

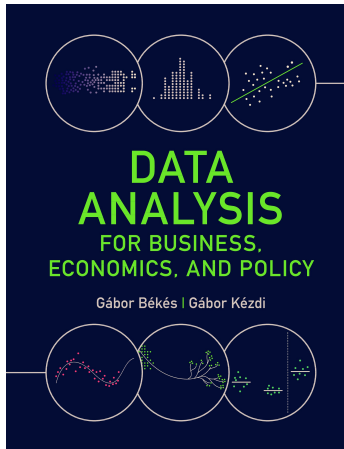
X. Regression and Matching with Observational Data

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Evaluation

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Slideshow for the Békés-Kézdi Data Analysis textbook



- ▶ Cambridge University Press, 2021
- ▶ gabors-data-analysis.com
 - ▶ Download all data and code:
gabors-data-analysis.com/data-and-code/
- ▶ These slides are for Chapter 21

Regression and causality

- ▶ Causality – is about interpretation
- ▶ You see a pattern in the data – revealed by regression analysis
- ▶ Then, you interpret it....
- ▶ unless...
 - ▶ you get to design your own experiment
 - ▶ in that case you have a causal effect in mind and you induce controlled variation a variable
 - ▶ if all goes fine you know how to interpret patterns

Causality and regression

- ▶ You have observational data for many possible reasons
- ▶ Experiments may be hard, expensive, unethical
- ▶ Look for great external validity
- ▶ Process of work?

Observational data approaches

- ▶ Thinking 1: Thought experiment
- ▶ Thinking 2: Variation in y - unobserved heterogeneity
- ▶ Thinking 3: Source of variation in x
- ▶ Tools 1: Regression with controlling on confounders
- ▶ Tools 2: Exact matching
- ▶ Tools 3: Matching on the propensity score

Thinking 1: Thought experiment

- ▶ Data analysts turn to observational data for answering causal questions when they can't run an appropriate experiment.
 - ▶ Often there is not enough time or resources
 - ▶ would require controlling for too many things that would make external validity too low.
 - ▶ impossible run due to ethical concerns.
- ▶ Even when no experiment, worth to think about an experiment that could uncover the effect we are after.
- ▶ *Thought experiments*: experiments that are designed in some detail but not carried out.

Thinking 1: Thought experiment

Thinking through a thought experiment when doing causal analysis on observational data has several advantages. It can:

- ▶ clarify the details of the *intervention* we want to examine and how it compares to the causal variable in the data.
- ▶ clarify the situations: what exactly it would mean for observations to be "treated" and "untreated".
- ▶ help understand the *mechanisms* through which the causal variable may affect the outcome.
- ▶ help understand how random assignment compares to the *source of variation* in the causal variable in our data.

Case study: Founder/Family Ownership and Quality of Management

- ▶ Though experiment
- ▶ We investigate whether the fact that a company is owned by its founder, or their family members, has an effect on the quality of management.
- ▶ Whether founder/family owned companies are better or worse managed than other firms, on average because of their ownership.
- ▶ This is a causal question: we are after an effect.
- ▶ Great way to understand what the intervention and the counterfactuals are.

Case study: Founder/Family Ownership and Quality of Management

- ▶ The subjects of this thought experiment are companies.
- ▶ The intervention is changing ownership of the company.
- ▶ For that we need a subject pool with the same ownership and randomly assign some of them to change their ownership.
 - ▶ To change ownership the owners would sell their stake to other investors, either directly or indirectly (stock market).
 - ▶ Effect of the intervention would be a form of ownership that can be the result of such sales.

Case study: Founder/Family Ownership and Quality of Management

- ▶ Take all founder/family owned companies.
- ▶ Randomly chose half of them and make them sell their stakes to whoever would want that.
 - ▶ assume perfect compliance: treated companies receive offers that they don't refuse
- ▶ As a result of the intervention, untreated companies remain in founder/family ownership, while treated companies have other forms of ownership
- ▶ After some time, measure the quality of management among treated and untreated firms.
- ▶ The difference between their average quality scores would show the average effect of giving up founder/family ownership.

