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by Mathias Drehmann and Nikola Tarashev

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Measuring the systemic importance of interconnected banks[1]

Mathias Drehmann and Nikola Tarashev[2]

Abstract

We develop a measure of systemic importance that accounts for the extent to which a bank propagates shocks across the banking system and is vulnerable to propagated shocks. Based on Shapley values, this measure gauges the contribution of interconnected banks to systemic risk, in contrast to other measures proposed in the literature. An empirical implementation of our measure reveals that systemic importance depends materially on the bank's role in the interbank network, both as a borrower and as a lender. We also find substantial differences between alternative measures, which implies that prudential authorities should be careful in choosing the underlying approach.

Keywords: Systemic risk, Shapley values, Interbank positions

JEL Classification: C15, G20, G28, L14.

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1 Introduction

A commonly held view is that interconnectedness is a key driver of systemic importance. Surprisingly, the literature has produced few concrete insights that underpin this view, even though it may shape regulatory requirements for systemically important institutions. In this paper, we provide a conceptual framework for analysing the relationship between interconnectedness and systemic importance and investigate this relationship empirically.

We explore two different approaches to measuring systemic importance: one related to banks' *participation* in systemic events and another one to their *contribution* to systemic risk. Even though the two approaches decompose the same quantum of system-wide risk, they allocate it differently across banks. In this sense, they adopt different concepts of systemic importance.

The first approach has been popularised by a number of recent articles that measure systemic importance as the expected losses generated by a bank in systemic events. These events are characterised by system-wide aggregate losses exceeding a critical level, thus leading to disruptions in the real economy. Since this approach equates systemic importance with the expected *participation* of individual banks in systemic events, we label it the participation approach (PA).

A bank's participation in systemic events is conceptually different from its contribution to systemic risk. For example, a bank that has small positions vis-à-vis non-banks will impose only small direct losses on the real economy and, thus, will participate little in systemic events. The same bank, however, might have large positions on the interbank market. It can thus contribute materially to systemic risk by transmitting distress from one bank to another.

The contribution approach (CA), which is rooted in the Shapley value methodology, targets directly banks' contribution to systemic risk. This methodology was first proposed by Shapley (1953) for the allocation of the value created in cooperative games across individual players. Tarashev et al (2010) showed how Shapley values can be used to measure systemic importance when banks are *not* connected in an interbank network.

In order to measure banks' contribution to systemic risk in the presence of an interbank network, it is necessary to modify CA. Namely, it is necessary to account explicitly for the fact that a bank contributes to systemic risk not only via its exposure to exogenous shocks but also by propagating such shocks through the system and by being itself vulnerable to propagated shocks. We propose to do this in a generalized contribution approach (GCA), which is the key methodological innovation of our paper.

Our empirical analysis of stylised banking systems and a system of 20 large globally active banks leads to two main conclusions. First, the structure of the interbank network and banks' role in this network are quantitatively important drivers of the systemic importance of individual banks. Taking the interbank network into account raises the measured levels of systemic importance. And the rise is greater for banks with greater interbank market activity. Importantly, systemic importance is influenced by direct but also by indirect interbank linkages, which are captured only by a holistic approach to the system.

Second, the choice of a particular approach to measuring systemic importance matters not only from a conceptual but also from an empirical point of view. Particularly pronounced in the presence of interbank linkages, this finding underscores how important it is that prudential authorities choose the approach that is in line with their concept of systemic importance. Concretely, PA assigns a higher (lower) degree of systemic importance to an interbank lender (borrower) than GCA. The reason for this is that PA attributes the risk associated with an interbank transaction entirely to the lending counterparty, ie the counterparty that bears this risk and can eventually transfer it onto its creditors in a systemic event. By contrast, GCA splits this risk equally between the two counterparties. In this way, GCA captures the idea that the systemic importance of an interconnected bank depends not only on the risk it imposes directly on the real economy, but also on the risk it imposes on other banks in the system.

The rest of the paper is organised as follows. In Section 2, we review briefly the related literature. Then, we present the analytic setup in Section 3: the measure of systemic risk, the Shapley value methodology, and alternative applications of this methodology. In Section 4, we describe empirical implementation of the analytic setup. In Section 5, we analyse stylised systems that help us build intuition for how the interbank network structure affects systemic risk and the systemic importance of individual banks. In Section 6, we use this intuition to analyse a system of 20 large banks. We conclude with Section 7.

2. Literature review

This paper bridges the literature on banks' systemic importance and that on interbank networks. A measure of systemic importance, proposed by Huang et al (2010), Acharya et al (2009) and Brownlees and Engle (2010), has recently gained in popularity.[3] These papers first compute the system-wide loss distribution and define a set of systemic events, which are states of the world that occur with a small probability but in which aggregate losses exceed a critical threshold. The systemic importance of a particular bank is then set equal to the expected losses it generates, conditional on systemic events. In other words, systemic importance is measured as the expected *participation* of individual institutions in systemic events. We thus will refer to this approach as the *participation approach*. The measure corresponding to this approach can be interpreted as the actuarially fair premium that each institution should pay to a (hypothetical) provider of insurance against system-wide losses in a systemic event.

In order to capture a different concept of systemic importance – the *contribution* of institutions to systemic risk – Tarashev et al (2010) propose an approach based on Shapley values. A bank contributes to systemic risk not only through losses that it imposes on non-banks but also by affecting the probability and severity of the losses generated by other banks in systemic events. And in contrast to the participation approach, the *contribution approach* proposed by Tarashev et al (2010) captures this idea directly. It does so by focusing on each subsystem – or subgroup of banks that belong to the entire system – and calculating the difference between the risk of this subsystem with and without a particular bank. Averaging such marginal risk contributions across all possible subsystems delivers the systemic importance of the bank under the contribution approach.

In their empirical application Tarashev et al (2010) highlight that bank size, institution-specific probabilities of default and exposures to common risk factors interact in a non-linear fashion to determine the systemic importance of financial institutions. In a related paper, Staum (2010) uses Shapley values to design a deposit insurance scheme in the presence of fire-sale externalities and mergers. The insights of these papers are however incomplete because, just like Acharya et al (2009) and Huang et al (2010), they do not consider explicitly the interbank *network* structure as a driver of systemic importance.

[3] Another strand of the literature gauges systemic importance by the impact that the failure of (or distress at) one bank has on the rest of the system. An often cited measure from this literature is CoVaR, which has been popularised by Adrian and Brunnermeier (2008). In this paper, we abstract from this and related measures as they do not attempt to allocate systemic risk to individual institutions and, thus, are not additive across institutions. That said, such measures can be easily implemented in the empirical framework we develop below. This is illustrated by Drehmann and Tarashev (2011).

By now, there is a large literature – surveyed recently by Upper (2011) and Allen and Babus (2009) – on how linkages in an interbank network influence systemic risk. A number of the theoretical and empirical findings of this literature, as well as some of the methodological challenges faced by it, re-emerge in our analysis below. An example is the finding, first shown by Allen and Gale (2000), that the interbank network determines the extent of contagion in the system and, thus, has a first-order impact on system-wide risk.

The large size of the interbank network literature notwithstanding, we are aware of only two papers that measure the systemic importance of interconnected institutions. In the first one, Gauthier et al (2009) use the approach of Tarashev et al (2010) in a system of five Canadian banks. But as we show below, this approach does not handle correctly the risk that banks impose on each other through interbank linkages. As a result, the approach needs to be modified materially in order to truly capture the extent to which interconnected banks contribute to systemic risk. In the second paper, Liu and Staum (2010) do tackle challenges related to the measurement of systemic importance in the presence of an interbank network. The approach they propose is different from the one we adopt below, requires complex linear programming techniques and is applied in extremely stylized settings.

3. Systemic risk and systemic importance

In this section, we lay out the analytic framework. First, we present the system and the notion of risk we use throughout the paper. Second, we define our measure of system-wide risk: expected shortfall. Third, we outline the Shapley value methodology as a general tool for attributing system-wide risk to individual institutions. Finally, we specify two alternative attribution procedures, which are special cases of the Shapley value methodology.

