Матем. ДЗ на 24.09.22

Письменный стр 76–78

№ 2.4.6

$$(x^2-\sin^2 y)\,dx+(x\sin 2y)\,dy=0 \quad (1)$$

1) $P'_y=-2\sin y\cos y=-\sin 2y$ $\neq$

$Q'_x=\sin 2y$

2) Найдем интегр. множитель

1. $k(x)=\dfrac{P'_y-Q'_x}{Q}=\dfrac{-\sin 2y-\sin 2y}{x\sin 2y}=-\dfrac{2}{x}$

2. $\int k(x)dx=-2\ln|x|+C_1,\ C_1=0$

3. $\mu(x)=e^{\int k(x)dx}=e^{-2\ln|x|}=\boxed{\dfrac{1}{x^2}},\ x\neq 0$

3) Получим ур-е в полных диф-ах.

$\bar{P}=\mu P=(x^2-\sin^2 y)\cdot\dfrac{1}{x^2}=1-\dfrac{\sin^2 y}{x^2}$

$\bar{Q}=\mu Q=(x\sin 2y)\cdot\dfrac{1}{x^2}=\dfrac{\sin 2y}{x}$

$$\left(1-\frac{\sin^2 y}{x^2}\right)dx+\left(\frac{\sin 2y}{x}\right)dy=0$$

$\bar{P}'_y=-\dfrac{\sin 2y}{x^2}$ $=$

$\bar{Q}'_x=-\dfrac{\sin 2y}{x^2}$

1. $\begin{cases}\dfrac{\partial u}{\partial x}=1-\dfrac{\sin^2 y}{x^2}\\ \dfrac{\partial u}{\partial y}=\dfrac{\sin 2y}{x}\end{cases}$

2. $u(x,y)=\int\dfrac{\partial u}{\partial x}dx=\int\left(1-\dfrac{\sin^2 y}{x^2}\right)dx=x+\dfrac{\sin^2 y}{x}+\varphi(y)$

$$\frac{\partial U}{\partial y} = \left(x + \frac{\sin^2 y}{x} + \varphi(y)\right)'_y = 0 + \frac{2\sin y\cos y}{x} + \varphi'(y) =$$

$$= \frac{\sin 2y}{x} + \varphi'(y) = \frac{\sin 2y}{x}$$

3. $\varphi'(y) = 0$

$\varphi(y) = C_1$

4. $u(x, y) = x + \frac{\sin^2 y}{x} + C_1 = 0$

$x + \frac{\sin^2 y}{x} = C \quad | \cdot x$

(2) $x^2 + \sin^2 y = Cx$

I. $x = 0$

(1) $0 + 0 = 0$

(2) $0 + \sin^2 y = 0$

при $\sin y = 0$

Ответ: $\mu(x) = \frac{1}{x^2}$ – интегр. множитель

$x^2 + \sin^2 y = Cx$ – общий интеграл ур-я

№ 2.4.11

$$(x^2+2xy+1)\,dx+\underbrace{(x^2+y^2-1)}_{\frac{\partial u}{\partial y}}\,dy=0$$

$P'_y=2x$ $\equiv$

$Q'_x=2x$

$$u(x,y)=\int(x^2+2xy+1)\,dx=\frac{x^3}{3}+\frac{2yx^2}{2}+x+\varphi(y)=$$

$$=\frac{x^3}{3}+yx^2+x+\varphi(y)$$

$$\frac{\partial u}{\partial y}=\left(\frac{x^3}{3}+yx^2+x+\varphi(y)\right)'_y=0+x^2+0+\varphi'(y)=$$

$$=x^2+\varphi'(y)=x^2+y^2-1$$

$$\varphi'(y)=y^2-1$$

$$\varphi(y)=\int(y^2-1)\,dy=\frac{y^3}{3}-y+C_1$$

$$u(x,y)=\frac{x^3}{3}+yx^2+x+\frac{y^3}{3}-y+C_1=0 \quad |\cdot 3$$

$$x^3+y^3+3x^2y+3x-3y=C$$

Ответ: $x^3+y^3+3x^2y+3x-3y=C$

№ 2.4.12

$\sin(x+y)dx + x\cos(x+y)(dx+dy) = 0$

$(\sin(x+y) + x\cos(x+y))dx + \underbrace{x\cos(x+y)}_{\frac{\partial u}{\partial y}}dy = 0$