Case study: Founder/Family Ownership and Quality of Management

- ▶ This thought experiment would identify the opposite of what the original question would imply.
- ▶ Instead of the "effect" of founder/family ownership it can measure the effect of giving up founder/family ownership.
 - ▶ effect identified in thought experiment = mirror image of the effect in our original question.
- ▶ Empirical work: the "effect" of founder/family ownership.
- ▶ Interpreting the results → relate to experiment of selling stake and compare outcomes.
- ▶ There cases of family taking firm private

Variables to Condition on, Variables Not to Condition On

- ▶ Investigate sources of variation in the causal variable, two types of variation in x
 - ▶ Exogenous sources are variables that are independent of potential outcomes.
 - ▶ Endogenous sources are variables that are related to potential outcomes.
- ▶ Use exogenous sources in x , while conditioning on all endogenous sources of variation = confounders.
- ▶ Collect potential sources = thinking exercise
- ▶ Endogenous sources of variation, to condition on (confounders):
 - ▶ Common cause: the variable affects x and y .
 - ▶ Mechanism of reverse causality: y affects x through this variable.
 - ▶ Unwanted mechanism: x affects y through this variable, but we don't want to consider it when estimating the effect of x on y .

Variables to Condition on, Variables Not to Condition On

- ▶ bad conditioners: variables that data analysts should not condition on when attempting to uncover the effect of x on y :
 - ▶ An exogenous source of variation in x .
 - ▶ A mechanism that we want to include in the effect to be uncovered.
 - ▶ Common consequence: both x and y affect the variable

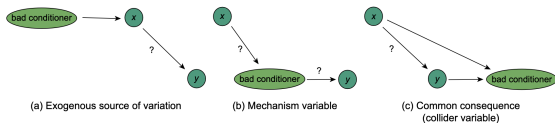


Figure 19.7 The three types of bad conditioning variables

Variables to Condition on, Variables Not to Condition On

- ▶ Look at variables we shall have, and what we have
- ▶ List and categories
- ▶ Causal map (DAG)
- ▶ Use tools to condition on those variable we shall
 - ▶ Multivariate regression
 - ▶ Matching
 - ▶ Use smart tricks in rare settings

Conditioning, ATE, ATET

- ▶ Our usual aim is to estimate ATE
- ▶ Sometimes we also care about ATET: the treatment effect on the treated
 - ▶ ATET focuses directly on participants - sometimes this is what policy cares about
 - ▶ ATE may be driven by selection or spillovers - sometimes you are interested in this
- ▶ If random assignment $ATET = ATE$
- ▶ With observational data, ATET may be different to ATE
 - ▶ No random assignment, treated and not treated subjects may be different (heterogeneously) in some unobserved way
 - ▶ Example: self-selection as unobserved confounder

Case study: Founder/Family Ownership and Quality of Management

- ▶ Observational cross-sectional data
- ▶ World Management Survey = cross-section of many firms in manufacturing from 21 countries
- ▶ The outcome variable is the management score
- ▶ The causal variable is founder/family ownership
- ▶ Several tasks before running regressions
 - ▶ Think about and identify sources of variation in ownership
 - ▶ Draw a causal map
 - ▶ Decide on observable variables to condition on

Case study: Sources of variation in ownership

- ▶ Let us look for variation in x , ownership. Think + identify + decide.
- ▶ Firm started as founder/family-owned?
 - ▶ Alternative: spin-offs, joint ventures, multinational affiliates of other firms, including multinationals.
- ▶ Products and technology affect ownership = sources of variation in x .
- ▶ Yet it's likely to be an endogenous source, technology correlated with management, too.

Case study: Sources of variation in ownership

- ▶ Let us look for variation in x , ownership. Think + identify + decide.
- ▶ Cultural and institutional factors, norms in a society. Affect cost of starting business, FDI. *How about y ?*
- ▶ Likely endogenous source, culture, norms correlated with management, too.
- ▶ How about family features. Children of founders, their interests, skills. Clearly affects if ownership may be passed on. *How about y ?*
- ▶ Likely exogenous - gender/number of kids not related to management quality
- ▶ This is the variation we need but not use as control!

