FinalExam

Viktors Djakonovs

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Originals variants

$$W = E[R] = \left(\frac{1}{1-\rho}\right) E[S], \quad W_q = \left(\frac{\rho}{1-\rho}\right) E[S], \quad L_q' = \frac{1}{1-\rho}, \tag{11.7}$$

where $E[S] = 1/\mu$ is the mean service time.

Example 11.7 Let us compute these performance measures for the oil change center of Example 11.6. Using the parameters $\lambda = 4$, $\mu = 5$, and $\rho = 4/5$, we have the following measures:

Mean number in system:	$\rho/(1-\rho) = 4$ cars,
Variance of number in system:	$\rho/(1-\rho)^2 = 20$ cars,
Average response time:	$1/(\mu-\lambda) = 1$ hour,
Average time in queue prior to service:	$\rho/(\mu-\lambda) = .8$ hours (or 48 minutes),
Average size of queue when it is not empty:	$\mu/(\mu-\lambda) = 5$ cars.

Throughput, Utilization, and Traffic Intensity

It is apparent from the definitions given in the previous section that the throughput of the *M/M/*1 queue is just equal to the arrival rate, i.e., $X = \lambda$, and that the utilization is equal to the traffic intensity and is given by $U = \lambda/\mu$.

Example 11.8 Consider a cable modem which is used to transmit 8-bit characters and which has a capacity of 4 megabits per second (Mbps). Thus the maximum rate is 500,000 characters per second (cps). Given that traffic arrives at a rate of 450,000 cps, let us compute some standard performance measures, when this system is modelled as an *M/M/*1 queue.

In this example, we have $\lambda = 450{,}000$ cps and $\mu = 500{,}000$ cps. Thus the utilization (and hence the traffic intensity) of the cable modem is $\rho = \lambda/\mu = 0.9$. The mean number of characters in the system L, and the mean number of characters waiting to be transmitted L_q, are

$$L = \frac{\rho}{1-\rho} = \frac{0.9}{0.1} = 9 \quad \text{and} \quad L_q = \frac{\rho^2}{1-\rho} = \frac{0.9^2}{0.1} = 8.1;$$

the average transmission time per character (the response time) is equal to the expected number in the system divided by λ (from Little's law) which gives

$$\frac{\rho}{1-\rho} \times \frac{1}{\lambda} = \frac{0.9}{0.1} \times \frac{1}{450{,}000} = .00002 \text{ seconds.}$$

The throughput is just equal to $\lambda = 450{,}000$ cps.

Distribution of Time Spent in an *M/M/*1 Queue (Response Time)

Most performance measures presented so far have been average values: the mean number of customers in the system, the mean time spent waiting prior to service, and so on. We may also wish to find the distributions of response time and queueing time, just as in $p_n, n = 0, 1, 2, \ldots,$ we have the distribution of customer population. Whereas queue length and p_n are unaffected by the scheduling policy employed by a queueing system, it should be apparent that the distribution of time that a customer spends in the system is a function of this policy. Here our concern is with the *M/M/*1 queue and the FCFS scheduling discipline.

We first consider the total time (response time) a customer has to spend in the system, which includes the time spent in the queue waiting for service plus the time spent actually receiving service. Denote this random variable, the response time, by R, its PDF by $W_r(t)$, its density by $w_r(t)$, and its expected value by $E[R]$. We shall show that

$$w_r(t) = (\mu - \lambda)e^{-(\mu-\lambda)t}, \quad t > 0,$$

$$W = E[R] = (\frac{1}{1-\rho})E[S], W_q = (\frac{\rho}{1-\rho})E[S], L_q^{'} = \frac{1}{1-\rho}, (11.7)$$

where $E[S] = \frac{1}{\mu}$ is the mean service time.

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The throughput is just equal to $\lambda = 450,000$ cps.

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```latex
    \documentclass{report}
\usepackage[utf8]{inputenc}
\usepackage{tikz}
\usepackage{tabu}
\usepackage{amssymb}
\usepackage{multicol}
\usepackage{latexsym}
\usepackage{wrapfig}
\usepackage{layout}
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\geometry{bottom=3.9cm}
\geometry{left=3.1cm}
\geometry{right=3.2cm}
\pagestyle{empty}

\begin{document}
\title{FinalExam}
\author{Viktors Djakonovs}
\date{May 2019}

\maketitle
\section*{Originals variants}
\includegraphics[scale=0.2, width=20cm,height=15cm,angle=+90]{IMG_7548.JPG}
\pagebreak


\begin{center}\section*{\fontsize{10}{15}
\selectfont 11.2 Birth-Death Processes: The M/M/1 Queue 409}\end{center}
{\color{black}\rule{\linewidth}{0.01mm}}
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\noindent where $E[S]=\frac{1}{\mu}$ is the mean service time. \\
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11.6. Using the parameters $\lambda=4, \mu=5, and
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\columnbreak \\
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\end{multicols}

\noindent \textbf{Throughput, Utilization, and Traffic Intensity} \\
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