3.1 System-wide risk

Let the system be a set N comprised of n banks, indexed by $i \in \{1,2,\cdots,n\}$. On the asset side of these banks' stylised balance sheets, there are claims on non-banks and other banks in the system. On the liability side, there are debt securities held by other banks or non-banks, as well as equity held by non-banks. The value of the bank's assets undergoes stochastic shocks that translate into changes of its equity. Once assets fall below debt liabilities, equity is zero and the bank defaults. In this case, debt holders incur credit losses.

Throughout the paper, we adopt the perspective of a prudential authority, who is only concerned about credit losses vis-à-vis the rest of the economy. Thus, we consider only the risk associated with losses incurred by the *non-bank* creditors of the *n* banks. Concretely, we denote by L_i^N the stochastic loss incurred by the non-bank creditors of bank i in system N. That said, we consider the risk faced by bank creditors and equity holders *indirectly*, as it affects the risk of non-bank creditors.

Not accounting for equity risk directly is reasonable from a *public policy* perspective because a core function of equity is to absorb losses. That said, we make the strong assumption that a positive equity value, no matter how small, allows the bank to function normally and does not imply any systemic repercussions. Clearly, shocks to equity are important from a bank's *own* risk management perspective, even if they are not a concern to a prudential authority.

In addition, our system-wide perspective implies that we *cannot* consider losses on interbank exposures directly. Since the interbank liabilities of one bank are the interbank assets of another, losses to the interbank creditors of one bank are ultimately incurred by the equity holders or non-bank creditors of one or more other banks in the system. As a result, including interbank losses in the definition of L_i^N would have involved double counting at the level of the overall system.

3.2 Measuring risk in the system

We adopt a popular measure of *tail* risk: expected shortfall (ES). At an intuitive level, ES is the expected value of aggregate losses above a certain level. When applied to any subsystem, ie a subset of the overall system, ES can be stated more formally as follows:

$$ES\left(N^{sub}\right) = E\left(\sum_{i\in N^{sub}} L_i^{N^{sub}} \mid \sum_{i\in N^{sub}} L_i^{N^{sub}} \geq q\right) \equiv E\left(\sum_{i\in N^{sub}} L_i^{N^{sub}} \mid e\left(N^{sub}\right)\right) \text{ for any } N^{sub} \subseteq N \quad (1)$$

where q is some, typically high, quantile of the distribution of aggregate losses in the system.[4] In turn, $e\left(N^{sub}\right)$ is the set of loss configurations that deliver aggregate losses equal to or greater than q. When the focus is on the whole system, this set of loss configurations, $e(N)$, is referred to as *systemic events*.

3.3 Shapley values

Used as measures of systemic importance, Shapley values are portions of system-wide risk that are attributed to individual institutions. And because Shapley values are additive, the sum of these portions across the banks in the system equals exactly the level of system-wide risk. In the next subsection, we outline the Shapley value methodology in its general form, which can be applied to a wide variety of settings and under very weak conditions. Then, we outline two specific applications of the methodology when system-wide risk is measured via ES. Even though they decompose the same quantum of system-wide risk, they attribute different portions of this risk to the same bank. In this sense, they adopt different concepts of the systemic importance of individual banks.

3.3.1 General specification[5]

At the heart of Shapley values is the so-called characteristic function ϑ. In the context of a banking system, ϑ maps any subsystem $N^{sub} \subseteq N$ into a measure of risk. This function needs to satisfy two weak conditions. First, it should be defined on each of the 2^n subsystems of banks.[6] Second, when ϑ is applied to the entire system, it should coincide with the chosen measure of system-wide risk. Thus, given equation (1), $\vartheta(N) = ES(N)$.

For a given function ϑ, the Shapley value of bank *i* is a weighted average of the increments of risk that this bank generates as it joins any possible subsystem comprised of other banks. Denote the risk of subsystem $N^{sub} \subseteq N$ by $\vartheta\left(N^{sub}\right)$ and the risk in that subsystem without bank *i* by $\vartheta\left(N^{sub} - i\right)$. Then, the formal expression for the Shapley value, $ShV_i(N)$, of bank *i* in system *N* is:

$$ShV_i(N) = \frac{1}{n} \sum_{n_S=1}^{n} \frac{1}{c(n_s)} \sum_{\substack{N^{sub} \supset i \\ |N^{sub}| = n_S}} \left(\vartheta\left(N^{sub}\right) - \vartheta\left(N^{sub} - i\right)\right) \quad (2)$$

In this expression, $N^{sub} \supset i$ are all the subsystems $N^{sub} \subseteq N$ that contain bank *i*, $|N^{sub}|$ stands for the number of banks in subsystem N^{sub}, and $c(n_s) = (n-1)!/(n-n_s)!(n_s-1)!$ is

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[4] Even though it specifies ES in intuitive terms, expression (1) is not precise enough to be applied to general stochastic settings. For our numerical analysis, we use the precise definition of Gordy (2003).

[5] This section draws heavily on Mas-Colell et al (1995). Shapley values were first introduced by Shapley (1953).

[6] These subsystems are: *Ø, {1}, {2}, {3}, …, {n}, {1,2}, {1,3}, …, {n-1,n}, …, {1,2,3,…,n}*.

the number of subsystems that contain bank *i* and are comprised of n_s banks. In addition, the empty set carries no risk: $\vartheta(\varnothing)=0$.

For a given characteristic function ϑ, the Shapley values form the *unique* set of measures of systemic importance that satisfy a number of appealing and important properties. We mention here two of these – the additivity and fairness properties – while the rest are discussed at length in Tarashev et al (2010). The additivity property states that the sum of Shapley values equals *exactly* the aggregate measure of systemic risk. Indeed, it can be verified that equation (2) implies that $\sum_{i=1}^{n} ShV_i(N) = \vartheta(N) = ES(N)$.

In turn, the fairness property states that the increment of the Shapley value of bank *i* that is due to the presence of bank *k* in the system equals the increment of the Shapley value of bank *k* that is due to the presence of bank *i*. Formally:

$$ShV_i(N) - ShV_i(N-k) = ShV_k(N) - ShV_k(N-i) \text{ for all } i,k \in N \tag{3}$$

While the fairness property in expression (3) holds for any underlying characteristic function ϑ, the *value* of the increments on each side of the expression does change with ϑ. In other words, the choice of a characteristic function determines the extent to which the Shapley value of bank *i* depends on the risk generated by bank *k* in the system (left-hand side of (3)). And, since banks are treated symmetrically, the particular characteristic function also determines the extent to which the Shapley value of bank *i* reflects the impact of this bank – through an interbank link, for example – on the Shapley value of bank *k* (right-hand side of (3)). The fairness property plays a key role in explaining our empirical results. Once we have introduced the specific characteristic functions we consider, we will therefore come back to this property and concretise it further.

We implement two different applications of the Shapley value methodology, which deliver two different measures of systemic importance when the metric of system-wide risk is ES. The differences between the two applications are entirely due to the underlying characteristic functions. The first function we consider defines the *participation approach*, which has been quite popular in the literature. The second characteristic function delivers Shapley values that gauge the *contribution* of individual banks to systemic risk. This function generalises the contribution approach presented in Tarashev et al (2010) to the case of interconnected banks, thus defining the *generalised contribution approach*.

A key difference between both approaches emerges in the presence of an interbank network. Take an interbank link that raises systemic risk by some increment. As we explain below, the participation approach essentially allocates this increment entirely to the interbank lender. The contribution approach, by contrast, treats the interbank lender and borrower symmetrically and splits the incremental increase in systemic risk equally between them. This example shows that the two approaches embed different concepts of systemic importance.

3.3.2 Participation approach (PA)

As discussed in Section 2, a number of recent papers measure a bank's systemic importance by the expected losses it generates in systemic events. In the notation of equation (1), systemic events, *e(N)*, are states of the world in which aggregate losses generated by the banks in the *whole* system exceed a particular threshold: $\sum_{i \in N} L_i^N > q$. Thus, the proposed measure of systemic importance can be expressed as: $E\left(L_i^N \mid \sum_{i \in N} L_i^N > q\right) = E\left(L_i^N \mid e(N)\right)$, which is the expected participation of bank *i* in systemic events, hence our label participation approach (PA). Note that $E\left(L_i^N \mid e(N)\right)$ is exactly the actuarially fair premium that bank i would have to pay to a scheme that insures non-bank creditors against losses in systemic events, *e(N).*

Interestingly, PA is a special application of the Shapley value methodology. To see this, let us define the characteristic function, ϑ^{PA} as

$$\vartheta^{PA}\left(N^{sub}\right) \equiv E\left(\sum_{i \in N^{sub}} L_j^N \mid e(N)\right) \text{ for any } N^{sub} \subseteq N \qquad (4)$$

Expressions (1) and (4) imply that $\vartheta^{PA}(N) = ES(N)$, as required.

