Д/з на 17.09.22 Математика
Письменный стр 67, 72

№ 2.2.17

$\sqrt{y}(2\sqrt{x}-\sqrt{y})dx + x\,dy = 0$

Проверим однородное ли ур-е: $(f(\lambda x, \lambda y) = \lambda^n f(x,y))$:

$$\begin{cases} P(x,y) = \sqrt{y}(2\sqrt{x}-\sqrt{y}) = 2\sqrt{x}\sqrt{y} - y \\ Q(x,y) = x \end{cases}$$

$$\begin{cases} P(\lambda x, \lambda y) = 2\sqrt{\lambda}\sqrt{x}\sqrt{\lambda}\sqrt{y} - \lambda y = \lambda(2\sqrt{x}\sqrt{y} - y) = \underline{\lambda^1}\cdot P(x,y) \\ Q(\lambda x, \lambda y) = \lambda x = \underline{\lambda^1}\cdot Q(x,y) \end{cases}$$

Ур-е однородное

Решение: 1) замена: $y = ux$

$u = \frac{y}{x}$

$y' = u'x + u$

$dy = x\,du + u\,dx$

$\sqrt{ux}(2\sqrt{x} - \sqrt{ux})dx + x^2 du + xu\,dx = 0$

$(2x\sqrt{u} \underline{- ux} \underline{+ xu})dx + x^2 du = 0 \quad \Big| \cdot \frac{1}{x^2 \cdot \sqrt{u}}$

$\frac{2\,dx}{x} + \frac{du}{\sqrt{u}} = 0$

$2\int \frac{dx}{x} + \int \frac{du}{u^{\frac{1}{2}}} = C$

$2\ln|x| + 2\sqrt{u} = 2\ln|C| \quad |:2$

$\ln\left|\frac{C}{x}\right| = \sqrt{\frac{y}{x}}$

$y = x\ln^2\left|\frac{C}{x}\right|$

I. $x^2=0$

$\sqrt{y}(2\sqrt{x}-\sqrt{y})dx+xdy=0$ (1)

$0+0=0$

$y=x\ln^2\left|\frac{c}{x}\right|,\ x\neq 0$ (2)

II. $\sqrt{u}=0$

$\sqrt{\frac{y}{x}}=0$

$y=0$

(1) $0+0=0$

(2) при $\frac{c}{x}=1$

Ответ: $\left[\begin{array}{l} y=x\ln^2\left|\frac{c}{x}\right| \\ x=0 \end{array}\right.$

№2.2.18

$y'=\frac{x+y}{x-y}$

$y=ux$

$y'=u'x+u$

$u'x+u=\frac{x+ux}{x-ux}$

$u'x+ux-u'ux^2-u^2x=x+ux$

$u'(x^2-ux^2)=u^2x+x$

$u'x^2(1-u)-x(1+u^2)=0\ \Big|\cdot\frac{1}{x},\ x\neq 0$

$$\frac{du}{dx}x(1-u)-(1+u^2)=0 \quad \Big| \cdot \frac{dx}{x(1+u^2)}$$

$$\frac{(1-u)du}{1+u^2}-\frac{dx}{x}=0$$

$$\int\frac{(1-u)du}{1+u^2}-\int\frac{dx}{x}=C$$

1) $\int\frac{du}{1+u^2}-\int\frac{udu}{1+u^2}=$

$\quad$ ① $\quad\quad$ ②

① $\int\frac{du}{1+u^2}=\operatorname{arctg}\frac{y}{x}+C_1$

② $\int\frac{udu}{1+u^2}=\begin{bmatrix}1+u^2=t\\ udu=\frac{dt}{2}\end{bmatrix}=\int\frac{dt}{2t}=\frac{\ln|1+u^2|}{2}+C_2$

$=\ln\left|\frac{\sqrt{y^2+x^2}}{x}\right|+C_2$

2) $\operatorname{arctg}\frac{y}{x}-\left(\ln\left|\frac{\sqrt{y^2+x^2}}{x}\right|+\ln|x|\right)=\ln|C|$

