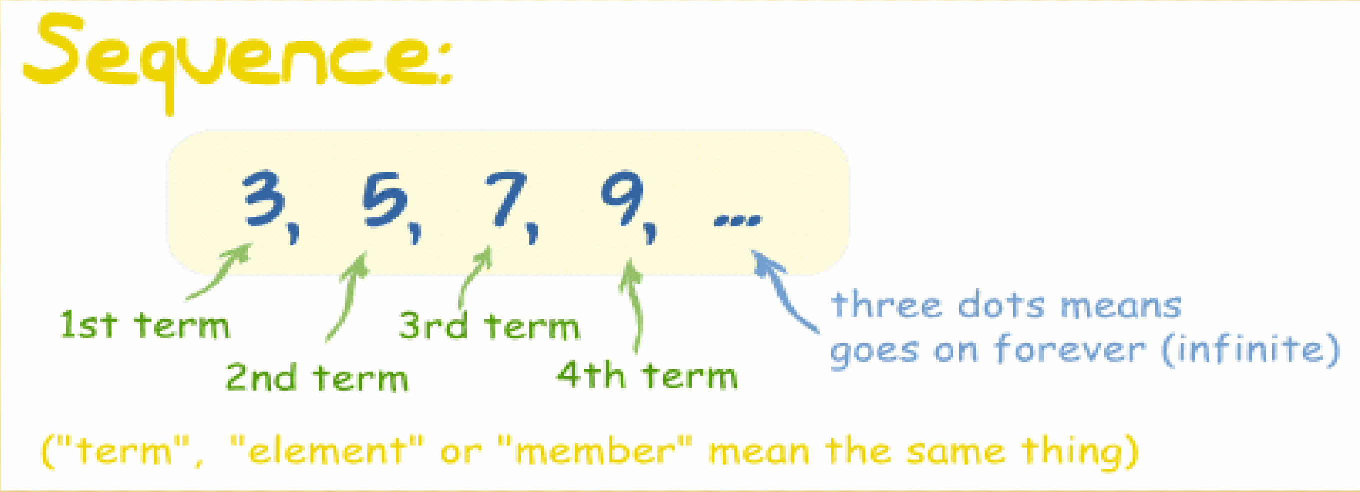
COUNTING

Mathematics for Computing
(IT 1030)

SEQUENCES

- What is a Sequence?

- A Sequence is a list of things (usually numbers) that are in order; Infinite or Finite



INFINITE OR FINITE SEQUENCES

- When the sequence goes on forever it is called an **infinite sequence, otherwise it is a finite sequence.**

Ex: {1,2,3,4,…} is an infinite sequence

{2,4,6} is a finite sequence with 3 terms

SET VS. SEQUENCE

Set	Sequence
Terms need not to be in order	Terms must be in order
Values cannot repeat	Values can repeat

Ex: {0, 1, 0, 1, 0, 1, ...} is the sequence of alternating 0s and 1s. The **set is just {0,1} or {1,0}** .

ARITHMETIC SEQUENCE

- It has a common difference between successive terms.

  Ex: 2, 4, 6, 8, …

term position

$$a_n = a_1 + (n-1)d$$

n^{th} term

first term

common difference

Sum of nth terms =

$$\frac{n}{2}(2a + (n-1)d)$$

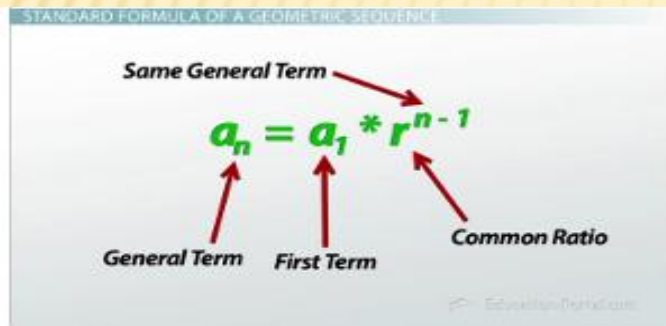
Q: Find the 10^{th} term and the sum of first 10 terms of the following sequence A_n.

A_n : {3, 8, 13, 18, 23,…}

GEOMETRIC SEQUENCE

- It has a common ratio between successive terms.

Ex: 2, 4, 8, 16, …



$$S_n = \frac{a_1(1-r^n)}{1-r}$$

Q: Find the 11th term and the sum of first 11 terms of the following sequence A_n.

A_n : {3, 6, 12, 24,…}

SIGMA NOTATION

- It represents **summation** of many similar terms.

Value of n in the final term (may be ∞)

The Greek letter *sigma* means "sum."

