

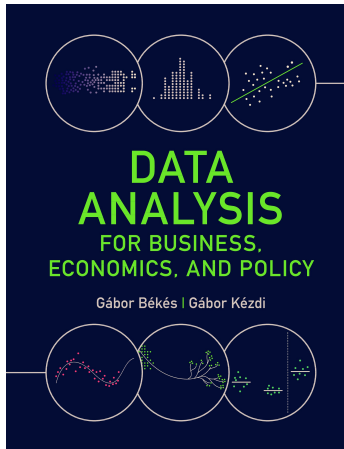
XI: Difference-in-Differences

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Evaluation

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Slideshow for the Békés-Kézdi Data Analysis textbook



- ▶ Cambridge University Press, 2021
- ▶ gabors-data-analysis.com
 - ▶ Download all data and code:
gabors-data-analysis.com/data-and-code/
- ▶ These slides are for Chapter 22

Basic Difference-in-Differences Analysis: Comparing Average Changes

- ▶ Observational data: treated and untreated groups tend to be different in terms of their potential outcomes
- ▶ Often, endogenous sources of variation in x and we cannot condition on all confounders.
- ▶ Close to causality IF we found a variable that captured a lot of the endogenous variation.
- ▶ Idea: the pre-intervention observation of the outcome variable may be such a variable.

Conditioning on Pre-intervention Outcomes

- ▶ Difference-in-differences analysis, (diff-in-diffs)
- ▶ repeated observations on many subjects.
 - ▶ cross-section time series (xt) panel data
- ▶ Observational data, but the outcome variable is observed not only after the intervention but also before it.
- ▶ Basic Difference-in-Differences Analysis: Comparing Average Changes

Basic Difference-in-Differences Analysis: Comparing Average Changes

- ▶ Use the pre-intervention observation of the outcome variable to soak up endogenous variation in x
- ▶ = untreated outcome at the time before the intervention.
- ▶ Comparing subjects that are similar in their untreated potential outcomes, as measured before the intervention.
- ▶ Not the same as conditioning on their untreated potential outcomes after the intervention, which we would like to do.
- ▶ So will help, closer to causality, but not there.

Difference-in-Differences Analysis: Comparing Average Changes

- ▶ Once again, focus on binary intervention
- ▶ Difference-in-differences analysis = comparing the average changes of the outcome among units in the treatment group and in the non-treatment group.
- ▶ Observe the outcome for same units before the intervention and after the intervention both in the treatment group and in the control group.
 - ▶ We'll relax this at the end of class

Basic diff-in-diffs

- ▶ The diff-in-diffs estimation = the difference in the changes (in the “differences”)
- ▶ Average of the changes in outcome among “treated” vs control.
- ▶ Δy_i : change in outcome y for observation i :
- ▶ Average change among treated, untreated observations: $\overline{\Delta y_{treated}}$, $\overline{\Delta y_{untreated}}$,
- ▶ The diff-in-diffs estimator of the effect of x on y as $\beta_{diff-in-diffs}$.

$$\Delta y_i = y_{i,after} - y_{i,before} \quad (1)$$

- ▶ The estimated $\beta_{diff-in-diffs}$ in the data is

$$\hat{\beta}_{diff-in-diffs} = \overline{\Delta y}_{treated} - \overline{\Delta y}_{untreated} \quad (2)$$

The difference-in-differences setup

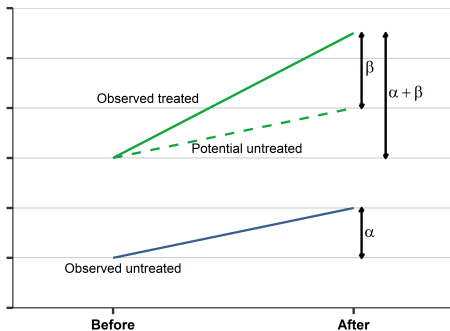
- We can summarize the four data points and the difference we are after:

	Untreated	Treated	Diff: Treated-Untreated
Before	$\bar{y}_{untreated,before}$	$\bar{y}_{treated,before}$	$\bar{y}_{treated,before} - \bar{y}_{untreated,before}$
After	$\bar{y}_{untreated,after}$	$\bar{y}_{treated,after}$	$\bar{y}_{treated,after} - \bar{y}_{untreated,after}$
Diff: After-Before	$\Delta \bar{y}_{untreated}$	$\Delta \bar{y}_{treated}$	$\Delta \bar{y}_{treated} - \Delta \bar{y}_{untreated}$

- The estimated $\beta_{diff-in-diffs}$ in the data is

$$\hat{\beta}_{diff-in-diffs} = \Delta \bar{y}_{treated} - \Delta \bar{y}_{untreated} \quad (3)$$

The diff-in-diff graph



- ▶ the changes are related to the coefficients of the regression.
- ▶ α is the average change among untreated subjects;
- ▶ $\alpha + \beta$ is the average change among treated subjects;
- ▶ β is the difference between the two.

The power of diff-in-diffs

- ▶ This is the key issue. Estimate the causal effect even when there is a change affecting all subjects
 - ▶ Demand may affect all markets, products equally, ie there is universal change
 - ▶ As captured by α
 - ▶ But some subjects (treated) change more or less.

Diff-in-diff in a regression

- ▶ Regression with Δy as the dependent variable and *treated* as the explanatory variable:

$$\Delta y^E = \alpha + \beta \textit{treated} \quad (4)$$

- ▶ In this regression, the estimated $\hat{\alpha}$ is the average change in y among untreated subjects in the data.
- ▶ The effect of the intervention is $\hat{\beta}$: it is the difference between the average change in y among treated subjects ($x = 1$) and untreated subjects ($x = 0$) in the data.
- ▶ This is the diff-in-diffs estimate $\hat{\beta} = \hat{\beta}_{\text{diff-in-diffs}}$.

Case study: Airline merger - setup

- ▶ USA airline merger
- ▶ American Airlines filed for bankruptcy in November 2011.
- ▶ US Airways, a competitor, announced its intent to take over American Airlines in 2012.
- ▶ Merger was allowed in April 2015.

Case study: Airline merger - raw data

- ▶ Airline Origin and Destination Survey (DB1B) collected by the Office of Airline Information of the Bureau of Transportation Statistics, US Dept Transport
- ▶ Six years of data. Before= year 2011 and After = year 2016
- ▶ Itinerary level, N = 3 million per quarter (10% random sample)
- ▶ Big Data: transaction level, collected automatically, 15GB

Case study: Airline merger - tidy data

- ▶ Tidy data
- ▶ Observations: airline X route X year X quarter level,
- ▶ n 600-700 thousand per quarter (total n=18,410,466)
- ▶ route: unique combination of airports in itinerary (e.g., DTW MSP DTW)

Case study: Airline merger - defining market

- ▶ Aggregate at market level
- ▶ Markets are defined by their origin airport and their destination airport, and whether they are one-way or return routes
- ▶ Defining the market in this setup is not straightforward:
 - ▶ Complicated trips.
- ▶ One-way tickets versus return tickets.
 - ▶ Return tickets = final destination is reasonably clear
 - ▶ Market one-way: final destination = last airport here.
 - ▶ Market return: selected routes with a clear middle airport only
 - ▶ We dropped all other return routes.
 - ▶ Affected less than 10% of the passengers.
 - ▶ Asymmetric routes – keep them.

Case study: Difference-in-differences table for the effect of AA–US merger on average log prices

	(1) Untreated	(2) Treated	(3) Difference: Treated - Untreated
Before	4.92	4.96	+0.04
After	5.08	4.94	-0.14
Difference: After - Before	+0.16	-0.02	-0.18

Note: Average log price, weighted by the number of passengers at baseline. Source: airline-tickets-usa dataset. N=112 632 markets.

