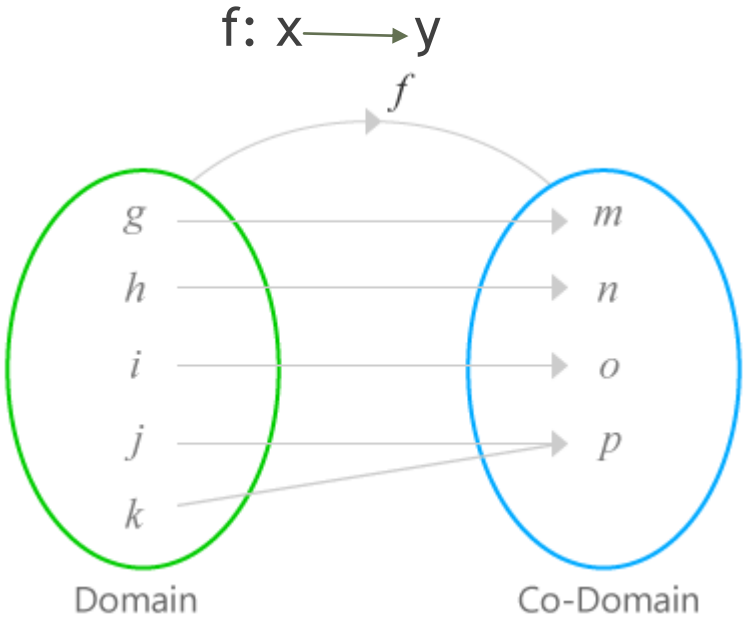
Functions

Mathematics for Computing (IT 1030)

Definition

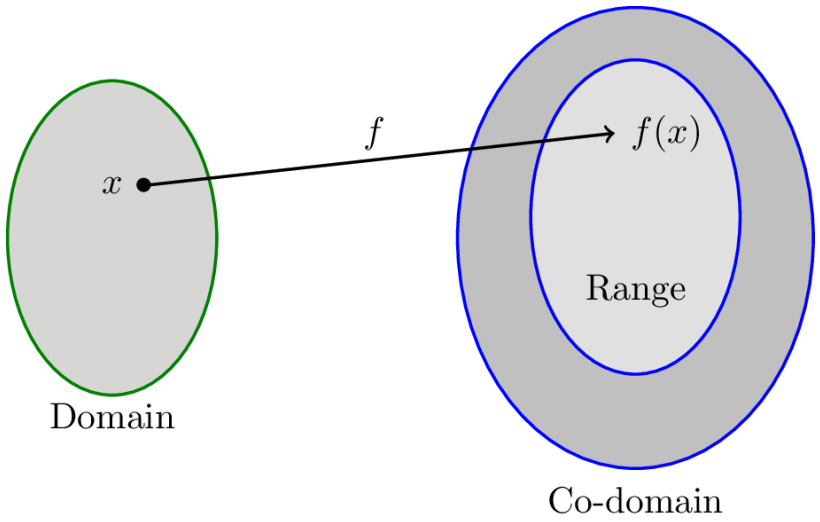
- A **function *f* from a set *X* to a set *Y*** is a relationship between elements of X and elements of Y with the property that <u>each element</u> of X is related to a <u>unique</u> element of Y.



Range/Image

- The unique element *y* to which *f* sends *x* is denoted by *f(x)* and is called ***f* of *x***, or the **value of *f* at *x***, or the **image of *x* under *f***.
- The set of all values of *f* taken together is called the **range of *f*** or the **image of *X* under *f***.

range of f = {y ϵ Y | y=f(x), for some x in X }



Examples

- Which of the following are functions ?