Review: Types of Confounders

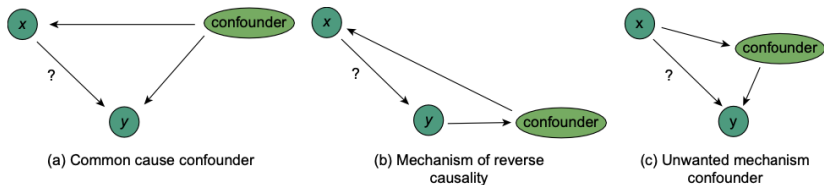
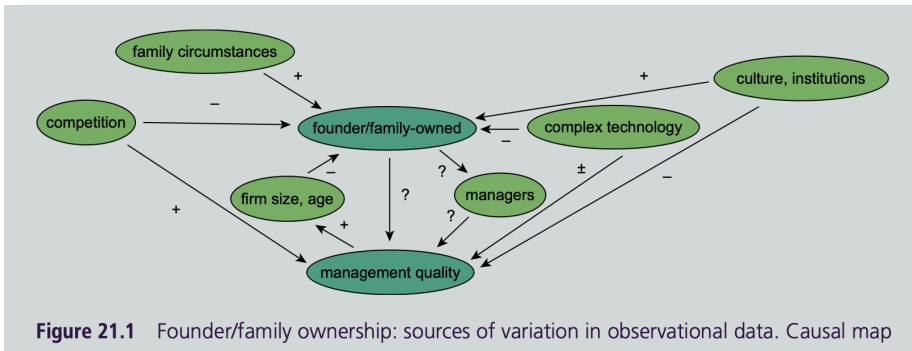


Figure 19.6 Three types of confounders

Case study: Founder/family ownership: sources of variation in observational data. Causal map



Case study: Sources of variation in ownership

- ▶ Family circumstances – exogenous variation in x
- ▶ Competition – common cause confounder
- ▶ Culture and institutions – common cause confounder
- ▶ Technology, product type – common cause confounder
- ▶ Firm size, firm age – hard – may be mechanisms of reverse causality
- ▶ Feature of managers (their age, experience) – mechanism
- ▶ which ones to control on?

Case study: Sources of variation in ownership

- ▶ Family circumstances – exogenous variation in x [NO Control]
- ▶ Competition – common cause confounder [Control]
- ▶ Culture and institutions – common cause confounder [Control]
- ▶ Technology, product type – common cause confounder [Control]
- ▶ Firm size, firm age – may be mechanisms of reverse causality [Maybe Control]
- ▶ Feature of managers (their age, experience) – mechanism [NO Control]
- ▶ Important: Try measuring pre-treatment and/or pre-outcome measurement (y), i.e. at “baseline” (with some time lag)

Conditioning on Confounders by Regression

- ▶ Linear regression to condition on other variables to estimate the effect of x on y , conditioning on observable confounder variables (z_1, z_2, \dots):

$$y^E = \beta_0 + \beta_1 x + \beta_2 z_1 + \beta_3 z_2 + \dots \quad (1)$$

- ▶ Note: β_1 always = estimate of average difference in y between observations that are different in x but have the same values for z_1, z_2, \dots . Even if not causal.
- ▶ If the z_1, z_2, \dots variables capture all endogenous sources of variation, x is exogenous in the regression.
 - ▶ Conditional on z_1, z_2, \dots , variation in x is exogenous.
 - ▶ OLS estimate of β_1 is a good estimate of ATE of x on y .

Conditioning on Confounders by Regression

- ▶ Conditioning on all relevant confounders - very unlikely in observational data.
- ▶ z_1, z_2, \dots capture some, but not all, of the endogenous sources of variation in x , x is endogenous in the regression
 - ▶ OLS estimate of β_1 is a not good estimate of the average effect of x on y .
- ▶ OLS is biased - omitted variables bias = difference between the true ATE of x on y and estimated ATE for the β_1 coefficient on x by this regression.
 - ▶ When x is exogenous in the regression, the omitted variable bias is zero.
 - ▶ [Chapter 10](#): bias depends on how the omitted confounders are related to x and y .