Terms of the sum

$$a = \sum_{n=1}^{10} a_n = a_1 + a_2 + \ldots + a_9 + a_{10}$$

n ranges from 1 up to 10, counting by 1

The index n labels each term. n = 1, 2, 3, ...

Q: Expand the followings

(i) $\sum_{n=4}^{8} 3n + 7$

(ii) $\sum_{i=1}^{2} \sum_{j=4}^{6} (3ij)$

PROPERTIES OF SIGMA NOTATION

- There are a couple of formulas for summation notation.

1. $\sum_{i=i_0}^{n} ca_i = c\sum_{i=i_0}^{n} a_i$ where c is any number. So, we can factor constants out of a summation.
2. $\sum_{i=i_0}^{n}(a_i \pm b_i) = \sum_{i=i_0}^{n} a_i \pm \sum_{i=i_0}^{n} b_i$ So we can break up a summation across a sum or difference.

Show that,

$$\sum_{r=1}^{n}(6r+5) = 6\sum_{r=1}^{n} r + 5\sum_{r=1}^{n} 1$$

PI NOTATION

- It represents **product** of many similar terms.

$$\prod_{i=1}^{n} a_i = a_1 * a_2 * \ldots\ldots\ldots * a_n$$

- And It also satisfies,

$$\left(\prod_{i=1}^{k} a_k\right)\left(\prod_{i=1}^{k} b_k\right) = \prod_{i=1}^{k}\left(a_k b_k\right)$$

Example,

$$\prod_{k=3}^{7} k$$

$$= (3)(4)(5)(6)(7)$$

FACTORIAL(N)

$n!$

$$n! = [1*2*3*4* \dots *n]$$

n! is "n factorial"

- Find the following values

  (i) 3!

  (ii) 5! * 2!

  (iii) 0!

${}^nC_r$ AND ${}^nP_r$ NOTATIONS

$$nCr = \frac{n!}{r!\ (n - r)!}$$

$$nPr = \frac{n!}{(n - r)!}$$

Find the followings,

(i) ${}^5C_3$

(ii) ${}^6c_1$

(iii) ${}^5P_3$

(iv) ${}^7P_0$

PERMUTATIONS

- A permutation is an arrangement of objects in specific order.
- The order of the arrangement is important!!
- Example:
  - How many distinct, 3 letter words can be arranged using {a, b, c} ?? (6 arrangements)

- For any integer $n \geq 1$, the number of permutation of n elements is n!

EXAMPLE

(i) How many ways can the letters in the word COMPUTER be arranged in a row?

All the eight letters are in the word COMPUTER are distinct, so the number of ways,

$$8! = 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 40{,}320$$

(ii) How many ways can the letters in the word COMPUTER be arranged if the letters "CO" must remain next to each other (in order) as a unit?

$$7! = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 5040.$$

PERMUTATIONS OF SELECTED ELEMENTS

- If n and r are integers and 1<=r<=n, then the number of r permutations of a set of n elements is given by the formula

$$^{n}P_{r} = \frac{n!}{(n - r)!}$$

- Example:

  A license plate begins with three letters. If the possible letters are A, B, C, D and E, how many different permutations of these letters can be made if no letter is used more than once? (Ans: 60)

COMBINATIONS

- The number of combinations of *n* things taken *r* at a time is given by:

$$^{n}C_{r} = \frac{n!}{r!(n-r)!}$$

- The order of the arrangement is not important!!
- Example: In how many ways can a coach choose three swimmers from among five swimmers? (10 ways)

QUESTION

(i) 16 teams enter a competition. They are divided up into four Pools (A, B, C and D) of four teams each.

Every team plays one match against the other teams in its Pool.

After the Pool matches are completed:

- the winner of Pool A plays the second placed team of Pool B
- the winner of Pool B plays the second placed team of Pool A
- the winner of Pool C plays the second placed team of Pool D
- the winner of Pool D plays the second placed team of Pool C

The winners of these four matches then play semi-finals, and the winners of the semi-finals play in the final.

How many matches are played altogether?

QUESTION

- How many “Mahajana sampatha” Tickets can be printed in a single draw ?? (numbers are selected from 0 to 9 and it can repeat)

BINOMIAL THEOREM

$$(a+b)^n = \sum_{k=0}^{n} {}^nC_k a^{n-k} b^k = a^n + {}^nC_1 a^{n-1} b + {}^nC_2 a^{n-2} b^2 + \ldots\ldots + {}^nC_{n-1} a^1 b + b^n$$

NOTE: $(r+1)^{th}$ term of this expansion is ${}^nC_r\, a^{n-r}\, b^r$

Question:

(i) What is the coefficient for x^3 in $(2x+4)^8$?

(ii) Expand $(3x-2y)^5$

The End