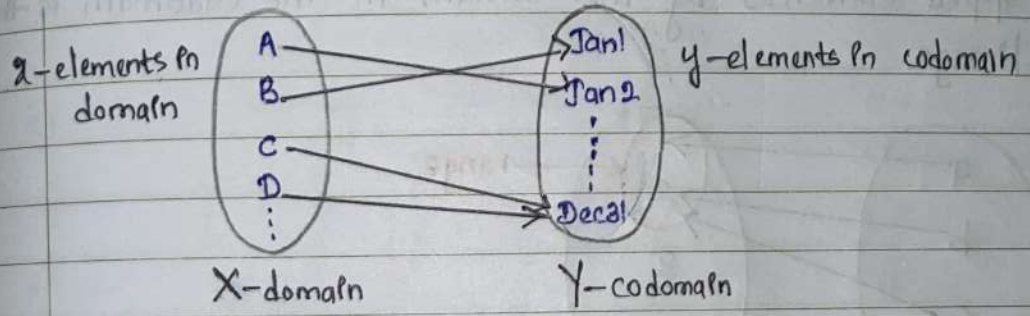
Date 10/10/2022

06. FUNCTIONS

* Function is a specific relation with a specific condition which defines a relation between domain (X) & codomain (Y).

$$f : X \longrightarrow Y$$

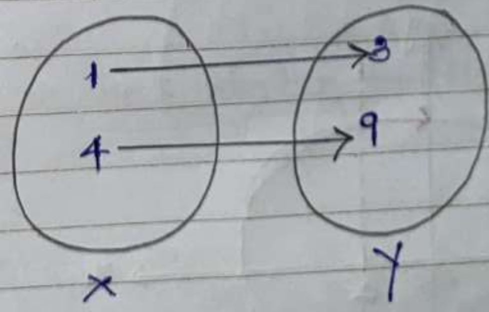


$$\underbrace{f(x) = \text{birthday of 'x'}}_{\text{a relation.}}$$

* Specific condition to be a function – Each element of ~~...~~ X is related to a unique element of Y. (one to one or many to one)

* $f : \mathbb{R} \longrightarrow \mathbb{R}$

(Q) $f(x) = 2x + 1$



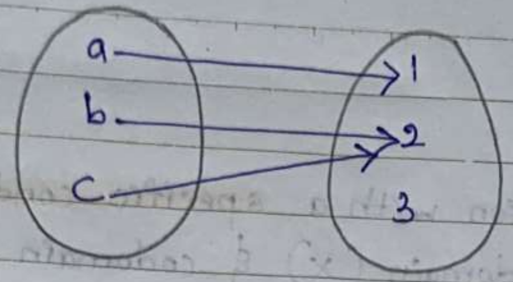
$f(1) = 2(1) + 1$

$\quad = 3$

$f(4) = 2(4) + 1$

$\quad = 9$

*



This is a function.

* The mapped elements by the domain in the codomain is the range/image.