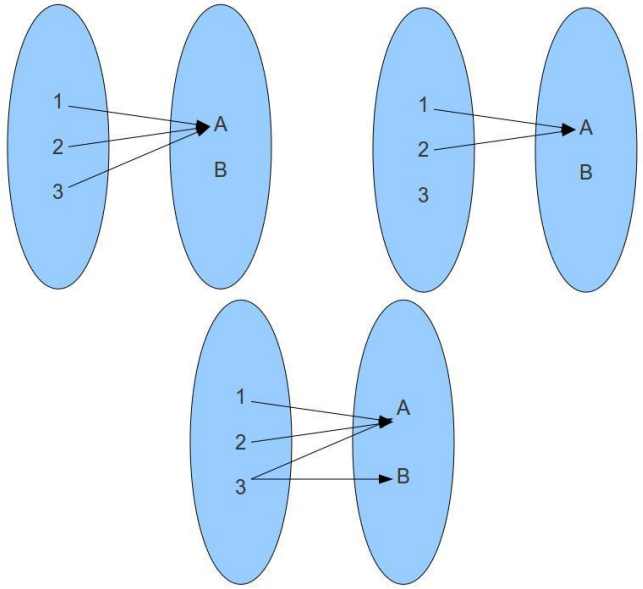


Figure 01

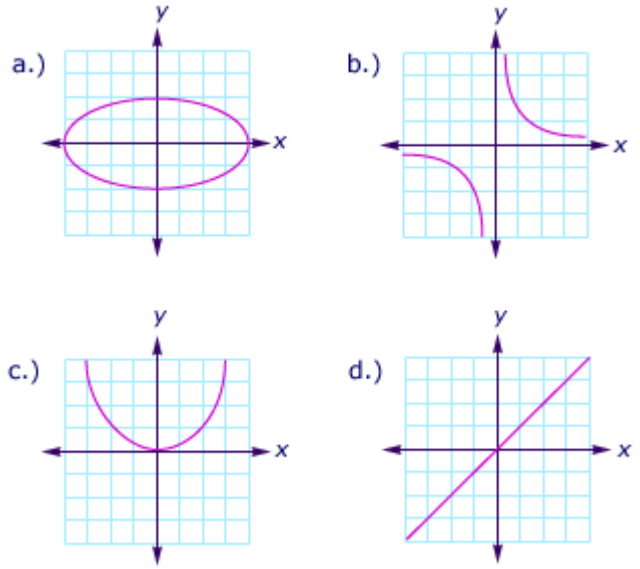


Figure 02

Equality of Functions

- Suppose f and g are functions from X to Y, Then f equals g, written f = g, if and only if,

$$f(x) = g(x) \quad \text{for all } x \in X.$$

Example :

Define $f: R \to R$ and $f: R \to R$ by the following formulas:

$f(x) = |x|$ for all $x \in R$,

$g(x) = \sqrt{x^2}$ for all $x \in R$.

Yes. Since the absolute value of a number equals the square root of its square,

$$|x| = \sqrt{x^2} \quad \text{for all } x \in R.$$

Hence $f = g$

One to One Functions

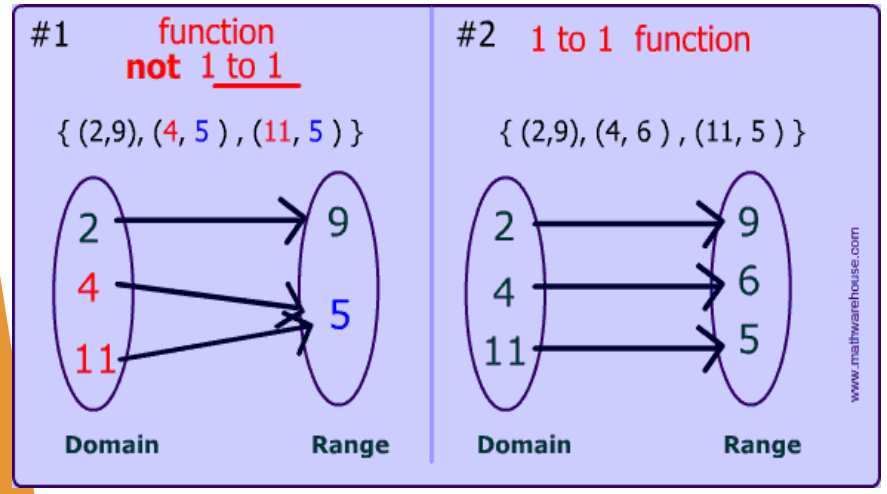
Definition

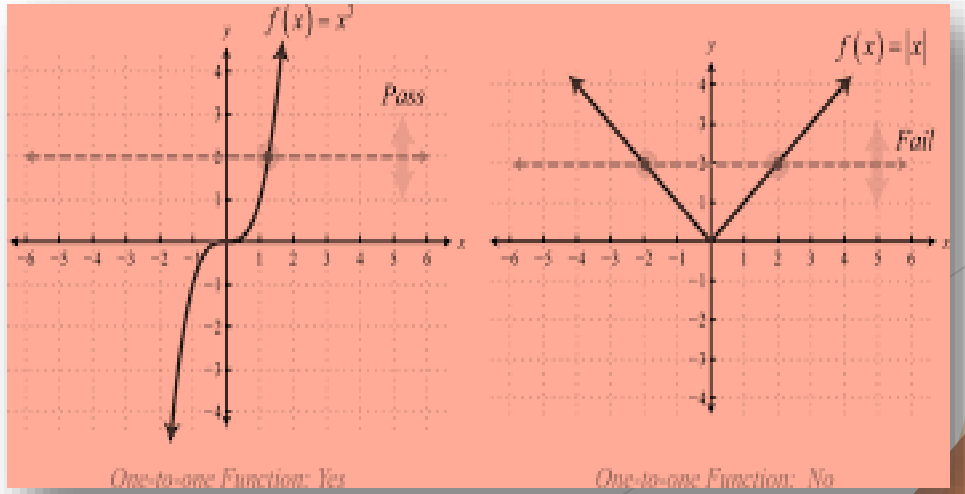
- Let f be a function from a set X to a set Y. f is one to one (or injective) if, and only if, for all elements x_1 and x_2 in X,

  if $f(x_1)=f(x_2)$, then $x_1 = x_2$.

Or, equivalently,

if $x_1 \neq x_2$, then $f(x_1) \neq f(x_2)$.