Case study: Airline merger - setup

- ▶ The outcome variable is change average price.
- ▶ The causal variable is being part of merger

$$(\Delta \ln p)^E = \alpha + \beta AAUS_{before} \quad (5)$$

Case study: Basic difference-in-differences estimate of the effect of the AA–US merger on log prices

	(1)	(2)	(3)
VARIABLES	All markets	Small markets	Large markets
$AAUS_{before}$	-0.18** (0.01)	-0.16** (0.01)	-0.26** (0.03)
Constant	0.16** (0.01)	0.14** (0.01)	0.24** (0.02)
Observations	112,632	111,745	887
R-squared	0.05	0.04	0.09

Source: `US-airlines` dataset. Note: Difference-in-differences estimate of the effect of AA-US merger on average prices. Observations: Markets Before period: 2011. After period: 2016. Outcome variable: log average price. Treated: both AA and US on market in 2011; untreated: neither on market. Weighted by the number of passengers at baseline.

Case study: Diff-in-diffs effect of merger on prices

- ▶ Intercept: prices increased by 16 percent, on average, on untreated markets between 2011 and 2016.
- ▶ Prices increased a lot more on large markets, by 24% compared to the 13% increase in small markets.
 - ▶ Note that prices are not adjusted for inflation here so some of the price increase is sort of natural.
- ▶ Prices on treated markets increased by 18% *less*, on average (column 1)
 - ▶ === Price on treated markets actually fell by 2%

The Parallel Trends Assumption

- ▶ Without the treatment, the outcome would have followed the same trend
 - ▶ Outcome would have changed the same way
- ▶ Assumption.
- ▶ Assumes no selection related to changes
 - ▶ Whichever unit is treated is not the result of decisions that are related to how outcome changes
- ▶ Assumes no reverse causality from changes
 - ▶ Whichever unit is treated is not caused by change of outcome (or its anticipation)

The Parallel Trends Assumption

- ▶ Taking differences takes care of problems related to levels.
- ▶ Allows for selection and reverse causality related to levels
 - ▶ Whichever unit is treated may be related to initial levels of outcome
 - ▶ As long as it is not related to changes

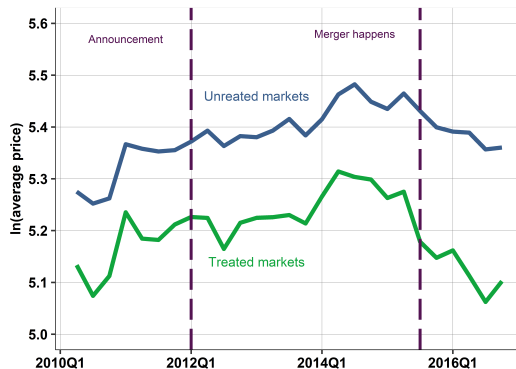
The Parallel Trends Assumption

- ▶ The parallel trends assumption is an assumption that is impossible to verify or falsify directly.
 - ▶ We can't know how outcomes would have changed on average in the treatment group without the intervention.
- ▶ Indirect evidence can support or contradict it.
- ▶ Look at observed trends before the intervention in the two groups when none were treated.
 - ▶ If observed outcomes have the same trend before the intervention the parallel trends assumption is more likely to be true than with very different pre-intervention trends.
- ▶ The parallel trends assumption cannot be tested directly

The Parallel Trends Assumption: ATET and ATE

- ▶ The parallel trends assumption: without the intervention, outcomes would have changed the same way, on average, in the treatment group and the non-treatment group.
- ▶ If the assumption is true, diff-in-diffs gives a good estimate of ATET.
- ▶ If, *in addition*, the average of outcomes in the non-treatment group would have changed the same way, had they been treated, as it changed in the treatment group, diff-in-diffs gives a good estimate of ATE, too.
- ▶ Speculation, no test
- ▶ Check by looking at pre-trends - helps a great deal, esp re ATET