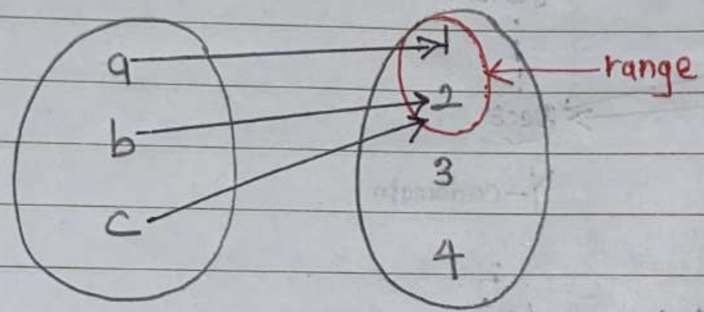


figure 01

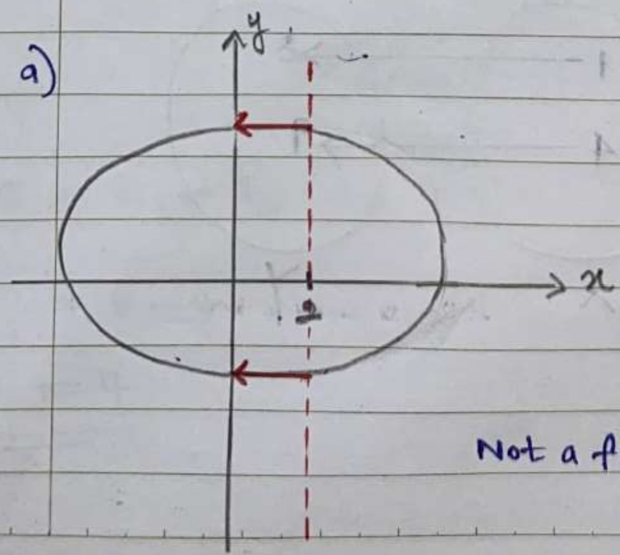
1) Function

2) Not a function

3) Not a function.

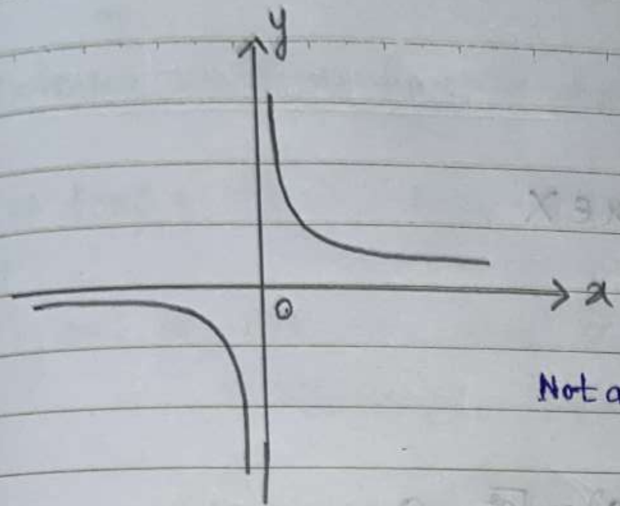
figure 02

a)



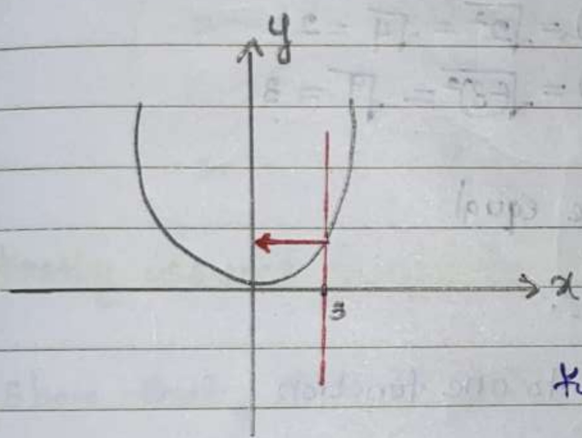
Not a function

b)



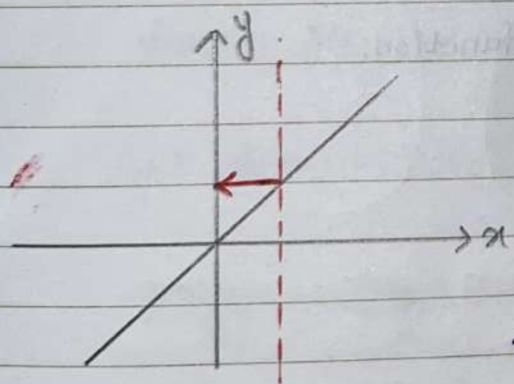
Not a function (0 has no any values of y
∴ not a f^n)

c)



function.

d)



function.

* We use **vertical line test** to check whether it is a function or not.
(when a graph is given).

Extra

(i) $\sqrt{16} = 4$

Here $+\sqrt{}$ is given

$x^2 = 16$ ← Here we take the square root so $\pm\sqrt{}$

$= \pm\sqrt{16}$

$= \pm 4$

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Equality of functions.

