investors) and banks (both, solvent and insolvent), and using deposit market clearing ($D^B_2 = \int_{\underline{\Delta}^{\psi}}^{\widetilde{\Delta}^{\psi}_E} d^{B,\psi}_{2i} g^{\psi}\left(\Delta_i\right) di = \int_j d^{H,\psi}_{2j} dj = D^H_2$), yields:

$$\begin{aligned} & C^{H,\psi}_1 + C^{B,\psi}_1 + X^{H,\psi}_2 + X^{B,\psi}_2 - X^H_1 - X^B_1 \\ \leq \quad & \epsilon^H_1 + \int_{\widetilde{\Delta}^{\psi}_E}^{\overline{\Delta}^{\psi}} \left[\mathbb{1}_{\mathcal{L}_i>0} \cdot \left((1-f)\,\tau \textstyle\sum_{\omega} \left[\Pr\left\{\omega\right\} F^{\psi,\omega}\left(K^B_1; \Delta_i\right)\right] - \gamma\right) - \mathbb{1}_{\mathcal{L}_i=0} \cdot \mathcal{M}_i\right] g^{\psi}\left(\Delta_i\right) di, \forall \psi, \end{aligned}$$

where the last summand captures the deadweight losses from bankruptcy, i.e. losses due to physical liquidation and the fixed bankruptcy cost. $\mathbb{1}_{\mathcal{L}_i>0}$ ($\mathbb{1}_{\mathcal{L}_i=0}$) is an indicator function that takes on a value of 1 if $\mathcal{L}_i$ from equation 1 is positive (zero). $\mathcal{M}_i \equiv f q^{\psi_2} \sum_{\omega} \Pr\left\{\omega\right\} \left[F^{\psi_2,\omega}\left(K^B_1; \Delta_i\right)\right]$ is the market value of divested assets by an insolvent bank of type Δ.

At $t = 2$. The payoffs from risk-less storage and the (risky) production technology are realized and are allocated to banks and households, either because they hold part of the productive technology, or because they agreed to share the proceeds through debt and/or state-contingent equity claims. Using the market clearing conditions, state-by-state aggregation across the budget constraints of solvent banks, household depositors, and household investors yields:

$$C_2^{B,\psi,\omega}+C_2^{H,\psi,\omega} \leq X_2^{B,\psi}+X_2^{H,\psi}+\int_{\underline{\Delta}^{\psi}}^{\widetilde{\Delta}_E^{\psi}} F^{\psi,\omega}\left(K_1^B;\Delta_i\right) g^{\psi}\left(\Delta_i\right) di+\int_{\widetilde{\Delta}_E^{\psi}}^{\overline{\Delta}^{\psi}} fF^{\psi,\omega}\left(K_1^B;\Delta_i\right) g^{\psi}\left(\Delta_i\right) di, \forall\psi,\omega,$$

where $\int_{\underline{\Delta}^{\psi}}^{\widetilde{\Delta}_E^{\psi}} F^{\psi,\omega}\left(K_1^B;\Delta_i\right) g^{\psi}\left(\Delta_i\right) di$ is the return on the original investment technology of solvent banks, and $\int_{\widetilde{\Delta}_E^{\psi}}^{\overline{\Delta}^{\psi}} fF^{\psi,\omega}\left(K_1^B;\Delta_i\right) g^{\psi}\left(\Delta_i\right) di$ is the return on the investment technologies of bankrupt banks that were liquidated on the market at $t=1$.

Prices. From depositor optimality at $t=1$ we know that $p_{d,2}^{\psi} > 1$ cannot be an equilibrium price since depositors would otherwise invest in storage. Moreover, from financial market participation, depositor optimality and deposit market clearing $p_{d,2}^{\psi} \leq 1$ holds with equality if and only if the aggregate supply of debt is at least as large as the refinancing need of banks at $t=1$, i.e. if:

$$N_1^{H,\psi} - C_1^H \geq D_1^{B,\psi} \geq D_2^{B,\psi}. \tag{22}$$

A sufficient condition for this to be true is given by $\epsilon_1^H \geq \epsilon_0^H$.

Next, from household optimality at $t=1$ we can see that if the total supply of debt exceeds banks' financing need at $t=0$, then:

$$\epsilon_0^H - C_0^H \geq p_{d,1} \sum_{\psi} \Pr\{\psi\}\, \vartheta\left(S^{\psi}\right) D_1^B, \tag{23}$$

A sufficient condition for this to be true is $\epsilon_0^H \geq h\left(\check{K}_1^B\right)$, where $\check{K}_1^B$ solves $h'\left(\check{K}_1^B\right)=1$.

Moreover, the indifference of households between storage and deposits requires:

$$p_{d,1} = \frac{1}{\sum_{\psi} \Pr\{\psi\}\, \vartheta\left(S^{\psi}\right)} \frac{\sum_{\psi} \Pr\{\psi\}\, \vartheta\left(S^{\psi}\right) \left[\Phi\left(\hat{\rho}^{\psi}\right)\ \lambda_1^{HI,\psi} + \left(1-\Phi\left(\hat{\rho}^{\psi}\right)\right)\ \lambda_1^{HD,\psi}\right]}{\sum_{\psi} \Pr\{\psi\} \left[\Phi\left(\hat{\rho}^{\psi}\right)\ \lambda_1^{HI,\psi} + \left(1-\Phi\left(\hat{\rho}^{\psi}\right)\right)\ \lambda_1^{HD,\psi}\right]} < 1. \tag{24}$$

Recall that we are interested in the case where households' collective endowments exceed banks' aggregate financing needs at $t=0$ and $t=1$, i.e. where banks' inability to refinance arises as an equilibrium phenomenon. This implies that $p_{d,2}^{\psi}=1$ and that inequalities (22) and (23) hold with equality. The household-specific condition characterizing indifference between financial market participation and remaining a depositor becomes:

$$\hat{\rho}_i^{\psi} = \sum_{\omega} \Pr\{\omega\} \left[\left(1 - p_{e,2}^{\psi,\omega}\right) e_i^{HI,\psi,\omega} \right] + \left(1 - q^{\psi}\right) a_i^{HI,\psi}. \tag{25}$$

Given $p_{e,2}^{\psi,\omega} = q^{\psi}$, we have $\partial \hat{\rho}_i^{\psi} / \partial p_{e,2}^{\psi,\omega} < 0$. Intuitively, a lower investor premium is–ceteris paribus–associated with fewer households willing to pay the financial market participation cost.

3.2 Equilibrium existence and uniqueness

To establish existence and uniqueness, we start by analyzing the continuation equilibrium at $t = 1$ in the aggregate state ψ for given state variables $S^{\psi} \equiv \left(K_1^B, N_1^B, N_1^{H,\psi}\right)$. Initially, this is done under the assumption that all groups of agents are identical before uncertainty realizes at the beginning of $t = 1$, i.e. $k_{1i}^B = K_1^B$, $n_{1i}^B = N_1^B$ and $n_{1j}^{H,\psi} = N_1^{H,\psi}$. We then show that the size of a bank at $t = 0$ is uniquely determined for given choices of other banks and that household choices are uniquely determined if the equilibrium exhibits bankruptcies. Proposition 4 summarizes.

Proposition 4. ***(Existence and uniqueness)*** *There exists a unique equilibrium provided* $\Pr\{\psi_2\}$ *is small and provided insolvent banks divest their assets on a market on which the supply of specialized investment capital has a sufficiently high price elasticity. If there is a positive incidence of bankruptcies, the equilibrium is characterized by symmetric* $t = 0$ *choices of banks. Absent bankruptcies, households'* $t = 0$ *choices are indeterminate on the individual level.*

Proof. See Appendix Section A.2.4. □

Notably, the equilibrium either exhibits no bankruptcies (if banks' portfolio risk in state ψ_2, i.e. $\overline{\Delta}^{\psi_2}$, is small) or a positive mass of bankruptcies in state ψ_2. The key sufficient condition for existence is that the aggregate state with a systematic capital shortfall is unlikely, i.e. that $\Pr\{\psi_2\}$ is small. The second sufficient condition related to the supply of specialized investment capital generated by financial market segmentation can be assured, e.g. for uniformly distributed financial market participation costs, if $\overline{\rho}$ is sufficiently small (see Proposition 1).

A quantitative illustration in Appendix A.1 reveals that equilibrium existence and uniqueness prevails for a range of $\Pr\{\psi_2\}$ and $\overline{\rho}$.

4 Efficiency analysis

Having established equilibrium existence and uniqueness, we proceed with the normative analysis. Following the literature (e.g., Geanakoplos and Polemarchakis (1986); Dávila and Korinek (2018)), we consider the problem of a constrained social planner who can only affect $t = 0$ choices of banks and is subject to the same constraints as in the decentralized equilibrium; all later decisions are left to households and banks with prices determined in markets. Formally, the planner maximizes the weighted sum of welfare of the two sets of agents for given Pareto weights, θ^H for households and θ^B for banks. We take a utilitarian approach and set $\theta^H = \theta^B = 1$.

The second-best allocation is obtained by solving the constrained planner problem. This is done for the baseline where liability side recapitalizations at $t = 1$ are optimal (Proposition 3):

$$\max_{\left\{\begin{array}{c} C_0^H, C_0^B, K_1^B \\ D_1, X_1^H, X_1^B \end{array}\right\}} \left(\begin{array}{c} u^H\left(C_0^H\right) + \sum_\psi \Pr\{\psi\} \left[\begin{array}{c} \Phi\left(\hat{\rho}^\psi\right) V^{HI,\psi}\left(S^\psi\right) - \int_0^{\hat{\rho}^\psi} \hat{\rho}^\psi \phi\left(\rho_j\right) d\rho_j \\ + \left(1 - \Phi\left(\hat{\rho}^\psi\right)\right) V^{HD,\psi}\left(S^\psi\right) \end{array} \right] \\ + \left[u^B\left(C_0^B\right) + \sum_\psi \Pr\{\psi\} \int_{\underline{\Delta}^\psi}^{\widetilde{\Delta}_E^\psi} V_i^{B,\psi}\left(S^\psi\right) g^\psi\left(\Delta_i\right) di \right] \end{array} \right) \tag{26}$$

$$\begin{aligned}
s.t.\quad [\nu_0]: \quad & C_0^H + C_0^B + h\left(K_1^B\right) + X_1^H + X_1^B \leq \epsilon_0^H \\
\left[\theta^H \eta_0^H\right]: \quad & C_0^H \geq 0 \quad \left[\theta^B \eta_0^B\right]: \; C_0^B \geq 0 \quad \left[\theta^B \eta_1^B\right]: \; X_1^B \geq 0 \\
& N_1^B = -D_1 + X_1^B \quad N_1^{H,\psi} = \epsilon_1^H + \vartheta\left(S^\psi\right) D_1 + X_1^H, \forall \psi \\
& \hat{\rho}^\psi = V^{HI,\psi}\left(S^\psi\right) - V^{HD,\psi}\left(S^\psi\right) \geq 0 \\
& \vartheta\left(S^\psi\right) = \int_{\underline{\Delta}^\psi}^{\widetilde{\Delta}_E^\psi} g^\psi\left(\Delta_i\right) di + \int_{\widetilde{\Delta}_E^\psi}^{\overline{\Delta}^\psi} \frac{\mathcal{L}\left(K_1^B, X_1^B, D_1, q^\psi; \Delta_i\right)}{D_1} g^\psi\left(\Delta_i\right) di \\
& \widetilde{\Delta}_E^\psi = \min\left\{ \max\left\{ \underline{\Delta}^\psi, \frac{n_1^B + R K_1^B}{\left(1 - p_{e,2}^{\psi,\omega_1} \frac{\epsilon+1}{2}\right) K_1^B} \right\}, \overline{\Delta}^\psi \right\} \\
& p_{e,2}^{\psi,\omega_1} = q^\psi \ solves \ \Phi\left(\hat{\rho}^\psi\right) N_1^{H,\psi} = \left(\begin{array}{c} \int_{\underline{\Delta}^\psi}^{\widetilde{\Delta}_E^\psi} \left(-N_1^B - \left(R - \Delta_i\right) K_1^B\right) g^\psi\left(\Delta_i\right) di \\ + q^\psi f \int_{\widetilde{\Delta}_E^\psi}^{\overline{\Delta}^\psi} \sum_\omega \Pr\{\omega\} F^{\psi,\omega}\left(K_1^B; \Delta_i\right) g^\psi\left(\Delta_i\right) di \end{array} \right).
\end{aligned}$$

The first inequality constraint in problem (26) is the aggregate resource constraint at $t = 0$. The financial market participation threshold, $\hat{\rho}^\psi$, and the expected repayment rate for $t = 0$ deposits, $\vartheta\left(S^\psi\right)$, have been derived earlier. To obtain $\widetilde{\Delta}_E^\psi$ we continue to focus on the case in which households' collective endowments are in principle sufficient to cover banks' financing needs, so that

$p_{d,2}^{\psi} = 1$. Finally, we eliminate prices from the planner problem by employing market clearing and household indifference, which implicitly yields $p_{e,2}^{\psi_2,\omega_1} = q^{\psi_2}$, appearing in $\vartheta\left(S^{\psi_2}\right)$ and $\widetilde{\Delta}_E^{\psi_2}$.

We analyze efficiency with the help of an envelope argument. This requires to calculate the welfare effects of changes in the state variables that are not internalized. To this end, we adopt the terminology used in Dávila and Korinek (2018). Notably, our model does not have a collateral externality as in Lorenzoni (2008) or Dávila and Korinek (2018), but a bankruptcy effect interacting with the pecuniary externality. While the direction of the inefficiency is in general ambiguous, it can be uniquely determined under the most plausible scenario when the endogenous investor premium is negatively associated with ex-ante bank leverage. Moreover, the inefficiency disappears when financial markets are not segmented. Proposition 5 summarizes the results formally and we rely on similar sufficient conditions as in the Proof of Proposition 4.

Proposition 5. ***(Efficiency)*** *The laissez-faire equilibrium is constrained*

1. *efficient when financial markets are not segmented,*

2. *inefficient with over-leveraging of banks at $t = 0$, provided $\Pr\{\psi_2\}$ is small and the supply of specialized investment capital has a sufficiently high price elasticity.*

Proof. See Appendix Section A.2.5. □

The sufficient conditions for $\Pr\{\psi_2\}$ and the price elasticity of the supply of specialized investment capital compare with Propositions 1 and 4. The first result is intuitive. Absent insolvencies and with zero financial market participation costs there are no pecuniary externalities and the equilibrium is constrained efficient. This is because the possibility of bankruptcies (and the associated costs) is fully internalized by banks and $t = 0$ debt holders. Incomplete markets for ex-ante risk-sharing and a contract space constrained to short-term debt have distributional implications, but no efficiency implications; even with costly bankruptcies, i.e. if $\vartheta\left(S^{\psi_2}\right) < 1$ and $\gamma > 0$.

Contrastingly, inefficiencies emerge when financial markets are segmented and the investor premium is positive, because the costs are no longer fully internalized. The second result of Proposition 5 stems from the combination of pecuniary externalities, incomplete markets for ex-ante risk-sharing, and the constrained contract space at $t = 0$. Absent insolvencies the inefficient over-leveraging is exclusively driven by the intensive recapitalization margin. Individual banks only internalize the direct effect of their ex-ante choice of the bank size on individual recapitalization volumes in the state ψ_2 but not the indirect effect that higher individual recapitalization volumes lead to higher aggregate volumes.