The key feature of ϑ^{PA} is that it keeps two important inputs constant across subsystems. First, irrespective of the subsystem, N^{sub}, ϑ^{PA} employs conditioning events that coincide with the systemic events $e(N)$, which are determined at the level of the whole system. Second, again irrespective of the subsystem, ϑ^{PA} considers the losses, L_i^N, that non-bank creditors of bank i experience when the whole system is in place. This is despite the fact that the risk of bank i depends on its interbank linkages and these change typically with the subsystem.

Equation (4) implies that, under ϑ^{PA}, the increment of risk that a bank generates as it joins a subsystem is the same across all subsystems, as $\vartheta^{PA}\left(N^{sub}\right) - \vartheta^{PA}\left(N^{sub} - i\right) = E\left(L_i^N \mid e(N)\right)$. Since a Shapley value is a weighted average of such increments, it follows that:

$$ShV_i\left(N; \vartheta^{PA}\right) = ShV_i\left(N^{sub}; \vartheta^{PA}\right) = E\left(L_i^N \mid e(N)\right) \quad \textit{for all } i \in N^{sub} \textit{ and all } N^{sub} \subseteq N \qquad (5)$$

which is the expected participation of bank i in systemic events (see above).

The first equality in equation (5) means that, under ϑ^{PA}, the Shapley value of a bank remains the same even if it is evaluated for a subsystem of banks. This leads to the following expression of the fairness property. For *each* $i, k \in N$:

$$ShV_i\left(N; \vartheta^{PA}\right) - ShV_i\left(N - k; \vartheta^{PA}\right) = ShV_k\left(N; \vartheta^{PA}\right) - ShV_k\left(N - i; \vartheta^{PA}\right) = 0 \qquad (6)$$

To see what this means more concretely, consider a case in which bank i has a credit exposure to bank k. Also assume that this interbank link creates risk for the non-bank creditors of bank i because of contagion from bank k to i. By the second equality in equation (6), however, ϑ^{PA} fails to associate this risk (even partly) with the interbank borrower, bank k. As a result, Shapley values under ϑ^{PA} attribute the entire risk created by the interbank link to the lender, bank i.

3.3.3 Generalised contribution approach (GCA)

It is possible to move away from PA and closer to the original idea behind the Shapley value methodology. In the light of Section 3.3.1, this means that we should measure the risk that a bank generates on its own as well as this bank's contributions to the risk in each subsystem of other banks. In terms of the general specification in equation (2), this suggests that, when we evaluate the risk of a subsystem from which bank i is excluded, we cannot simply consider the risk that this bank generates when the *entire* system is in place. We thus resort to the characteristic function of the second, generalised contribution approach (GCA):

$$\vartheta^{GCA}\left(N^{sub}\right) \equiv E\left(\sum_{i \in N^{sub}} L_i^{N^{sub}} \mid e\left(N^{sub}\right)\right) \text{ for any } N^{sub} \subseteq N \qquad (7)$$

At the level of the entire system N, ϑ^{GCA} and ϑ^{PA} coincide, as $\vartheta^{GCA}(N) = \vartheta^{PA}(N) = ES(N)$. But the allocation of this system-wide risk differs between the two approaches for two important reasons. First, ϑ^{GCA} allows the stochastic losses incurred by the non-bank creditors of bank i to depend on the subsystem considered: $L_i^{N^{sub}}$. Of course, such dependence is redundant if there are no interbank links and a bank can fail only because of shocks coming from outside the system. This is the setting in which Tarashev et al (2010) propose their contribution approach. However, when there is an interbank network, the absence of some banks from a particular subsystem implies the absence of some interbank links and, ultimately, the absence of some sources of risk for the banks in the subsystem. This implies that bank-specific losses do depend on which other banks are in the subsystem, as reflected in ϑ^{GCA}. Gauthier et al (2010) abstract from this issue and, as a result, fail to equate the removal of a bank from a (sub)system with the removal of the entire risk that this bank generates. In Annex 1, we formulate their approach with our notation and show that it produces measures of systemic importance that can differ materially from those implied by GCA.

To capture the dependence of bank-level losses on the subsystem, we implement GCA as follows. First, we assume that banks inside any given subsystem replace their exposures to banks outside this subsystem with a *risk-free* asset. Second, we make a similar assumption on the liability side: banks inside any given subsystem replace their liabilities to banks outside this subsystem by borrowing from outside the system. Thus, excluding a bank from a (sub)system removes the entire risk that bank generates in this (sub)system.

The second difference between ϑ^{GCA} and ϑ^{PA} is due to the fact that ϑ^{GCA} incorporates conditioning events, $e\left(N^{sub}\right)$, that change with the subsystem. Thus, in contrast to ϑ^{PA}, ϑ^{GCA} measures risk as the expected shortfall in each subsystem: $\vartheta^{GCA}\left(N^{sub}\right) = ES(N^{sub})$. This leads to the following special case of the general Shapley value formula in (2):

$$ShV_i\left(N, \vartheta^{GCA}\right) = \frac{1}{n}\sum_{n_S=1}^{n}\frac{1}{c\left(n_s\right)}\sum_{\substack{N^{sub}\ni i \\ |N^{sub}|=n_S}}\left(ES\left(N^{sub}\right) - ES\left(N^{sub} - i\right)\right) \tag{8}$$

The two distinctive features of ϑ^{GCA} lead to a insightful fairness property of Shapley values under GCA. This comes to the fore if the simultaneous presence of two banks $i, k \in N$ raises systemic risk (which occurs when these banks have strictly positive sizes and PDs and positively correlated assets). ϑ^{GCA} then assigns a higher Shapley value to either bank in the entire system than in a system excluding the other bank. Moreover, the increment of systemic risk that is caused by the joint presence of the two banks is split into equal halves, which *GCA* attributes to each of these banks. Or formally:

$$ShV_i\left(N; \vartheta^{GCA}\right) - ShV_i\left(N-k; \vartheta^{GCA}\right) = ShV_k\left(N; \vartheta^{GCA}\right) - ShV_k\left(N-i; \vartheta^{GCA}\right) > 0 \tag{9}$$

The strict inequality indicates that, in contrast to ϑ^{PA}, ϑ^{GCA} does convey the extent to which one bank affects the riskiness of another. And in order to see a concrete implication of this, we again consider the case in which bank *i* has a credit exposure to bank *k*. Equation (9) implies that ϑ^{GCA} captures the contribution of this interbank link to systemic risk and splits it equally between banks *i* and *k*. By extension, a Shapley value under ϑ^{GCA} attributes the risk created by an interbank link equally to the interbank lender and the interbank borrower.

4 Empirical implementation

The methodology outlined so far can be applied to any probability distribution of losses, as long as it is well-defined for each subsystem. To specify a particular distribution, we start with the following expression for of the stochastic losses associated with bank *i* in N^{sub}:

$$L_i^{N^{sub}} = s_i \cdot LGD_i \cdot I_i \tag{10}$$

In this expression, LGD_i, or loss given default loss, is the share of non-banks' credit exposure to bank *i* that is lost if this bank defaults. In turn, s_i denotes the size of bank *i*. In line with the discussion in Section 3.1, s_i equals the debt liabilities of bank i to non-banks, which we normalise by the aggregate level of non-bank liabilities in the system. This in turn implies that all our measures of systemic risk and banks' importance are also expressed per unit of aggregate system size. Finally, $I_i = 1$ if bank i defaults and $I_i = 0$ otherwise. As stated above, a bank defaults if and only if its total assets are lower than its non-equity liabilities.

We distinguish two types of bank defaults. First, a *fundamental* default is triggered solely by adverse shocks to a bank's claims on nonbanks, which we refer to as "exogenous shocks". Exogenous shocks, X_i, are driven by common and idiosyncratic factors. We assume that the factors are mutually independent normal variables, denoted by M_j and Z_i, respectively:

$$X_i = \rho_i \cdot M + \sqrt{1-\rho_i^2} Z_i \qquad \text{for all } i \in \{1,2,\cdots,n\} \tag{11}$$

We refer to the probability of a fundamental default as the *fundamental PD*. Higher common factor loadings, $\rho_i \in [0,1]$, lead to a higher probability of joint fundamental defaults.