$f(x) = g(x)$; for all $x \in X$

$f(x) = |x|$

$g(x) = \sqrt{x^2}$

ex: $f(0) = |0| = 0$ $\quad$ $g(0) = \sqrt{0^2} = 0$

$f(2) = |2| = 2$ $\quad$ $g(2) = \sqrt{2^2} = \sqrt{4} = 2$

$f(-3) = |-3| = 3$ $\quad$ $g(-3) = \sqrt{(-3)^2} = \sqrt{9} = 3$

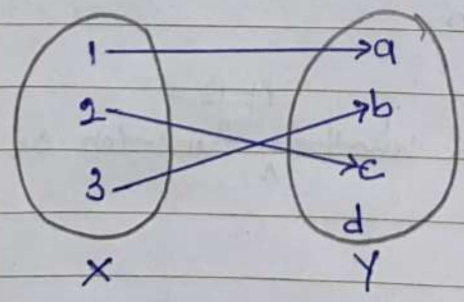
$\therefore$ the two f^ns are equal

$f(x) = g(x)$

* Special functions → One to one function
→ On to function.

One to one function

$f: X \rightarrow Y$

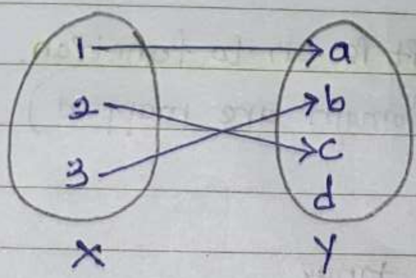


_ one to one f^n _

Defining one to one function mathematically

if $f(x_1) = f(x_2)$, then $x_1 = x_2$

ex: if $f(x_1) = b$ then $x_1 = 3$
$f(x_2) = b$ $\quad x_2 = 3$



* Firstly assume $f(x_1) = f(x_2)$, then $x_1 = x_2 \Longrightarrow$ one to one f^n.

(Q) Show that, $f: \mathbb{R} \longrightarrow \mathbb{R}$

$$f(x) = \frac{3x+1}{5} \text{ is one to one.?}$$

Let $f(x_1) = f(x_2)$; $\underbrace{x_1, x_2 \in \mathbb{R}}_{\text{elements of real numbers}}$

$$\frac{3x_1+1}{5} = \frac{3x_2+1}{5}$$

$$3x_1 + 1 = 3x_2 + 1$$

$$3x_1 = 3x_2$$

$$x_1 = x_2$$ $\therefore f^n$ is one to one.

(Q) $f: \mathbb{R} \longrightarrow \mathbb{R}$

$f(x) = x^2$

$f(2) = 2^2 = 4$

$2, -2 \in \mathbb{R}$

$f(-2) = \cancel{f}(-2)^2 = 4$

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$f(2) = f(-2)$

but

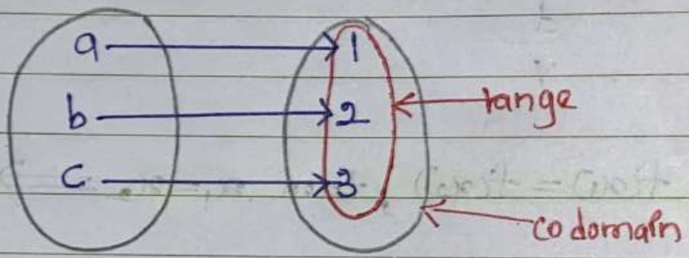
$2 \neq -2$

$\therefore$ function is not one to one

On to function

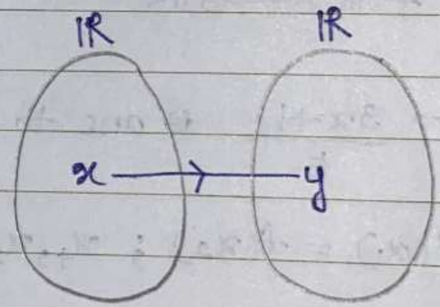
* Range = Codomain, then it is on to function.

(All the elements in the codomain are mapped)



(Q) $f: \mathbb{R} \longrightarrow \mathbb{R}$

$f(x) = 2x+1$



$\underbrace{\text{Let, } y \in \mathbb{R}}$ (codomain)

we select any element from codomain & it says whatever the value from y it is mapped to the domain, i.e. f^n is onto is proved finally.