Conditioning on Confounders by Regression

- ▶ OVB is positive (estimated ATE $>$ true ATE) when the omitted confounders are correlated in the same direction with x as with y .
 - ▶ OVB negative when omitted confounders associated in the opposite direction with x and y .
- ▶ If we can speculate well, we can sign the omitted variable bias
 - ▶ Sometimes we can.
- ▶ Signing OVB is often the key task - could help a great deal to see where we are re causality.

Selection of Variables in a Regression for Causal Analysis

- ▶ In practice, key question is: variable selection
 - ▶ Which z variables to add -all observed confounders or only some? Which ones?
 - ▶ What functional form? Interactions?
- ▶ Variable selection matters IF choices impact estimated ATE (coefficient estimates on x).
 - ▶ When equal: prefer simplest model, with the fewest variables, the simplest functional forms, and the fewest interactions.
- ▶ IF different regressions give substantially different coefficient estimates on x . pick one that includes more variables.
 - ▶ More variables, more flexible functional forms, or more interactions.
 - ▶ Still make sure to avoid bad conditioning variables.
- ▶ Adding variables that don't matter - usually no big deal.
 - ▶ But, in smaller dataset, it can make the effect estimates imprecise.
- ▶ Often sample size determines what we can do.

Case study: data

- ▶ Observational cross-sectional data
- ▶ World Management Survey.
- ▶ It is a cross-section of many firms in manufacturing from 21 countries. Representative sample of firms within countries.
- ▶ Consider a cross-section, each firm is just once in sample

Case study: outcome and causal variable

- ▶ The outcome variable is the management score.
 - ▶ Average of 18 scores that measure the quality of specific management practices.
 - ▶ Each score is measured on a 1 through 5 scale, with 1 for worst practice and 5 for best practice.
- ▶ The causal variable is founder/family ownership.
 - ▶ The ownership variable detailed
 - ▶ binary variable 1: firm is founder owned or family owned
- ▶ Other types of ownership we are interested in = could be the result of founders or their family selling their shares.
 - ▶ Drop observations that were owned by the government or a foundation or the employees. Why?
 - ▶ We also dropped observations with missing ownership data and "other" ownership type.

Case study: Summary of confounders

- ▶ List of confounders: suggested by causal map + available data
- ▶ Technology - industry dummy; share of college-educated workers (outside senior management).
- ▶ Customs, law - country dummy, product competition
- ▶ Firm size - not sure if confounder or bad control.
 - ▶ will try with and without
- ▶ Other variables that we'll use in our analysis: employment, college share, competition, industry, country

Exact matching

- ▶ Linear regression is an approximation
 - ▶ the difference in average y between observations with different x but the same values for the other right-hand-side variables z_1, z_2, \dots .
- ▶ Why do an approximation, when we can compare observations with the same z_1, z_2, \dots values?
- ▶ Couldn't we take those variables and find observations with the exact same values?
- ▶ This is idea of matching: compare the outcomes between observations that have the same values of all of the other variables and different values of the x variable.

Exact matching

- ▶ Ideal case exact matching - not an approximation.
- ▶ It matches observations on exact values
- ▶ Aggregation: observations = different value-combinations of all confounders
- ▶ With z_1, z_2, \dots variables, each cell would have a particular value-combination $z_1 = z_1^*, z_2 = z_2^*, \dots$
- ▶ Within each cell, compute the average y for all treated observations and the average y for all untreated observations, and we take their difference:

$$E[y|x = 1, z_1 = z_1^*, z_2 = z_2^*, \dots] - E[y|x = 0, z_1 = z_1^*, z_2 = z_2^*, \dots] \quad (2)$$

Exact matching

- ▶ ATET = number of treated observations in the cells as weights
- ▶ Matching gives a good estimate of ATET when selection is based on observables
 - ▶ This is often the default
- ▶ ATE = can calculate by some re-weighting - average of differences weighted by the number of observations in cells.
- ▶ If ATE and ATET is very different - something problematic is going on.
 - ▶ Strong self-selection, a confounder we did not take into account.