When considering the case with insolvencies, a second pecuniary externality arises via the risk-adjusted deposit rate. Inefficient over-leveraging occurs under sufficient conditions akin to the ones we used to establish equilibrium existence and uniqueness. Appendix A.1 provides a numerical illustration of the results in Proposition 5. We also show that the results generalize to the case of asset side recapitalizations under conditions that revert the pecking order.

From a policy perspective, the results derived in this section have immediate implications for macroprudential regulation. More specifically, the constrained efficient allocation can be implemented with a leverage requirement at $t = 0$ that sets a maximum value for d_1^B such that over-leveraging is ruled out. This policy instrument is, in sprit, identical to a tier 1 leverage ratio but based on aggregate conditions (i.e. the probability of a systemic capital shortfall event $\Pr\{\psi_2\}$) and not only on banks individual risk-taking.[23] Alternatively, one could envision a tax on leverage that is set such that banks internalize the externality, effectively eliminating the wedge between the optimality conditions of banks and the constrained social planner.

5 Discussion

We sketch relevant extensions and discuss the implications for the pecking order for bank recapitalization established in Proposition 3, for crowdedness in the capital market and for risk-sensitive capital regulation. Finally, we discuss testable implications.

5.1 Extension 1: Corporate governance friction

Consider a variation of our model - motivated by the corporate finance literature - where liability side recapitalization at $t = 1$ is costly for bank managers (e.g. because they lose a private control benefit). To fix ideas, we assume a private benefit $B > 0$ for bank managers that is lost during the issuance of state-contingent equity claims or when the bank becomes insolvent.

[23] In fact, a variation of our model where banks have an endowment of *inside equity* at $t = 0$ lends itself to the definition of a tier 1 leverage ratio as *inside equity* over total assets (k_1^B).

Formally, we append the problem in (12) with the additive term $B \cdot \mathbb{1}_{e_{2i}^{B,\psi_2,\omega_1}=0} \geq 0$, where the indicator function equals one if the bank does not issue equity. We find in a corollary to Proposition 3 that there exists a threshold $\widehat{B}\left(\Delta_i\right)$ such that banks of type $\Delta_i \leq \widetilde{\Delta}_A^{\psi_2}$ optimally recapitalize via the asset side whenever $B > \widehat{B}\left(\Delta_i\right)$, i.e. if the loss of the control benefit is sufficiently costly. For $B \leq \widehat{B}\left(\Delta_i\right)$ the same banks continue to prefer liability side recapitalizations despite the loss of control. Instead, banks of type $\Delta_i \in \left(\widetilde{\Delta}_A^{\psi_2}, \widetilde{\Delta}_E^{\psi_2}\right]$ always recapitalize by issuing equity. For $B \leq \widehat{B}\left(\Delta_i\right)$ they do so voluntarily and for $B > \widehat{B}\left(\Delta_i\right)$ they would prefer asset sales but the high level of portfolio risk forces them to conduct liability side recapitalizations to avoid default.

Corollary 1. ***(Corporate Governance and the Pecking Order)*** *Suppose the premium for specialized investment capital is positive, $q^{\psi_2} < 1$, and let $\widehat{B}\left(\Delta_i\right) \equiv \frac{\left(-n_1^B-(R-\Delta_i)k_1^B\right)\left(1-q^{\psi_2}\right)(R-\Delta_i)k_1^B}{\Pr\{\omega_1\}q^{\psi_2}\left[q^{\psi_2}\sum_\omega \Pr\{\omega\}F^{\psi_2,\omega}\left(k_1^B;\Delta_i\right)-(R-\Delta_i)k_1^B\right]} > 0,\ \forall \Delta_i < R$. Then banks with $\Delta_i \in \left[\underline{\Delta}^{\psi_2}, \widetilde{\Delta}_A^{\psi_2}\right]$ conduct asset side recapitalizations if $B > \widehat{B}\left(\Delta_i\right)$ and liability side recapitalizations if $B \leq \widehat{B}\left(\Delta_i\right)$. Banks with $\Delta_i \in \left(\widetilde{\Delta}_A^{\psi_2}, \widetilde{\Delta}_E^{\psi_2}\right]$ conduct voluntary liability side recapitalizations if $B > \widehat{B}\left(\Delta_i\right)$ and forced liability side recapitalizations if $B \leq \widehat{B}\left(\Delta_i\right)$.*

Proof. See Appendix Section A.2.6. □

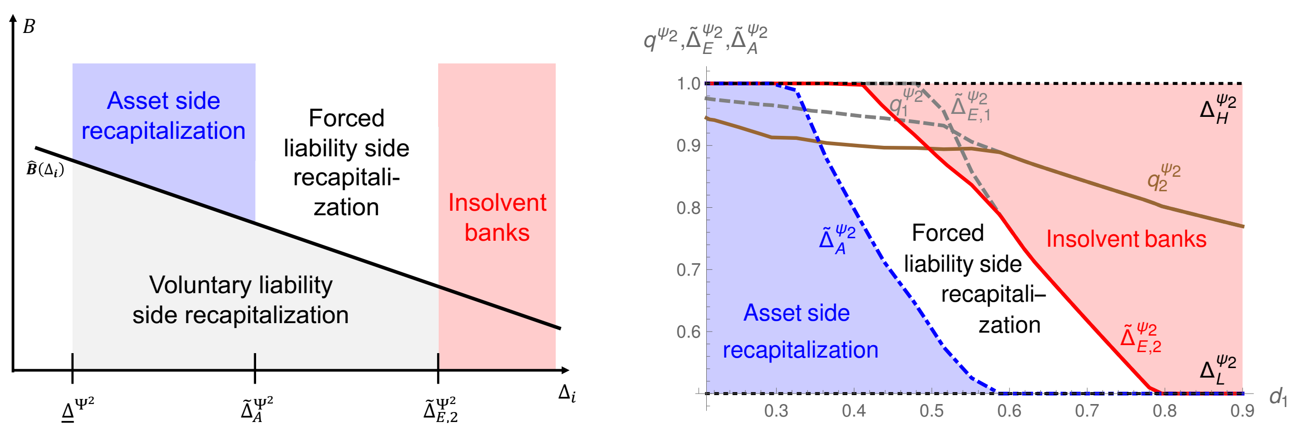

Figure 4: Left panel: Schematic partition into regions with different recapitalization strategies and default as a function of the bank's level of portfolio risk Δ_i and the private benefit B. Right panel: We replicate in gray color (dashed lines) the model with $B = 0$ illustrated in the left panel of Figure 3 and compare it with the model where $B > \widehat{B}(\Delta_i),\ \forall \Delta_i \leq \overline{\Delta}^{\psi_2}$, meaning that banks prefer asset side recapitalizations whenever feasible.

The result from Corollary 1 is illustrated in the left panel of Figure 4, where we assume that $\widehat{B}$ is a continuously decreasing function of Δ_i (which is guaranteed to hold for high values of Δ_i).

Graphically, one can see that there exists a range of the private control benefit B for which low risk banks conduct voluntary liability side recapitalizations, medium risk banks conduct asset side recapitalizations and high risk banks conduct forced liability side recapitalizations.

Previously, we discussed in Section 2.4 how market-based divestments of assets by insolvent banks can have a destabilizing effect by increasing the crowdedness in the capital market. Next, we show a result that extends the same logic to asset side recapitalizations by solvent banks. The result is summarized formally in Corollary 2, which mirrors Proposition 1.

Corollary 2. ***(Corporate Governance and Destabilization)*** *Suppose there is a positive mass of bankruptcies in state ψ_2 and that insolvent banks are fully divested on the market, $f = 1$. For a given level of leverage, d_1^B , and for a given exogenous supply schedule with a sufficiently high price elasticity (as in Proposition 1), the market-clearing price, q^{ψ_2}, is strictly lower and the incidence of bankruptcies is strictly higher when comparing the scenario where some solvent banks conduct asset side recapitalizations to the scenario where all solvent banks conduct liability side recapitalizations.*

Proof. See Appendix Section A.2.7. □

The right panel of Figure 4 visualizes the key insights of Corollary 2. The illustration printed in light grey shows the case with $B = 0$ and liability side recapitalizations; the portfolio threshold of risk is $\widetilde{\Delta}^{\psi_2}_{E,1}$. The illustration printed in dark colors depicts the case where $B > \widehat{B}(\Delta_i), \forall \Delta_i$ so that all solvent banks attempt to recapitalize via the asset side; the portfolio thresholds of risk are $\widetilde{\Delta}^{\psi_2}_{A}$ and $\widetilde{\Delta}^{\psi_2}_{E,2}$. For a given d_1^B all solvent banks of type $\Delta_i \in \left[\underline{\Delta}^{\psi_2}, \widetilde{\Delta}^{\psi_2}_{A}\right]$ conduct asset side operations (area colored in light blue), while all solvent banks of type $\Delta_i \in \left[\widetilde{\Delta}^{\psi_2}_{A}, \widetilde{\Delta}^{\psi_2}_{E,2}\right]$ are unable to do so and conduct forced liability side operations (white area).

When comparing with the baseline printed in gray (dashed lines), we can see that asset side recapitalizations increase fragility. Whenever a positive mass of solvent banks divest assets, the solvency threshold $\widetilde{\Delta}^{\psi_2}_{E,2}\left(d_1^B\right)$ is lower than the corresponding threshold $\widetilde{\Delta}^{\psi_2}_{E,1}\left(d_1^B\right)$ for exclusive liability side recapitalizations. The destabilizing effect operates via downward pressure on the price of capital, meaning that with asset side recapitalizations also the $q_2^{\psi_2}$ shown in dark brown is lower than the $q_1^{\psi_2}$ for the baseline depicted as a dashed grey line. Taken together, we identify a destabilizing role of asset side recapitalizations at the bank-specific and sector-wide levels. In our quantitative illustration in Appendix A.1 the described effects carry through in general equilibrium, as can be seen by comparing scenarios $BL5$ and CG in Table 2. Interestingly, social welfare is higher if a planner mandates liability side recapitalizations, despite a loss of the bank managers' private benefit. Please refer to the Appendix for a discussion.

From a policy perspective, this result calls for regulators to specify how a recapitalization of the banking system should be conducted. Corollary 2 and Figure 4 show that policy interventions in the context of mandatory recapitalizations and macroprudential leverage regulation are intertwined and both affect the incidence of bankruptcies. Since the threshold $\widehat{B}\left(\Delta_i\right)$ is a function of the price of specialized investment capital, banks also fail to internalize the effect of their recapitalization choice on other banks. As a result, regulators may want to manipulate the recapitalization choice even if the private benefit of the banker is included in the social welfare measure.[24]

5.2 Extension 2: Asymmetric information

Next, consider a variant of the model in Section 5.1 where banks' portfolio risk is not fully revealed. Suppose the exact level of risk on the asset-by-asset level is observed privately, while the market has only partial information about each bank i 's portfolio. More specifically, assume that a fraction $1-\kappa_i$ of bank i's portfolio comprises transparent assets while a fraction $0 < \kappa_i < 1$ of assets is opaque. We assume that individual assets of bank i vary in the expected return they deliver, but that the average return is known. In this modified environment each bank i has an incentive to trade on its private information and offload opaque assets with the lowest expected return first (Malherbe, 2014). Hence, banks' opaque assets trade at a discount relative to transparent assets, $q_\kappa^{\psi_2} < q^{\psi_2}$; due to an Akerlof-type adverse selection problem. The discount arises since buyers of individual opaque assets in an over-the-counter market where mutual trades are unobserved and the value of individual assets cannot be inferred. They therefore assign the average value of opaque assets based on the volume traded and their knowledge of the distribution of κ_is.

If bank i can recapitalize by selling the fraction $1-\kappa_i$ of transparent assets, the asymmetric information problem does not matter much. The bank may sell some of its opaque assets with the lowest value first to gain from trading on private information and finance the remainder of its recapitalization need by selling transparent assets. Instead, if bank i has a high recapitalization need

[24] This could, for instance, be done via a tax-subsidy scheme that subsidizes liability side recapitalizations. A plausible interpretation for the compensation of the owners under a tax-subsidy scheme is to think of a 'golden parachute' clause that allows corporations and regulators to adequately deal with corporate governance frictions.

and a high κ_i that would require to sell many opaque assets at a steep discount alongside all of its transparent assets, then bank i may only prefer an assets sales over equity issuance for a higher level of private benefit, say if $B > \widehat{B}_{\kappa}\left(\Delta_i\right) > \widehat{B}\left(\Delta_i\right)$. This occurs because the lemons problem has bite for asset side recapitalizations, while it does not have bite for liability side recapitalizations since equity investors can participate in the potential upside of banks' average portfolios.

In practice, asymmetric information frictions between banks and the market–which are arguably bigger on the asset-by-asset level than on the bank level–would work in favor of recapitalizations via the issuance of state-contingent equity claims because of the destabilizing role of divesting opaque assets at potentially steep discounts.[25] This is particularly true for bank stress test scenarios where adverse selection problems on the bank level that could impede liability side recapitalizations à la Myers and Majluf (1984) are likely to be muted (Hanson et al., 2011).

Notably information asymmetries on the asset side are conceivably less pronounced between banks than between banks and investors from outside the banking system. This would imply that a higher loss-bearing capacity inside the banking system is associated with a reduction in the severity of adverse selection problems on the asset side because banks themselves could buy assets on an interbank market that are considered as excessively opaque by outside investors.

5.3 Extension 3: Interbank market

Next, we sketch an extension of our baseline model with an interbank market. So far, we have restricted attention to the role of specialized investment capital supplied from outside the banking system, and ignored the possibility for well-capitalized banks to provide capital to undercapitalized banks. One could argue that an interbank market is not essential for a model designed to capture dynamics that occur under unfavorable *aggregate* economic conditions. Yet, for completeness, we argue below that our key insights extend to the case with interbank trade. More specifically, we consider the following modification (which nests our baseline model where $B = 0$): In state ψ_2, let a fraction $0 \leq z \leq 1$ of banks be characterized by a portfolio risk drawn from $\Delta_j \sim U\left[\underline{\Delta}^{\psi_2}, \overline{\Delta}^{\psi_2}\right]$, as before. Instead, let a fraction $(1-z)$ of banks be of type $\Delta_i = 0$ with a risk-less portfolio. The latter banks are the natural suppliers of *inside capital*.

[25] Note that the notion of destabilization differs from Corollary 2 in that it hinges on asymmetric information.

Given the result in Proposition 3 we can partition the banking sector as follows:

$$\Delta_i = \begin{cases} > \tilde{\Delta}_E^{\psi_2} & \textit{insolvent banks} \\ \in \left(\hat{\Delta}, \tilde{\Delta}_E^{\psi_2}\right] & \textit{recapitalizing banks} \\ 0 \leq \hat{\Delta} & \textit{banks without recapitalization need / suppliers of inside capital}, \end{cases}$$

where $\hat{\Delta} \in (0, \underline{\Delta})$ solves $\mathcal{C}_i\left(\hat{\Delta}\right) = 0$ for all $\Delta_i \leq \hat{\Delta}$.