* First select y value from codomain & check the value in domain corresponding to it.

suppose,

$y = 2x+1$

$2x = y-1$

$x = \frac{y-1}{2} \in \mathbb{R}$

for all $y \in \mathbb{R}$ (codomain)

there exists $x \in \mathbb{R}$ (domain)

Such that,

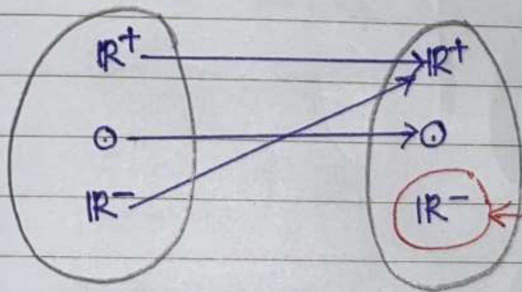
$$f(x) = f\left(\frac{y-1}{2}\right) = y$$

$\therefore f$ is on to

(i) $f(x) = x^2$

ex: $f(2) = 2^2 = 4$

$f(-2) = -2^2 = 4$



bcoz not mapped f^n is not on to f^n

Not on to function

Assume f^n is onto,

$-2 \in \mathbb{R}$ (codomain)

Suppose,

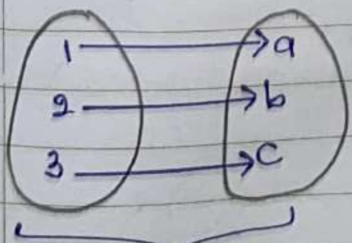
$x^2 = -2$

$x = \pm\sqrt{-2} \notin \mathbb{R}$

$\therefore f^n$ is not onto

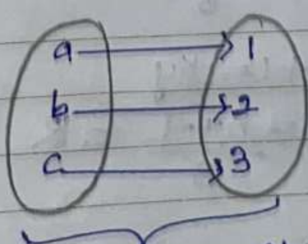
Bijection | One to One correspondance: (f^n which is one to one & onto)

$f: X \longrightarrow Y$



onto & one toone f^n

$f: Y \longrightarrow X$



inverse f^n

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* Inverse function exists only if function is one to one & onto.

* We have to prove the function is one to one & onto to find inverse function.

Inverse function

ex: $f : \mathbb{R} \longrightarrow \mathbb{R}$

$$f(x) = \frac{3x+1}{5}$$

find f^{-1} ?

One-to-one

Let $x_1, x_2 \in \mathbb{R}$

suppose $f(x_1) = f(x_2)$

$$\frac{3x_1+1}{5} = \frac{3x_2+1}{5}$$

$$3x_1+1 = 3x_2+1$$

$$3x_1 = 3x_2$$

$$x_1 = x_2$$

$\therefore$ one to one

Onto

Let $y \in \mathbb{R}$ (codomain)

Suppose,

$$y = \frac{3x+1}{5}$$

$$3x+1 = 5y$$

$$x = \frac{5y-1}{3} \in \mathbb{R} \text{ (domain)}$$

e. & on to.