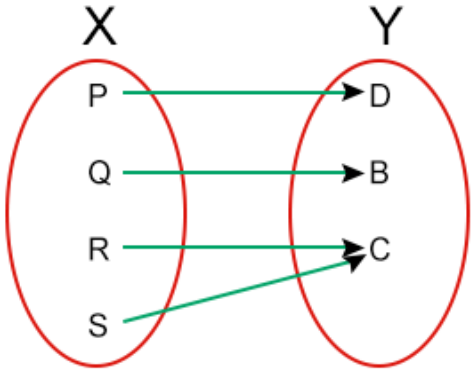




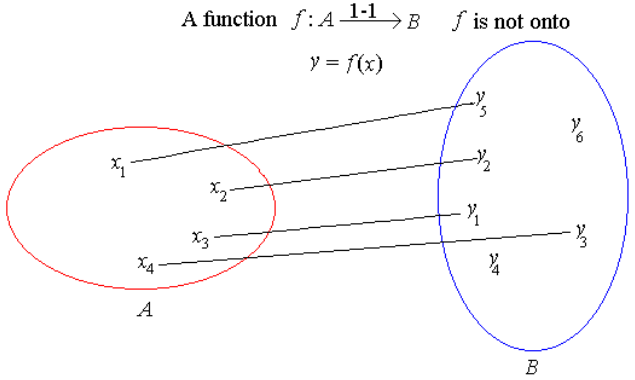
Onto function

- Let f be a function from a set X to a set Y. f is onto (or surjective) if, and only if, given any element y in Y, it is possible to find an element x in X with the property that y = f(x).

  Example:



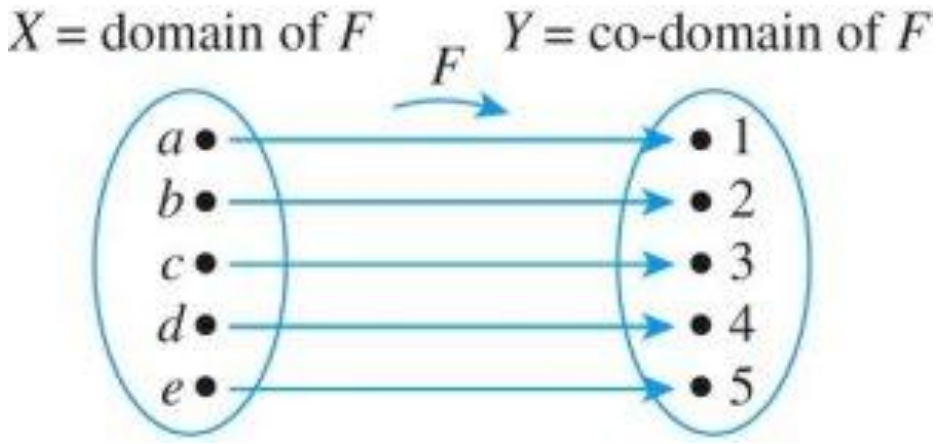
Onto function



Not onto function

One-to-one Correspondence

- A one to one correspondence (or bijection) from a set X to a set Y is a function f: x → y that is both one to one and onto.



Inverse Function

- If f is one-to-one and onto then f^{-1} exists.

Definition :-

Suppose $f : X \to Y$ is a one to one correspondence; that is f is one to one and onto. Then, there is a function $f^{-1} : Y \to X$.

Given any element y in Y,

$f^{-1}(y)$ = that unique element x in X such that $f(x)$ equals y.

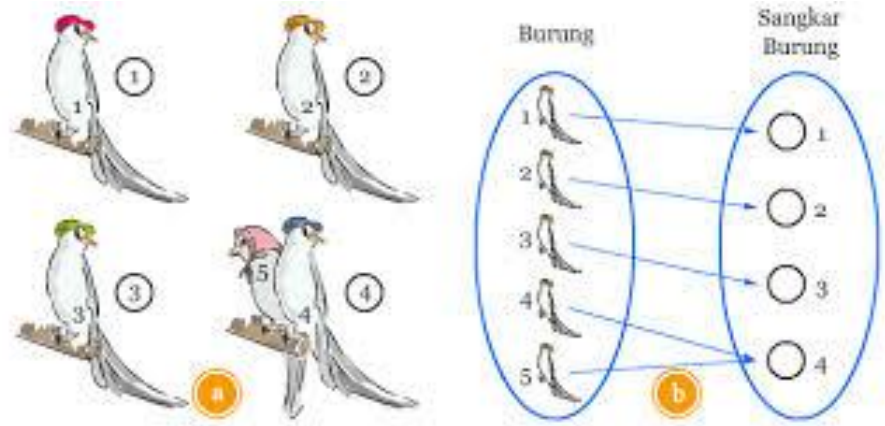
Example:

The function $f : R \to R$ is defined by the formula $f(x) = 4x - 3$ for all real numbers x.
Show that f is a one-to-one correspondence and find its inverse function.

Pigeonhole Principle

Definition

- A function from one finite set to a smaller finite set cannot be one-to-one. There must be at least two elements in the domain that have the same image in the co-domain.



The End