Exact matching

- ▶ It is feasible when many observations, few variables or variables with few values.
- ▶ In practice, exact matching is rarely feasible.
 - ▶ unlikely to find exact matches for all z values.
- ▶ In practice, in some cells have $x = 1$ observations only, others, $x = 0$ only.
- ▶ For ATE: both are problem
 - ▶ For ATET, need cells in which we have $x = 1$ observations

Exact matching

- ▶ In practice, in some cells have $x = 1$ observations only, others, $x = 0$ only. Two possible reasons:
- ▶ Substantive problem: $x = 1$ and $x = 0$ observations differ so much that some values of some confounder variables exist only in one of the two groups in the population.
- ▶ Data problem. A value combination is not there in our sample, but could be, and could very well be in the population
 - ▶ Larger sample can help
- ▶ Can we know which one we face?

Coarsened exact matching

- ▶ Coarsening qualitative variables means joining categories to fewer, broader ones and creating binary variables for those broader categories (e.g., groups of countries, less refined industry categories).
- ▶ Coarsening quantitative variables means creating bins (e.g., bins for age of individuals or size of organizations).
- ▶ Fewer binary variables and fewer bins of quantitative variables make matches more likely by reducing the number of variables.
- ▶ Coarsening is based on a trade-off: it makes exact matches more likely but it reduces variation in the confounder variables used for the matching

Exact matching: summary

- ▶ The interpretation of this estimate is intuitive: it is the average difference in y between treated and untreated observations that have the exact same z_1, z_2, \dots
- ▶ Recall that the linear regression gives an approximation to this average difference.
- ▶ In contrast, exact matching is not an approximation.
- ▶ If matching is successful for all $x = 1$ observations, it gives exactly the average difference in the data.
- ▶ The key problem is feasibility: could be too many values. Aggregation is arbitrary.

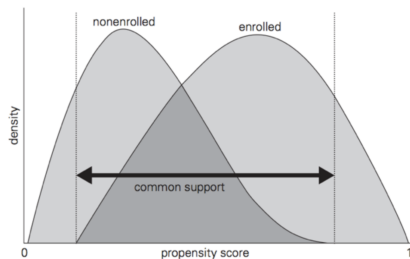
The idea of the common support

- ▶ Exact matching may fail for a substantive reason = there is a lack of common support.
 - ▶ "Support" = the set of values a variable can take.
- ▶ Common support = confounders can take the same values among treated and untreated observations.
- ▶ In the population or general pattern, our data represents.
- ▶ When we don't have common support, we can't estimate the effect for all subjects in the data.

The idea of the common support

- ▶ Consequence is general not just for matching
- ▶ We shouldn't (cannot) estimate ATE when have no common support.
- ▶ Instead, we shall estimate the effect of x on the part of the dataset with common support
- ▶ Compare distributions with histograms, tabulate key categorical variables, even interactions
- ▶ Drop ranges of observations when no common support

Common support: Illustration with probability to be treated



Matching on the Propensity Score

- ▶ Idea = creating a single quantitative variable from the many confounder variables.
- ▶ Matching is then done by finding similar observations in terms of this single quantitative variable.
- ▶ Similar observations = nearest neighbors.
- ▶ Most widely used method is called matching on the propensity score.
- ▶ The propensity score is a conditional probability: it is the probability of an observation having $x = 1$ as opposed to $x = 0$, conditional on all the confounder variables z .
- ▶ The propensity score is a single quantitative variable (the probability) that combines all confounder variables (the conditioning variables)

Matching on the Propensity Score

- ▶ The propensity score is not something we know. It is something we need to estimate it.
- ▶ That means estimating, or, more precisely, predicting, the probability of $x = 1$ for each and every observation in the data, based on what values they have for the z variables.
- ▶ The usual procedure is to estimate a probability model, most often a logit, for the probability of $x = 1$, as a function of the confounder variables.

Using a logit, we get the propensity score, *pscore*,

$$pscore = P[x = 1 | z_1, z_2, \dots] = x^P = \Lambda(\gamma_0 + \gamma_1 z_1 + \gamma_2 z_2 + \dots) \quad (3)$$

Matching on the Propensity Score

- ▶ With the propensity score at hand, we can match $x = 1$ and $x = 0$ observations that are close to each other.
- ▶ The most widely used matching procedure is *nearest neighbor matching on the propensity score*.
- ▶ This procedure takes each $x = 1$ observation, matches it to the $x = 0$ observation with the nearest value of the propensity score.
- ▶ If many $x = 0$ observations are nearest neighbors, all are picked and average outcome taken.
- ▶ Once a match is found, take difference of y values between the matched $x = 1$ and the $x = 0$ observation.