For $z = 1$ we are in the baseline model discussed in Sections 2-4. If $z < 1$, there is a positive mass of banks with loss-bearing capacity in state ψ_2. These banks can offer asset swaps, that is to trade some of their safe assets against risky assets of undercapitalized banks at a rate that reflects the price of specialized investment capital, q^{ψ_2}. This reduces the aggregate capital need by under-capitalized banks and thereby the aggregate demand for specialized investment capital from outside the banking system. In the extreme, if the supply of *inside capital* is sufficiently high, then recapitalizing banks do not face crowded markets since the financial market segmentation does not have bite, i.e. $q^{\psi_2} = 1$.[26] Conversely, the imperfectly elastic capital supply from outside the banking system matters when inside supply is limited, which is precisely the case that we want to emphasize. Here banks with strong balance sheets that supply capital receive the same compensation as household investors.[27]

The explicit modeling of an interbank market leaves the essence of our model unaltered, suggesting that our insights regarding the recapitalization strategy and efficiency carry over. In Table 2 of our quantitative illustration in Appendix A.1 we show that this is indeed the case. Scenario IB considers a variation of the baseline model in scenario $BL2$ where a fraction $(1 - z) = 1/5$ of banks do not have to recapitalize and can supply capital to other banks. The introduction of an interbank market is associated with an increase in q^{ψ_2} due to the additional supply of inside capital. As in the baseline model, the equilibrium is characterized by inefficient over-leveraging, but the incidence of bankruptcies is lower and leverage is higher.

[26]See, e.g., Walther (2016) for a banking model with asset fire sales that uses a similar assumption.

[27]Notably potential information asymmetries on the asset side are arguably less pronounced between banks than between banks and investors from outside the banking system. This would imply that a higher loss-bearing capacity inside the banking system is associated with lower premia on specialized investment capital.

5.4 Extension 4: Risk-sensitive capital regulation

While banks are ex-ante identical in the baseline model, we may also consider a modification with two groups of banks that have observably different portfolio characteristics at $t = 0$. In the baseline model the optimal and efficient levels of bank leverage are tied to deep model parameters such as $\overline{\Delta}^{\psi_2}$ and ϵ, which is negatively associated with the incidence of bankruptcies since $d\widetilde{\Delta}_E^{\psi}/d\epsilon > 0$.

As a result, risk-sensitive capital regulation can play a relevant role in mitigating market crowdedness when there is an observable difference in banks' ex-ante risk profiles. To sketch this idea, we compare scenarios $BL2$ and $BL3$ in Table 2, Appendix A.1. We find that a lower bank profitability in the crisis state ψ_2, as measured by a lower upside in $BL3$ relative to $BL2$ ($\epsilon = 2/5 < 1/2$), translates into a reduction in the efficient level of bank leverage. In this quantitative example optimal capital regulation prescribes lower leverage for banks with a lower upside.

5.5 Testable implications

Our theory generates several testable implications. This section summarizes these implications and puts them in the context of existing empirical work.

Liability side vs. asset side bank recapitalizations. There is an empirical literature analyzing banks' recapitalization strategies during normal times (De Jonghe and Öztekin, 2015; Dinger and Vallascas, 2016) and in response to regulatory interventions (Lambertini and Mukherjee, 2016; Eber and Minoiu, 2016; Gropp et al., 2016). Our model provides guidance on how to interpret some of these results and directions for future research. Specifically, our theory suggests there is a fundamental tendency for banks to prefer liability side recapitalizations (Proposition 3), and links deviations from this tendency to measurable bank properties.

If bank managers and/or the controlling shareholders are more averse to the loss of control, asset-side recapitalizations become more attractive (Corollary 1). A higher B in our model would, for example, be associated with environments in which managers or majority shareholders are more likely to extract private rents, or in which a dilution of control leads to less efficient operations. On the other hand, banks with more opaque assets, a higher κ_i in our model, are predicted to be even more inclined to conduct liability side recapitalizations (Section 5.2). Empirically, opaque assets typically include asset-backed securities or own-named covered bonds. In addition, asymmetric information frictions related to asset quality are more pronounced when the interbank market capacity is exhausted and liquidated assets are bought by non-bank investors (Section 5.3).

Different to the Fed's SCAP in May 2009, in which equity issuance played an important role, the European Central Bank's (ECB's) Comprehensive Assessment in 2013/14 was associated with mostly asset side recapitalizations (Bank for International Settlements, 2016; Eber and Minoiu, 2016; Gropp et al., 2016). Our theoretical framework suggests that these differences can at least partly be explained by lower private costs of control loss or higher asset opacity in the US. At the same time, we also acknowledge the importance of stress test design (i.e. toughness and transparency) and the potential role played by regulatory forbearance (Acharya et al., 2020). In fact, liability side operations have been shown to be more attractive after tough bank stress tests when effects related to informational asymmetries (Myers and Majluf, 1984; Colliard and Gromb, 2018) are smaller (Hanson et al., 2011). Conversely, a recapitalization strategy of reduced asset growth over an extended period may be attractive depending on the treatment of non-performing loans (NPLs) in the central bank collateral framework.[28]

The COVID-19 pandemic with its negative implications for bank capitalization also provides a laboratory to test these implications. It constitutes a systemic capital shortfall event as modeled in our theoretical framework and a scenario in which asymmetric information problems appear (at least at the onset) to play a subordinate role. Instead, central bank interventions and responses by financial regulators are likely to be paramount in shaping bank recapitalizations.

Asset prices and recapitalization costs. In our model, asset prices are inversely related to banks' recapitalization costs, which implies a negative relationship between asset prices and bank leverage (Proposition 5). If this is indeed the case empirically, then our theory has implications for the relationship between ex-ante equity buffers in the banking system and asset price volatility. More specifically, it predicts that lower leverage in the banking system should be associated with lower volatility in asset prices and in the cost of specialized investment capital, i.e. $d\left(q^{\psi_1} - q^{\psi_2}\right)/dK_1^B > 0$. This is because asset prices are more sensitive in the bad state if leverage is higher ($dq^{\psi_2}/dK_1^B < 0$), while they are unaffected in the good state.

[28]NPLs in the context of central bank collateral frameworks have received renewed attention (e.g. ECB (2017)).

Destabilizing role of asset side recapitalizations. Our theory suggests that the frequency of bankruptcies is higher for asset side recapitalizations due to their destabilizing role relative to equity issuance (Propositions 3, 2 and Corollary 2, as well as the right panel of Figure 4). When taking this prediction at face value, we would expect asset side recapitalizations to have a more negative effect on banks' share prices than liability side recapitalizations, everything else equal. In line with our model this prediction is best tested in the context of bank stress tests during episodes of financial distress, where potential opposing effects stemming from information asymmetries between investors and banks à la Myers and Majluf (1984) are likely to be less relevant.

6 Conclusion

This paper develops a general equilibrium model of bank capitalization with segmented financial markets. After system-wide capital shortfalls many banks face simultaneous recapitalization needs to protect against potential withdrawals of uninsured deposits. We study privately and socially optimal bank capitalization, as well as the recapitalization choice, and how it is affected by a variety of observable factors. We draw lessons for macroprudential regulation and present a set of novel testable implications that may be of particular interest for an empiricist studying bank recapitalizations and stress tests during episodes of systemic distress such as the Great Recession or the COVID-19 crisis.

In our model banks are inefficiently under-capitalized under plausible parameter conditions. Key to the efficiency analysis is an imperfectly elastic supply of specialized investment capital in the short-term, which emerges due to financial market segmentation. Banks' ability to recapitalize depends on endogenous future market conditions. Constrained inefficiency arises due to a pecuniary externality in banks' recapitalization constraints and, hence, provides a new rational for macroprudential regulation. In the presence of governance frictions banks may prefer asset side recapitalizations despite a destabilizing effect on both the bank-specific and systemic level. As a result, optimal regulation calls for a combination of leverage requirements and tax-subsidy schemes or other instruments to induce banks to take the socially optimal recapitalization choice.

Since the pecuniary externality in our model generates systemic implications, our results also provide a rationale for the cooperation of national financial regulators (or the creation of supranational regulators such as the European Systemic Risk Board) in the context of mandatory recapitalizations and macroprudential leverage regulation. Moreover, we offer insights for public bank stress test design, as our results may be interpreted as an argument for staggered stress tests and against extensive simultaneous testing exercises. Our tractable two-period model allows us to contribute to a better understanding of banks' recapitalization choices, as well as their implications on stability and efficiency. Importantly, the main qualitative results can be generalized beyond competitive deposit markets and do not hinge on the assumption that deposits are uninsured. Conditional on correlated portfolios (Acharya and Yorulmazer, 2007; Farhi and Tirole, 2012), for example, private recapitalization efforts may also be triggered when asset risk translates into a required level of capital, and therefore in the presence of risk-sensitive capital regulation. We leave regulatory capital requirements as an alternative trigger for recapitalizations and the study of optimal bank specific crisis management tools (liquidity assistance) and market-based crisis management tools (market-maker of last resort) for future research.

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A Appendix

A.1 Numerical example

To illustrate some of the key insights we support the equilibrium and efficiency analysis of our baseline model, as well as the discussion of extensions, with numerical examples. Using the parameters in Table 1, the model generates a need for bank recapitalizations with a probability of $\Pr\{\psi_2\}$ = 1-10%, meaning that *crises are rare*. The technology is parameterized as $h\left(k_1^B\right) = k_1^B + \frac{1}{2}\left(k_1^B\right)^2$ and we consider uniformly distributed downside risk $\Delta_i \sim U\left[\underline{\Delta}^{\psi_2}, \overline{\Delta}^{\psi_2}\right]$. By varying the upside that banks can offer to investors, ϵ, we can influence the incidence of bankruptcies. Financial market participation costs are also uniformly distributed, $\rho_j \sim U\left[\underline{\rho}, \overline{\rho}\right]$, and we allow for both elastic ($\overline{\rho} = \underline{\rho}$) and imperfectly elastic supply of capital, ($\overline{\rho} > \underline{\rho}$).

Parameter	$\Pr\{\psi_1\}$	$\Pr\{\psi_2\}$	R	ϵ	$\overline{\Delta}^{\psi_2}$	$\underline{\Delta}^{\psi_2}$	$\varepsilon_0^H = \varepsilon_1^H$
Value	$\in [0,1]$	$1 - \Pr\{\psi_1\}$	$3/2$	$\in [2/5, 1]$	1	$1/2$	1
Parameter	$\underline{\rho}$	$\overline{\rho}$	f	τ	γ	z	B
Value	0	$\in \left\{\underline{\rho}, 3/2\right\}$	$\in \{2/3, 1\}$	0	$\in \{0, 1/5\}$	$\in \{4/5, 1\}$	$\in \{0, 1/25\}$

Table 1: Model parameters

Table 2 summarizes the numerical results. The chosen parameters allow us to focus on an interior solution to the baseline model and the extensions. Scenario $NB1$ starts with the model without bankruptcies. Absent financial market segmentation, i.e. if $\overline{\rho} = \underline{\rho} = 0$, we have that the equilibrium is efficient with $\left(k_1^B\right)^* = \left(k_1^B\right)^{SP} = 1/2$ and an ex-ante level of social welfare of 2.125. This result holds for all $\Pr\{\psi_2\} \in [0,1]$ and speaks to result 1 of Proposition 5.

Scenario $BL1$ serves as a baseline with financial market segmentation and bankruptcies, i.e. $\vartheta_2^{\psi} < 1$. Bankruptcies occur due to a combination of costly recapitalizations and a reduced ability of banks to offer upside to investors because we set $\epsilon = 1/2$. In the baseline banks optimally conduct liability side recapitalizations, as derived in Proposition 3. The banking sector is overleveraged since there is a discrepancy in optimal and efficient levels of leverage, $\left(k_1^B\right)^* = 0.459 > \left(k_1^B\right)^{SP} = 0.438$, which stems from a general equilibrium price effect and speaks to result 2 of Proposition 5. In fact, the lower leverage chosen by the planner is associated with a lower premium for investment capital, i.e. a higher q^{ψ_2}, and fewer bankruptcies. Conditional on the crisis state, the incidence of bankruptcies is reduced from 42.6% to 28.4%. From an ex-ante perspective this is equivalent to a reduction of the bankruptcy probability from 2.1% to 1.4%. Notably, the inefficiency emerges despite the absence of bankruptcy costs, since all assets are divested on the market, $f = 1$, and the additive bankruptcy cost is set to zero, $\gamma = 0$.

Next, we set $\gamma = 1/5$ in scenario $BL2$. Banks reduce their leverage due to the precautionary motive. Still, the expected depositor repayment is lower than in $BL1$ with $\vartheta^{\psi_2} = 0.853 < 0.965$.

Scenario IB considers the extension with an interbank market discussed in Section 5.3. Specifically, a fraction $z = 1/5$ of banks has no recapitalization need and supplies investment capital at the market price. As a result, the market pressure is reduced which crystalizes in a significantly higher market clearing price, $q^{\psi_2} = 0.933 > 0.901$, despite the higher level of leverage. The less crowed capital market makes it easier for banks to recapitalize and the equilibrium incidence of bankruptcies conditional on the crisis state is reduced to 21.4%.

Scenario $BL3$ illustrates the effect of a lower upside, $\epsilon = 2/5 < 1/2$. The incidence of bankruptcies increases drastically when compared to scenarios $BL1$ and $BL2$. Conditional on the crisis state it is 66.6% in the decentralized equilibrium and the expected repayment of depositors drops to $\vartheta^{\psi_2} = 0.657$. Scenario $BL4$ changes the bankruptcy protocol. We set the additive bankruptcy cost to zero, $\gamma = 0$, and, instead, assume that a fraction $1 - f = 1/3$ of assets owned by bankrupt institutions become worthless. Relative to scenario $BL3$, we can see that the lower volume of divested assets reduces the market pressure, leading to a higher market clearing price, $q^{\psi_2} = 0.911 > 0.868$. As a result, the incidence of bankruptcies is reduced.

To examine the extension with the corporate governance friction discussed in Section 5.1, we consider a less elastic supply schedule of investment capital that allows for a sufficiently high market clearing price to assure that asset side recapitalizations are feasible. In scenario CG asset side recapitalizations are privately optimal because of the private benefit $B = 1/25$. Comparing scenarios $BL5$ and CG illustrates how asset side recapitalizations can have a destabilizing effect not only on the bank specific, but also on the systemic level, which speaks to Proposition 2. Given that crisis are very rare, $\Pr\{\psi_2\} = 1\%$, leverage is high in both scenarios. For our model parametrization we find that despite a somewhat lower level of leverage in CG due to a precautionary motive, the incidence of bankruptcies is higher than in $BL5$. Intuitively, asset side recapitalization exert additional market pressure, reducing q^{ψ_2} and making it harder for banks to recapitalize.

In terms of social welfare, we find that mandatory liability side recapitalizations can be welfare improving. For the chosen model parameters there is a wedge between the individually and socially optimal recapitalization choice. Specifically, for the constrained planner solution (CP), where the planner only restricts ex-ante leverage (as in Section 4), ex-ante social welfare (as measured in (26)) in Scenario CG is 2.122, while it is 2.123 and, therefore, higher in Scenario $BL5$. Conditional on being in the rare crisis state, the welfare is 1.945 in Scenario CG and 1.953 in $BL5$.

Finally, scenario $NB2$ revisits the economy without bankruptcies from $NB1$. Different to before, we add the general equilibrium price effect with imperfectly elastic capital supply, $\overline{\rho} = 3/2 > \underline{\rho} = 0$. In spite of the absence of bankruptcies, an inefficient over-leveraging emerges with a small difference in optimal and efficient levels of leverage, $\left(k_1^B\right)^* = 0.483 > \left(k_1^B\right)^{SP} = 0.480$. This speaks to result 2 of Proposition 5 and highlights that while bankruptcies are fueling the magnitude of the inefficiency, they are not essential for the existence of inefficient over-leveraging.