Ответ: $\operatorname{arctg}\frac{y}{x}=C\sqrt{y^2+x^2}$

№ 2.2.19

$y'\cos\frac{y}{x}-\frac{y}{x}\cos\frac{y}{x}+1=0$

$\cos\frac{y}{x}dy-\left(\frac{y}{x}\cos\frac{y}{x}+1\right)dx=0$

$y=ux$

$u=\frac{y}{x}$

$y'=u'x+u$

$(u'x+u)\cos u-u\cos u+1=0$

$u'x\cos u+u\cos u-u\cos u+1=0$

$u' x \cos u + 1 = 0$

$\frac{du\, x \cos u}{dx} + 1 = 0 \;/ \cdot \frac{dx}{x}$

$\cos u \, du + \frac{dx}{x} = 0$

$\int \cos u \, du + \int \frac{dx}{x} = c$

Ombani $\sin \frac{y}{x} + \ln|x| = c$

Nº 2.2.20

$xy' + x \, tg \frac{y}{x} = y \;/ \cdot \frac{1}{x}$

$y' + tg \frac{y}{x} = \frac{y}{x}$ (1)

$y = ux$

$u = \frac{y}{x}$

$y' = u'x + u$

$x \frac{du}{dx} + tg\, u = 0 \;/ \cdot \frac{dx}{x \cdot tg\, u}$

$\frac{du}{tg\, u} + \frac{dx}{x} = 0$

$\int \frac{du}{tg\, u} + \int \frac{dx}{x} = C$

(*)

(*) $\int \frac{du}{tg\, u} = \int \frac{\cos u \, du}{\sin u} = \begin{bmatrix} \sin u = t \\ \cos u \, du = dt \end{bmatrix} = \int \frac{dt}{t} = \ln|\sin u| + C_1$

$\ln|\sin \frac{y}{x}| + \ln|x| = \ln|C|$

$x \sin \frac{y}{x} = C$ (2)

I. при $u=0$

$\sin u = \sin\frac{y}{x} = 0$ $\quad \frac{y}{x} = 2\pi k,\ k\in\mathbb{Z}$

(1) $0+0=0$

(2) $x\cdot 0 = C$ при $C=0$

Ответ: $x\sin\frac{y}{x} = C$

Б-2.2.21

$$\frac{dy}{dx} - \frac{y}{x}(1+\ln y - \ln x) = 0$$

$$y' - \frac{y}{x}\left(1+\ln\frac{y}{x}\right) = 0 \quad (1)$$

$y = ux$

$u = \frac{y}{x}$

$y' = u'x + u$

$u'x + u - u(1+\ln u) = 0$

$u'x - u\ln u = 0$

$$\frac{du}{dx}x - u\ln u = 0 \ \Big/ \cdot \frac{dx}{x\,u\ln u}$$

$$\frac{du}{u\ln u} - \frac{dx}{x} = 0$$

$$\int\frac{du}{u\ln u} - \int\frac{dx}{x} = C$$

$$① \int\frac{du}{u\ln u} = \begin{bmatrix}\ln u = t\\ \frac{du}{u} = dt\end{bmatrix} = \int\frac{dt}{t} = \ln\left|\ln\left|\frac{y}{x}\right|\right| + C_1$$

$$\ln\left|\ln\left|\frac{y}{x}\right|\right| - \ln|x| = \ln|C|$$

$\ln\left|\frac{y}{x}\right| = cx$

$\frac{y}{x} = e^{cx}$

$y = x e^{cx}$ (2)

I. $u = 0$

$y = 0$

y(1) $y \neq 0$

II. $\ln u = 0$

$u = 1$

$\frac{y}{x} - 1 = 0 \Rightarrow y = x$

(1) $0 - 1/(1+0) = 0$

$1 - 1 - 0 \neq 0$

(2) $y = x$ при $c = 0$

$1 = e^{0x}$

Ответ: $y = xe^{cx}$

№ 2.2.22

$(3x^2 - y^2)y' = 2xy \quad |\cdot \frac{1}{x^2}$

$\left(3 - \frac{y^2}{x^2}\right)y' = 2\frac{y}{x}$

$y = ux$

$u = \frac{y}{x}$

$y' = u'x + u$

$(3 - u^2)(u'x + u) = 2u$

$$u'x + u = \frac{2u}{3-u^2}$$

$$u'x + u\left(1 - \frac{2}{3-u^2}\right) = 0$$

$$x\frac{du}{dx} + u\left(\frac{3-u^2-2}{3-u^2}\right) = 0 \quad \Big/ \cdot \frac{dx}{x\, u\left(\frac{1-u^2}{3-u^2}\right)}$$

