① а) $a_n = \frac{(-1)^n}{n} \xrightarrow[n\to\infty]{} 0$, но не е монотона

б) Не постои. Секоја монотоно опаѓачка и ограничена низа е конвергентна.

② $f(x) = \frac{3x^4+2}{4x^3-1}$, $g(x) = \sin x$

а) $\lim\limits_{x\to\infty} \frac{3x^4+2}{4x^3-1} \cdot \sin\frac{1}{x} = \lim\limits_{x\to\infty} \overbrace{\left(\frac{3x^4+2}{(4x^3-1)x}\right)}^{\to\frac{3}{4}} \cdot \overbrace{\left(\frac{\sin\frac{1}{x}}{\frac{1}{x}}\right)}^{\to 1} = \frac{3}{4}$

б) $h(x) = \frac{1}{g(x)} = \frac{1}{\sin x}$

$D_h:\ \sin x \neq 0 \Rightarrow x \neq k\pi \Rightarrow D_h = \mathbb{R} \setminus \{k\pi \mid k\in\mathbb{Z}\}$

$h'(x) = -\frac{\cos x}{\sin^2 x} = 0 \Rightarrow \cos x = 0 \Rightarrow \boxed{x = \frac{\pi}{2} + k\pi,\ k\in\mathbb{Z}}$ – во овие точки има хоризонтални тангенти

$k=1 \Rightarrow x_0 = \frac{\pi}{2} + \pi = \frac{3\pi}{2} \in [\pi, 2\pi]$ $\boxed{t:\ y=-1}$ – тангента во точката $(\frac{3\pi}{2}, -1)$

$y_0 = \frac{1}{\sin x_0} = \frac{1}{\sin\frac{3\pi}{2}} = \frac{1}{-1} = -1$

③ $f(x) = \begin{cases} 4-x, & 0 \le x < 3 \\ \ln(x^2-8)+1 & 3 \le x \le 4 \end{cases}$

а) $x=3$ – можен прекин

$f(3) = \ln(9-8)+1 = \ln 1 + 1 = 1$

$f(3^+) = \lim\limits_{x\to3^+}\left(\ln(x^2-8)+1\right) = 1$

$f(3^-) = \lim\limits_{x\to3^-}(4-x) = 1$

$f(3) = f(3^+) = f(3^-) \Rightarrow f$ е непр. во $x=3$

б) $f'_+(3) = \lim\limits_{\Delta x\to0^+} \frac{f(\overbrace{3+\Delta x}^{>3}) - f(3)}{\Delta x} = \lim\limits_{\Delta x\to0^+} \frac{\ln((3+\Delta x)^2-8) + \cancel{1} - \cancel{1}}{\Delta x} =$

$= \lim\limits_{\Delta x\to0^+} \frac{\ln(9+6\Delta x+(\Delta x)^2-8)}{\Delta x} =$

$= \lim\limits_{\Delta x\to0^+} \frac{\ln(1+(6\Delta x+(\Delta x)^2))}{6\Delta x+(\Delta x)^2}^{\to 1} \cdot \frac{6\Delta x+(\Delta x)^2}{\Delta x}$

$= \lim\limits_{\Delta x\to0^+} \frac{\cancel{\Delta x}(6+\Delta x)}{\cancel{\Delta x}} = 6$

$f'_-(3) = \lim\limits_{\Delta x\to0^-} \frac{f(\overbrace{3+\Delta x}^{<3}) - f(3)}{\Delta x} = \lim \frac{4-(3+\Delta x)-1}{\Delta x}$

$= \lim\limits_{\Delta x\to0^-} \frac{-\Delta x}{\Delta x} = -1$

$f'_+(3) \neq f'_-(3) \Rightarrow f$ не е дифер. во $x=$

f не ги задоволува условите на Т. на Рол бидејќи f не е дифер. во $x = 3 \in [0,4]$ и $4 = f(0) \neq f(4) = \ln 8 + 1$

5) а) $I=\int \frac{dx}{x(\sqrt{x-4}+2)} = \left\{ \begin{matrix} x-4=t^2 \\ dx=2t\,dt \end{matrix} \right\} = 2\int \frac{t\,dt}{(t^2+4)(t+2)}$

$$\frac{t}{(t+2)(t^2+4)} = \frac{A}{t+2} + \frac{Bt+C}{t^2+4} \quad / \cdot (t+2)(t^2+4)$$

$$t = A(t^2+4) + (Bt+C)(t+2)$$

$$t = \underline{At^2} + 4A + \underline{Bt^2} + 2Bt + Ct + 2C$$

$A+B=0 \qquad A=-\frac{1}{4},\ B=\frac{1}{4},\ C=\frac{1}{2}$

$2B+C=1$

$4A+2C=0$

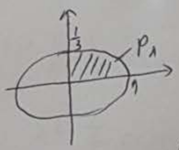
$$I = 2\int \frac{t\,dt}{(t^2+4)(t+2)} = 2\left[\int \frac{-\frac{1}{4}}{t+2}dt + \int \frac{\frac{1}{4}t+\frac{1}{2}}{t^2+4}dt\right]$$

$$= -\frac{1}{2}\int \frac{dt}{t+2} + 2\cdot\frac{1}{8}\int \frac{2t+4}{t^2+4}dt$$

$$= -\frac{1}{2}\ln|t+2| + \frac{1}{4}\int \underbrace{\frac{2t\,dt}{t^2+4}}_{\text{смена: } t^2+4=s} + \frac{1}{4}\cdot 4\int \frac{dt}{t^2+4} =$$

$$= -\frac{1}{2}\ln|t+2| + \frac{1}{4}\ln(t^2+4) + \frac{1}{2}\,\mathrm{arctg}\,\frac{t}{2} + C, \quad t=\sqrt{x-4}$$

б) $x^2+9y^2=1 \Rightarrow \frac{x^2}{1} + \frac{y^2}{\left(\frac{1}{3}\right)^2} = 1 \Rightarrow a=1,\ b=\frac{1}{3}$



$\left\{ \begin{matrix} x = a\cos t = \cos t \\ y = b\sin t = \frac{1}{3}\sin t \end{matrix} \right.,\ t\in[0,4\pi)$

$$P = 4P_1 = 4\int_{\pi/2}^{0} y\cdot\dot{x}\,dt = 4\frac{1}{3}\int_{\pi/2}^{0}\sin t\cdot(-\sin t)\,dt = -\frac{4}{3}\int_{\pi/2}^{0}\sin^2 t\,dt = \frac{4}{3}\int_{0}^{\pi/2}\sin^2 t\,dt$$

$$= \frac{4}{3}\int_0^{\pi/2}\frac{1-\cos 2t}{2}dt = \frac{2}{3}\left[t - \frac{\sin 2t}{2}\right]\Big|_0^{\pi/2} = \frac{2}{3}\left[\frac{\pi}{2} - \frac{1}{2}\underset{0}{\underset{\shortparallel}{\sin\pi}}\right] = \frac{\pi}{3}$$

4) иста со задача 2 од јулскиот испит (10.07.2020)