Tutorial 06 (Answers)

08.) (a) $f : \mathbb{R} - \{-5/3\} \to \mathbb{R} - \{2/3\}$, $f(x) = \dfrac{2x+3}{3x+5}$

One to one.

Let $x_1, x_2 \in \mathbb{R} - \{-5/3\}$.

Let assume $f(x_1) = f(x_2)$.

$$\frac{2x_1+3}{3x_1+5} = \frac{2x_2+3}{3x_2+5}$$

$$(2x_1+3)(3x_2+5) = (2x_2+3)(3x_1+5)$$

$$6x_1x_2 + 10x_1 + 9x_2 + 15 = 6x_1x_2 + 10x_2 + 9x_1 + 15$$

$$\therefore x_1 = x_2.$$

$\therefore$ For all $x_1, x_2 \in \mathbb{R} - \{-5/3\}$, if $f(x_1) = f(x_2)$, then $x_1 = x_2$.

Hence f is a one to one function $\to$ ①

Onto.

Let $y \in \mathbb{R} - \{2/3\}$.

Then $y = f(x)$

$$y = \frac{2x+3}{3x+5}$$

$$3xy + 5y = 2x+3$$

$$3xy - 2x = 3 - 5y$$

$$x(3y-2) = 3-5y$$

$$x = \frac{3-5y}{3y-2} \in \mathbb{R} - \{-5/3\}$$

$\therefore$ For all $y \in \mathbb{R} - \{2/3\}$, there exists $x \in \mathbb{R} - \{-5/3\}$ such that $x = \dfrac{3-5y}{3y-2}$.

Hence f is an onto function $\to$ ②

Thus, by ① & ② f is a bijection. $\therefore$ $f^{-1}(x)$ exists.

Hence $f^{-1}(x) = \dfrac{3-5x}{3x-2}$; $f^{-1} : \mathbb{R} - \{2/3\} \to \mathbb{R} - \{-5/3\}$.