Second, a *contagion* default occurs if a bank has survived the shocks to its nonbank assets but is pushed into default by the default of one (or more) of its bank obligors in subsystem $N^{sub} \subseteq N$. We call the probability of such a default *contagion PD*. Thus, the overall PD of a bank equals its fundamental PD plus its contagion PD.

When banks fail, positions are not netted. We assume that there are bankruptcy costs, which reduce the value at default of a bank's debt liabilities, vis-à-vis both non-banks and other banks, by a fraction α.[7] We also assume that nonbank creditors are senior to bank creditors.[8]

This setup defines the probability distribution of system-wide losses for our empirical analysis. In order to build intuition for the role of the interbank network as a driver of banks' systemic importance, we first analyse hypothetical and highly stylised systems. We then consider networks based on actual data for 20 large internationally active banks. The two types of banking systems differ in terms of the calibrated parameters, such as bank size and PD (which we discuss in detail below), but are similar in terms of the simulation procedure.

We implement the simulation procedure in the following steps. In the light of equation (11), we start by drawing a set of exogenous shocks (to claims on non-banks), which determines whether one or several banks experience a fundamental default. Then, we account for contagion defaults via the "clearing algorithm" of Eisenberg and Noe (2001). In line with the discussion in Section 3.3, we apply this algorithm only once in the case of the PA (i.e. when the network of the entire system is in place) and 2^n times (i.e. for each subsystem in a system of n banks) in the case of GCA. To construct a probability distribution of losses, we draw one million sets of exogenous shocks. For the calculation of ES, we set q to the 99th percentile of this distribution. Finally, we set bankruptcy costs to α = 20%.[9]

7 Bankruptcy costs are likely to affect non-bank and bank creditors differently. This can be easily implemented in the model without changing the main insights of the paper.

8 Nonbank creditors are senior to banks in several countries, such as Germany (see Upper, 2009).

9 The insights of the paper do not depend on the choice of α, as long as bankruptcy costs are not extreme. For a sample of failed US banks, James (1990) estimates that losses on bank assets amount to around 30% on average. This contains direct losses as well as loss of charter value and at least 10 percentage points of administrative and legal expenses.

5 Hypothetical Networks

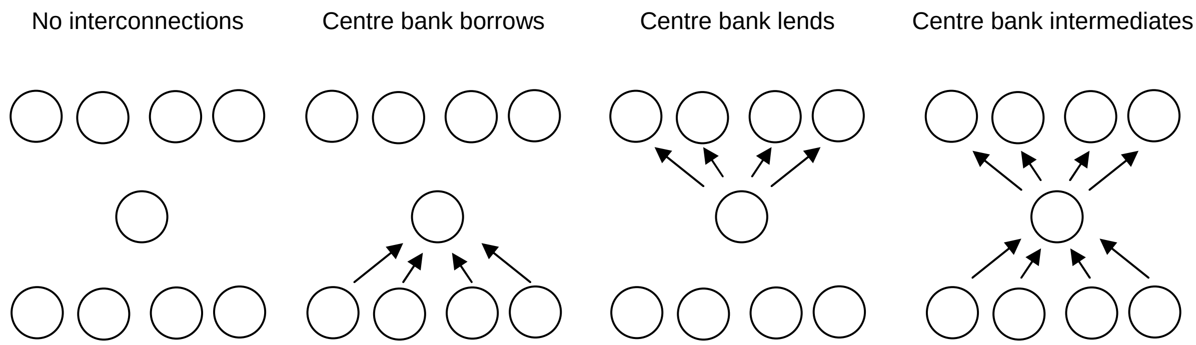
In this section, we first outline the design of hypothetical and highly stylised interbank networks. Then, we study how these networks affect the systemic importance of individual banks under the generalised contribution approach. Finally, we compare the implications of GCA and PA in the stylized settings.

5.1 Design of hypothetical networks

We assume that a hypothetical system is comprised of 9 banks and we explore four different interbank networks: a system with no interbank connections as well as three different network structures. The four setups are portrayed in Graph 1. In each of the three setups with interbank connections there is one bank with a central role, henceforth the centre bank. In the first two of these networks, four periphery banks either borrow from or lend to the centre bank, while the remaining four banks do not participate in the interbank market. The last network captures in a stylised fashion the real-life phenomenon in which the centre bank intermediates between periphery banks, four of which borrow and four of which lend to it.

Graph 1

Hypothetical interbank networks



Balance sheets in the different systems are shown in Table 1. In all cases, banks have 5 units of equity and borrow 87 units from nonbanks. This means that they are of the same size (recall Section 3.1). Periphery banks have 8 units of interbank liabilities (if they borrow from the centre bank) or 8 units of interbank assets (if they lend), which fully determines the interbank positions of the centre bank. The resulting share of interbank positions in a periphery (centre) bank's balance sheet is close to the mean (maximum) of the corresponding shares in our sample of 20 large banks (see Section 6.1 below).

We draw exogenous shocks (to non-bank assets) in line with the calibration scheme outlined in Section 4. The common factor loading ρ_i is the same across all banks and corresponds to the average common factor loading of 0.67 in our data on 20 large banks (see below). In addition, shocks are calibrated such that the (fundamental) PD of each bank in the system with no interbank linkages is 0.42%, halfway between the median and mean PD estimates for the same 20 banks. The fundamental PD of each bank is the same in each of the other stylised systems.

Table 1

Balance sheet of hypothetical banks

			No interbank connections			Centre bank borrows			Centre bank lends			Centre bank intermediates		
	EQ	NBL	IBL	IBA	TA	IBL	IBA	TA	IBL	IBA	TA	IBL	IBA	TA
No IB connection	5	87	0	0	92	0	0	92	0	0	92			
PB lender	5	87				0	8	92	0	0	92	0	8	92
PB borrower	5	87				0	0	92	8	0	100	8	0	100
CB	5	87				32	0	124	0	32	92	32	32	124
CB as CCP	3	0										32	32	35

Note: IB: interbank market; PB: periphery bank; CB: centre bank; CCP: central counterparty; EQ: equity; NBL: non-bank liabilities; IBL: interbank liabilities; IBA: interbank assets; TA: total assets.

5.2 Systemic importance under different network structures

A priori, the network structure should affect both the absolute and relative levels of systemic importance of individual banks. If all banks are ex ante the same, the network that is more vulnerable to contagion defaults should lead to higher levels of system-wide ES and thus to higher (uniform) Shapley values. In a system of heterogeneous banks, however, it seems intuitive that the most interconnected bank – in the stylised examples, the centre bank – has the highest Shapley value.

Table 2 shows the results related to the stylised systems. Not surprisingly, the system without an interbank network does not experience contagion defaults and, as a result, has the lowest ES. In comparison, ES is higher in the two systems where the centre bank either borrows from or lends to the periphery and, thus, contagion defaults are possible. In the first case, the centre bank is the only source of contagion risk, whereas in the second it is exposed to multiple sources of such risk from the periphery. Since all else is kept the same, the multiple sources of contagion risk give rise to a higher "aggregate" probability of contagion defaults (0.51% vs. 4 x 0.10%) and, ultimately, to a higher system-wide ES. Finally, ES is highest when the centre bank acts as an intermediary between periphery banks. In this case there is an additional channel of shock propagation, as the default of one (or several) borrowing banks in the periphery can be transmitted, via the default of the centre bank, to lending banks in the periphery.

More interestingly, Table 2 also shows how the network structure and banks' position in it affect their systemic importance. For concreteness, we focus only on GCA Shapley values in this subsection (second to last column in the table). As anticipated above, all banks feature their lowest Shapley values in the system without an interbank network, when system-wide ES is at its lowest. Across the three setups with interbank linkages, the Shapley value of the centre bank is always higher than that of any periphery bank. While this is intuitive, it is not obvious when looking purely at contagion PDs. The system in which the centre bank only borrows from the periphery is a case in point. Here, the centre bank is a source of risk for periphery banks but can never default because another bank has defaulted, i.e. its contagion PD is zero. Nonetheless, the GCA Shapley value of the centre bank is more than 60% larger than the Shapley value of each interbank lender.