Matching on the Propensity Score

- ▶ Matching and then difference taking is repeated for all $x = 1$ observations.
- ▶ The estimated effect of x on y is then the average of those differences.
- ▶ If all confounders are included, the propensity score incorporates all endogenous sources of variation in the causal variable.
- ▶ In practice, many possible decisions...

Case study: variables

- ▶ The outcome variable is the management score: range in the data is 1 to 4.9, its average is 2.88, standard deviation 0.64
- ▶ The causal variable is whether the firm is owned by its founder or their family: 45%
- ▶ Direct comparison: 2.68 vs 3.05
- ▶ Founder/family owned firms – management score is -0.37 points lower, on average.
 - ▶ Difference a little more than half SD of outcome variable (0.64) - so large in magnitude
- ▶ Causal statement would be like: The quality of management in founder/family owned firms would increase by 0.37 points, on average, if the ownership of their firm were transferred to other investors.
 - ▶ Transferring ownership away from founder/family would make management quality improve

Case study: Estimates of the effect of founder/family ownership on the quality of management. Multiple regression results

Variables	(1) No confounders	(2) With confounders	(3) With confounders interacted
Founder/family owned	-0.37** (0.01)	-0.19** (0.01)	-0.19** (0.01)
Constant	3.05** (0.01)	1.75** (0.05)	1.46** (0.22)
Observations	8,440	8,439	8,439
R-squared	0.08	0.29	0.37

Note: Outcome variable: management quality score. Robust standard error estimates in parentheses.** $p < 0.01$,
 * $p < 0.05$. Source: wms-management-survey dataset.

Case study: Add variables

- ▶ When adding confounders, coefficient drops from -0.37 to -0.19
- ▶ The quality of management is lower, on average, by 0.19 points or about 30% of a standard deviation, in founder/family-owned firms than other firms of the same country, industry, size, age, with the same proportion of college-educated workers, and with a similar number of competitors.
- ▶ Adding confounders with interactions, quadratic forms, does not matter
 - ▶ causal variable + up to 745 variables in the regression

Case study: Causality and signing the bias

- ▶ When adding confounders, coefficient is -0.19.
- ▶ Biased? Yes. **But how?**
- ▶ Most omitted confounders are correlated with founder/family ownership and the quality of management in opposite directions.
- ▶ the estimated effect of founder/family ownership is biased in the negative direction.
- ▶ Thus the true effect is probably weaker (less negative).
 - ▶ As did confounders we have already added.
- ▶ True effect could be zero. Or even positive.
- ▶ **What can we do to increase belief in causality?**

Comparing Linear Regression and Matching

- ▶ ATE (and ATET) make sense only with common support.
- ▶ Regression and matching uncover, deal lack of common support differently.
- ▶ Exact matching automatically drops observations (no matching).
- ▶ Matching on the propensity score, also detects the lack of common support.
 - ▶ If PS close to 0 or 1 – not be matched by nearest neighbor matching.
- ▶ Linear regression not detect the lack of common support. Uses all observations to produce its coefficients.
 - ▶ This would include observations without common support.
- ▶ Lack of common support -> estimate a biased average effect of x on y .
 - ▶ Estimated regression line affected by observations that are not supposed to count.

Comparing Linear Regression and Matching

- ▶ When estimating ATE by regression, we need to make sure that the support is common before the estimation.
- ▶ The lack of common support means OLS may under or over-estimate the effect of x on y .
- ▶ Extra step of data analysis.

