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REBCO4

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Viktorija Maskalenko REBCO4

▶ Prove that the expression (18.112) yields the *n*th Laguerre polynomia.

Evaluating the nth derivative in (18.112) using Leibnitz' theorem we find

$$L_n(x) = \frac{e^x}{n!}\sum_{r=0}^{n} {}^nC_r \frac{d^r x^n}{dx^r}\frac{d^{n-r}e^- x}{dx^n - r}$$

$$= \frac{e^x}{n!}\sum_{r=0}^{n} \frac{n!}{r!(n-r)!}\frac{n!}{(n-r)!}x^{n-r}(-1)^{n-r}e^{-x}$$

$$= \sum_{r=0}^{n}(-1)^{n-r}\frac{n!}{r!(n-r)!(n-r)!}x^{n-r}.4$$

Relabeling the summation using the index m = n - r, we obtain

$$L_n(x) = \sum_{m=0}^{n}(-1)^m \frac{n!}{(m!)^2(n-m)!}x^m,$$

which is precisely the expression (18.111) for the nth Languerre polynomal. ◀

Matual orthogonality

In sectin 17.4, we noted than Laguarre's aquation could be put into Sturm-Liouville from whith $p = xe^{-x}$, $q = 0$, $\lambda = v$ and $\rho = e^{-x}$, and its natural interval is thus $[0, \infty]$. Since the Laguerre polynomials $L_n(x)$ ara solutions of the equation and are regular at the end-points, they must be mutually orthogonal over this interval with respect tothe weight function $\rho = e^{-x}$ i.e

$$\int_0^\infty L_n(x)e^{-x}dx = 0 \text{ if } n \neq k$$

This result may also be proved directly using the Rodrigues' formula (18.112).
Indeed, the normalisation, when $k = n$, is most easly found using this method.

Show that

$$I = \int_0^\infty L_n(x)L_n(x)e{-}xdx = 1$$

Using the Rodrigues' formula (18.112), we may write

$$I = \frac{1}{n!}\int_0^\infty L_n(x)\frac{d^n}{dx^n}(x^n e^{-x})dx = \frac{(-1)^n}{n!}\int_0^\infty \frac{d^n L_n}{dx^n}x^n e^{-x}dx,$$

where, in the second equality, we have integrated by parts n timse and used the fact that the boundaty terms all vanish. When $d^n L_n/dx^n$ is evaluted using (18.111), only the derivate of the $m = n$ term survives and that has value $[(-1)^n n!n!]/[(n!)^2 0!] = (-1)^n$. Thus we have

$$I = \frac{1}{n!}\int_0^\infty x^n e^{-x}dx = 1$$

where, in the section equary, we use the expression (18.153) defining the gamma function (see section 18.12). ◀

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\documentclass{article}
\usepackage[utf8]{inputenc}
\usepackage[english]{babel}
\usepackage{xcolor}
\usepackage{setspace}
\usepackage[utf8]{inputenc}
\usepackage[english]{babel}
\usepackage[a4paper,total={6in,9in}]{geometry}
\usepackage[papersize={21cm,29.7cm}]{geometry}
\usepackage{amssymb}
\title{REBCO4}
\author{vikamaskalenko }
\date{March 2019}
\maketitle
\begin{document}
Viktorija Maskalenko REBCO4

\includegraphics[width=\textwidth,height=10cm]{1819_108_C4_734.jpg}

\newpage
\hspace{110pt}
28.7 LANGUAGE FUNCTION
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\parindent=0cm
\hrule



\vspace{5mm}
\fcolorbox{black!100}{gray!15}{\begin{minipage}{40em}
\blacktriangleright
\textbf{ Prove that the expression (18.112) yields the $n$th Laguerre polynomia.}
\end{minipage}}
\vspace{0,5mm}
\headsep = 5pt
\par Evaluating the nth derivative in (18.112) using Leibnitz' theorem we find

$$L_n(x) = \frac{e^x}{n!}\sum_{r=0}^n ^nC_r\frac{d^rx^n}{dx^r}\frac{d^\text{n-r} e^-x}{dx^n-r}$$
$$= \frac{e^x}{n!}\sum_{r=0}^n\frac{n!}{r!(n-r)!}\frac{n!}{(n-r)!}x^\text{n-r}(-1)^\text{n-r}e^\te
$$=\sum_{r=0}^n(-1)^\text{n-r}\frac{n!}{r!(n-r)!(n-r)!}x^\text{n-r}.4$$
Relabeling the summation using the index m = n - r, we obtain
$$L_n(x)=\sum_{m=0}^n (-1)^m\frac{n!}{(m!)^2(n-m)!}x^m,$$
which is precisely the expression (18.111) for the $n$th Languerre polynomal.
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\blacktriangleleft

\vspace{4mm}
\hspace{160pt}
\onehalfspacing
\textit{Matual orthogonality}

In sectin 17.4, we noted than Laguarre's aquation could be put into Sturm-Liouville from whith $p =

\\
\vspace{4mm}
\begin{center}
$\int^\infty_0 L_n(x)e^\text{-x}dx=0$  if $n\neq k$
\end{center}
\vspace{4mm}


This result may also be proved directly using the Rodrigues' formula (18.112).

Indeed, the normalisation, when $k=n$, is most easly found using this method.

\fcolorbox{black!100}{gray!15}{\begin{minipage}{40em}
\textit{Show that}
\textbf{$$I = \int^\infty_0 L_n(x)L_n(x)e\text{-x}dx=1$$}
\end{minipage}}
\vspace{3mm}

Using the Rodrigues' formula (18.112), we may write
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where, in the second equality, we have integrated by parts $n$ timse and used the fact that the bo
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where, in the section equary, we use the expression (18.153) defining the gamma function (see secti
\blacktriangleleft
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