Scenario	Description	Parameters	Sol.	k_1^B	q^{ψ_2}	$\hat{\Delta}_E^{\psi_2}$	$\hat{\Delta}_A^{\psi_2}$	ϑ^{ψ_2}
$NB1$	No bankruptcies w/o segmentation ($\overline{\rho} = \underline{\rho} = 0$)	$\Pr\{\psi_2\} \in [0,1], \epsilon = z = 1,$ $f = 1, \tau = \gamma = B = 0$	LF	0.500	1.000	$\overline{\Delta}^{\psi_2}$	n/a	1.000
			CP	0.500	1.000	$\overline{\Delta}^{\psi_2}$	n/a	1.000
$BL1$	Bankruptcies w/ price effect ($\overline{\rho} = \frac{3}{2}, \underline{\rho} = 0$)	As before in Scen. $NB1$ but $\epsilon = \frac{1}{2}$, $\Pr\{\psi_2\} = 5\%$	LF	0.459	0.890	0.787	n/a	0.965
			CP	0.438	0.908	0.858	n/a	0.983
$BL2$	Bankruptcies w/ price effect ($\overline{\rho} = \frac{3}{2}, \underline{\rho} = 0$)	As before in Scen. $BL1$ but $\gamma = \frac{1}{5}$	LF	0.424	0.901	0.837	n/a	0.853
			CP	0.404	0.921	0.951	n/a	0.956
IB	As before but w/ interbank market	As before in Scen. $BL2$ but $z = \frac{4}{5}$	LF	0.446	0.933	0.893	n/a	0.930
			CP	0.435	0.942	0.941	n/a	0.962
$BL3$	Bankruptcies w/ price effect ($\overline{\rho} = \frac{3}{2}, \underline{\rho} = 0$)	As before in Scen. $BL2$ but $\epsilon = \frac{2}{5}$	LF	0.441	0.868	0.667	n/a	0.657
			CP	0.399	0.882	0.711	n/a	0.703
$BL4$	Bankruptcies w/ price effect ($\overline{\rho} = \frac{3}{2}, \underline{\rho} = 0$)	As before in Scen. $BL1$ but $f = \frac{2}{3}, \gamma = 0$	LF	0.424	0.911	0.742	n/a	0.799
			CP	0.401	0.923	0.805	n/a	0.851
$BL5$	Bankruptcies w/ price effect ($\overline{\rho} = \frac{3}{4}, \underline{\rho} = 0$)	As before in Scen. $BL2$ but $\Pr\{\psi_2\} = 1\%$	LF	0.487	0.942	0.866	n/a	0.900
			CP	0.485	0.943	0.870	n/a	0.904
CG	As before but w/ governance friction	As before in Scen. $BL5$ but $B = \frac{1}{25} > \widehat{B}(\overline{\Delta}^{\psi_2})$	LF	0.480	0.923	0.833	0.611	0.870
			CP	0.470	0.924	0.852	0.640	0.883
$NB2$	No bankruptcies w/ price effect ($\overline{\rho} = \frac{3}{2}, \underline{\rho} = 0$)	As before in Scen. $NB1$ with $\Pr\{\psi_2\} = 10\%$	LF	0.483	0.816	$\overline{\Delta}^{\psi_2}$	n/a	1.000
			CP	0.480	0.815	$\overline{\Delta}^{\psi_2}$	n/a	1.000

Table 2: Results from numerical simulation using model parameters from Table 1. LF stands for *laissez-faire* solution and CP for *constrained planner* solution. Note that our focus in CG is on the socially optimal level of bank leverage and not on the socially optimal recapitalization choice.

A.2 Proofs

A.2.1 Proof of Proposition 3

Banks have to recapitalize in state ψ_2 whenever $\mathcal{C}_i > 0$, so that either equity issuance or asset sales have to be strictly positive. If $\widetilde{\Delta}_E^{\psi_2} < \underline{\Delta}^{\psi_2}$, we know from Lemma 1 that neither liability nor asset side recapitalizations are feasible. If $\widetilde{\Delta}_E^{\psi_2} > \underline{\Delta}^{\psi_2}$, instead, all banks with $\Delta_i \leq \widetilde{\Delta}_E^{\psi_2}$ can recapitalize.

For these banks, we know from equations (14) and (15) that:

$$\lambda_{1i}^{B,\psi_2}\left(q^{\psi_2} - \frac{R-\Delta_i}{R+\frac{\epsilon-1}{2}\Delta_i}\right) - \lambda_{2i}^{B,\psi_2,\omega_1}\frac{\frac{1+\epsilon}{2}\Delta_i}{R+\frac{\epsilon-1}{2}\Delta_i} + \left(\xi_{a,1i}^{B,\psi_2} - \xi_{a,2i}^{B,\psi_2}\right) = 0 \quad \lambda_{1i}^{B,\psi_2} - \frac{\lambda_{2i}^{B,\psi_2,\omega} - 2\xi_{e,i}^{B,\psi_2,\omega}}{p_{e,2}^{\psi_2,\omega}} = 0.$$

Together with household investors' indifference ($q^{\psi_2} = p_{e,2}^{\psi_2,\omega}, \forall\omega$), it follows that:

$$\lambda_{1i}^{B,\psi_2}\left(q^{\psi_2} - 1\right)\frac{R-\Delta_i}{R+\frac{\epsilon-1}{2}\Delta_i} = \xi_{e,i}^{B,\psi_2,\omega_1}\frac{(1+\epsilon)\,\Delta_i}{R+\frac{\epsilon-1}{2}\Delta_i} - \left(\xi_{a,1i}^{B,\psi_2} - \xi_{a,2i}^{B,\psi_2}\right). \tag{27}$$

Since $q^{\psi_2} > 1$ cannot be an equilibrium due to investors' access to storage, we know that $q^{\psi_2} \leq 1$ and show by contradiction that $a_i^{B,\psi_2} = 0$ if $q^{\psi_2} < 1$. Suppose $a_i^{B,\psi_2} > 0$, which implies that $\xi_{a,1i}^{B,\psi_2} = 0$. Since $\xi_{e,i}^{B,\psi_2,\omega_1} \geq 0$ and $\xi_{a,2i}^{B,\psi_2} \geq 0$, equation (27) is violated. Hence, a positive level of asset sales requires $q^{\psi_2} = 1$. At this price banks are indifferent how to recapitalize. For $q^{\psi_2} < 1$ instead it has to be that $\xi_{a,1i}^{B,\psi_2} > 0$ and therefore that $a_i^{B,\psi_2} = 0$. This concludes the proof.

A.2.2 Proof of Proposition 1

The proof analyses the comparative static with respect to f, taking the remuneration of initial debt, $\overline{p}_{d,1}$, bank leverage, $d_1^B = h\left(k_1^B\right)/\overline{p}_{d,1}$, and the household net worth n_1^{H,ψ_2} as given. n_1^{H,ψ_2} determines the exogenously given supply schedule for specialized investment capital:

$$\chi_{\mathcal{S}}{}^{\psi_2}\left(p_{e,2}^{\psi_2,\omega_1}; n_1^{H,\psi_2}\right) \equiv \Phi\left(\hat{\rho}^{\psi_2}\right) n_1^{H,\psi_2} = \Phi\left(\frac{1-p_{e,2}^{\psi_2,\omega_1}}{p_{e,2}^{\psi_2,\omega_1}} n_1^{H,\psi_2}\right) n_1^{H,\psi_2}. \tag{28}$$

The supply increases in the premium for specialized investment capital, $d\chi_{\mathcal{S}}{}^{\psi_2}/dp_{e,2}^{\psi_2,\omega_1} < 0$, where $p_{e,2}^{\psi_2,\omega_1} = q^{\psi_2}$. Observe that the supply schedule is monotone and continuous, starting at zero for $q^{\psi_2} \in \left[\frac{n_1^{H,\psi_2}}{n_1^{H,\psi_2}+\underline{\rho}}, 1\right]$ and reaching n_1^{H,ψ_2} for $q^{\psi_2} = \frac{n_1^{H,\psi_2}}{n_1^{H,\psi_2}+\overline{\rho}}$.

We next, examine the demand for capital. Recall that the interest of Proposition 1 is in an economy with a positive mass of bankruptcies, i.e. with interior solutions to the threshold levels of portfolio risk. Ultimately solvent banks always conduct liability side recapitalizations (Proposition 3), while insolvent banks (partially) divest their assets on the market:

$$\chi_D^{\psi_2}\left(q^{\psi_2}; k_1^B, \overline{p}_{d,1}, f\right) \equiv \int_{\underline{\Delta}^{\psi_2}}^{\widetilde{\Delta}_E^{\psi_2}} \mathcal{C}\left(\Delta_i\right)\; g^{\psi_2}\left(\Delta_i\right) di + \int_{\widetilde{\Delta}_E^{\psi_2}}^{\overline{\Delta}^{\psi_2}} \mathcal{M}\left(f; \Delta_i\right)\; g^{\psi_2}\left(\Delta_i\right) di. \tag{29}$$

The capital shortfall is $\mathcal{C}\left(\Delta_i\right) = \max\left\{0, h\left(k_1^B\right)/\overline{p}_{d,1} - \left(R - \Delta_i\right) k_1^B\right\}$. Recall that we defined the market value of divested assets by insolvent banks as $\mathcal{M}\left(f; \Delta_i\right) \equiv f q^{\psi_2} \sum_\omega \Pr\left\{\omega\right\}\left[F^{\psi_2,\omega}\left(k_1^B; \Delta_i\right)\right]$. We have $\frac{d\chi_D^{\psi_2}}{df} = \int_{\widetilde{\Delta}_E^{\psi_2}}^{\overline{\Delta}^{\psi_2}} \frac{d\mathcal{M}(f;\Delta_i)}{df}\; g^{\psi_2}\left(\Delta_i\right) di > 0$.

We next consider market clearing. To this end, first observe that $d\chi_D^{\psi_2}/dq^{\psi_2} \lessgtr 0$. Intuitively, a higher premium (lower q^{ψ_2}) decreases demand if the reduction in market pressure via a reduced number of bankruptcies dominates (extensive recapitalization margin; $d\widetilde{\Delta}_E^{\psi_2}/dq^{\psi_2} > 0$), while it increases demand if the effect that divested assets become more expensive dominates:

$$\frac{d\chi_D^{\psi_2}}{dq^{\psi_2}} = \int_{\widetilde{\Delta}_E^{\psi_2}}^{\overline{\Delta}^{\psi_2}} \frac{d\mathcal{M}_i\left(f\right)}{dq^{\psi_2}}\; g^{\psi_2}\left(\Delta_i\right) di + \frac{d\widetilde{\Delta}_E^{\psi_2}}{dq^{\psi_2}}\; \theta\left(q^{\psi_2}; k_1^B, \overline{p}_{d,1}, f, \widetilde{\Delta}_E^{\psi_2}\right)\; g^{\psi_2}\left(\widetilde{\Delta}_E^{\psi_2}\right), \tag{30}$$

where $\theta\left(q^{\psi_2}; k_1^B, \overline{p}_{d,1}, f, \Delta_i\right) \equiv \mathcal{C}_i\left(\Delta_i\right) - \mathcal{M}_i\left(f; \Delta_i\right)$. Note that $d\chi_D^{\psi_2}/dq^{\psi_2} > 0$ if $f = 0$ and $d\chi_D^{\psi_2}/dq^{\psi_2} \lessgtr 0$ if $f = 1$ for which $\theta < 0$. Thus, $d\chi_D^{\psi_2}/dq^{\psi_2} < 0$ may occur for large f.

Since the demand function is continuous and has a finite slope, a single-crossing can be assured for any $f \in [0, 1]$ if the price elasticity of supply is sufficiently high.[29] This concludes the proof.

A.2.3 Proof of Proposition 2

Having established single-crossing in Proposition 1, we next study the response of the bankruptcy incidence when f increases. Given interiority, i.e. a positive mass of bankruptcies, as assumed in Proposition 1, we have $\frac{d\widetilde{\Delta}_E^{\psi_2}}{df} = \frac{d\widetilde{\Delta}_E^{\psi_2}}{dq^{\psi_2}} \frac{dq^{\psi_2}}{df} < 0$ provided the price elasticity of the supply schedule is sufficiently high, where $dq^{\psi_2}/df < 0$ follows from $d\chi_D^{\psi_2}/df > 0$. This concludes the proof.

[29] For uniformly distributed participation costs and arbitrarily steep supply schedule this can be achieved if $\overline{\rho} \to \underline{\rho}$:

$$\lim_{\overline{\rho}\to\underline{\rho}} \frac{d\chi_S{}^{\psi_2}\left(q^{\psi_2}; n_1^{H,\psi_2}\right)}{dq^{\psi_2}} = \lim_{\overline{\rho}\to\underline{\rho}} \frac{q^{\psi_2} - 1}{\overline{\rho} - \underline{\rho}} \left(\frac{n_1^{H,\psi_2}}{q^{\psi_2}}\right)^2 = -\infty.$$

A.2.4 Proof of Proposition 4

The proof consists of three parts. In parts one and two we establish existence and uniqueness of symmetric equilibria for the cases without and with bankruptcies. Thereafter, we derive in part three conditions that ensure these equilibria do not co-exist and, thereby, establish overall equilibrium uniqueness. Below, we present the proof for our baseline model. This simplifies the exposition, but the results readily extend to the modified models in Section 5.

Part 1: The case without bankruptcies. Taking symmetric household choices at $t = 0$ as given, we first establish the existence of a continuation equilibrium with uniquely determined quantities of specialized investment capital at $t = 1$, and characterize it separately for state ψ_1, where bank payoffs are safe (*Step 1*), and state ψ_2, where payoffs are risky (*Step 2*). Next, taking the continuation equilibrium as given, we characterize banks' and households' optimal choices at $t = 0$ and verify their symmetry (*Step 3*). Finally, we use market clearing at $t = 1$ and optimality conditions at $t = 0$ to establish equilibrium existence and uniqueness (*Step 4)*.

Step 1: Consider state ψ_1 where banks do not face return risk. Taking outstanding net debt as given, all banks need to roll over $-n_{1i}^B = d_{1i}^B - x_{1i}^B$; at the same time, their predetermined loan portfolio allows them to guarantee (safe) repayments of Rk_{1i}^B. For $Rk_{1i}^B < -n_{1i}^B$, the ability to offer safe claims is thus strictly smaller than the refinancing need and neither depositors nor investors will provide the required funding; as a result, banks go bankrupt and their assets are divested. For $Rk_{1i}^B \geq -n_{1i}^B$, instead, depositors are willing to roll over the entire outstanding debt; additional recapitalization is not necessary. In the corresponding continuation equilibrium the market for (risk-free) deposits clears at $p_{d,2}^{\psi_1} = 1$. Banks' demand and households' supply respectively satisfy $d_{2i}^{B,\psi_1} - x_{2i}^{B,\psi_1} \geq -n_{1i}^B, \forall i$ and $d_{2j}^{HD,\psi_1} + x_{2j}^{HD,\psi_1} \leq n_{1j}^{H,\psi_1}, \forall j$. The capital market clears at $p_{e,2}^{\psi_1} \in [n_{1j}^{H,\psi_1}/(n_{1j}^{H,\psi_1} + \underline{\rho}), 1]$ and with zero volumes. To see this, observe that at $p_{e,2}^{\psi_1} < n_{1j}^{H,\psi_1}/(n_{1j}^{H,\psi_1} + \underline{\rho})$, it would be strictly preferable for households with the lowest participation costs to invest their entire net worth on financial markets, while banks would strictly prefer issuing equity over borrowing if $p_{e,2}^{\psi_1,\omega} > 1$. As a result, prices outside of $[n_{1j}^{H,\psi_1}/(n_{1j}^{H,\psi_1}+\underline{\rho}), 1]$ would create excess demand or excess supply and cannot be sustained in equilibrium. Finally, notice that the allocation of consumption over $t = 1, 2$ – and therefore storage and the exact level of borrowing

– are irrelevant for our purposes and remain indeterminate. Provided that $Rk_{1i}^B \geq -n_{1i}^B$, the continuation equilibrium in state ψ_1 is thus characterized by $c_{1i}^{B,\psi_1} + c_{2i}^{B,\psi_1} = n_{1i}^B + Rk_{1i}^B \geq 0, \forall i$ and by $c_{1j}^{HD,\psi_1} + c_{2j}^{HD,\psi_1} = n_{1j}^{H,\psi_1} > 0, \forall j$.

