[redacted]

IT1030 – Mathematics for Computing
Lecture 01 – Number Systems

Number Systems

[redacted]

- Mathematical notation/symbols for representing values (numbers).
- In different systems, different symbols are used.

0123456789

٠١٢٣٤٥٦٧٨٩

I II III IV V VI VII VIII IX X

০১২৩৪৫৬৭৮৯

൦൧൨൩൪൫൬൭൮൯

๐๑๒๓๔๕๖๗๘๙

〇一二三四五六七八九

Positional Number System

[redacted]

- We are used to dealing with numbers in the **decimal** system.
- This is probably a result of having ten fingers.
- It is a is a **positional** number system.
  - Roman number system is not such a system.
- The position of the symbol denotes the magnitude.
- A positional number system uses a **base** (aka **radix**).
- A number system with base *b* is a system that uses distinct symbols for *b* digits.

Positional Number System (cont'd.)

[redacted]

- The most common base used in everyday activities is 10 (Decimal System).
- Different bases are used in other situations.
- The base can be written as a subscript to the number for easy identification.
  - Example: 1265_{10} & 010000001_2
- 4 types of positional number systems are discussed.
  - Decimal (Base = 10, {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}).
  - Binary (Base = 2, {0, 1}).
  - Octal (Base = 8, {0, 1, 2, 3, 4, 5, 6, 7, 8}).
  - Hexadecimal (Base = 16, {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F}).

Positional Number System (cont'd.)

[redacted]

- To determine the quantity, it is necessary to multiply each digit by an integer power of the base and then form the sum of all weighted digits.

Example (7241_{10}):-

7		2		4		1
×		×		×		×
10^3		10^2		10^1		10^0
=		=		=		=
7000	+	200	+	40	+	1
			7241			

Binary Number System

[redacted]

- Mainly used in computers and computer-based devices.
- A computer contains electronic components that uses voltages.
- Therefore numbers are represented in a computer with a base of 2.
- All other information is represented using a binary code as well.
  - Letters of the alphabet and punctuation marks
  - Microprocessor instruction
  - Graphics/Video
  - Sound

Number Base Conversion

[redacted]

- Should be able to convert a number in one base to another base.
- Examples will be discussed in converting,
  - Decimal ↔ Binary
  - Decimal ↔ Octal
  - Decimal ↔ Hexadecimal
  - Binary ↔ Octal

Repeated Division Method

[redacted]

- Can be easily used to convert a decimal number to another base.

1. Divide the number successively by the base.
2. After each division record the remainder.
3. The result is read from the last remainder upwards.

Repeated Division Method

[redacted]

- Convert 123_{10} to a binary representation.
  - 123/2 ⇒ q = 61 & r = 1
  - 61/2 ⇒ q = 30 & r = 1
  - 30/2 ⇒ q = 15 & r = 0
  - 15/2 ⇒ q = 7 & r = 1
  - 7/2 ⇒ q = 4 & r = 1
  - 4/2 ⇒ q = 2 & r = 0
  - 2/2 ⇒ q = 1 & r = 0
- $123_{10} = 0011011_2$

Repeated Subtraction Method

[redacted]

- Can be used to convert a decimal number to binary.

1. Starting with the 1s place, write down all of the binary place values in order until you get to the first binary place value that is GREATER THAN the decimal number you are trying to convert.
2. Mark out the largest place value (it just tells us how many place values we need).
3. Subtract the largest place value from the decimal number. Place a "1" under that place value.

Repeated Subtraction Method

[redacted]

4. For the rest of the place values, try to subtract each one from the previous result.
   - If you can, place a “1” under that place value.
   - If you can’t, place a “0” under that place value.
5. Repeat Step 4 until all of the place values have been processed.

- Convert 123_{10} to binary using the repeated subtraction method.

Other Base Conversions (cont'd.)

[redacted]

- Binary/Octal/Hexadecimal to Decimal

1. Take the left most none zero bit,
2. Multiply by the base and add it to the bit on its right.
3. Now take this result, multiply by the base it and add it to the next bit on the right.
4. Continue in this way until the right-most bit has been added in.

The fundamental setup of positional number systems can be used as well.

Other Base Conversions (cont'd.)

[redacted]

- Binary to Octal/Hexadecimal

1. Form the bits into groups of **three/four** starting at the right and move leftwards.
2. Replace each group of three bits with the corresponding octal/hexadecimal digit.

- Octal/Hexadecimal to Binary

The opposite of the above process is used.

Conversion of Fractions

[redacted]

- Decimal Fractions to Binary Fractions

1. Begin with the decimal fraction and multiply by 2. The whole number part of the result is the first binary digit to the right of the point.
2. Disregard the whole number part of the previous result and multiply by 2 once again. The whole number part of this new result is the second binary digit to the right of the point.
3. Continue this process until we get a zero as our decimal part or until we recognize an infinite repeating pattern.