Case study: Airline merger - Pre-intervention trends in log average price

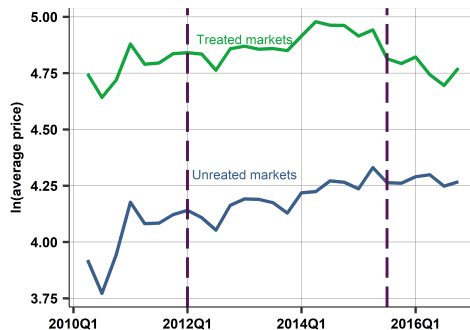
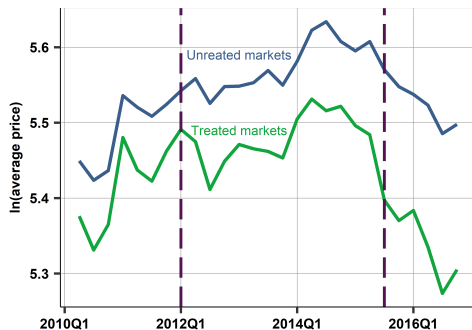


Pre-intervention trends in log average price. Treated versus untreated markets; all markets. Note: Weighted averages by the number of passengers. Source: `airlines` dataset.

Case study

- ▶ The two lines appear to move together but there are some differences. Not huge.
- ▶ Eyeballing the differences in 2011 versus 2016 confirms the the -10% + difference in differences of our regression estimate. Mostly happening in 2015.
- ▶ Are pre-intervention trends *parallel*?
 - ▶ Not exactly, but close
 - ▶ Larger patterns are similar across the two lines until 2015.
 - ▶ Good if not perfect evidence.
- ▶ May conclude that pre-intervention trends were quite similar.

Case study: Pre-intervention trends, small versus large markets



Note: Airline markets in the United States (origin–final destination airport-pairs separately for one way and return routes). Weighted averages by the number of passengers.

Conditioning on Additional Confounders in Diff-in-Diffs Regressions

- ▶ If the parallel trend assumption is not true we have a problem.
- ▶ This is a problem of endogeneity - we do not have the ATET with simple regression.
- ▶ The usual indirect strategy to mitigate the problem is conditioning on potential confounders.
- ▶ We observe observations both before and after the intervention → new opportunities for conditioning.

Conditioning on potential confounders

- ▶ With outcome y , binary treatment variable *treatment* and potential confounder variables z_1, z_2, \dots
- ▶ the general form of the diff-in-diffs regression with additional right-hand-side variables is

$$\Delta y^E = \alpha + \beta \textit{treatment} + \gamma_1 z_1 + \gamma_2 z_2 + \dots \quad (6)$$

- ▶ if z variables include all potential confounders – β is a good estimator for the average treatment effect.
- ▶ In the diff-in-diffs framework those conditions ensure that the parallel trends assumption holds conditional on the z variables.

Conditioning on potential confounders

- ▶ Main difference is nature of z
- ▶ z can be level measured at "before"
- ▶ z can be difference between after and before. This is the key novelty.
- ▶ Other considerations same as before.
 - ▶ Look for correlation with y and x
 - ▶ Avoid bad conditioning variable, ie mechanism

Conditioning on potential confounders

- ▶ Start thinking about the sources of variation in the causal variable.
- ▶ why both AA and US were present on some markets before the intervention and why they weren't present on other markets.
- ▶ Our treatment variable is defined for baseline. Thus we should worry about confounders at baseline.
- ▶ The size of the market.
 - ▶ Both airlines are more likely to be present in larger markets. Larger markets may be more expensive if city is larger or richer.
- ▶ Competition at markets (number of companies)
- ▶ Route features

Case study: Difference-in-differences conditioning on confounders

	(1)	(2)	(3)
Variables	All markets	Small markets	Large markets
$AAUS_{before}$	-0.11** (0.01)	-0.10** (0.01)	-0.13** (0.03)
$\ln no.passengers_{before}$	-0.00 (0.00)	0.00 (0.00)	0.06** (0.02)
Return route	0.19** (0.01)	0.20** (0.01)	0.17** (0.03)
Number of stops	-0.03** (0.01)	0.00 (0.01)	-0.07** (0.02)
Share of largest carrier	0.26** (0.02)	0.21** (0.02)	0.43** (0.07)
Constant	-0.15** (0.03)	-0.17** (0.02)	-0.74** (0.21)
Observations	112,632	111,745	887
R-squared	0.14	0.11	0.23

Source:

US-airlines

dataset.