to find

for all $y \in \mathbb{R}$ (codomain) there exists $y = \mathbb{R}$ domain

$y \in \mathbb{R}$ (domain) such that

$$f(x) = f\left(\frac{5y-1}{3}\right) = y$$

$\therefore$ f^n is on to

$\therefore$ f^n is bijection

i.e f^{-1} exists

ex: $f : \mathbb{R} \longrightarrow \mathbb{R}$

$$f(x) = \frac{3x+1}{5}$$

$$y = \frac{3x+1}{5}$$

Lets,

$$x = \frac{3y+1}{5}$$

$$3y = 5x - 1$$

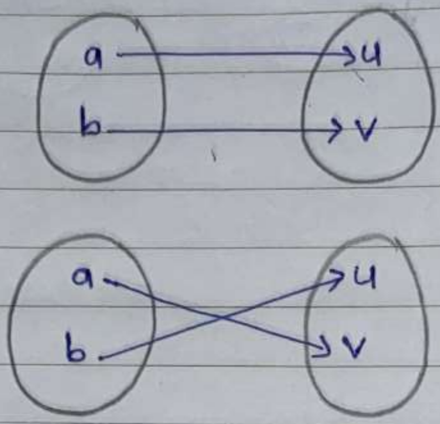
$$y = \frac{5x-1}{3}$$

$$f^{-1} : \mathbb{R} \longrightarrow \mathbb{R}$$

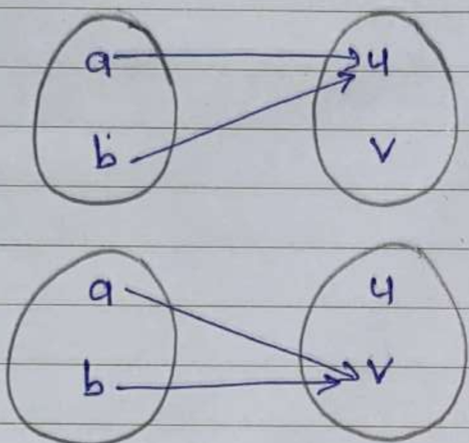
$$f^{-1}(x) = \frac{5x-1}{3}$$

Tutorial

(Q1) a. $x = \{a, b\}$ to $Y = \{u, v\}$

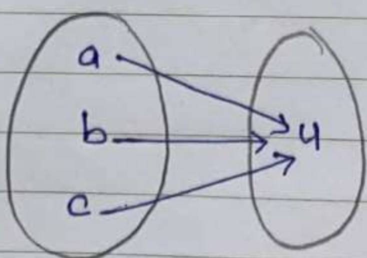


one to one f^n
&
on to f^n

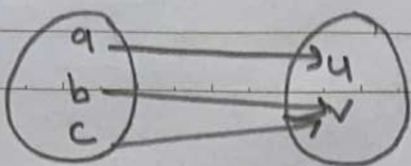


not one to one f^n
but
on to f^n

b. $x = \{a, b, c\}$ to $Y = \{u\}$

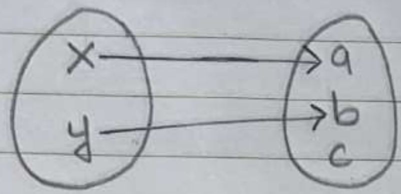


* when co-domain small than domain we can't define one to one, only on to is possible.



on to f^n.

* When codomain big than domain we can't define onto. Only one to one is possible.



One to one f^n

(03) $f(x) = 2x$

$$g(x) = \frac{2x^3 + 2x}{x^2+1}$$

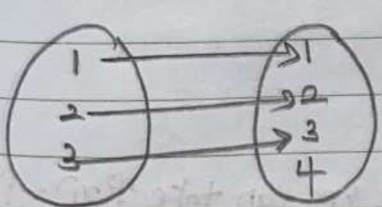
$$= \frac{2x\cancel{(x^2+1)}}{\cancel{(x^2+1)}} \; ; \; x^2+1 \neq 0$$

$g(x) = 2x$

$g(x) = f(x)$

(04) $X = \{1, 2, 3\}$ $Y = \{1, 2, 3, 4\}$ $Z = \{1, 2\}$

a. ~~one~~ $f: X \longrightarrow Y$

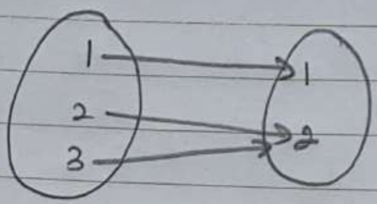


or $f: \{1, 2, 3\} \longrightarrow \{1, 2, 3, 4\}$

$f(x) = x$

one to one not onto

b. $g: x \longrightarrow z$



One to one not onto

(05) $f: \mathbb{R} \longrightarrow \mathbb{R}$

$f(x) = \frac{x+1}{x}$; $x \neq 0$

Let $x_1, x_2 \in \mathbb{R} - \{0\}$

Suppose,

$f(x_1) = f(x_2)$

$\frac{x_1+1}{x_1} = \frac{x_2+1}{x_2}$

$x_2(x_1+1) = x_1(x_2+1)$

$\cancel{x_2 x_1} + x_2 = \cancel{x_1 x_2} + x_1$

$x_2 = x_1$

$\therefore$ One to one

* When proving something is one to one we can take $f(x_1) = f(x_2)$. But if function is not one to one we can put 2 values and prove it.

