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Mathematics for Computing – IT1030

Lecture 04 - Differentiation

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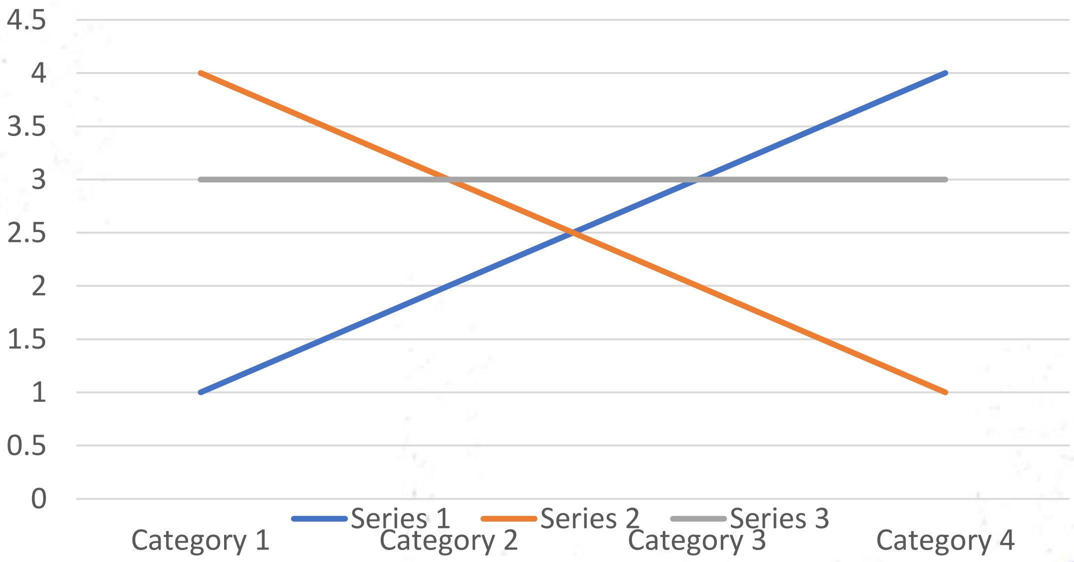
Introduction to Differentiation

- Slope of a line indicates the rate at which a line rises or falls
- For a line, this rate (or slope) is the same at every point on the line



Tangent Line to a Graph

- To determine the rate at which a graph rises or falls at a *single point*, you can find the slope of the tangent line at the point.

- In simple terms, **the *tangent line* to the graph of a function at a point *P* is the line that best approximates the graph at that point**.

Slope of a Graph

- Because a tangent line approximates the graph at a point, the problem of finding the slope of a graph at a point becomes one of finding the slope of the tangent line at the point.

- **Example 1.1** Use the graph to approximate the slope of the graph of $f(x) = x^2$ at the point (1 ,1).
- From the graph of $f(x) = x^2$, you can see that the tangent line at (1,1) rises approximately two units for each unit change in *x*. Thus, the slope of the tangent line at (1,1) is given by

$$\text{Slope} = \frac{\text{change in } y}{\text{change in } x} \approx \frac{2}{1} = 2.$$

The Derivative of a Function

- The derivative of $f(x)$ at x is given by

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

provided this limit exists.

- A function is differentiable at x if its derivative exists at x.
- The process of finding derivatives is called differentiation.
- $f'(x)$ is also denoted by,

$$\frac{dy}{dx}, y', D_x[y], \frac{d\,[f(x)]}{dx}$$

Some Rules for Differentiation

- The Constant Rule
- The Power Rule
- The Constant Multiple Rule
- The Sum and Difference Rules
- The Product Rule
- The Quotient Rule
- The Chain Rule

The Constant Rule

The Constant Rule

The derivative of a constant function is zero. That is,

$$\frac{d}{dx}[c] = 0, \qquad c \text{ is a constant.}$$

Example 1.2

Find the derivative of $f(x) = 3$

$f'(x) = 0$

The Power Rule

The (Simple) Power Rule

$$\frac{d}{dx}[x^n] = nx^{n-1}, \qquad n \text{ is any real number.}$$

- **Example 1.3**
- Find the derivative of $f(x) = x^3$
- $f'(x) = 3x^2$

The Constant Multiple Rule

The Constant Multiple Rule

If f is a differentiable function of x, and c is a real number, then

$$\frac{d}{dx}[cf(x)] = cf'(x), \qquad c \text{ is a constant.}$$

- **Example 1.4**
- Find the derivative of $f(x) = 3x^2$
- $f'(x) = 6x$

The Sum and Difference Rules

If f and g are both differentiable, then

The sum rule $\quad \frac{d}{dx}[f(x) + g(x)] = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$

The difference rule $\quad \frac{d}{dx}[f(x) - g(x)] = \frac{d}{dx}f(x) - \frac{d}{dx}g(x)$

Example 1.5

Find the derivative of $f(x) = 3x^2 + 3$

$f'(x) = 6x + 0 = 6x$

Find the derivative of $f(x) = 3x - 2x^2$

$f'(x) = 3 - 4x$

The Product Rule

$$\frac{d}{dx}[\underbrace{f(x)}_{\text{First function}}\underbrace{g(x)}_{\text{Second function}}] = \underbrace{f(x)}_{\text{First function}}\underbrace{g'(x)}_{\text{The derivative of second function}} + \underbrace{g(x)}_{\text{Second function}}\underbrace{f'(x)}_{\text{The derivative of first function}}$$

- **Example 1.6**
- Find the derivative of $f(x) = 3x^2$(2x-5)
- $f'(x) = 3x^2 * 2 + (2x - 5) * 6x = 18x^2 - 30x$

The Quotient Rule

$$\boxed{\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}; g(x) \neq 0}$$

Example 1.7

Find the derivative of $f(x) = \frac{2x}{(x-4)}$

$$f'(x) = \frac{(x-4)*2 - 2x*1}{(x-4)^2} = \frac{-8}{(x-4)^2}$$

The Chain Rule

If $y = f(u)$ is a differentiable function of u, and $u = g(x)$ is a differentiable function of x, then $y = f(g(x))$ is a differentiable function of x, and then

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$$

- Example 1.8
- Find the derivative of $f(x) = (3x - 5)^2$
- $f'(x) = 2(3x-5) * (3 * 1 - 0) = 6(3x-5)$

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End of Lecture 04

Next Lecture : Integration

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