Conversion of Fractions (Example)

[redacted]

- Convert 0.625_{10} to binary.
- Convert 0.1_{10} to binary.
- Convert 1.625_{10} to binary.

Conversion of Fractions (cont'd.)

[redacted]

- Binary Fractions to Decimal Fractions

The fundamental setup of positional number systems used in converting binary integers to decimals can be used here.

- Represent 10.01101_2 as a decimal number.

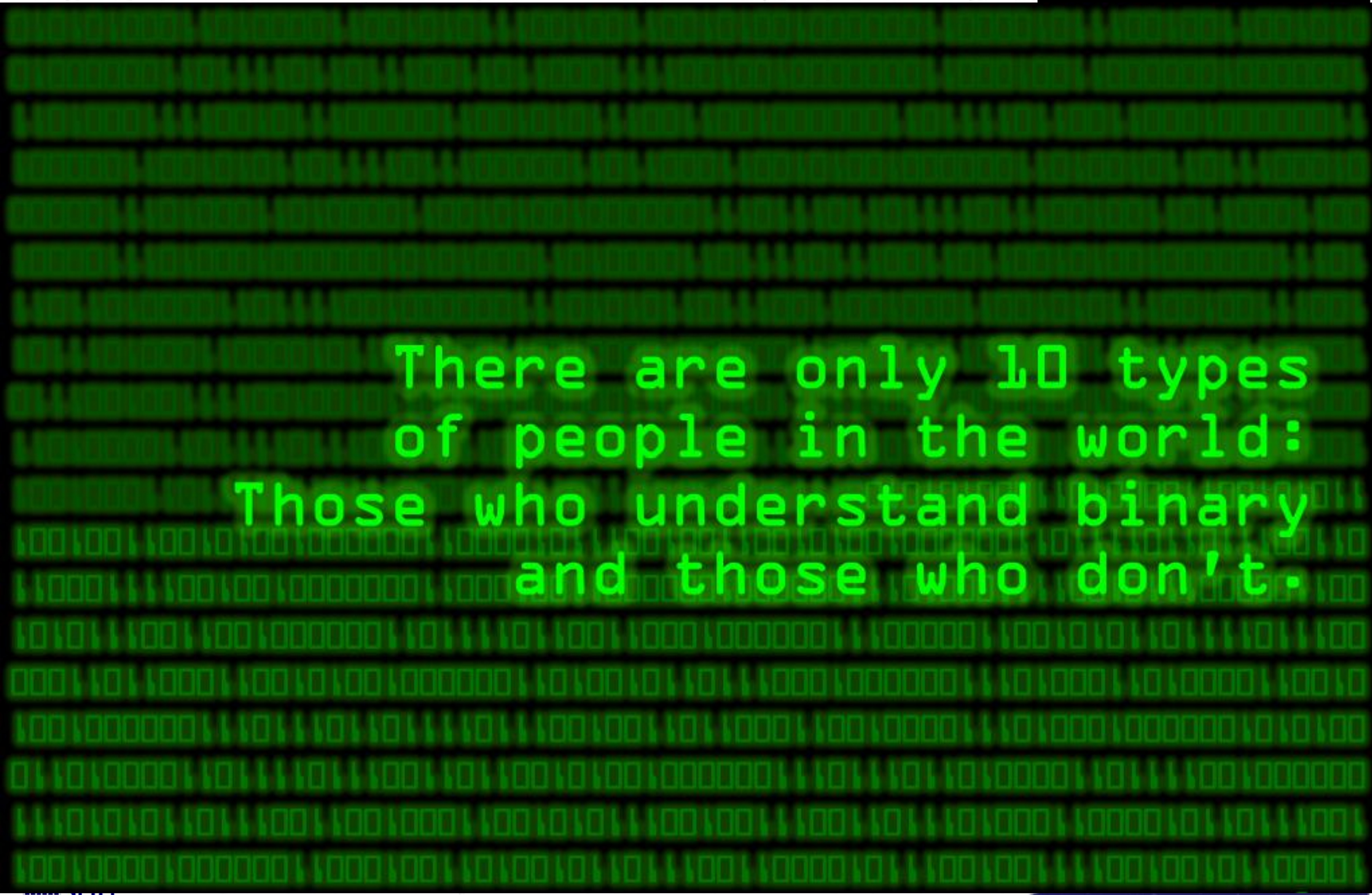
Summary

[redacted]

- Students should be able to,
  - Understand the numerical system.
  - Explain why computer designers chose to use the binary system for representing information in computers.
  - Explain different number systems.
  - Translate numbers between number systems.

- Understanding the pattern in each set of conversions will make it easier to remember the methods.

[redacted]



There are only 10 types of people in the world: Those who understand binary and those who don't.

[redacted]

End of Lecture 01

Next Lecture:- Computer Arithmetic