This result is driven by the fairness property of Shapley values. As discussed in Section 3.3.3, this property implies that, when an interbank link raises systemic risk, GCA splits the rise equally between the borrower and the lender. This can be easily seen in the stylised systems, where PDs, correlations and size are constant across banks. For example, switching from the system without a network to the system in which the centre bank only borrows from the periphery raises the system-wide ES by 0.94 percentage points (from 4.01% to 4.95%). Attributing this rise equally to borrowers and lenders would mean attributing 0.5*0.94 = 0.47 to the centre bank and (0.5*0.94)/4 = 0.1175 to each of the four lending periphery banks. This matches almost perfectly the actual increases in Shapley values, which are 0.45 (= 0.9-0.45) for the centre bank and 0.11 (= 0.56-0.45) for the periphery banks.[10] The same intuition holds if the centre bank acts only as an interbank lender or if it intermediates.

Table 2

Results for the hypothetical banking[1]

				Shapley Values	
	ES	**f.PD[2]**	**c.PD[2]**	**GCA**	**PA**
No interconnections	4.01				
All (9 banks)		0.42	0	0.45	0.45
Centre bank borrows	4.95				
Centre bank		0.42	0	0.90	0.64
PB lender (4 banks)		0.42	0.10	0.56	0.66
No IB connection (4 banks)		0.42	0	0.45	0.42
Centre bank lends	5.16				
Centre bank		0.42	0.51	1.06	1.63
PB borrower (4 banks)		0.42	0	0.57	0.48
No IB connection (4 banks)		0.42	0	0.45	0.40
Centre bank intermediates	7.73				
Centre bank		0.42	0.51	2.06	1.78
PB lender (4 banks)		0.42	0.29	0.71	1.00
PB borrower (4 banks)		0.42	0	0.71	0.49
Centre bank as CCP	4.85				
Centre bank		0	0.19	0.22	0
PB lender (4 banks)		0.42	0.07	0.58	0.70
PB borrower (4 banks)		0.42	0	0.58	0.52

Note: [1] All values are in per cent. ES and Shapley values are expressed per unit of system size. ES pertains to the system as a whole. All other values pertain to a bank in the particular group. [2] f.PD and c.PD are fundamental and contagious PDs, respectively

When the centre bank intermediates between periphery banks (Table 2, fourth panel), its Shapley value is larger than the *sum* of its Shapley values when it only lends or borrows. The reason is that an intermediating centre bank creates indirect links between periphery banks. As a result, an adverse shock to an interbank borrower in one part of the system can cause the default of an interbank lender in another part. This highlights that the systemic

[10] The match is not exact because we calculate the rise in Shapley values by comparing two systems that differ in the underlying interbank network and thus are underpinned by different loss distributions.

importance of an institution can be correctly measured only when both direct and indirect linkages are taken into account in a holistic perspective on the system.

5.3 Comparing GCA and PA: Borrowers vs. lenders

PA and GCA can generate materially different Shapley values in the presence of an interbank network because of the different underlying concepts of systemic importance. And the differences are due to the different treatment by the two approaches of the counterparties in interbank links. As explained in Section 3.3.2 above, PA tends to attribute the risk generated by an interbank link to the lender. By contrast, GCA treats the two counterparties of an interbank link symmetrically and, thus, splits the associated risk equally between them. This lowers (raises) the GCA Shapley value of an interbank lender (borrower) relative to the corresponding PA value.

The last two columns in Table 2 illustrate this point. In *all* the stylised systems, an interbank borrower (lender) has a higher (lower) Shapley value under GCA than under PA. This does not only affect the relative size of Shapley values across approaches but also the rank ordering of systemic importance of different banks. For example, if the centre bank only borrows from the periphery, PA Shapley values underplay this bank's role as a propagator of shocks and attribute lower systemic importance to it than to interbank lenders in the periphery (0.64% vs. 0.66%). By contrast, the centre bank has the highest GCA Shapley value in that system (0.9%). This reflects the fact that, by being the *only* propagator of shocks, this bank is the main contributor to systemic risk.

It is instructive to note that the stylised interbank networks we consider may *depress* the Shapley values of banks that *do not* participate in the interbank market. Under PA, the Shapley value of each bank in the system without interbank linkages is higher (0.45% of the system size) than that of banks that do not participate in the interbank market when the centre bank lends (0.40%) or borrows (0.42%). To see why, note that the introduction of interbank linkages raises the probability of extreme losses because the probability of joint defaults of interconnected banks is higher than that of unconnected banks. The impact of introducing a network is then twofold: a higher probability of extreme losses in systemic events, and greater participation in these events by banks that end up with interbank linkages. The flipside is that banks that stay unconnected participate less in the systemic events when a network is introduced, which lowers these banks' PA Shapley values (see Section 3.3.2). That said, the introduction of an interbank network does not seem to affect the contribution of unconnected banks to systemic risk and their GCA Shapley values remain virtually the same.

5.4 Comparing GCA and PA: Intermediaries

It is also of interest to examine the relative sizes of the Shapley values that GCA and PA attribute to a centre bank that *intermediates*. Even though both approaches allocate risk differently to lenders and borrowers, the results from the previous subsection suggest that the two Shapley values could be similar, as the intermediating bank lends and borrows exactly the same amounts in the interbank market. However, the system in question possesses a feature that was absent from the systems examined above. Namely, the centre bank *creates* indirect exposures between periphery lenders and periphery borrowers. Then, by the fairness property in equation (9), part of the risk stemming from these exposures raises the GCA Shapley value of the centre bank. By contrast, and in line with the analysis in the previous section, PA attributes this risk entirely to periphery lenders. In the end, the PA Shapley value of the intermediating bank is lower than the GCA value (1.78% vs. 2.06%).

The difference between PA and GCA becomes particularly stark, when we assume that the centre bank acts as a central counterparty (CCP) and thus *only intermediates* in the interbank market but neither lends to nor borrows from nonbanks. By the definition in Section 3.1, the centre bank is then of zero size.[11]

Table 2 (bottom panel) reports risk measures for the system and individual banks when the centre bank acts as a CCP. Since it does not lend to nonbanks, the centre bank has a fundamental PD of zero, which lowers system-wide ES from 7.73% to 4.85%. In addition, since it does not borrow from nonbanks, the centre bank does not contribute directly to losses by non banks and, thus, does not *participate* in systemic events. This results in a PA Shapley value of zero. But, at an intuitive level, this bank does *contribute* to systemic risk because it creates indirect links between lending and borrowing periphery banks. This underpins a positive GCA Shapley value of 0.22%.

While this is a stylised example, it highlights a key conceptual difference between the approach capturing *participation* in systemic events and that capturing *contribution* to systemic risk. The example also illustrates that there could be substantial quantitative differences between the measurements of the systemic importance of individual institutions under the two approaches. In turn, this emphasises the need for policy makers to carefully choose a measure, which is in line with their concept of systemic importance.

6 Analysing the systemic importance of 20 large banks

In this section, we build on the insights of the stylized hypothetical networks in order to analyse the systemic importance of 20 large internationally active banks. Before delving into the analysis, we first describe our data.

6.1 Data for the system of 20 large banks

To calibrate the model for the system of 20 large banks, we rely on two sources. First, we obtain balance sheet information from Bankscope. Second, Moody's KMV provide us with the PDs of individual banks and pair-wise correlations of banks' asset returns.

All balance sheet data are from end-2009. Table 3 reports that the largest bank in our system (bank C) is 3.5 times larger than the smallest (bank G). The table also shows the shares of each bank's interbank assets (*IBA* = loans and advances to banks) and interbank liabilities (*IBL* = deposits from banks) in the bank's total assets and liabilities, respectively. Both *IBA* and *IBL* average approximately 10% across banks.

However, these shares provide incomplete information about the network of interlinkages among the 20 banks. First, total interbank positions of any of these banks include positions vis-à-vis banks that we abstract from. Second, part of the actual interbank links stem from off-balance sheet positions, which cannot be identified in our data. When interpreting the results below, it should be kept in mind that the former implies that we may overstate the importance of the network, whereas the latter implies that we may understate it.