Case study: Common support

- ▶ We argued that common support is needed to avoid biased ATE
- ▶ While matching is designed to do that, we can check it with regressions
- ▶ Checked statistics of the distributions of each included confounder among founder/family owned vs other ownership.
- ▶ Concluded: common support assumption OK in our data
- ▶ Main reason why similar results from regression and matching

Case study conclusions

- ▶ We estimated an average treatment effect, fairly precisely.
- ▶ Is this the "true" effect of founder/family ownership of a company on the quality of management?
- ▶ Probably not, more likely an upper bound in magnitude
 - ▶ Most likely other confounders, negative bias - overestimated size of the effect

Case study conclusions

- ▶ Did conditioning on observable confounders matter?
 - ▶ Yes
 - ▶ When we conditioned on what we could, the difference halved
- ▶ Did the way we condition on them matter?
 - ▶ No
 - ▶ Regression estimates were essentially the same as the estimates from matching on the propensity score
 - ▶ Including many interactions among the confounder variables didn't matter, either
- ▶ What matters is what we can condition on
 - ▶ The causal map helped outline what we would want to condition on
 - ▶ Our data had a small subset of those variables
- ▶ If we want a better estimate need to measure more of those potential confounders
- ▶ Or isolate exogenous variation in x in some other way

The ultimate PSM problem



Prince Charles

- Male
- Born in 1948
- Raised in the UK
- Married twice
- Lives in a castle
- Wealthy & famous



Ozzy Osbourne

- Male
- Born in 1948
- Raised in the UK
- Married twice
- Lives in a castle
- Wealthy & famous

Review of advanced methods to help read papers

- ▶ Introduce two ways to isolate exogenous variation in x to uncover its effect on y
 - ▶ instrumental variables
 - ▶ regression-discontinuity.
- ▶ Alternative to condition on all confounders
- ▶ Make sure that we use only the exogenous part of variation in x for estimating its effect.
- ▶ Can be used under specific circumstances.

Instrumental variables

- ▶ Instrumental variables (IV) is a method to estimate the effect of x on y
- ▶ By directly isolating an exogenous source of variation in x
- ▶ Under ideal circumstances the IV method can give a good estimate of the effect
- ▶ In observational data
- ▶ Even if there are endogenous sources of variation in x , too

Instrumental variables main idea

- ▶ There is a variable in the data that is an exogenous source of variation in x
- ▶ This is called the instrumental variable, IV, or simply the instrument
 - ▶ The IV is independent of potential outcomes
 - ▶ The IV affects x
 - ▶ The IV has no direct effect on y
- ▶ Compare y across observations that are different in the IV
 - ▶ If there is a difference in observed y
 - ▶ That must be the effect of the IV
 - ▶ Because the IV is exogenous (independent of potential outcomes)
 - ▶ And the effect of the IV is only through x
 - ▶ Thus, that difference in observed y is because of the effect of x on y

Instrumental variables example

- ▶ What is the effect of having more than two children (x , binary) on whether the mother works for pay (y , binary), in the USA?
- ▶ The IV is whether the first two children have the same sex
 - ▶ It's one of the many sources of variation in x
 - ▶ It does affect x : the proportion of women with more than two children is 6 percentage points higher (+0.06) if the first two children have the same sex (USA).
 - ▶ The IV is likely exogenous
 - ▶ The IV likely has no effect on y except through x
- ▶ Women whose first two children have the same sex are less likely to work for pay
 - ▶ Difference is 0.8 percentage point (−0.008)
- ▶ That difference must be the effect of those women being more likely to have more than two children

Instrumental variables example

- ▶ So we established that having more than two children leads to a lower likelihood of work for pay
- ▶ But by how much?
- ▶ Answer: adjust the effect of same-sex first children on y (-0.008) by its effect on x (0.06)
- ▶ The effect of having more than two children (x) on working for pay (y) is then negative 13 percentage points
 - ▶ $-0.008/0.06 = -0.13$

