Mathematics for Computing (IT1030)

Lecture 3.2 - Computer Arithmetic

INTRODUCTION

- *Recap*:
  - Binary numbers are a number system with base 2.
  - Information represented inside a computer takes binary values.
  - Previous lecture dealt with the conversions between different number systems.

- This lecture deals with basic mathematical operations (such as addition, subtraction, multiplication and division) for binary numbers.

BINARY ADDITION

- Addition in the decimal number system.
  - Add values rightmost position (least significant).
  - If this addition is grater than 10, 1 is carried to the 2nd position and added.
  - This process is carried for all the positions.

- Binary addition follows the same set of rules.
  - If the addition is greater that 2, 1 is carried to the 2nd next position.

EXAMPLES

- Evaluate the following.
  - $101_2 + 101_2$
  - $00011010_2 + 00001100_2$
  - $10001 + 11101$
  - $1110 + 1111$
  - $101101 + 11001$
  - $10111 + 110101$
  - $1011001 + 111010$
  - $11011 + 1001010$
- Compare the above results by converting them to decimal numbers.

BINARY SUBTRACTION

- Similar to subtraction in the decimal number system.
- Inverse of addition.
- If the values cannot be subtracted, borrow from the next position.
- Subtraction table,
  - $0 - 0 = 0$
  - $1 - 0 = 1$
  - $1 - 1 = 0$
  - $0 - 1 = 1$ with a borrow of 1.

EXAMPLES

- Evaluate the following.
  - $10110 - 10010$
  - $1011011 - 10010$
  - $100010110 - 1111010$
  - $1010110 - 101010$
  - $101101 - 100111$
  - $1000101 - 101100$
  - $1110110 - 1010111$
- Compare the above results by converting them to decimal numbers.

MULTIPLICATION & DIVISION

- Similar to multiplication and division in the decimal number system.
- Rules of binary multiplication,
  - $0 \times 0 = 0$
  - $0 \times 1 = 0$
  - $1 \times 0 = 0$
  - $1 \times 1 = 1.$
- Rules of binary division,
  - $0 \div 1 = 0$
  - $1 \div 1 = 1.$

EXAMPLES

- Evaluate the following.
  - 1100×1010
  - 1111×101
  - 0011×11
  - 1100110×1000
  - $1000 \div 10$
  - $1010 \div 11$
  - $1111 \div 111$
- Compare the above results by converting them to decimal numbers.

SELF STUDYING

- How these values are represented in a computer.
- How much space each value takes when storing.
- What happens if a binary operation provides a result which exceeds the allocated space (Overflow)?
- What happens if an operation provides a result that is too small for the allocated space (Underflow)?
- How are negative numbers represented in a computer?

COMPLIMENTARY ARITHMETIC

- Complements are used in digital computers for simplifying,
  - the subtraction operation
  - the logical manipulation.
- Two types of compliments for each base b system.
  - r's compliment
  - $(r-1)$'s compliment
  - Example: For binary numbers, 2's complement and 1's complement.

Complimentary Arithmetic (Cont'd.)

- Given a number N in base r having n digits, $(r-1)$'s complement of N,

$$N' = (r^n - 1) - N$$

- Given a number N in base r having n digits, r's complement of N,

$$N' = r^n - N \text{ for } N \neq 0;\ 0 \text{ otherwise}$$

- Comparing $(r-1)$'s compliment, r's compliment can be obtained by adding 1 to the $(r-1)$'s compliment.

EXAMPLES

- Obtain 9's compliment and 10's compliment of 246700.

- Get the 1's and 2's compliments of the following binary numbers.
  - 1100011
  - 0001111
  - 1010100
  - 1111011


BINARY
IT'S AS EASY AS
01.10.11

SUMMARY

- Students should be able to,
  - Perform basic binary operations (addition, subtraction, multiplication and division).
  - Explain overflow and underflow.
  - Explain how signs works in binary representations in a computer.
  - To perform complement operations (r's complement and (r-1)'s complement.

END OF LECTURE 3.2

Next Lecture:-
Differentiation