The derivation of the interbank network is further complicated by the lack of publicly available information on bilateral interbank exposures. As most of the literature, we address this issue by constructing a maximum entropy (ME) matrix of bilateral exposures. An ME matrix satisfies two conditions simultaneously. First, the sum of the entries in each row / column corresponding to a particular bank equals the aggregate level of this banks' liabilities / assets

[11] The balance sheet of the centre bank acting as CCP is reported in Table 1 (last row).To satisfy the balance sheet identity, we assume in this case that the centre bank invests 3 units in a risk-free assets. The balance sheets of the periphery banks remain the same as in the network where the centre bank intermediates.

vis-à-vis the other banks in the system.[12] Second, the interbank assets and liabilities of each bank are distributed as uniformly as possible across the other banks in the system.

Table 3

Descriptive statistics of the 20 large internationally active banks

	Bank																			
	A	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**	**J**	**K**	**L**	**M**	**N**	**O**	**P**	**Q**	**R**	**S**	**T**
Size	5.6	5.5	8.2	5	3.1	6.4	2.3	6.6	6.8	4.9	5.1	4.7	4.8	7.7	4.4	4.1	3.8	3.9	3.5	3.6
IBA	9.6	13.4	4.3	21	12.6	21.7	20.8	3.1	7.9	3.7	18.6	3.4	3.8	5.4	7.2	6.6	5.8	10.3	8.4	3.2
IBL	13	20.8	11.4	9.2	17.3	9	25.1	3.1	5.9	7.5	14.2	8.4	4	8.9	8.3	9.5	1.9	5	12.4	2.3

Note: size: liabilities to nonbanks divided by total non-bank liabilities, in per cent; IBA: interbank assets divided by total assets, in per cent; IBL: interbank liabilities divided by total liabilities, in per cent.

The second assumption is clearly ad hoc. Therefore, we also randomly simulate other interbank matrixes that are consistent with the observed data.[13] In particular, we consider 225 random perturbations around the ME matrix and choose the one that differs the most (in terms of the 2-norm distance) from the ME matrix. Since 287 out of the 380 off-diagonal entries of this matrix are equal to zero, we refer to it as a high-concentration (HC) matrix. Similar to the hypothetical networks, we also consider the zero matrix, which rules out interbank positions.

We set bank-level PDs to the average value of each bank's monthly *one-year* expected default frequency (EDF), as estimated by Moody's KMV for 2006-2009. These PDs average 0.6% in the cross section, have a median of 0.25% and a standard deviation of 0.8%.

When working with the correlation matrix of banks' asset returns, we make two choices. First, we treat this matrix as equal to the correlation matrix of the returns on banks' non-bank assets. In principle, the correlation matrix should reflect the fact that the co-movement of banks' asset values would be caused by (i) exogenous common factors affecting assets vis-à-vis non-banks but *also* by (ii) interbank linkages. That said, since we model the latter linkages as driven exclusively by interbank *credit* exposures and banks' PDs are quite low, the impact of these linkages on asset-return correlations turns out to be negligible.[14] Second, we impose a single-common-factor structure on the correlation matrix (see Tarashev and

[12] In our system of 20 banks the sum of interbank assets is not equal to the sum of interbank liabilities as these banks have interbank exposures vis-à-vis banks that we do not consider. In order to work with an internally consistent matrix of interbank positions, we create a "sink bank" that absorbs excess amount of interbank assets or liabilities. We assume that this bank does not default and abstract from its potential losses on the interbank market.

[13] We use the RAS algorithm to derive the ME solution, which in turn uses the relative entropy matrix as a prior. As explained in Upper (2011), this matrix assumes that the exposure of bank i to bank j is $x_{i,j}=a_i{*}l_j$ if i≠j and 0 for i=j, where a_i (l_j) are the normalised total interbank assets (liabilities). We generate random interbank matrices by treating each entry of the prior matrix as a uniform variable distributed between zero and twice its initial value. In addition, we randomly restrict off-diagonal entries to be equal to zero. We then apply the RAS algorithm to this modified prior and only consider matrices for which the algorithm converges.

[14] While asset-return correlations estimated by Moody's KMV range between 0.30 and 0.60, network interlinkages increase correlations by roughly 0.01.

Table 4

Systemic risk and systemic importance in a system of 20 large banks

In per cent

Bank	f.PD rank	No interbank network ES = 3.2 median PD = 0.15			ME matrix ES = 4.3 median PD = 0.25			HC matrix ES = 4.4 median PD = 0.30		
			Shapley values			Shapley values			Shapley values	
		c.PD	GCA	PA	c.PD	GCA	PA	c.PD	GCA	PA
A	6	0	0.28	0.29	0.05	0.35	0.27	0.05	0.36	0.32
B	13	0	0.07	0.07	0.14	0.22	0.22	0.08	0.21	0.17
C	16	0	0.04	0.05	0.06	0.09	0.14	0.05	0.12	0.12
D	3	0	0.43	0.45	0.11	0.53	0.47	0.17	0.52	0.46
E	4	0	0.16	0.11	0.18	0.27	0.21	0.14	0.27	0.20
F	19	0	0.00	0.00	0.14	0.13	0.19	0.16	0.13	0.21
G	20	0	0.00	0.00	0.12	0.06	0.06	0.05	0.02	0.03
H	7	0	0.32	0.34	0.07	0.35	0.39	0.11	0.36	0.45
I	17	0	0.02	0.03	0.06	0.05	0.11	0.06	0.06	0.11
J	9	0	0.10	0.10	0.07	0.13	0.14	0.11	0.15	0.17
K	12	0	0.06	0.07	0.09	0.15	0.15	0.22	0.25	0.26
L	15	0	0.04	0.05	0.05	0.06	0.08	0.03	0.06	0.07
M	2	0	0.53	0.57	0.05	0.54	0.53	0.03	0.53	0.44
N	5	0	0.51	0.51	0.07	0.60	0.58	0.07	0.56	0.57
O	18	0	0.01	0.01	0.04	0.02	0.04	0.04	0.03	0.04
P	10	0	0.09	0.09	0.09	0.14	0.15	0.04	0.11	0.11
Q	1	0	0.39	0.36	0.07	0.38	0.32	0.04	0.37	0.32
R	11	0	0.05	0.04	0.13	0.09	0.12	0.26	0.13	0.19
S	14	0	0.03	0.03	0.06	0.06	0.07	0.04	0.05	0.05
T	8	0	0.10	0.07	0.02	0.10	0.07	0.02	0.10	0.08

Note: f.PD: fundamental PD; c.PD: contagion PD; GCA: generalised contribution approach; PA: participation approach; ME: maximum entropy; HC: high concentration. Shapley values are expressed per unit of system size.

Zhu (2008)). This structure is not necessary for measuring systemic risk and systemic importance but has the important expositional advantage of allowing us to describe the commonality of banks' non-bank exposures via 20 bank-specific common-factor loadings. For our numerical results, we start with three correlation matrices – for 2006, 2007 and 2009

– which lead to three sets of common-factor loadings.[15] Then, we assign to each bank its average common-factor loading, corresponding to ρ_i in equation (11). The common factor loadings we work with have a mean of *0.67*, which corresponds roughly to the average correlation of $0.67^2 = 0.45$.

The final element of our calibration scheme pertains to the exogenous shocks in equation (11). We calibrate the mean and variance of these shocks to be such that the frequencies of first-round defaults plus the frequencies of contagion defaults, caused by the propagation of shocks to non-bank assets through the interbank network *under the ME matrix*, match *exactly* the PDs we derive from the Moody's KMV data. Keeping the distribution of the shocks fixed allows us to study how different network structures, as implied by the zero or the HC matrix, affect banks' PDs.

For each of the three interbank networks, Table 4 reports fundamental and contagion PDs. As we are not allowed to freely publish bank specific PDs from Moody's KMV, we only report the rank of fundamental PDs (1 = highest and 20 = lowest). The values of these PDs remain the same across networks. By contrast, contagion PDs, for which we report actual values, do change with the structure of the network.

6.2 Drivers of systemic importance

Before we explore the impact of the three different interbank networks, we briefly look at how other risk parameters affect a bank's systemic importance. It turns out that, in the absence of an interbank network, the fundamental PD is the main driver of systemic importance in the system we consider. To illustrate this, we rank-order the 20 banks according to the values of each of the three drivers and according to their Shapley values (Table 5). The orderings of fundamental PDs and Shapley values are almost identical. By contrast, the other two drivers provide virtually no information about systemic importance. It should be emphasized, however, that this is a feature of the particular network we consider, rather than a general result (Tarashev et al (2010)).