Instrumental variables formula

$$\hat{x}^E = \hat{\pi}_0 + \hat{\pi}_1 IV \quad (4)$$

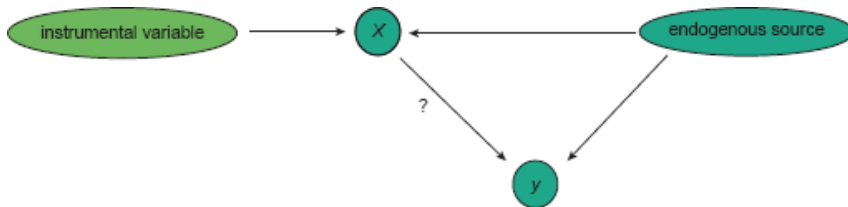
$$\hat{y}^E = \hat{\phi}_0 + \hat{\phi}_1 IV \quad (5)$$

$$\hat{\beta}_{IV} = \hat{\phi}_1 / \hat{\pi}_1 \quad (6)$$

- ▶ First equation is the effect of the IV on x
 - ▶ Called the first stage
 - ▶ In the example $\hat{\pi}_1 = 0.06$
- ▶ Second equation is the effect of the IV on y
 - ▶ Called the reduced form
 - ▶ In the example $\hat{\phi}_1 = -0.008$
- ▶ Third equation is the instrumental variables estimate of the effect of x on y
 - ▶ In the example $\hat{\beta}_{IV} = -0.13$

Causal map with an instrumental variable

- ▶ This causal map illustrates a situation in which the IV works even though there is endogenous source of variation in x
- ▶ As long as the IV is an exogenous source



Instrumental variables summary

- ▶ When applicable, IV is a powerful method to estimate the effect of x on y
- ▶ When is it applicable?
- ▶ The key assumption is exogeneity
 - ▶ The IV should be independent of potential outcomes
 - ▶ It can affect y only through x
 - ▶ This is an assumption that we can't verify
- ▶ The other assumption is that the IV should affect x
 - ▶ This we can easily check in the data
- ▶ It's usually difficult to find an IV that fits the requirements
- ▶ When the requirements are not met, the IV estimate is biased
 - ▶ And the IV estimate doesn't necessarily get us closer to the true effect

Regression-discontinuity

- ▶ Regression-discontinuity (RD) is another method to estimate the effect of x on y
- ▶ By directly isolating an exogenous source of variation in x even in the presence of endogenous variation, too
- ▶ It is applicable under very specific circumstances
- ▶ When there is a threshold value of a variable that determines treatment
 - ▶ This is called the running variable
 - ▶ For example, an age threshold (age is the running variable)
- ▶ Main idea: subjects on the two sides of the threshold are very similar to each other
 - ▶ The closer they are to the threshold the more similar they are
 - ▶ In their potential outcomes, too
- ▶ So it's almost like random assignment

Regression-discontinuity example

- ▶ Subjects are unemployed people
- ▶ Intervention is a compulsory program that helps job search (x)
- ▶ Outcome is whether they find a job in 3 months (y)
- ▶ Subjects below age 25 are required to participate in the program
- ▶ Subjects 25 or older cannot participate in the program
- ▶ Compare the outcome of 24-year-old subjects and 25-year-old subjects
 - ▶ If average y differs between the two groups that's because of the effect of the program
 - ▶ Because the job finding rate with or without the program (potential outcomes) should be similar

Regression-discontinuity extensions and caveats

- ▶ A version of RD allows for both sides of the threshold to be treated with some probability
 - ▶ In the simple version above the probability was one for one group and zero for the other
 - ▶ In the general version all is needed is a noticeable difference in the treatment probabilities at the threshold of the running variable
- ▶ Caveats
 - ▶ The threshold of the running variable would determine the intervention probability only
 - ▶ Nothing else related to potential outcomes
 - ▶ Subjects should not be able to manipulate the running variable
 - ▶ The method can give a good estimate of the effect for the group of subjects around the threshold value of the running variable

Main takeaways

- ▶ We need exogenous variation in x to uncover its effect on y , but that's hard to achieve with cross-sectional observational data
 - ▶ We can rarely condition on all confounders, so our effect estimates are almost always biased
 - ▶ By conditioning on what we can, we may decrease this bias
 - ▶ We may be able to sign the bias
- ▶ Linear regression and matching on the propensity score are alternative ways to condition on observable confounders
- ▶ With common support, regression and matching tend to give similar results
- ▶ With experience and luck, we may find another, more direct way to isolate exogenous variation in x
 - ▶ Instrumental variables method
 - ▶ Regression-discontinuity design