Using quantitative causal variable

- ▶ A quantitative causal variable determined by $x_{baseline}$, its baseline features
- ▶ With outcome y , change in outcome Δy , and quantitative causal variable x , the regression formula looks the same as with a binary x variable.

$$\Delta y^E = \alpha + \beta x_{baseline} \quad (7)$$

- ▶ Interpretation different from binary
- ▶ α : how y is expected to change when $x_{baseline}$ is zero.
- ▶ β : the difference in the expected change in y between subjects that are different in $x_{baseline}$ by one unit at baseline.
 - ▶ comparing two subjects that are different in x at baseline by one unit, we can expect y to change by β more units for the subject with the larger x value.

Changes Regressed On Changes

- ▶ Diff-in-diffs with change in continuous RHS variable:

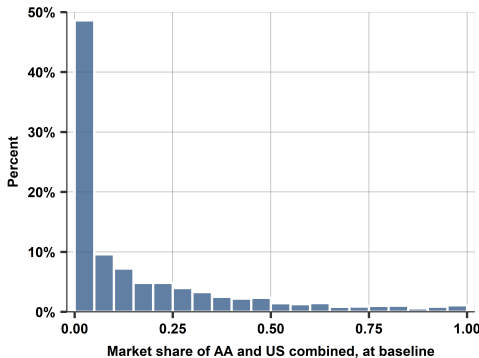
$$\Delta y^E = \alpha + \beta \Delta x \quad (8)$$

- ▶ α : how y is expected to change when x does not change.
- ▶ β shows the difference in the expected change in y between subjects with different change in x .
 - ▶ Comparing two subjects that are different in how much x changes, by one unit, we can expect y to change by β more units for the subject with the larger change in x .

Case study: Airlines - Quantitative causal variable

- ▶ Binary - AA and US were present on the market at baseline or not
- ▶ Better: define the causal variable as their combined market share at baseline.
- ▶ Could capture heterogeneity - [Why?](#)

Case study: Heterogeneity in market share of AA + US at baseline



- ▶ Airline markets in the United States (origin–final destination airport-pairs separately for one-way and return routes).
- ▶ Baseline is year 2011. The histogram is weighted by the number of passengers at baseline.

Case study: Airlines - Quantitative causal variable

- ▶ Presumably, markets in which the two airlines had a small share were less affected than markets in which they had a larger share.
- ▶ Define the causal variable as their combined market share at baseline. This share variable is between zero and one.
- ▶ The regression formula is the following:

$$\begin{aligned}
 (\Delta \ln p)^E = & \beta_0 + \beta_1 AAUS_{before} + \beta_2 \ln passengers_{before} \\
 & + \beta_3 return + \beta_4 stops + \beta_5 sharelargest_{before}
 \end{aligned}
 \tag{9}$$

Case study: Diff in diffs with market share of the two airlines at baseline

	(1)	(2)	(3)
Variables	All markets	Small markets	Large markets
Market share before	-0.27** (0.02)	-0.17** (0.02)	-0.42** (0.05)
$\ln no. passengers_{before}$	-0.01** (0.00)	-0.01** (0.00)	0.05* (0.02)
Return route	0.21** (0.01)	0.21** (0.01)	0.19** (0.02)
Number of stops	-0.03** (0.01)	0.00 (0.01)	-0.07** (0.02)
Share of largest carrier	0.31** (0.02)	0.26** (0.02)	0.47** (0.05)
Constant	-0.12** (0.03)	-0.15** (0.02)	-0.72** (0.20)
Observations	112,632	111,745	887
R-squared	0.15	0.11	0.30

Source:

US-airlines

dataset.