Table 5

Shapley values and drivers of systemic importance: no network

	Bank																			
	A	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**	**J**	**K**	**L**	**M**	**N**	**O**	**P**	**Q**	**R**	**S**	**T**
GCA	6	11	14	3	7	19	20	5	17	9	12	15	1	2	18	10	4	13	16	8
f.PD	6	13	16	3	4	20	19	7	18	9	12	15	2	5	17	10	1	11	14	8
size	6	7	1	9	19	5	20	4	3	10	8	12	11	2	13	14	16	15	18	17
CF	15	4	3	17	13	1	6	9	7	10	11	8	16	5	14	2	20	19	12	18

Note: GCA Shapley values and drivers are ranked from 1 = highest to 20 = lowest within each category. GCA: generalised contribution approach; f. PD: fundamental PD; CF: loading on the common risk factor.

[15] Moody's KMV does not provide correlations for 2008.

6.2.1 The impact of the network

In this section, we examine how system-wide risk and the systemic importance of individual banks change when we replace the zero matrix of interbank exposures with the ME matrix. As with the stylised systems, we initially focus on GCA Shapley values only.

Not surprisingly, the introduction of interbank linkages gives rise to positive contagion PDs, raising system-wide ES by 30%, from 3.2% under the zero matrix to 4.3% (Table 4). Since the interbank network is a material driver of systemic risk, it elevates the systemic importance of almost all the banks in the system (compare left to middle and right-hand panels of Table 4). And the impact is strongest on banks that are most active in the interbank market. Concretely, banks B, F and K, which feature the highest levels of interbank positions (see Table 3), also experience the greatest rises in Shapley values when the zero interbank matrix is replaced by the ME matrix.

At the other extreme is the *negative* impact of the interbank network on the Shapley values of banks M and Q, which have the lowest interbank positions in the system. Featuring the two highest PDs, these banks have the highest and fourth highest Shapley values in the absence of a network. When a network is introduced, however, the ES of the system rises and the rise is underpinned mostly by losses stemming from the joint failures of the highly interconnected banks. This squeezes out banks M and Q as participants in systemic events, which depresses their PA Shapley values. The impact on GCA Shapley values is more muted. The same result, arising for similar reasons, was recorded in Section 5.3 for the case of non-connected banks in stylised systems.

As already frequently pointed out, the fairness property implies in this case that the increase in ES due to the interbank network should be split equally between borrowers and lenders on the interbank market. Thus, the quantum of risk attributed to the group of borrowers / lenders should be allocated pro rata across individual banks in each group. This insight can be captured by a simple network impact indicator *NII*:

$$NII_{ME,i} = 0.5 * \left(ES_{ME} - ES_0\right) * \frac{IBL_i}{\sum_{j=1}^{20} IBL_j} + 0.5 * \left(ES_{ME} - ES_0\right) * \frac{IBA_i}{\sum_{j=1}^{20} IBA_j} \tag{12}$$

The subscript of ES indicates the network matrix under which the measure is derived; IBL and IBA denote interbank liabilities and assets, respectively.

The first two column in Table 6 shows the results of a simple OLS regression in which *NII* is used as an explanatory variable for the difference between banks' GCA Shapley values under the ME matrix and those under the zero matrix.[16] In both cases, the coefficient of *NII* is positive and highly significant, implying that a bank's Shapley value is higher when this bank participates more in the interbank market, either as a borrower or as a lender.

The fit of these regressions is high but far from perfect. As suggested by the analysis of the stylized systems in Section 5, the fit would have been much better had banks differed only in terms of the amount of their interbank lending and borrowing. In the system of 20 banks that we are considering, however, banks also differ in terms of their size, riskiness, the riskiness of their counterparties in the interbank market and the concentration of their interbank positions.

[16] We use OLS regressions despite the nonlinear impact of different drivers on Shapley values. Our goal is simply to check for significant relationships with the expected sign.

Table 6

Regression results

	GCA_{ME}-GCA_0	GCA_{HC}-GCA_0	GCA_{ME}-GCA_0	GCA_{HC}-GCA_0	GCA_{ME}-GCA_{HC}	PA_{HC}-GCA_{HC}
NII	1.21***	1.32***				
LI_{nwk} [(1)]			12.14***	7.81***		8.16***
BI_{nwk} [(1)]			8.36***	12.32***		-9.38***
LI_{ME} - LI_{HC}					0.15**	
BI_{ME} - BI_{HC}					0.42***	
R^2	0.78	0.71	0.85	0.91	0.66	0.76
# obs	20	20	20	20	20	20

Note: All regressions include a constant, which is sometimes significant but is not reported. The regression of PA_{HC}-GCA_{HC} also includes f.PD as a control variable, which is significant. [(1)] *nwk* is either ME for the regression of GCA_{ME}-GCA_0 or HC for the other two regressions.

6.2.2 The impact of alternative network structures

A priori, the sign of the difference between the system-wide ES under the ME matrix and that under the HC matrix is ambiguous. To understand why, first recall that the ME and HC matrix distribute the aggregate interbank lending and borrowing of each bank differently across potential counterparties. Whereas the ME matrix assumes that banks aim to spread their interbank positions as widely as possible, the HC matrix is constructed so that these positions are as concentrated as possible. Thus, if two banks are connected under both matrices, the shock caused by the default of one bank would tend to be propagated more strongly to the other under the HC matrix, as the particular bilateral exposure would tend to be larger. On its own, this would increase the probability of joint defaults under the HC matrix relative to that under the ME matrix. In turn, this would make the difference between the respective values of ES positive. On the other hand, each shock is propagated to more institutions under the ME than under the HC matrix. On its own, this would have the opposite effect on the relative levels of ES. In our particular case, the two effects roughly balance each other out, resulting in an ES of 4.30% of the system size under the ME and 4.37% under the HC matrix.

Despite the similar levels of system-wide risk, the two network structures differ materially in their implications for the systemic importance of individual banks. For example, the Shapley value of bank K is 66% higher when interbank markets are captured by the HC matrix (0.25) rather than the ME matrix (0.15) (Table 4, last two last two panels). Largely, this increase is largely driven by the substantial rise in the bank's contagion PD (from 0.09% to 0.2%), which in turn is caused by the greater concentration of its interbank exposures under the HC matrix. In contrast, the HC matrix concentrates the exposures of bank G to low-PD counterparties, which lowers this bank's overall PD and, ultimately, its Shapley value.

In line with the results for the hypothetical systems, the impact of the network structure goes beyond banks' PDs. The systemic importance of a bank may change from one network structure to another not because of a change in its individual riskiness but because of a change in how it affects the riskiness of other banks. This is illustrated by bank C, which would experience a 30% *rise* in its GCA Shapley value (from 0.09% of the system size to 0.12%), even though its contagion PD would *fall* from 0.06% to 0.05% if the interbank network changed from the ME to the HC matrix. Having the second largest level of interbank liabilities, this bank is a key propagator of shocks through the system. In turn, the GCA

Shapley value captures the fact that the impact of bank C on the riskiness of other banks is stronger in a more concentrated system.

In order to study the impact of the interbank network on systemic importance more formally, we construct two additional indicators.[17] The first indicator gauges how the interbank network affects the extent to which a bank is a *direct* source of risk for its non-bank creditors. Given that the contagion PD of bank *i* reflects the impact of the network on this bank's riskiness and that the impact of this bank's default on non-banks is measured by its size, we construct the following lending indicator (*LI*):

$$LI_{nwk,i} = c.PD_{nwk,i} * s_i, \tag{13}$$

where *nwk* indicates that the network is either defined by the ME or HC matrix. We expect that the difference between the Shapley values under the ME and the HC matrices are related positively to the difference in the respective lending indicators, $LI_{ME,i} - LI_{HC,i}$.