$$\frac{du}{u\left(\frac{1-u^2}{3-u^2}\right)} + \frac{dx}{x} = 0$$

1

1) $$\int \frac{du}{u\left(\frac{u^2-1}{u^2-3}\right)} = \int \frac{u^2-3}{u(u^2-1)}\, du = \int \frac{u^2\, du}{u^3-u} - 3\int \frac{du}{u^3-u} =$$

1

$=$

1 $$\int \frac{u^2 du}{u^3-u} = \int \frac{u\, du}{u^2-1} = \left[\begin{matrix} u^2-1 = t \\ 2u\, du = dt \end{matrix}\right] =$$

$$= \int \frac{dt}{2t} = \frac{\ln|u^2-1|}{2} + C_1$$

2 $$\int \frac{du}{u^3-u} = \int \frac{du}{u(u-1)(u+1)}$$

5.2.2.23

$y' - 1 = e^{\frac{y}{x}} + \frac{y}{x} \qquad y(1) = 0$

$y = ux$

$u = \frac{y}{x}$

$y' = u'x + u$

$y' = e^{\frac{y}{x}} + \frac{y}{x} + 1$

$u'x + u = e^{u} + u + 1$

$\frac{x\,du}{dx} = e^{u} + 1 \Big| \cdot \frac{dx}{x(e^{u}+1)}$

$\frac{du}{e^{u}+1} - \frac{dx}{x} = 0$

$\underset{①}{\int \frac{du}{e^{u}+1}} - \int \frac{du}{x} = C$

① $\int \frac{du}{e^{u}+1} = \begin{bmatrix} e^{u} = t \\ du = \frac{dt}{e^{u}} \end{bmatrix} = \int \frac{dt}{t(t+1)} = \int \frac{dt}{t^2(1+\frac{1}{t})} =$

$= \begin{bmatrix} 1 + \frac{1}{t} = k \\ \frac{dt}{t^2} = -dk \end{bmatrix} = \int -\frac{dk}{k} = -\ln\left|\frac{t+1}{t}\right| + C_1 =$

$= u - \ln|e^{u}+1| + C_1$

$e^{\frac{y}{x}} = Cx(e^{\frac{y}{x}} + 1)$

$e^{0} = C \cdot 1 (e^{0} + 1)$

$2C = 1$

$C = \frac{1}{2}$

$$e^{\frac{y}{x}} = \frac{\cancel{x}}{2}\left(e^{\frac{y}{x}}+1\right)$$

$$e^{\frac{y}{x}}\left(\frac{\cancel{2}-\cancel{x}}{2}\right) = \frac{x}{2}$$

Ответ: $e^{\frac{y}{x}} = \frac{x}{2-x}$

№ 2.3.20

$$y' + \frac{x}{1-x^2}y = 2$$

$y = uv$

$y' = u'v + uv'$

$$u'v + uv' + \frac{x}{1-x^2}uv = 2$$

$$u'v + u\left(v' + \frac{xv}{1-x^2}\right) = 2$$

$$\begin{cases} v' + \frac{xv}{1-x^2} = 0 \\ u'v = 2 \end{cases}$$

$$\frac{dv}{dx} + \frac{xv}{1-x^2} = 0 \Big/ \cdot \frac{dx}{v}$$

$$\frac{dv}{v} + \frac{x\,dx}{1-x^2} = 0$$

$$\int \frac{dv}{v} + \int \underset{①}{\frac{x\,dx}{1-x^2}} = C$$

① $\int \frac{x\,dx}{1-x^2} = \begin{bmatrix} 1-x^2 = t \\ x\,dx = -\frac{dt}{2} \end{bmatrix} = -\int \frac{dt}{2t} = -\frac{\ln|1-x^2|}{2} + C_1$