Step 2: In state ψ_2, banks also go bankrupt if $Rk_{1i}^B < -n_{1i}^B$. To see this, notice that bank i can only guarantee $(R - \Delta_i)k_{1i}^B$ in safe claims. To cover the capital shortfall, investors would thus need to provide specialized capital (equity) of at least $\mathcal{C}_i = -n_{1i}^B - (R - \Delta_i)k_{1i}^B$. A bank that has fully exhausted its borrowing capacity, however, can offer a residual claimant no more than $(R + \epsilon\Delta_i)k_1^B - (R - \Delta_i)k_1^B = (1+\epsilon)\Delta_i k_{1i}^B$ in state ω_1 and zero in state ω_2; since $Rk_{1i}^B < -n_{1i}^B$ implies $(1+\epsilon)\Delta_i k_{1i}^B < 2\mathcal{C}_i$, this is unattractive for investors and bank i is unable to refinance.

Different from state ψ_1, however, $Rk_{1i}^B \geq -n_{1i}^B$ does not guarantee that banks can roll over their outstanding debt in state ψ_2. Successful refinancing now also requires them to attract sufficient specialized investment capital. Taking symmetric choices at $t = 0$, with $k_1^B = k_{1i}^B > 0, \forall i$, as given, we have $-n_1^B = h\left(k_1^B\right)$ (because we consider the case without bankruptcies) and $\mathcal{C}_i > 0, \forall i$ (since $h(0) = 0$, $h'(0) > 1$, $h'' > 0$, and $R - \underline{\Delta}^{\psi_2} < 1$). Thus, for liability side recapitalizations aggregate demand for specialized investment capital absent bankruptcies is given by $\chi_D^{\psi_2}$ from equation (29) with $\widetilde{\Delta}_E^{\psi_2} = \overline{\Delta}^{\psi_2}$. If all banks recapitalize, this aggregate demand can in equilibrium never be larger than the total amount that was borrowed at $t = 0$, and thus than households' aggregate endowments; as a consequence, $\chi_D^{\psi_2} < \varepsilon_0^H$. Finally, since $\mathcal{C}_i > 0, \forall i$ and because each bank requires a fixed amount of capital to prevent bankruptcy, the individual and aggregate demand are independent of the market clearing price ($d\chi_D^{\psi_2}/dp_{e,2}^{\psi_2,\omega_1} = 0$).

Aggregate supply of specialized investment capital, instead, is given by equation (28), where we recognize that households that pay the fixed participation cost optimally invest their entire net worth in financial assets. Since any unit that is not consumed at $t = 0$ can potentially be invested in specialized investment capital at $t = 1$, while there is no cost of delaying consumption, these households optimally set $c_0^H = 0$ at $t = 0$. This implies $d_1^H + x_1^H = \varepsilon_0^H$ and, given that there are no bankruptcies, $n_1^{H,\psi_2} = \varepsilon_0^H + \varepsilon_1^H$. In the household problem we showed that once they decide to participate as investors, they also set $c_1^{HI,\psi_2} = 0$ to maximize their equity investment. As a result, they consume $c_2^{HI,\psi_2,\omega_1} = e^{HI,\psi_2,\omega_1} = 2n_1^{H,\psi_2}/p_{e,2}^{\psi_2,\omega_1}$ in state ω_1 and nothing in state ω_2.

With the continuation utility of a depositor equal to $c_1^{HD,\psi_2} + c_2^{HD,\psi_2} = n_1^{H,\psi_2} > 0$ (as in *Step*

1), the indifference condition of the marginal household $\hat{\rho}^{\psi_2}$ is: $\hat{\rho}^{\psi_2} = \left(1/p_{e,2}^{\psi_2,\omega_1} - 1\right)\left(\varepsilon_0^H + \varepsilon_1^H\right)$. This implies that $\hat{\rho}^{\psi_2}$ is continuous and monotonically decreasing in $p_{e,2}^{\psi_2,\omega_1}$. Together with the continuous distribution of financial market participation costs it follows that $d\chi_S^{\psi_2}/dp_{e,2}^{\psi_2,\omega_1} < 0$, as well as $\chi_{\mathcal{S}}{}^{\psi_2} \to 0$ for $p_{e,2}^{\psi_2,\omega_1} \to \frac{\epsilon_1^H + \epsilon_0^H}{\epsilon_1^H + \epsilon_0^H + \underline{\rho}}$ and $\chi_{\mathcal{S}}{}^{\psi_2} \to (\epsilon_1^H + \epsilon_0^H)$ for $p_{e,2}^{\psi_2,\omega_1} \to 0$ and, hence, that a unique market-clearing price exists (see Proposition 1 for details). At this price, all banks are able to recapitalize and investors consume $c_{1j}^{HI,\psi_2}, c_{2j}^{HI,\psi_2,\omega_2} = 0$ and $c_{2j}^{HI,\psi_2,\omega_1} = 2n_{1j}^{H,\psi_2}/p_{e,2}^{\psi_2,\omega_1}$; new deposit financing is risk-less, so that the deposit market clears at $p_{d,2}^{\psi_2} = 1$ and depositors consume $c_{1j}^{HD,\psi_2} + c_{2j}^{HD,\psi_2} = n_{1j}^{H,\psi_2}$. Similar to *Step 1*, the exact allocation of consumption between periods $t = 1, 2$ and thus the choices of storage and borrowing are indeterminate and irrelevant for our purposes. Bankers consume $c_{1i}^{B,\psi_2} = c_{2i}^{B,\psi_2,\omega_2} = 0$ and $c_{2i}^{B,\psi_2,\omega_1} = (1+\epsilon)\Delta_i k_1^B - 2\mathcal{C}_i/p_{e,2}^{\psi_2,\omega_1}$.

Step 3: Having characterized the continuation equilibria for both states at $t = 1$, we now turn to $t = 0$. Building on the results from *Steps 1* and *2*, and taking the choices of all other banks as given, we can simplify an individual bank's problem as follows:

$$V_{0i}^B \equiv \max_{k_1^B} \sum_{\psi} \Pr\{\psi\} \int_{\underline{\Delta}^{\psi}}^{\overline{\Delta}^{\psi}} V_i^{B,\psi}\left(n_1^B, k_1^B; S^{\psi}\right) g^{\psi}\left(\Delta_i\right) di \qquad s.t. \qquad n_1^B = -h\left(k_1^B\right). \tag{31}$$

The first-order necessary condition of this problem is:

$$\sum_{\psi} \Pr\{\psi\} \int_{\underline{\Delta}^{\psi}}^{\overline{\Delta}^{\psi}} \left(\frac{\partial V_i^{B,\psi}}{\partial k_1^B} - \frac{\partial V_i^{B,\psi}}{\partial n_1^B} h'\left(k_1^B\right)\right) g^{\psi}\left(\Delta_i\right) di = 0.$$

For $k_1^B \to 0$, we have no capital shortfall and can write the left-hand side of the first-order condition as $R - \Pr\{\psi_2\}\mathbb{E}[\Delta_i] - h'(0)$. Since $R > \underline{h} \Rightarrow R > h'(0)$, continuity implies the existence of a $\Upsilon_1 > 0$, such that the left-hand side of the first-order condition, evaluated at $k_1^B = 0$, is strictly positive for all $\Pr\{\psi_2\} < \Upsilon_1$. This eliminates a corner solution with $k_1^{B*} = 0$ for sufficiently small values of $\Pr\{\psi_2\}$. Together with the strictly negative second-order condition, it follows that there exists a unique $k_1^{B*} > 0$ that solves the $t = 0$ problem, and thus a symmetric equilibrium at $t = 0$.

Next, we turn to households' portfolio choices at $t = 0$. For the case without bankruptcies equation (24) simplifies since bank debt is always repaid in full. Households are then indifferent between bank debt and storage so that their individual portfolio choice is indeterminate. At the aggregate, however, market clearing dictates $D_1^{H*} = D_1^{B*} = h\left(K_1^{B*}\right)$ and $X_1^{H*} = \epsilon_0^H - D_1^{H*}$.

Step 4: To establish uniqueness of the equilibrium on the aggregate level for the case without bankruptcies, it remains to be shown that there exists only one combination of aggregate capital and a market clearing price that is consistent with the equilibrium choices we have characterized in *Steps 1-3*. To do this, we first analyze how the premium on specialized investment capital varies with aggregate capital from the market clearing condition at $t = 1$:

$$\frac{dp_{e,2}^{\psi_2,\omega_1}}{dK_1^B} = \frac{R - \int_{\underline{\Delta}^{\psi_2}}^{\overline{\Delta}^{\psi_2}} \Delta_i \, g^{\psi_2}\left(\Delta_i\right) di - h'\left(K_1^B\right)}{\phi\left(\left(\frac{1}{p_{e,2}^{\psi_2,\omega_1}} - 1\right)\left(\epsilon_1^H + \epsilon_0^H\right)\right)\left(\frac{\epsilon_1^H + \epsilon_0^H}{p_{e,2}^{\psi_2,\omega_1}}\right)^2} < 0,$$

where $p_{e,2}^{\psi_2,\omega_1} \in \left[\frac{\epsilon_1^H + \epsilon_0^H}{\underline{\rho} + \epsilon_1^H + \epsilon_0^H}, 1\right]$ if $K_1^B = 0$ and $p_{e,2}^{\psi_2,\omega_1} \in \left[0, \frac{\epsilon_1^H + \epsilon_0^H}{\overline{\rho} + \epsilon_1^H + \epsilon_0^H}\right]$ if $K_1^B = \overline{K}_1^B$, with $\overline{K}_1^B$ being the highest feasible scale of the bank satisfying the resource constraints.

Next, the representative bank's optimality at $t = 0$ implies:

$$\frac{dp_{e,2}^{\psi_2,\omega_1}}{dK_1^B} = h''\left(K_1^B\right) \frac{\left[\frac{1+\epsilon}{2} + \frac{\Pr\{\psi_1\}}{\Pr\{\psi_2\}}\right] \int_{\underline{\Delta}^{\psi_2}}^{\overline{\Delta}^{\psi_2}} \Delta_i g^{\psi_2}\left(\Delta_i\right) di}{\left[-\frac{1}{p_{e,2}^{\psi_2,\omega_1}}\left(R - h'\left(K_1^B\right) - \int_{\underline{\Delta}^{\psi_2}}^{\overline{\Delta}^{\psi_2}} \Delta_i g^{\psi_2}\left(\Delta_i\right) di\right)\right]^2} > 0.$$

If $K_1^B = 0$, then:

$$p_{e,2}^{\psi_2,\omega_1} \in \left[0, \frac{\int_{\underline{\Delta}}^{\overline{\Delta}} \Delta_i g^{\psi_2}\left(\Delta_i\right) di - \left(R - h'(0)\right)}{\frac{\Pr\{\psi_1\}}{\Pr\{\psi_2\}}\left(R - h'(0)\right) + \frac{1+\epsilon}{2} \int_{\underline{\Delta}^{\psi_2}}^{\overline{\Delta}^{\psi_2}} \Delta_i g^{\psi_2}\left(\Delta_i\right) di}\right]. \tag{32}$$

Instead, $p_{e,2}^{\psi_2,\omega_1} \to 1$ if $K_1^B \to \widehat{K}_1^B$, with:

$$\widehat{K}_1^B \equiv h'^{-1}\left(R - \frac{\Pr\{\psi_2\}(1-\epsilon)}{2\left(\Pr\{\psi_1\} - \Pr\{\psi_2\}\right)} \int_{\underline{\Delta}^{\psi_2}}^{\overline{\Delta}^{\psi_2}} \Delta_i g^{\psi_2}\left(\Delta_i\right) di\right)$$

being the highest possible bank scale satisfying the first-order condition for a permissible premium for investment capital. We know that $\widehat{K}_1^B < \overline{K}_1^B$ by assumption. This is because $\overline{K}_1^B$ can be arbitrarily large when increasing household endowments, which does not affect the deposit rate.

A sufficient condition for $K_1^{B*} > 0$ and the existence of a unique $p_{e,2}^{\psi_2,\omega_1*}$ solving the system is:

$$\underline{\rho} \quad < \frac{\frac{1}{\Pr\{\psi_2\}}(R-h'(0))-\frac{1-\epsilon}{2}\int_{\underline{\Delta}^{\psi_2}}^{\overline{\Delta}^{\psi_2}} \Delta_i g^{\psi_2}(\Delta_i)di}{\int_{\underline{\Delta}^{\psi_2}}^{\overline{\Delta}^{\psi_2}} \Delta_i g^{\psi_2}(\Delta_i)di-(R-h'(0))} \left(\epsilon_1^H + \epsilon_0^H\right), \tag{33}$$

which holds, for example, for $\Pr\{\psi_2\} \to 0$. Hence, by continuity, there exists a $\widetilde{\Upsilon}_1 > 0$ such that inequality (33) holds for all $\Pr\{\psi_2\} \leq \widetilde{\Upsilon}_1$. To conclude, for the case without bankruptcies, there exists a unique equilibrium characterized by symmetric $t = 0$ bank choices provided $\Pr\{\psi_2\} \leq \Upsilon_1$ and a unique $p_{e,2}^{\psi_2,\omega_1*}$ under the sufficient condition $\Pr\{\psi_2\} \leq \max\left\{\Upsilon_1, \widetilde{\Upsilon}_1\right\}$.

Part 2: The case with bankruptcies. We extend our existence and uniqueness results to the case with bankruptcies. The continuation equilibrium remains unchanged in state ψ_1. In state ψ_2, instead, only some banks are able to recapitalize while others go bankrupt. Taking choices at $t = 0$ as given, capital demand at $t = 1$ is thus given by equation (29) where banks with $\Delta_i > \widetilde{\Delta}_E^{\psi_2}$ cannot promise investors a sufficient upside under ω_1 to compensate for the loss under ω_2.