Case study: Airlines - Quantitative causal variable

- ▶ Results suggests that this may be a useful specification
- ▶ let us compare markets where the number of passengers and the market share of the largest carrier were the same in 2011 and that are similar in whether they are return routes and how many stops they have.
- ▶ Prices decreased by 27% more, on average, in markets where the pre-merger share of AA and US was 100% instead of 0%.
- ▶ = Prices decreased by 2.7% more, on average, in markets where the pre-merger share of AA and US was 10% more

Case study: Airlines - Quantitative causal variable

- ▶ Results suggests that this may be a useful specification
- ▶ let us compare markets where the number of passengers and the market share of the largest carrier were the same in 2011 and that are similar in whether they are return routes and how many stops they have.
- ▶ Prices decreased by 27% more, on average, in markets where the pre-merger share of AA and US was 100% instead of 0%.
- ▶ = Prices decreased by 2.7% more, on average, in markets where the pre-merger share of AA and US was 10% more
- ▶ Allow heterogeneity, market shares matter – larger coefficient.

Repeated Observation of Same Units

- ▶ Recap. This is what we were doing
- ▶ Same units
 - ▶ Individuals, families, stores, firms, regions, countries
- ▶ Observed both before and after treatment period
 - ▶ “Panel” or “longitudinal” data
- ▶ Simplest: observed once before, once after
 - ▶ Each unit observed twice
 - ▶ It is the simplest panel data
 - ▶ basic diff-in-diffs applies to two-period panel data

Diff-in-diffs with pooled cross-sections

- ▶ Basic diff-in-diffs is based on observing the same units both before and after the treatment
- ▶ Sometimes we don't have such data
- ▶ But have data on different units before and after
 - ▶ There is a way to do a kind of a diff-in-diffs analysis
 - ▶ Only it works with additional assumptions
- ▶ Look at four groups and pool them
 - ▶ Treated and untreated units; before and after
 - ▶ Not the same units, but similar
 - ▶ Efforts to show this similarity

Example: Building a large factory close to houses

- ▶ Same units: observe the same houses sold before and after in treated and control neighborhoods
- ▶ Different units: observe *any* house ...
- ▶ Use features of houses as controls to mitigate

Diff-in-diffs with pooled cross-sections

- ▶ Difference-in-differences with pooled cross sections can be computed by measuring average outcomes in the two groups in the two time periods.

$$\beta_{diff-in-diff} = (\bar{y}_{treatment,after} - \bar{y}_{treatment,before}) - (\bar{y}_{non-treatment,after} - \bar{y}_{non-treatment,before}) \quad (10)$$

- ▶ However, here the before and after averages are computed from different units.
 - ▶ Index “treated” in the averages means units in the group that would become treated in the “after” period.
- ▶ This difference-in-differences is thus impossible to represent as a regression on changes within units as we cannot compute the change in the outcome, for the same units.
- ▶ Instead, one can represent this difference-in-differences in a regression using all units as separate cross-sectional observations.

Diff-in-diffs with pooled cross-sections

- ▶ Back to binary treatment setup
- ▶ The first binary variable, *treatment*, shows whether the observation belongs to the treatment group.
- ▶ The second binary variable, *after*, shows whether the observation is observed in the after period

$$y^E = \alpha + \beta treatment + \gamma after + \delta treatment \times after \quad (11)$$

- ▶ The coefficient of the interaction term, δ , difference-in-differences estimate: how much larger the after–before difference of average outcomes is in the treatment group than in the non-treatment group.

Diff-in-diffs with pooled cross-sections

Balanced panel, we can estimate this pooled xsec diff-in-diffs regression, and we get back the same results as with the basic diff-in-diffs regression that has the change in y on its left-hand side.

$$\delta = (y_{treatment=1,after=1}^E - y_{treatment=1,after=0}^E) - (y_{treatment=0,after=1}^E - y_{treatment=0,after=0}^E) \quad (12)$$

- ▶ Here δ is the difference-in-differences coefficient.
 - ▶ The advantage here, is that we could add control variables in a more transparent way

Diff-in-diffs with pooled cross-sections -example

- ▶ When subjects not the same
- ▶ New part: selection problem
 - ▶ Before and after observations are different
 - ▶ Assume that the samples observed before and after represent the same groups
 - ▶ Case study: same routes before/after. What do you think?