(Q) $f : \mathbb{R} - \{0\} \longrightarrow \mathbb{R}$

$$f(x) = x + \frac{1}{x}$$

is this one to one ?

$f(a) = f(b)$ similarly ;
$a \neq b$

$$2, \frac{1}{2} \in \mathbb{R} - \{0\}$$

$$f(2) = 2 + \frac{1}{2} = \frac{5}{2}$$

$$f\left(\frac{1}{2}\right) = \frac{1}{2} + \frac{1}{(1/2)} = \frac{1}{2} + 2 = \frac{5}{2}$$

$$f(2) = f\left(\frac{1}{2}\right)$$

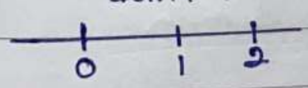
but

$2 \neq \frac{1}{2}$ $\therefore$ not one to one

06) $f : (0, 2) \longrightarrow (-1, 2)$

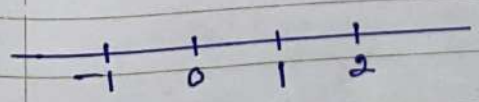
$$f(x) = \sqrt{x} - 1$$

domain



$\wedge$

codomain



$-1 \in \underbrace{[-1, 2]}_{\text{codomain}}$

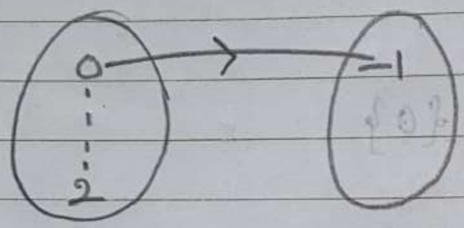
Always choose a value not mapped in codomain by the domain.

Suppose,

$-1 = \sqrt{x} - 1$

$\sqrt{x} = 0$

$x = 0$ ~~when~~ ← wrong bcoz it is mapped already.



$2 \in [-1, 2]$

Suppose,

$2 = \sqrt{x} - 1$

$3 = \sqrt{x}$

$x = 9 \notin \underbrace{[0, 2]}_{\text{domain}}$

$\therefore f^n$ is not onto

* when saying not onto f^n always start from a value in the codomain which is not mapped by the domain.

(07) $f : \mathbb{R} \longrightarrow \mathbb{R}$

$f(x) = |x|$

$f(a) = f(b)$

$a \neq b$

One to one

$-2, 2 \in \mathbb{R}$

$f(-2) = 2$

$f(2) = 2$

$\therefore f(-2) = f(2)$

$-2 \neq 2$

$\therefore$ not one to one

$\therefore$ Inverse doesn't exist

(08)

* If no x^2 terms in the expression, then f^n is one to one

a) $f(x) = -3x + 4$

one to one

Let $x_1, x_2 \in \mathbb{R}$

suppose,

$$f(x_1) = f(x_2)$$
$$-3x_1 + 4 = -3x_2 + 4$$
$$-3x_1 = -3x_2$$
$$x_1 = x_2$$

$\therefore$ one to one

On to

Let $y \in \mathbb{R}$

suppose,

$$y = -3x + 4$$
$$-3x = y - 4$$
$$x = \frac{y-4}{-3} \in \mathbb{R} \quad \text{(domain)}$$

for all $y \in \mathbb{R}$ ~~then~~ there exists $y = \mathbb{R}$ domain.

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$f(x) = f\left(\frac{y-4}{-3}\right) = y$

$\therefore$ f^n is on to

$\therefore$ f^n is a bijection

$y = -3x + 4$

$x = -3y + 4$

$-3y = x - 4$

$y = \frac{x-4}{-3}$

$y = \frac{4-x}{3}$

$f^{-1} : \mathbb{R} \longrightarrow \mathbb{R}$

$f^{-1}(x) = \frac{4-x}{3}$