Since recapitalizing banks continue to require a fixed amount of capital, the dependence of aggregate demand on the price of specialized investment capital is driven by $d\widetilde{\Delta}_E^{\psi_2}/dp_{e,2}^{\psi_2,\omega_1}$, which is strictly positive as long as $\widetilde{\Delta}_E^{\psi_2} < \overline{\Delta}^{\psi_2}$, and by the volume of market-based liquidations of insolvent banks.[30] If $f = 0$ the latter equals zero and $\chi_D^{\psi_2} < \epsilon_0^H$, as before. The aggregate supply is unchanged and there exists again a unique market clearing price in state ψ_2. Instead, if $f > 0$, the demand is potentially non-monotone in the price of investment capital due to the effect of the price on the extensive recapitalization margin (see equation (30) in the proof of Proposition 1). Despite the ambiguous sign, single crossing can be assured if the price elasticity of supply is sufficiently high. Taken together, *Steps 1-2* from part 1 carry through with minor modifications.

Step 3: Different from before, banks now take their potential bankruptcy and (partial) default into account at $t = 0$. From the problem in (18) we have:

$$V_0^B \equiv \quad \max_{k_1^B} \sum_\psi \Pr\{\psi\} \int_{\underline{\Delta}^\psi}^{\widetilde{\Delta}_E^\psi} V_i^{B,\psi}\left(n_1^B, k_1^B; S^\psi\right) g^\psi(\Delta_i)\, di \quad s.t.\ n_1^B = -\frac{h\left(k_1^B\right)}{p_{d,1}\left(\Pr\{\psi_1\}+\Pr\{\psi_2\}\vartheta\left(\cdot;S^{\psi_2}\right)\right)},$$

[30] Note that $\widetilde{\Delta}_E^{\psi_2} \leq \underline{\Delta}^{\psi_2}$ would correspond to the trivial case of no bank needing to recapitalize or going bankrupt.

and the corresponding first-order necessary condition:

$$\sum_{\psi} \Pr\{\psi\} \left(\int_{\underline{\Delta}^{\psi}}^{\widetilde{\Delta}_E^{\psi}} \left(\frac{\partial V_i^{B,\psi}}{\partial k_1^B} + \frac{\partial V_i^{B,\psi}}{\partial n_1^B} \frac{\partial n_1^B}{\partial k_1^B} \right) g^{\psi}(\Delta_i)\, di + \frac{d\widetilde{\Delta}_E^{\psi}}{dk_1^B} V_i^{B,\psi} g^{\psi}\left(\widetilde{\Delta}_E^{\psi}\right) \right) = 0. \tag{34}$$

Similar to Part 1, continuity implies the existence of a $\Upsilon_2 > 0$, such that the left-hand side of equation (34), evaluated at $k_1^B = 0$, is strictly positive for all $\Pr\{\psi_2\} < \Upsilon_2$. This rules out a corner solution. Since one can further show the existence of a $\Upsilon_2 > 0$ such that also the second-order condition continues to be strictly negative for all $\Pr\{\psi_2\} < \Upsilon_2$, it follows that there exists a unique $k_1^{B*} > 0$ solving the $t = 0$ problem of all banks i. Next, Lemmas 2 and 3 establish the symmetry of households' portfolio choices at $t = 0$.

Lemma 2. *If households' portfolio choices at $t = 0$ are interior ($d_{1j}^H, x_{1j}^H > 0, \forall j$), they are necessarily symmetric, i.e. $d_{1j}^H = d_1^H$, $\forall j$ and $x_{1j}^H = x_1^H, \forall j$.*

Proof. Taking interiority as given, equations (10) and (11) imply that $p_{d,1}$ is determined by equation (24). All households take $p_{d,1}$ as given and $-$ because of the premium for specialized investment capital at $t = 1$ $-$ find it optimal not to consume at $t = 0$. Since banks do not offer risky deposits at $t = 1$ we further have $p_{d,2}^{\psi} = 1$, which implies $\lambda_{1j}^{HD,\psi} = 1, \forall j, \psi$ independently of $t = 1$ portfolio choices. Next, we inspect whether $\hat{\rho}_j^{\psi}$ and $\lambda_{1j}^{HI,\psi}$ depend on the portfolio.

In the safe state ψ_1 banks roll over their deposits by issuing safe claims and we have no financial market participation, so that $\lambda_{1j}^{HI,\psi_1} = 1, \forall j$ and $\hat{\rho}_j^{\psi_1} = \hat{\rho}^{\psi_1} \leq 0,\ \forall j$. In the risky state ψ_2, instead, we have $\lambda_{1j}^{HI,\psi_2} = 1/q^{\psi_2}, \forall j$ because all investors optimally invest all their resources after incurring the financial market participation cost. It follows that λ_{1j}^{HI,ψ_2} depends on the market-clearing price for specialized investment capital but not on households' individual portfolio choices. This in turn implies $\hat{\rho}_j^{\psi_2} = \hat{\rho}^{\psi_2}, \forall j$ from equation (11). Moreover, symmetric portfolio choices at $t = 0$ follows from equation (8) because resources and, hence, portfolios of households need to be identical, which concludes the proof. □

Lemma 3. *An interior portfolio choice, i.e. $d_{1j}^H, x_{1j}^H > 0, \forall j$ is optimal for households at $t = 0$.*

Proof. The proof is by contradiction. Suppose households find it optimal to either invest all their resources in debt or storage, and let the types of households that exclusively invest in deposits or storage be identified by the subscripts $H1$ and $H2$, respectively. Households take the deposit rate

$p_{d,1}$ and banks' default probability as given and find it optimal not to consume at $t=0$. The first necessary condition for the existence of a mixed equilibrium characterized by an asymmetric portfolio choice at $t=0$ is that households must be indifferent between investing all their resources in debt or in storage. Formally, $V_0^H\left(0,\epsilon_0^H,0\right)=V_0^H\left(0,0,\epsilon_0^H\right)$, which is equivalent to:

$$\begin{aligned} & \Pr\{\psi_1\}\left[\frac{\epsilon_0^H}{p_{d,1}\sum_\psi \Pr\{\psi\}\vartheta\left(\cdot;S^{\psi_2}\right)}+\epsilon_1^H\right]-\Pr\{\psi_2\}\int_0^{\hat{\rho}_{H1}^{\psi_2}}\rho\phi\left(\rho\right)d\rho \\ & +\Pr\{\psi_2\}\frac{\hat{\rho}_{H1}^{\psi_2}+\left(1-\hat{\rho}_{H1}^{\psi_2}\right)\Phi\left(\hat{\rho}_{H1}^{\psi_2}\right)}{\hat{\rho}^{H1,\psi_2}}\left[\frac{\epsilon_0^H\vartheta\left(\cdot;S^{\psi_2}\right)}{p_{d,1}\sum_\psi \Pr\{\psi\}\vartheta\left(\cdot;S^{\psi_2}\right)}+\epsilon_1^H\right] \\ = & \left(\Pr\{\psi_1\}+\Pr\{\psi_2\}\frac{\hat{\rho}_{H2}^{\psi_2}+\left(1-\hat{\rho}_{H2}^{\psi_2}\right)\Phi\left(\hat{\rho}_{H2}^{\psi_2}\right)}{\hat{\rho}_{H2}^{\psi_2}}\right)\left[\epsilon_0^H+\epsilon_1^H\right]-\Pr\{\psi_2\}\int_0^{\hat{\rho}_{H2}^{\psi_2}}\rho\phi\left(\rho\right)d\rho, \end{aligned} \tag{35}$$

where $\hat{\rho}_{H2}^{\psi_2}>\hat{\rho}_{H1}^{\psi_2}$ from equation (8) since the fixed financial market participation cost supports a larger investment in capital. Equation (35) then describes how the return on deposits and capital has to adjust in order to maintain household indifference. The second necessary condition is that no household has an incentive to deviate to a mixed portfolio choice with $d_{1j}^H>0\wedge x_{1j}^H>0$. In other words, both types must find their extreme portfolio choices optimal. This requires to revisit the first-order necessary conditions. For households investing exclusively in debt we have:

$$\lambda_0^{H1}=\frac{\sum_\psi \Pr\{\psi\}\vartheta\left(\cdot;S^{\psi_2}\right)\left[\ \Phi\left(\hat{\rho}^{H1,\psi}\right)\ \lambda_1^{HI,\psi}+\left(1-\Phi\left(\hat{\rho}^{H1,\psi}\right)\right)\ \lambda_1^{HD,\psi}\ \right]}{p_{d,1}\sum_\psi \Pr\{\psi\}\vartheta\left(\cdot;S^{\psi_2}\right)} \tag{36}$$

$$\lambda_0^{H1}=\sum_\psi \Pr\{\psi\}\left[\ \Phi\left(\hat{\rho}^{H1,\psi}\right)\ \lambda_1^{HI,\psi}+\left(1-\Phi\left(\hat{\rho}^{H1,\psi}\right)\right)\ \lambda_1^{HD,\psi}\ \right]+\xi_x^H. \tag{37}$$

Since $\hat{\rho}^{H2,\psi}>\hat{\rho}^{H1,\psi}$ and $\lambda_1^{HI,\psi_2}>\lambda_1^{HD,\psi_2}$, $\lambda_0^{H1}<\lambda_0^{H2}$, with $\lambda_0^{H1}=\frac{dV_0^{H1}}{d\epsilon_0^H}$ confirming equation (36). A combination of equations (36) and (37) gives:

$$1=\frac{\sum_\psi \Pr\{\psi\}\vartheta\left(\cdot;S^{\psi_2}\right)\left[\ \Phi\left(\hat{\rho}^{H1,\psi}\right)\ \lambda_1^{HI,\psi}+\left(1-\Phi\left(\hat{\rho}^{H1,\psi}\right)\right)\ \lambda_1^{HD,\psi}\ \right]-\xi_x^H}{\sum_\psi \Pr\{\psi\}\left[\ \Phi\left(\hat{\rho}^{H1,\psi}\right)\ \lambda_1^{HI,\psi}+\left(1-\Phi\left(\hat{\rho}^{H1,\psi}\right)\right)\ \lambda_1^{HD,\psi}\ \right]},$$

leading to a contradiction since $\xi_x^H<0$ is not permitted. As a result, extreme portfolio choices are not optimal and the result in Lemma 3 follows. $\square$

Step 4: To establish uniqueness of the equilibrium on the aggregate level for the case with bankruptcies, what remains to be shown is that there again exists only one combination of K_1^B and $p_{e,2}^{\psi_2,\omega}$ that is consistent with the equilibrium choices we have characterized in Steps 1-3. We proceed as in part 1. From $t=1$ market clearing in Proposition 1 the price $p_{e,2}^{\psi_2,\omega}$ solves:

$$\begin{aligned} Z \equiv \; & \Phi\left(\frac{1-p_{e,2}^{\psi_2,\omega}}{p_{e,2}^{\psi_2,\omega}} V^{HD,2\psi}\left(N_1^{H,\psi_2}; S^{\psi_2}\right)\right) V^{HD,\psi_2}\left(N_1^{H,\psi_2}; S^{\psi_2}\right) \\ & - \left(\begin{array}{c} \int_{\underline{\Delta}^{\psi_2}}^{\widetilde{\Delta}_E^{\psi_2}} \left(\frac{h\left(k_1^B\right)}{\overline{p}_{d,1}\left(\cdot;\widetilde{\Delta}_E^{\psi_2}\right)} - \left(R-\Delta_i\right) K_1^B\right) g^{\psi_2}\left(\Delta_i\right) di \\ + p_{e,2}^{\psi_2,\omega} f \int_{\widetilde{\Delta}_E^{\psi_2}}^{\overline{\Delta}^{\psi_2}} \left(R + \frac{\epsilon-1}{2}\Delta_i\right) K_1^B g^{\psi_2}\left(\Delta_i\right) di \end{array}\right) = 0. \end{aligned}$$

We apply the implicit function theorem. Given $k_1^B = K_1^B$, the total derivative is:

$$\frac{dp_{e,2}^{\psi,\omega_1}}{dK_1^B} = \frac{\left(\begin{array}{c} \phi\left(\cdot\right) \frac{1-p_{e,2}^{\psi_2,\omega}}{p_{e,2}^{\psi_2,\omega}} V_{k_1^B}^{HD,\psi_2} V^{HD,\psi_2} + \Phi\left(\cdot\right) V_{k_1^B}^{HD,\psi_2} \\ - \int_{\underline{\Delta}^{\psi_2}}^{\widetilde{\Delta}_E^{\psi_2}} \left(\frac{h'\left(k_1^B\right) - \frac{h\left(k_1^B\right)}{\overline{p}_{d,1}} \frac{\partial \overline{p}_{d,1}}{\partial k_1^B}}{\overline{p}_{d,1}} - \left(R-\Delta_i\right)\right) g^{\psi_2}\left(\Delta_i\right) di \\ - p_{e,2}^{\psi_2,\omega_1} \; f \int_{\widetilde{\Delta}_E^{\psi_2}}^{\overline{\Delta}^{\psi_2}} \left(R + \frac{\epsilon-1}{2}\Delta_i\right) g^{\psi_2}\left(\Delta_i\right) di \\ + p_{e,2}^{\psi_2,\omega_1} f \frac{\partial \widetilde{\Delta}_E^{\psi_2}}{\partial k_1} \left(R + \frac{\epsilon-1}{2}\widetilde{\Delta}_E^{\psi_2}\right) K_1^B g^{\psi_2}\left(\widetilde{\Delta}_E^{\psi_2}\right) \end{array}\right)}{\left(\begin{array}{c} -\phi\left(\cdot\right)\left(\frac{1-p_{e,2}^{\psi_2,\omega}}{p_{e,2}^{\psi_2,\omega}} V_{p_{e,2}^{\psi_2,\omega_1}}^{HD,\psi_2} - \frac{V^{HD,\psi_2}}{\left(p_{e,2}^{\psi_2,\omega}\right)^2}\right) V^{HD,\psi_2} - \Phi\left(\cdot\right) V_{p_{e,2}^{\psi_2,\omega_1}}^{HD,\psi_2} \\ - \int_{\underline{\Delta}^{\psi_2}}^{\widetilde{\Delta}_E^{\psi_2}} \frac{h\left(k_1^B\right)}{\left(\overline{p}_{d,1}\right)^2} \frac{\partial \overline{p}_{d,1}}{\partial p_{e,2}^{\psi_2,\omega_1}} g^{\psi_2}\left(\Delta_i\right) di \\ + \; f \int_{\widetilde{\Delta}_E^{\psi_2}}^{\overline{\Delta}^{\psi_2}} \left(R + \frac{\epsilon-1}{2}\Delta_i\right) K_1^B g^{\psi_2}\left(\Delta_i\right) di \\ - p_{e,2}^{\psi_2,\omega} f \frac{\partial \widetilde{\Delta}_E^{\psi_2}}{\partial p_{e,2}^{\psi_2,\omega_1}} \left(R + \frac{\epsilon-1}{2}\widetilde{\Delta}_E^{\psi_2}\right) K_1^B g^{\psi_2}\left(\widetilde{\Delta}_E^{\psi_2}\right) \end{array}\right)}, \tag{38}$$

where $V_{k_1^B}^{HD,\psi_2} < 0$ and $\frac{\partial \overline{p}_{d,1}}{\partial k_1^B} < 0$ under the sufficient condition that $\Pr\{\psi_2\}$ is small. As a result, there exists by continuity a $\widetilde{\Upsilon}_2 > 0$ such that the nominator is negative for all $\Pr\{\psi_2\} < \widetilde{\Upsilon}_2$. Next, we study the denominator. For $\Pr\{\psi_2\} \to 0$ the denominator can be written as:

$$\phi\left(\cdot\right)\left(\frac{V^{HD,\psi_2}}{p_{e,2}^{\psi_2,\omega}}\right)^2 - \frac{\partial \vartheta}{\partial p_{e,2}^{\psi_2,\omega_1}} D_1 \left[\Phi\left(\cdot\right) + \frac{1-p_{e,2}^{\psi_2,\omega}}{p_{e,2}^{\psi_2,\omega}} V^{HD,\psi_2}\right] + \; f \int_{\widetilde{\Delta}_E^{\psi_2}}^{\overline{\Delta}^{\psi_2}} \left(R + \frac{\epsilon-1}{2}\Delta_i\right) K_1^B g^{\psi_2}\left(\Delta_i\right) di \;,$$

which is positive if the price elasticity of supply is sufficiently high as in Proposition 1. Recall that for uniformly distributed participation costs an arbitrarily steep supply schedule can be achieved if $\overline{\rho} \to \underline{\rho}$. Observe, however, that $\lim_{\Pr\{\psi_2\}\to 0} \frac{dp_{e,2}^{\psi_2,\omega_1}}{dK_1^B}$ generically takes on a finite value, which will prove to be useful. As before, we have $p_{e,2}^{\psi_2,\omega_1} \in \left[\frac{\epsilon_1^H+\epsilon_0^H}{\underline{\rho}+\epsilon_1^H+\epsilon_0^H}, 1\right]$ if $K_1^B = 0$ and $p_{e,2}^{\psi_2,\omega_1} \in \left[0, \frac{\epsilon_1^H+\epsilon_0^H}{\underline{\rho}+\epsilon_1^H+\epsilon_0^H}\right]$ if $K_1^B \to \widetilde{K}_1^B$, with $\widetilde{K}_1^B$ being the highest feasible scale of the bank satisfying the resource constraints.