$\ln|v| - \frac{\ln|1-x^2|}{2} = \ln|C|$, $\ln|C| = 1$

$v = \sqrt{1-x^2}$

$u'\sqrt{1-x^2} = 2$

$\frac{du}{dx}\sqrt{1-x^2} = 2 \quad \Big| \cdot \frac{dx}{\sqrt{1-x^2}}$

$du - \frac{2\,dx}{\sqrt{1-x^2}} = 0$

$\int du - 2\int\frac{dx}{\sqrt{1-x^2}} = C$

$u = 2\arcsin x + C$

$y = uv = \sqrt{1-x^2}\,(2\arcsin x + C)$

Ответ: $\sqrt{1-x^2}\,(2\arcsin x + C)$

№ 2.3.21

$y' - \frac{y}{\sin x} = tg\frac{x}{2}$

$y = uv$

$y' = u'v + uv'$

$u'v + uv' - \frac{uv}{\sin x} = tg\frac{x}{2}$

$u'v + u\left(v' - \frac{v}{\sin x}\right) = tg\frac{x}{2}$

$$\begin{cases} v' - \frac{v}{\sin x} = 0 \\ u'v = tg\frac{x}{2} \end{cases}$$

$$\frac{dv}{dt} - \frac{v}{\sin x} = 0 \quad \Big/ \frac{dx}{v}$$

$$\frac{dv}{v} - \frac{dx}{\sin x} = 0$$

$$\int \frac{dv}{v} - \int \frac{dx}{\sin x} = C$$

(1)

(1) $$\int \frac{dx}{\sin x} = \int \frac{dx}{2\cos\left(\frac{x}{2}\right)\sin\left(\frac{x}{2}\right)} = \left[\begin{matrix} \frac{x}{2} = t \\ dx = 2dt \end{matrix}\right] = \int \frac{dt}{\cos t \sin t} =$$

$$= \int \frac{dt}{\cos^2 t \, tg\, t} = \left[\begin{matrix} tg\, t = k \\ \frac{dt}{\cos^2 t} = dk \end{matrix}\right] = \int \frac{dk}{k} = \ln|tg(t)| + C_1$$

$$\ln|v| - \ln|tg\, t| = \ln|C|, \quad \ln|C| = 0$$

$$v = tg\frac{x}{2}$$

$$u' \, tg\frac{x}{2} = tg\frac{x}{2}$$

$$\frac{du}{dx} = 1$$

$$\int du = \int dx = C$$

$$u = x + C$$

$$y = uv = tg\frac{x}{2}(x + C)$$

Ответ: $y = tg\frac{x}{2}(x+C)$

№ 2.3.9

$$(1+x^2)y' + 2xy = 3x^2 \quad \Big| \cdot \left(\frac{1}{1+x^2}\right)$$

$$y' + \frac{2xy}{1+x^2} = 0 \quad (1)$$

$$\frac{dy}{dx} + \frac{2xy}{1+x^2} = 0 \quad \Big| \cdot \frac{dx}{y}$$

$$\frac{dy}{y} + \frac{2x}{1+x^2}dx = 0$$

$$\int \frac{dy}{y} + 2\underbrace{\int \frac{x}{1+x^2}dx}_{(2)} = C$$

(2) $\int \frac{x}{1+x^2}dx = \begin{bmatrix} 1+x^2 = t \\ 2xdx = dt \end{bmatrix} = \int \frac{dt}{2t} = \frac{\ln|1+x^2|}{2} + C_1$

$$\ln|y| + \ln|1+x^2| = \ln|C|$$

$$y = \frac{C(x)}{1+x^2}$$

$$y' = \frac{C'(x)(1+x^2) - C(x)(1+x^2)'}{(1+x^2)^2} = -\frac{C(x) \cdot x \cdot 2}{(1+x^2)^2} +$$

$$+ \frac{C'(x)}{(1+x^2)} = \frac{C'(x)}{1+x^2} - \frac{C(x)2x}{(1+x^2)^2}$$

(1) $C'(x) - \frac{(1+x^2)\cdot(-2xC(x))}{(1+x^2)^2} + \frac{2x \cdot C(x)}{1+x^2} = 3x$

$$C'(x) = 3x^2$$

$$C(x) = x^3 + \bar{C}$$

Ответ: $y = \frac{x^3 + \bar{C}}{1 + x^2}$