Case study: Airline merger – selection

- Instead of working with differences, we reverse back to observations as we have them.

New 1 We can now have a different sample with more observations

New 2 We can add Z variables to capture relevant differences between units observed before and after - tackle selection as confounder

$$(\ln p)^E = \alpha + \beta AAUS_{before} + \gamma after + \delta AAUS_{before} \times after \quad (13)$$

- Without confounders, same result as before
- Unbalanced nature did not matter much

Case study: Airline merger – selection

- ▶ Extended sample.
 - ▶ markets that are observed both before and after
 - ▶ Includes markets that are observed in the before time period only [new]
- ▶ Run same regression as we started with. Different format.
- ▶ Row are now levels not differences (twice as many observations) + include markets dropped before
- ▶ results do not change in this case

Case study: Diff-in-diff estimates, pooled cross-sections

- ▶ Pooled cross sections - unbalanced panel
- ▶ Unbalanced panel, also including markets observed at baseline only. AAUSbefore: binary variable for both AA and US on market in 2011.

Case study: Diff-in-diff estimates, pooled cross-sections

Variables	(1) All markets	(2) Small markets	(3) Large markets
$AAUS_{before} \times after$	-0.11** (0.01)	-0.10** (0.01)	-0.14** (0.03)
$AAUS_{before}$	0.43** (0.02)	0.29** (0.01)	0.69** (0.05)
$after$	-0.19** (0.04)	-0.14** (0.03)	-0.82** (0.22)
$\ln no. passengers_{before}$	-0.37** (0.01)	-0.29** (0.00)	-0.54** (0.05)
Return route	0.84** (0.02)	0.84** (0.01)	0.95** (0.05)
Number of stops	0.07** (0.02)	0.14** (0.01)	0.03 (0.04)
Share of largest carrier	-1.45** (0.05)	-1.40** (0.03)	-1.69** (0.13)
$\ln passengers_{before} \times after$	-0.00 (0.00)	-0.00 (0.00)	0.06* (0.02)
Return route $\times after$	0.20** (0.01)	0.20** (0.01)	0.19** (0.03)
Number of stops $\times after$	0.01 (0.02)	-0.00 (0.02)	0.01 (0.05)
Share of largest carrier $\times after$	0.30**	0.23**	0.44**

Case study: Diff-in-diff estimates, pooled cross-sections

- ▶ Practically the same as the corresponding diff-in-diffs coefficients
- ▶ In this case, we managed to control on confounders to tackle selection
 - ▶ Was not a big deal here

Case study: Diff-in-diff estimates, pooled cross-sections

- ▶ Practically the same as the corresponding diff-in-diffs coefficients
- ▶ In this case, we managed to control on confounders to tackle selection
 - ▶ Was not a big deal here

Case study: Airline merger - summary

- ▶ Estimated that the merger led to a decrease in prices.
- ▶ How close it is to ATET: close but not certain: pre-trends, not the same, but close, and the divergence in prices after the merger was a lot stronger.
- ▶ true effect of the merger was smaller than our estimate, but it was likely negative.
- ▶ Including baseline confounder variables confirmed: led to smaller negative estimates
- ▶ Heterogeneity was important: pre-treatment share of the two airlines – obtained stronger negative effect estimates.
- ▶ Only a few years
- ▶ Don't know quality, just price.
- ▶ Fairly convincing that in SR not anti-competitive in price.

Main takeaways

- ▶ Difference-in-differences (diff-in-diffs) estimates the effect of an intervention by comparing how the outcome variable changes among treated subjects versus untreated subjects
 - ▶ Diff-in-diffs uses observational panel data and has a better chance to estimate the effect of an intervention than using cross-sectional data, because it conditions on pre-intervention outcomes
 - ▶ Diff-in-diffs gives a good estimate of the effect if the parallel trends assumption holds
 - ▶ We can't test the parallel trends assumption directly, but examining pre-intervention trends can give indirect evidence