By borrowing on the interbank market, each bank is also an *indirect* source of risk for non-bank creditors. Our second indicator captures the impact of the interbank network structure on this type of risk via two components. The first component captures the impact of each bank *i* on the riskiness of each other bank *j*. Ideally, we should measure this impact by assigning a portion of the contagion PD of bank *j* to bank *i*. Given that there could be several rounds of defaults in the system, this portion is extremely difficult to derive. As a proxy, we use the product of the share of bank *i* in the total interbank lending of bank *j* and the contagion PD of bank *j*: $(IBA_{j,i} / IBA_j)*c.PD_j$. The second component of the indicator reflects the fact that the indirect impact of bank *i* on nonbank creditors would be higher if this bank borrows from a bank *j* that itself borrows heavily from non-banks, ie if bank *j* has a large size, s_j. Putting the two components together and summing them across all counterparties of bank *i*, we obtain the following borrowing indicator (BI_i):

$$BI_{nwk,i} = \sum_{j \neq i} \frac{IBA_{j,i}^{nwk}}{IBA_j} * c.PD_{nwk,j} * s_j \; . \tag{14}$$

As with the lending indicator, we expect that the difference between the Shapley values under the ME matrix and those under the HC matrix are related positively to the difference between the borrowing indicators, $BI_{ME,i} - BI_{HC,i}$.

The third and forth column of Table 6 show that BI_{nwk} and LI_{nwk} can indeed explain the impact of the network structure on the systemic importance of individual banks. Considering only HC or ME at a time, the corresponding indicators have positive and highly significant coefficients. In addition, the fit of the regression is now better than that when NII_{nwk} is used as an explanatory variable. More importantly, the differences $LI_{ME,i} - LI_{HC,i}$ and $BI_{ME,i} - BI_{HC,i}$ help explain the difference between ME and HC Shapley values (fifth column, Table 6). Both coefficients are with the expected positive sign, are highly significant and imply a good fit.

6.2.3 Comparing GCA and PA

Our analysis of hypothetical networks (above) reveals that the Shapley values under GCA and under PA can lead to substantially different conclusions as regards the systemic importance of individual institutions. Concretely, PA Shapley values are larger than GCA

[17] Even though we construct NII_{HC} *as well, it is roughly equal to* NII_{ME} and, thus, cannot help us distinguish the implications of the ME matrix from those of the HC matrix. NII_{HC} and NII_{ME} are roughly equal for two reasons. First, the two network structures give rise to similar ES. Second, *NII* reflects *aggregate* interbank positions, which are the *same* under both the ME and HC matrix.

ones for an interbank lender, and smaller for an interbank borrower. In this section, we obtain a similar result for the system of 20 real-world banks.

We start the comparison between PA and GCA Shapley values by zooming onto two banks and the HC matrix. The first one is bank E, which has an average *LI* in the cross-section but the third highest *BI*. Thus, this bank imposes an average level of risk on its direct nonbank creditors. But, being a significant source of contagion in the interbank market, it imposes significant risk to other banks. Since only GCA captures the latter characteristic of bank K, it leads to a Shapley value that is by one-third higher than the bank's PA Shapley value (0.27% vs. 0.20%). The relationship between GCA and PA Shapley values is reversed for the second bank, R, which has the third highest *LI* and the third lowest *BI*. Its high *LI* indicates that it is quite vulnerable to risk from the interbank market. This makes bank K participate a lot in systemic events, boosting its PA Shapley value. GCA, however, attributes part of this risk to banks borrowing from bank R. The upshot is that PA attributes a 50% higher level of systemic importance to bank R than GCA: 0.19% of the system size, as opposed to 0.13%.

To systematize such results across all 20 banks, we focus on the HC matrix and use *BI* and *LI* as explanatory variables in a regression of the difference between PA and GCA Shapley values.[18] Table 6 reveals that the two indicators do possess statistically significant explanatory power.[19] The positive sign of the *LI* coefficient indicates that, as anticipated, a PA Shapley value tends to be higher than the corresponding GCA Shapley value when the bank creates systemic risk mainly via its role as a lender on the interbank market. Likewise, the negative coefficient of *BI* confirms that GCA tends to assign a higher Shapley value than PA to an interbank borrower.

7 Conclusion

In this paper we provide a framework for analysing the systemic importance of interconnected banks. We explore two approaches to measuring systemic importance, one reflecting a bank's participation in systemic events and the other its contribution to systemic risk. Both approaches decompose the same quantum of system-wide risk. But they allocate it differently across banks, thus revealing different underlying concepts of systemic importance.

A key difference between the participation and contribution approach is the way in which they allocate risk associated with an interbank transaction. The participation approach assigns this risk to the lending counterparty, which is the ultimate absorber of the risk. In contrast, the general contribution approach splits the risk equally between the two counterparties. In this way, the general contribution approach captures the idea that the systemic importance of an interconnected bank depends not only on the risk it imposes directly on the real economy, but also on the risk it generates by both borrowing from and lending to other banks.

Our findings highlight that different approaches can lead to materially different measures of systemic importance. This indicates that, when designing regulatory requirements for systemically important institutions prudential authorities should be careful in choosing the approach that corresponds to their desired concept of systemic importance.

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18 The results are qualitatively the same under the ME matrix, but regression coefficients are less significant.

19 The difference between PA and GCA can, in principle, also be affected by banks' PD, size and exposure to common exogenous risk factors (see Tarashev et al (2010)). Fundamental PDs are included as a control variable in the regressions because they exhibit statistically significant explanatory power. The associated coefficient is, however, not reported in the table. The other two drivers do not exhibit statistical significance and are, therefore, dropped from the regressions.

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Annex 1: Mechanical contribution approach

It is possible to design Shapley values that decompose system-wide ES differently from the participation approach (PA) and the generalised contribution approach (GCA), presented in Section 3.3. Gauthier et al (2010) do so by adopting the following characteristic function:

$$\vartheta^{MCA}\left(N^{sub}\right) \equiv E\left(\sum_{i \in N^{sub}} L_i^N \mid e\left(N^{sub}\right)\right). \tag{A1}$$

This characteristic function has two important features. First, like ϑ^{GCA}, ϑ^{MCA} allows the conditioning events, $e\left(N^{sub}\right)$, to change across subsystems. Second, like ϑ^{PA}, ϑ^{MCA} considers only the losses that bank i generates in the entire system, L_i^N. We refer to this approach as the mechanical contribution approach (MCA).

The second feature of ϑ^{MCA} has two implications. First, MCA is computationally more efficient than GCA. This is because the former approach requires that the probability distribution of losses be calculated only once (for the overall system), whereas the latter requires the calculation of a loss distribution for each of the 2^n subsystems. Second, MCA does not capture fully the true contribution of a bank to systemic risk. The reason is that, by keeping losses fixed across subsystems, ϑ^{MCA} cannot equate the removal of a bank from a (sub)system with the removal of the entire risk that this bank generates. And this creates a potential for material differences between the implications of MCA and those of GCA.

We use the hypothetical networks outlined in Section 5.1 to investigate the differences between MCA Shapley values and Shapley values under PA and GCA. The findings, reported in Table A1, reveal significant differences between the GCA and MCA Shapley values. Concretely, when the centre bank intermediates, the absolute differences between the MCA and GCA Shapley values equal on average one-third of GCA Shapley values. And the results are similar across all hypothetical systems. That said, MCA Shapley values are much better aligned with PA Shapley values. In the light of equations (4), (7) and (A.1), this indicates that the assumption about stochastic losses (L_i^N vs. $L_i^{N^{sub}}$) has a larger impact on the implications of characteristic functions than the assumption about conditioning events ($e(N)$ vs. $e\left(N^{sub}\right)$).

Table A1

MCA results for the hypothetical banking systems

	No	**CB borrows**			**CB lends**			**CB intermediates**			**CB as CCP**		
	All	CB	PB lend	No	CB	PB borrow	No	CB	PB lend	PB borrow	CB	PB lend	PB borrow
GCA	0.45	0.9	0.56	0.45	1.06	0.57	0.45	2.06	0.71	0.71	0.22	0.58	0.58
PA	0.45	0.64	0.66	0.42	1.63	0.48	0.4	1.78	1	0.49	0	0.7	0.52
MCA	0.45	0.62	0.64	0.44	1.36	0.51	0.44	1.56	1.02	0.52	0	0.87	0.34

Note: All values are in per cent. CB = Centre bank; PB = periphery bank; No = no interbank market linkages. “lend” and “borrow” indicate whether periphery banks are interbank lenders or borrowers. All systems are the same as those in Table 2.