The representative bank's optimality at $t = 0$ implies:

$$\lim_{\Pr\{\psi_2\}\to 0} \frac{dp_{e,2}^{\psi_2,\omega_1}}{dK_1^B} = -\frac{h'\left(K_1^B\right)\overbrace{\frac{\partial p_{d,1}}{\partial K_1^B}}^{<0} - p_{d,1}h''\left(K_1^B\right)}{h'\left(k_1^B\right)\underbrace{\frac{\partial p_{d,1}}{\partial p_{e,2}^{\psi_2,\omega_1}}}_{>0}}\Big|_{\Pr\{\psi_2\}\to 0} = +\infty,$$

since $\frac{\partial p_{d,1}}{\partial p_{e,2}^{\psi_2,\omega_1}} \searrow 0$ if $\Pr\{\psi_2\} \to 0$. Moreover, for $K_1^B \to 0$ we have no bankruptcies and can use the result from equation (32). Instead, when allowing for bankruptcies when K_1^B grows large, then $p_{e,2}^{\psi_2,\omega_1} \to 1$. As before, single-crossing can be established. By continuity, there exists a $\check{\Upsilon}_2 > 0$ such that the result holds for all $\Pr\{\psi_2\} \leq \check{\Upsilon}_2$. To conclude, for the case with bankruptcies there exists a unique equilibrium characterized by symmetric $t = 0$ choices provided $\Pr\{\psi_2\}$ is sufficiently small. Formally, if $\Pr\{\psi_2\} \leq \min\left\{\Upsilon_2, \widetilde{\Upsilon}_2, \check{\Upsilon}_2\right\}$.

Part 3: Ruling out coexistence. To conclude the overall proof, we argue that for a given set of model parameters there cannot exist at the same time an equilibrium without bankruptcies and an equilibrium with bankruptcies. This requires us to recognize how bankruptcies come about and to take into account their effect on the demand for specialized investment capital and banks' $t = 0$ problem. To do so, we start by comparing the first-order conditions to the banks' $t = 0$ problem under the conjecture that there are no bankruptcies and under the conjecture that there are bankruptcies. Assuming that there exists an equilibrium without bankruptcies for a certain set of model parameters that is characterized by $\left(K_1^{B*}, p_{e,2}^{\psi_2,\omega_1 *}\right)$, we can first show that for sufficiently small $\Pr\{\psi_2\}$ banks optimally select a $k_1^{B*} < K_1^{B*}$ for a given $p_{e,2}^{\psi_2,\omega_1 *}$ when expecting a positive incidence of bankruptcies. Second, it is the case that the levels of k_1^B solving the first-order conditions under the two conjectures are arbitrarily close and approach $h'^{-1}(R)$. As a result, we can concentrate on the market-clearing conditions under the two conjectures to see whether equilibrium co-existence can be ruled out. Turning to the supply-side, we have that the supply of specialized investment capital under the conjecture of no bankruptcies is higher since $n_{1j}^{H,\psi_2*} > \epsilon_1^H + d_{1j}^H > \epsilon_1^H + \vartheta\left(S^{\psi_2}\right) d_{1j}^H$ with $\epsilon_1^H + \vartheta\left(S^{\psi_2}\right) d_{1j}^H \to n_{1j}^{H,\psi_2*}$ for $\Pr\{\psi_2\} \to 0$. In addition, for any sufficiently small value of $\Pr\{\psi_2\}$ we can show that the market-clearing price for specialized investment capital is weakly lower under the conjecture of no bankruptcies, i.e. $p_{e,2}^{\psi_2,\omega_1} < p_{e,2}^{\psi_2,\omega_1*}$ and $p_{e,2}^{\psi_2,\omega_1} \to p_{e,2}^{\psi_2,\omega_1*}$, provided the price elasticity of supply is sufficiently high (a sufficient condition also used in the Proof of Proposition 2). As a result, there are two effects implying that $k_1^{B*} < K_1^{B*}$ for a given $p_{e,2}^{\psi_2,\omega_1} < p_{e,2}^{\psi_2,\omega_1*}$ By continuity, there exists a $\Upsilon_3 > 0$ such that the result holds for all $\Pr\{\psi_2\} \leq \Upsilon_3$ given a sufficiently high price elasticity of supply. Taken together, market-clearing can only be consistent with the solution to one of the two first-oder conditions and we can rule out co-existence. This concludes the overall proof of Proposition 4.

A.2.5 Proof of Proposition 5

Below, we present the derivations for the baseline model. This allows us to simplify the exposition and the results can be readily extended to the modified models discussed in Section 5. We start with some preliminaries. First we derive the optimally condition of the simplified bank problem at $t = 0$ where we use $x_1^B = c_0^B = 0$ and $d_1^B = h\left(k_1^B\right)/\overline{p}_{d,1}\left(\cdot;\widetilde{\Delta}_E^{\psi_2}\right)$, with $\overline{p}_{d,1}\left(\cdot;\widetilde{\Delta}_E^{\psi_2}\right) \equiv p_{d,1}\left(\Pr\{\psi_1\} + \Pr\{\psi_2\}\vartheta\left(\cdot;S^{\psi_2}\right)\right)$. The first-order necessary condition is:

$$\Pr\{\psi_1\}\left(R - \frac{dd_1^B}{dk_1^B}\right) + \Pr\{\psi_2\}\int_{\underline{\Delta}^{\psi_2}}^{\widetilde{\Delta}_E^{\psi_2}}\left(\frac{R - \Delta_i - \frac{dd_1^B}{dk_1^B}}{p_{e,2}^{\psi_2,\omega_1}} + \frac{\epsilon+1}{2}\Delta_i\right) g^{\psi_2}\left(\Delta_i\right) = 0, \tag{39}$$

where we took the derivative with respect to k_1^B and where:

$$\frac{dd_1^B}{dk_1^B} = \left(h'\left(k_1^B\right) - p_{d,1}d_1^B\sum_{\psi}\Pr\{\psi\}\frac{d\vartheta\left(\cdot;S^{\psi}\right)}{dk_1^B}\right)/\overline{p}_{d,1}\left(\cdot;\widetilde{\Delta}_E^{\psi_2}\right).$$

Recall that $\mathcal{L}_i/d_1^B < 1$. Hence, $\frac{d\vartheta\left(\cdot;S^{\psi}\right)}{dk_1^B} = \frac{d\widetilde{\Delta}_E^{\psi}}{dk_1^B}\left(1 - \frac{\mathcal{L}_i}{d_1^B}\right) g\left(\widetilde{\Delta}_E^{\psi}\right) + \int_{\underline{\Delta}^{\psi}}^{\widetilde{\Delta}_E^{\psi}}\frac{d(\mathcal{L}_i/d_1^B)}{dk_1^B} g^{\psi}\left(\Delta_i\right) di \leq 0$ holds with strict inequality when $\widetilde{\Delta}_E^{\psi} < \overline{\Delta}^{\psi}$ and provided that *(a)* $d\widetilde{\Delta}_E^{\psi}/dk_1^B < 0$ and *(b)* $d(\mathcal{L}_i/d_1^B)/dk_1^B < 0$. The latter two inequalities indeed hold if $d\vartheta\left(\cdot;S^{\psi}\right)/dk_1^B < 0$. However, it remains to rule out a solution where the opposite is true, i.e. where $d\vartheta\left(\cdot;S^{\psi}\right)/dk_1^B > 0$. This may be possible for extreme parameter values since both $\widetilde{\Delta}_E^{\psi}/dk_1^B$ and $d(\mathcal{L}_i/d_1^B)/dk_1^B$ are a function of $d\vartheta\left(\cdot;S^{\psi}\right)/dk_1^B$. By continuity we can show that there exists a $\Upsilon_4 > 0$ such that both $d\widetilde{\Delta}_E^{\psi_2}/dk_1^B < 0$ and $d(\mathcal{L}_i/d_1^B)/dk_1^B < 0$ hold for all $\Pr\{\psi_2\} \leq \Upsilon_4$ independent of the sign of $d\vartheta\left(\cdot; S^{\psi}\right)/dk_1^B$. This is because $d\overline{p}_{d,1}/dk_1^B \to 0$ for $\Pr\{\psi_2\} \to 0$. Hence, $d\vartheta\left(\cdot; S^{\psi_2}\right)/dk_1^B$ must be strictly negative if $\Pr\{\psi_2\} \leq \Upsilon_4$ and there exists a unique solution to equation (39).

For the simplified planner problem with $\theta^H = \theta^B = 1$ the first-order necessary condition is:

$$\begin{aligned}\left[K_1^B\right]: \quad & \Pr\{\psi_1\}\left(R - h'\left(k_1^B\right)\right) + \Pr\{\psi_2\}\int_{\underline{\Delta}^{\psi_2}}^{\widetilde{\Delta}_E^{\psi_2}} \Pr\{\omega_1\}\left(1+\epsilon\right)\Delta_i g^{\psi_2}\left(\Delta_i\right)di \\ & + \Pr\{\psi_2\}\left(\frac{d\vartheta\left(\cdot; S^{\psi_2}\right)}{dk_1^B} D_1 + \vartheta\left(\cdot; S^{\psi_2}\right)\frac{dD_1}{dk_1^B} + \frac{d\vartheta\left(\cdot; S^{\psi_2}\right)}{dK_1^B} D_1 + \vartheta\left(S^{\psi_2}\right)\frac{dD_1}{dK_1^B} - h'\left(k_1^B\right)\right) \\ & - \frac{\Pr\{\psi_2\}}{p_{e,2}^{\psi_2,\omega_1}}\int_{\underline{\Delta}^{\psi_2}}^{\widetilde{\Delta}_E^{\psi_2}}\left[\left(\frac{dD_1}{dk_1^B} + \frac{dD_1}{dK_1^B} - \left(R - \Delta_i\right)\right) - \frac{D_1 - (R-\Delta_i)K_1^B}{p_{e,2}^{\psi_2,\omega_1}}\frac{dp_{e,2}^{\psi_2,\omega_1}}{dK_1^B}\right] g^{\psi_2}\left(\Delta_i\right)di = 0,\end{aligned} \tag{40}$$

where:

$$\frac{dD_1}{dK_1^B} = -D_1\left(\frac{\frac{dp_{d,1}}{dK_1^B}}{p_{d,1}} + \frac{\Pr\{\psi_2\}\frac{d\vartheta\left(\cdot; S^{\psi}\right)}{dp_{e,2}^{\psi,\omega}}\frac{dp_{e,2}^{\psi,\omega}}{dK_1^B}}{\Pr\{\psi_1\} + \Pr\{\psi_2\}\vartheta\left(\cdot; S^{\psi_2}\right)}\right). \tag{41}$$

To analyze efficiency we use an envelope argument and evaluate equation (40) at the decentralized optimum by plugging in from equation (39):

$$\left(\begin{array}{c}\Pr\{\psi_1\}\left(\frac{dD_1}{dk_1^B} - h'\left(k_1^B\right)\right) \\ + \Pr\{\psi_2\}\left[\left(\frac{d\vartheta}{dk_1^B} + \frac{d\vartheta}{dK_1^B}\right)D_1 + \vartheta\left(\frac{dD_1}{dk_1^B} + \frac{dD_1}{dK_1^B}\right) - h'\left(k_1^B\right)\right] \\ - \frac{\Pr\{\psi_2\}}{p_{e,2}^{\psi_2,\omega_1}}\int_{\underline{\Delta}^{\psi_2}}^{\widetilde{\Delta}_E^{\psi_2}}\left[\frac{dD_1}{dK_1^B} - \frac{D_1 - (R-\Delta_i)K_1^B}{p_{e,2}^{\psi_2,\omega_1}}\frac{dp_{e,2}^{\psi_2,\omega_1}}{dK_1^B}\right] g^{\psi_2}\left(\Delta_i\right)di\end{array}\right)\Big|_{K_1^B = k_1^{B*}} \gtreqless 0. \tag{42}$$

We proceed by analyzing (42). Whenever the left-hand side is positive (negative) the planner prefers a higher (lower) level of investment and initial debt funding. We prove four results.

Result (1): Absent bankruptcies and financial market segmentation, the cost of specialized capital is independent of $t = 0$ choices, i.e. $dp_{e,2}^{\psi_2,\omega_1}/dK_1^B = dp_{d,1}/dK_1^B = 0$. Equation (42) simplifies:

$$\left(\Pr\{\psi_1\}\left(\frac{dD_1}{dk_1^B} - h'\left(k_1^B\right)\right) + \Pr\{\psi_2\}\left[\frac{d\vartheta}{dk_1^B}D_1 + \vartheta\frac{dD_1}{dk_1^B} - h'\left(k_1^B\right)\right]\right)\Big|_{K_1^B = k_1^{B*}} \gtreqless 0. \tag{43}$$

No bankruptcies implies that $\vartheta = 1$ and, hence, $dD_1/dk_1^B = h'\left(k_1^B\right)$ so that the left-hand side of (43) is zero. The laissez-faire equilibrium is constrained efficient.

Result (2): Absent bankruptcies but with financial market segmentation equation (42) writes:

$$\left(\frac{\Pr\{\psi_2\}}{\left(p_{e,2}^{\psi_2,\omega}\right)^2} \int_{\underline{\Delta}^{\psi_2}}^{\overline{\Delta}_E^{\psi_2}} \left[D_1 - (R - \Delta_i) K_1^B\right] \frac{dp_{e,2}^{\psi_2,\omega_1}}{dK_1^B} g^{\psi_2}(\Delta_i)\, di \right) |_{K_1 = k_1^{B*}} \gtreqless 0. \tag{44}$$

The laissez-faire equilibrium is generically constrained inefficient. In the plausible scenario where the premium on capital is positively associated with the size of the banking sector, $dp_{e,2}^{\psi_2,\omega_1}/dK_1^B < 0$, banks inefficiently over-leverage. Below we derive conditions such that $dp_{e,2}^{\psi_2,\omega_1}/dK_1^B < 0$.

