№ 2.6.29 Мосалова Виктория Кс-24

$y''+y'\sqrt{(y')^2-1}=0$

Замена:

$y''=p',\ y'=p$

$y(\pi)=0$
$y'(\pi)=-1$

(1) $p'+p\sqrt{p^2-1}=0$

$\frac{dp}{dx}+p\sqrt{p^2-1}=0 \ \Big|\cdot \frac{dx}{p\sqrt{p^2-1}}$

$\frac{dp}{p\sqrt{p^2-1}}+dx=0$

$\int\frac{dp}{p\sqrt{p^2-1}}+\int dx=C$

1) $\int\frac{dp}{p\sqrt{p^2-1}}=\begin{bmatrix}p^2-1=t^2\\ p^2=t^2+1\\ p\,dp=t\,dt\end{bmatrix}=\int\frac{t\,dt}{p^2\cdot\sqrt{t^2}}=\int\frac{dt}{t^2+1}=arctg(t)+C'=$
$=arctg(\sqrt{p^2-1})+C'$

$arctg(\sqrt{p^2-1})+x=C$

$C=arctg(\sqrt{(-1)^2-1})+\pi=0+\pi=\pi$

$\sqrt{p^2-1}=tg(C-x)$

$\sqrt{p^2-1}=tg(\pi-x)=-tg(x)$

Подставим в (1): $p'-p\cdot tg\,x=0 \ \Big|\cdot\frac{dx}{p}$

$\frac{dp}{p}-tg\,x\,dx=0$

$\int\frac{dp}{p}-\int tg\,x\,dx=C_1$

2) $-\int tg\,x\,dx=-\int\frac{\sin x}{\cos x}dx=\int\frac{d(\cos x)}{\cos x}=\ln|\cos x|+C_1'$

$\ln|p|+\ln|\cos x|=\ln|C_1|$

$y'=p=\frac{C_1}{\cos x}$

3) $y=\int\frac{C_1\,dx}{\cos x}=\left[\cos x=\sin(x+\frac{\pi}{2})\right]=C_1\int\frac{dx}{\sin(x+\frac{\pi}{2})}=\begin{bmatrix}x+\frac{\pi}{2}=2t\\ dx=2dt\end{bmatrix}=C_1\int\frac{2dt}{\sin 2t}=$

$=C_1\int\frac{2dt}{2\sin t\cos t}=C_1\int\frac{dt}{tg\,t\cos^2 t}=\begin{bmatrix}tg\,t=k\\ \frac{dt}{\cos^2 t}=dk\end{bmatrix}=C_1\int\frac{dk}{k}=C_1\cdot\ln|k|+C_2=$

$=C_1\ln\left|tg\left(\frac{x}{2}+\frac{\pi}{4}\right)\right|+C_2$

$\begin{cases}y'=\frac{C_1}{\cos x}\\ y=C_1\ln\left|tg\left(\frac{x}{2}+\frac{\pi}{4}\right)\right|+C_2\end{cases}$ $\begin{cases}-1=\frac{C_1}{\cos\pi}\Rightarrow C_1=1\\ 0=\underbrace{\ln\left|tg\left(\frac{\pi}{2}+\frac{\pi}{4}\right)\right|}_{\ln|-1|=0}+C_2\end{cases}$ $\left[tg\frac{3\pi}{4}=-1\right]\Rightarrow C_2=0$

$C_1 = 1 \quad C_2 = 0$

Ответ: $y = \ln\left|tg\left(\frac{x}{2} + \frac{\pi}{4}\right)\right|$