1) $P'_y = \cos(x+y) - x\sin(x+y)$ $\neq$

$Q'_x = \cos(x+y) - x\sin(x+y)$

2) $u(x,y) = \int(\sin(x+y) + x\cos(x+y))dx =$

$= \int\sin(x+y)dx + \int x(\cos(x+y))dx =$

$= \begin{bmatrix} u = x & dv = \cos(x+y)d(x+y) \\ du = dx & v = \sin(x+y) \end{bmatrix} =$

$= \int\sin(x+y)dx + x\sin(x+y) - \int\sin(x+y)dx =$

$= x\sin(x+y) + \varphi(y)$

3) $\frac{\partial u}{\partial y} = (x\sin(x+y) + \varphi(y))'_y = x\cos(x+y) + \varphi'(y) =$

$= x\cos(x+y)$

$\varphi'(y) = 0$

$\varphi(y) = C_1$

4) $u(x,y) = x\sin(x+y) + C_1 = 0$

$C = x\sin(x+y)$

Ответ: $x\sin(x+y) = C$

№ 2.4.13

$(3x^2+3x^2\ln y)dx-\underbrace{\left(2y-\frac{x^3}{y}\right)}_{\frac{\partial u}{\partial y}}dy=0$

1) $P'_y=\frac{3x^2}{y}$ ⊜

$Q'_x=\frac{3x^2}{y}$

2) $u(x,y)=\int(3x^2+3x^2\ln y)dx=x^3+x^3\ln y+\varphi(y)$

3) $\frac{\partial u}{\partial y}=(x^3+x^3\ln y+\varphi(y))'_y=0+\frac{x^3}{y}+\varphi'(y)=$

$=-2y+\frac{x^3}{y}$

$\varphi'(y)=-2y$

$\varphi(y)=\int-2y\,dy=-\frac{2y^2}{2}+\cancel{C}=-y^2+C_1$

4) $u(x,y)=x^3+x^3\ln y-y^2+C_1=0$

$x^3+x^3\ln y-y^2=C$

Ответ: $x^3+x^3\ln y-y^2=C$

№2.4.14

$3x^2y + \sin x = (\cos y - x^3)y' \mid \cdot dx$

$(3x^2y + \sin x)dx \underbrace{-(\cos y - x^3)dy}_{\frac{\partial u}{\partial y}} = 0$

1) $P'_y = 3x^2$
$Q'_x = 3x^2$ $\ominus$

2) $u(x,y) = \int(3x^2y + \sin x)dx = \frac{3yx^3}{3} - \cos x +$
$+ \varphi(y) = x^3y - \cos x + \varphi(y)$

3) $\frac{\partial u}{\partial y} = (x^3y - \cos x + \varphi(y))'_y = x^3 + \varphi'(y) = -\cos y + x^3$

$\varphi'(y) = -\cos y$

$\varphi(y) = -\int \cos y\, dy = -\sin y + C_1$

4) $u(x,y) = x^3 \cdot y - \cos x - \sin y + C_1 = 0$

$x^3y - \cos x - \sin y = C$

Ответ: $x^3y - \cos x - \sin y = C$

№ 2.4.28 (л)

$y^2 dx - (2xy + 3) dy = 0$

$P'_y = 2y$ $\neq$

$Q'_x = -2y$

$n(y) = \frac{Q'_x - P'_y}{P} = \frac{-2y - 2y}{y^2} = -\frac{4}{y}$

$\mu(y) = e^{\int n(y) dy} = e^{-4\int \frac{dy}{y}} = e^{-4 \ln y} =$

$= \frac{1}{y^4}$

$\bar{P} = \mu P = \frac{1}{y^2}$

$\bar{Q} = \mu Q = -\left(\frac{2x}{y^3} + \frac{3}{y^4}\right)$

$\left(\frac{1}{y^2}\right) dx - \left(\frac{2x}{y^3} + \frac{3}{y^4}\right) dy = 0$

$\bar{P}'_y = -\frac{2}{y^3}$ $=$

$\bar{Q}'_x = -\frac{2}{y^3}$

Ответ: $y^2 dx - (2xy + 3) dy = 0$
ур-е приводящееся к ур-ю
в полных дифференциалах