Result (3): When allowing for bankruptcies absent financial market segmentation, i.e for constant $p_{e,2}^{\psi_2,\omega_1}$, we have $\vartheta < 1$ and $d\vartheta/dk_1^B \neq 0$. Equation (42) now writes:

$$\frac{1 - p_{d,1}}{p_{d,1}} h'\left(k_1^B\right) |_{K_1 = k_1^{B*}} \leq 0. \tag{45}$$

Observe that the expression in (45) is zero if $\underline{\rho} = \overline{\rho} = 0$, which implies that $p_{e,2}^{\psi_2,\omega} = p_{d,1} = 1$. In this case the laissez-faire equilibrium is constrained efficient. Instead, if $\underline{\rho} = \overline{\rho} > 0$, that is with an elastic supply of specialized investment capital and a positive investor premium due to financial market segmentation, there is inefficient over-leveraging of the banking sector at $t = 0$.

Result (4): Finally, we consider the case with bankruptcies and financial market segmentation. A key sufficient condition is again that $\Pr\{\psi_2\}$ is small and that the price elasticity of capital supply is sufficiently high as in Proposition 1. Formally, if $dp_{e,2}^{\psi,\omega}/dK_1^B < 0$ and $dp_{d,1}/dK_1^B < 0$ the expression in equation (42) is negative. To see this we re-write (42) as:

$$\left(\begin{array}{c} \Pr\{\psi_1\} \frac{1-\overline{p}_{d,1}}{\overline{p}_{d,1}} h'\left(k_1^B\right) + \Pr\{\psi_2\} D_1 \frac{d\vartheta}{dk_1^B} \\ + \Pr\{\psi_2\} \frac{\Pr\{\psi_1\} D_1}{\Pr\{\psi_1\} + \Pr\{\psi_2\}\vartheta\left(\cdot; S^{\psi_2}\right)} \frac{d\vartheta}{dK_1^B} \\ + \Pr\{\psi_2\} \left(\frac{\vartheta}{\overline{p}_{d,1}} \left(h'\left(k_1^B\right) - D_1 \frac{dp_{d,1}}{dK_1^B} \left(\Pr\{\psi_1\} + \Pr\{\psi_2\}\vartheta\right)\right) - h'\left(k_1^B\right)\right) \\ - \frac{\Pr\{\psi_2\}}{p_{e,2}^{\psi_2,\omega_1}} \int_{\underline{\Delta}^{\psi_2}}^{\widetilde{\Delta}_E^{\psi_2}} \left[\frac{dD_1}{dK_1^B} - \frac{D_1 - (R-\Delta_i)K_1^B}{p_{e,2}^{\psi_2,\omega_1}} \frac{dp_{e,2}^{\psi_2,\omega_1}}{dK_1^B} \right] g^{\psi_2}(\Delta_i)\, di \end{array} \right) |_{K_1^B = k_1^{B*}} \gtreqless 0. \tag{46}$$

The first and last line are negative, while the second and third line are negative provided $\Pr\{\psi_2\}$ is small. By continuity there exists a $\Upsilon_5 > 0$ such that the result holds for all $\Pr\{\psi_2\} \leq \Upsilon_5$.

Price effects: It remains to analyze $dp_{e,2}^{\psi_2,\omega_1}/dK_1^B$ and $dp_{d,1}/dK_1$, which play a key role for Result (2) and (4). From household indifference we know that it suffices to analyze one of the prices since $q^{\psi_2} = p_{e,2}^{\psi_2,\omega},\ \forall\omega$. First, if $dp_{e,2}^{\psi_2,\omega_1}/dK_1 < 0$ we find that $dp_{d,1}/dK_1 < 0$ under the sufficient condition that $\Pr\{\psi_2\}$ is small. Second, we show that $dp_{e,2}^{\psi_2,\omega_1}/dK_1 < 0$ from market-clearing at $t = 1$.

For Result (2) we have $\partial\vartheta/\partial p_{e,2}^{\psi_2,\omega_1} = \partial\vartheta/\partial k_1^B = \partial\overline{p}_{d,1}/\partial p_{e,2}^{\psi_2,\omega_1} = 0$. Equation (38) simplifies:

$$\frac{dp_{e,2}^{\psi,\omega_1}}{dK_1^B} = - \frac{\int_{\underline{\Delta}^{\psi_2}}^{\overline{\Delta}^{\psi_2}} \left[\frac{h'\left(k_1^B\right)}{\overline{p}_{d,1}} - (R-\Delta_i) + f p_{e,2}^{\psi_2,\omega_1}\left(R + \frac{\epsilon-1}{1}\Delta_i\right)\right] g^{\psi_2}(\Delta_i)\, di}{\frac{\phi(\cdot)}{\left(p_{e,2}^{\psi_2,\omega}\right)^2}\left(V^{HD,\psi_2}\right)^2 + \ f\int_{\underline{\Delta}^{\psi_2}}^{\overline{\Delta}^{\psi_2}}\left(R + \frac{\epsilon-1}{1}\Delta_i\right) K_1^B g^{\psi_2}(\Delta_i)\, di} < 0.$$

The general equilibrium price effect is purely driven by the intensive recapitalization margin.

For Result (4) we revisit the argument in Part 2 of the Proof of Proposition 4 where we argue that $dp_{e,2}^{\psi_2,\omega_1}/dK_1 < 0$ for a sufficiently steep supply schedule, which ensures that the denominator of equation (38) is positive. This concludes the proof of Proposition 5.

A.2.6 Proof of Corollary 1

After appending the bank problem in (12) by the additive term $B \cdot \mathbb{1}_{e_{2i}^{B,\psi_2,\omega_1}=0} \geq 0$, we compare the bank manager's payoff under asset and liability side recapitalizations. Note that the formulation implicitly assumes that the manager's control benefit is lost entirely whenever any equity issuance occurs. From Proposition 3, and because the Euler equations from banks' $t = 1$ problem (equations (14) and (15)) are unaltered for $e_{2i}^{B,\psi_2,\omega}, a_i^{B,\psi_2} > 0$, we know that banks will never find it optimal to simultaneously conduct asset *and* liability side recapitalizations. It therefore remains to be shown under which conditions banks find it optimal to *exclusively* recapitalize via the asset side. To this end, we compare the bank manager's payoffs under exclusive asset and liability side recapitalization. If $q^{\psi_2} = p_{e,2}^{\psi,\omega_1} < 1$ and both forms of recapitalization are feasible (i.e., if $\Delta_i \leq \widetilde{\Delta}_E^{\psi_2}, \widetilde{\Delta}_A^{\psi_2}$), banks with $\mathcal{C}_i > 0$ prefer asset side recapitalizations if and only if:

$$V_{1i}^{B,\psi_2}\left(a_i^{B,\psi_2} > 0, e_{2i}^{B,\psi_2,\omega_1} = 0\right) > V_{1i}^{B,\psi_2}\left(a_i^{B,\psi_2} = 0, e_{2i}^{B,\psi_2,\omega_1} > 0\right) \ .$$

Using *(1)* that undercapitalized banks have no spare resources for consumption in state ω_2, i.e. $c_{2i}^{B,\psi_2,\omega_2} = 0$, and *(2)* that $c_{1i}^{B,\psi_2} = 0$ for $q^{\psi_2} < 1$ (because $q^{\psi_2} < 1 \Rightarrow \lambda_{1i}^{B,\psi_2} > 1 \Rightarrow \eta_{1i}^{B,\psi_2} > 0$ from equation (15)), this can be expressed as:

$$
\begin{aligned}
& c_{2i}^{B,\psi_2,\omega_1}\left(a_i^{B,\psi_2}>0, e_{2i}^{B,\psi_2,\omega_1}=0\right)+B>c_{2i}^{B,\psi_2,\omega_1}\left(a_i^{B,\psi_2}=0, e_{2i}^{B,\psi_2,\omega_1}>0\right) \\
\Leftrightarrow \quad & B>\widehat{B}\left(\Delta_i\right) \equiv \tfrac{-n_1^B-(R-\Delta_i)k_1^B}{\Pr\{\omega_1\}p_{e,2}^{\psi_2,\omega_1}} \tfrac{(R-\Delta_i)k_1^B\left(1-q^{\psi_2}\right)}{q^{\psi_2}\sum_\omega \Pr\{\omega\}F^{\psi_2,\omega}\left(k_1^B;\Delta_i\right)-(R-\Delta_i)k_1^B}>0,\ \forall \Delta_i<R
\end{aligned}
\tag{47}
$$

where we can show that:

$$
a_i^{B,\psi_2}=\frac{-n_1^B-\left(R-\Delta_i\right)k_1^B}{q^{\psi_2}-\frac{(R-\Delta_i)k_1^B}{\sum_\omega \Pr\{\omega\}F^{\psi_2,\omega}\left(k_1^B;\Delta_i\right)}} \tag{48}
$$

in the case of asset side recapitalizations.

The distinction described in Section 5.1 and in Corollary 1 follows. This concludes the proof.

A.2.7 Proof of Corollary 2

The proof compares two scenarios. In scenario 1 all banks conduct liability side recapitalizations, e.g. if $B=0$. In scenario 2 some banks conduct asset side recapitalizations, i.e. if $\exists\Delta_i \in \left[\underline{\Delta}^{\psi_2}, \widetilde{\Delta}_A^{\psi_2}\right]$ such that $B>\widehat{B}(\Delta_i)$. We focus on the case where $B>\widehat{B}(\Delta_i)$ holds for all $\Delta_i \in \left[\underline{\Delta}^{\psi_2}, \widetilde{\Delta}_A^{\psi_2}\right]$, but the result generalizes. We take the remuneration of initial debt, $\overline{p}_{d,1}$, bank leverage, $d_1^B=h\left(k_1^B\right)/\overline{p}_{d,1}$, and household net worth, n_1^{H,ψ_2}, as given. Household net worth determines the exogenously given capital supply schedule in equation (28) with $d\chi_{\mathcal{S}}{}^{\psi_2}/dp_{e,2}^{\psi_2,\omega_1}<0$.

We next, examine the capital demand for both scenarios in turn. Recall that the interest of Proposition 1 is in an economy with a positive mass of bankruptcies, i.e. with interior solutions to the threshold levels of portfolio risk. If solvent banks always conduct liability side recapitalizations, while insolvent banks' assets are entirely divested on the market (i.e. $f=1$) we have:

$$
\chi_{D,1}^{\psi_2}\left(k_1^B, p_{e,2}^{\psi_2,\omega_1}\right) \equiv \int_{\underline{\Delta}^{\psi_2}}^{\widetilde{\Delta}_{E,1}^{\psi_2}} \mathcal{C}_i\ g^{\psi_2}\left(\Delta_i\right) di+\int_{\widetilde{\Delta}_{E,1}^{\psi_2}}^{\overline{\Delta}^{\psi_2}} \mathcal{M}_i\ g^{\psi_2}\left(\Delta_i\right) di. \tag{49}
$$

Recall that the capital shortfall is defined as $\mathcal{C}_i=\max\left\{0, h\left(k_1^B\right)/\overline{p}_{d,1}-\left(R-\Delta_i\right)k_1^B\right\}$ and that the market value of divested assets by insolvent banks is $\mathcal{M}_i \equiv q^{\psi_2}\sum_\omega \Pr\{\omega\}\left[F^{\psi_2,\omega}\left(k_{1i}^B;\Delta_i\right)\right]$ with $q^{\psi_2}=p_{e,2}^{\psi_2,\omega_1}$. Demand in this scenario increases as more banks fail, $d\chi_{D,1}^{\psi_2}/d\widetilde{\Delta}_{E,1}^{\psi_2}<0$.

In the alternative scenario all solvent banks attempt asset side recapitalizations if feasible and otherwise resort to liability side recapitalizations. The modified capital demand reads:

$$\begin{aligned}\chi_{D,2}^{\psi_2}\left(k_1^B, p_{e,2}^{\psi_2,\omega_1}\right) \equiv & \int_{\underline{\Delta}^{\psi_2}}^{\widetilde{\Delta}_A^{\psi_2}} \mathcal{M}_i \frac{d_1^B-(R-\Delta_i)k_1^B}{\mathcal{M}_i-(R-\Delta_i)k_1^B}\, g^{\psi_2}\left(\Delta_i\right) di \\ & + \int_{\widetilde{\Delta}_A^{\psi_2}}^{\widetilde{\Delta}_{E,2}^{\psi_2}} \mathcal{C}_i\, g^{\psi_2}\left(\Delta_i\right) di + \int_{\widetilde{\Delta}_{E,2}^{\psi_2}}^{\overline{\Delta}^{\psi_2}} \mathcal{M}_i\, g^{\psi_2}\left(\Delta_i\right) di.\end{aligned} \tag{50}$$

The first summand in equation (50) takes into account that solvent banks only sell the necessary fraction of their assets. We can show that $\mathcal{C}_i < \mathcal{M}_i \frac{d_1^B-(R-\Delta_i)k_1^B}{\mathcal{M}_i-(R-\Delta_i)k_1^B} = q^{\psi_2} a_i^{B,\psi_2}$ for all $\Delta_i \leq \widetilde{\Delta}_{E,2}^{\psi_2}$, where a_i^{B,ψ_2} is derived in equation (48). As in the scenario with liability side recapitalizations, this implies that demand increases as more banks fail, $d\chi_{D,2}^{\psi_2}/d\widetilde{\Delta}_{E,2}^{\psi_2} < 0$. Importantly, demand also increases if more solvent banks conduct asset side recapitalizations, $d\chi_{D,2}^{\psi_2}/d\widetilde{\Delta}_A^{\psi_2} > 0$, and demand is lower for liability side recapitalizations, $\chi_{D,1}^{\psi_2} < \chi_{D,2}^{\psi_2}$.

We next consider market clearing. Observe that $d\chi_{D,1}^{\psi_2}/dp_{e,2}^{\psi_2,\omega_1} \lesseqgtr 0, d\chi_{D,2}^{\psi_2}/dp_{e,2}^{\psi_2,\omega_1} \lesseqgtr 0$. Intuitively, a higher premium (lower $p_{e,2}^{\psi_2,\omega_1}$) decreases demand if the reduction in market pressure via a reduced number of bankruptcies (extensive recapitalization margin; $d\widetilde{\Delta}_{E,1}^{\psi_2}/dp_{e,2}^{\psi_2,\omega_1}, d\widetilde{\Delta}_{E,2}^{\psi_2}/dp_{e,2}^{\psi_2,\omega_1} > 0$) dominates, while it increases demand if the higher asset valuations by insolvent banks dominate. Following the Proof of Proposition 1 we can show there exists a single-crossing and a unique market-clearing price if the price elasticity of supply is sufficiently high. Given that $\chi_{D,1}^{\psi_2} < \chi_{D,2}^{\psi_2}$ whenever some solvent banks can conduct asset side recapitalizations at the prevailing price $p_{e,2}^{\psi_2,\omega_1}$, we have that the market clearing price is strictly lower in the second scenario. Given interiority, i.e. a positive mass of bankruptcies as assumed in Proposition 1, we have that $\widetilde{\Delta}_{E,1}^{\psi_2} > \widetilde{\Delta}_{E,2}^{\psi_2}$ since $d\widetilde{\Delta}_E^{\psi_2}/dp_{e,2}^{\psi_2,\omega_1} > 0$. This concludes the proof.

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