Integration and its Applications

MATHEMATICS FOR COMPUTING (IT1030)

Indefinite Integration

Introduction

- Calculus involves two basic operations:
  - differentiation
  - integration (or anti differentiation)

- The two operations (integration & differentiation) are inverses of each other.

Definition of Anti derivative

- A function F is an anti derivative of a function f if for every x in the domain of f, it follows that

$F'(x) = f(x)$

- If $F(x)$ is an anti derivative of $f(x)$, then $F(x) + c$, where c is any constant, $F(x)$ is also an anti derivative of $f(x)$.

Notation for Anti derivatives and Indefinite Integrals

$$\int f(x)\ dx = F(x) + C$$

Integrand

Anti derivative

Integral Sign

Finding Anti-Derivatives

- The inverse relationship between the operations of integration and differentiation can be shown symbolically, as follows.

$$\frac{d}{dx}\left[\int f(x)\right] = f(x)$$

$$\int f'(x)dx = f(x) + C$$

Basic Integration Rules

1. $\int k dx = kx + C \quad ; \quad k \text{ is a constant}$
2. $\int kf(x)dx = k \int f(x)dx$
3. $\int [f(x) + g(x)]\, dx = \int f(x)\, dx + \int g(x)dx$
4. $\int [f(x) - g(x)]\, dx = \int f(x)\, dx - \int g(x)dx$
5. $\int x^n\, dx = \frac{x^{n+1}}{n+1} + C \quad , n \neq -1$

Basic Integration Rules

Example **Finding Indefinite Integrals**

Find each indefinite integral.

a. $\displaystyle\int \frac{1}{2}\, dx$ **b.** $\displaystyle\int 1\, dx$ **c.** $\displaystyle\int -5\, dt$

SOLUTION

a. $\displaystyle\int \frac{1}{2}\, dx = \frac{1}{2}x + C$ **b.** $\displaystyle\int 1\, dx = x + C$ **c.** $\displaystyle\int -5\, dt = -5t + C$

Basic Integration Rules

Example **Finding an Indefinite Integral**

Find $\int 3x\,dx$

$$\int 3x\,dx = 3\int x\,dx \qquad \text{Constant Multiple Rule}$$

$$= 3\int x^1\,dx \qquad \text{Rewrite } x \text{ as } x^1.$$

$$= 3\left(\frac{x^2}{2}\right) + C \qquad \text{Simple Power Rule with } n = 1$$

$$= \frac{3}{2}x^2 + C \qquad \text{Simplify.}$$

Basic Integration Rules

Example **Finding an Indefinite Integral**

Original Integral	*Rewrite*	*Integrate*	*Simplify*
a. $\displaystyle\int \frac{1}{x^3}\,dx$	$\displaystyle\int x^{-3}\,dx$	$\displaystyle\frac{x^{-2}}{-2} + C$	$\displaystyle -\frac{1}{2x^2} + C$
b. $\displaystyle\int \sqrt{x}\,dx$	$\displaystyle\int x^{1/2}\,dx$	$\displaystyle\frac{x^{3/2}}{3/2} + C$	$\displaystyle\frac{2}{3}x^{3/2} + C$

Basic Integration Rules

Example **Integrating Polynomial Functions**

Find (a) $\int (x + 2)\,dx$ and (b) $\int (3x^4 - 5x^2 + x)\,dx$.

SOLUTION

a.
$$\begin{aligned}\int (x + 2)\,dx &= \int x\,dx + \int 2\,dx\\ &= \frac{x^2}{2} + C_1 + 2x + C_2\\ &= \frac{x^2}{2} + 2x + C\end{aligned}$$

b.
$$\begin{aligned}\int (3x^4 - 5x^2 + x)\,dx &= 3\left(\frac{x^5}{5}\right) - 5\left(\frac{x^3}{3}\right) + \frac{x^2}{2} + C\\ &= \frac{3}{5}x^5 - \frac{5}{3}x^3 + \frac{1}{2}x^2 + C\end{aligned}$$

Definite Integration

Definition of a Definite Integral

- Let f be nonnegative and continuous on the closed interval $[a, b]$. The area of the region bounded by the graph of f, the x – axis, and the lines $x = a$ and $x = b$ Is denoted by,

$$Area = \int_a^b f(x)$$

- The expression $\int_a^b f(x)$

is called the definite integral from a to b, where a is the lower limit of integration and b is the upper limit of integration.

The Fundamental Theorem of Calculus

- If is f is nonnegative and continuous on the closed interval [a, b], then,

$$\int_a^b f(x) = F(b) - F(a)$$

where f is any function such that $\mathrm{F}'(x) = f(x)$ for all $x$in [a, b].

Properties of definite integrals

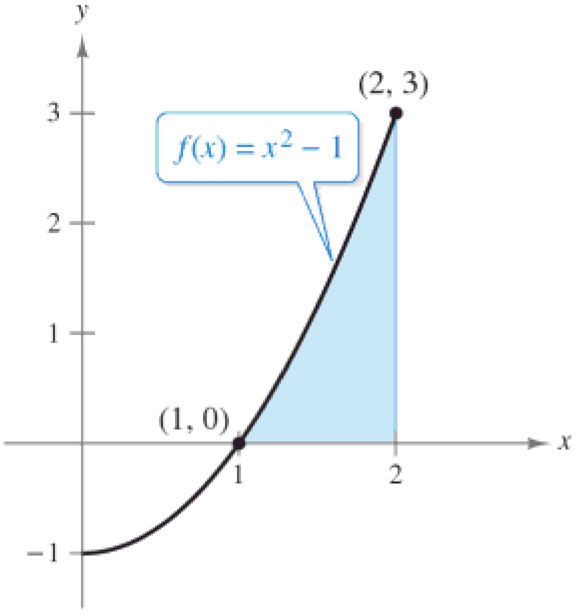
1. $\int_a^b kf(x)\,dx \ = k\int_a^b f(x)\,dx \quad , \ k \ is \ a \ constant.$
2. $\int_a^b [f(x) \pm g(x)]\,dx \ = \int_a^b f(x)\,dx \ \pm \int_a^b g(x)\,dx$
3. $\int_a^b f(x)\,dx \ = \int_a^c f(x)\,dx \ + \int_c^b f(x)\,dx \quad , \ a < c < b$
4. $\int_a^a f(x)\,dx \ = 0$
5. $\int_a^b f(x)\,dx \ = -\int_b^a f(x)\,dx$

Example

Example **Finding Area by the Fundamental Theorem**

Find the area of the region bounded by the x-axis and the graph of

$$f(x) = x^2 - 1, \quad 1 \leq x \leq 2.$$



$$\text{Area} = \int_1^2 (x^2 - 1)\, dx$$

Example

SOLUTION Note that $f(x) \geq 0$ on the interval $1 \leq x \leq 2$, as shown in Figure 5.9. So, you can represent the area of the region by a definite integral. To find the area, use the Fundamental Theorem of Calculus.

$$\text{Area} = \int_1^2 (x^2 - 1)\,dx \qquad \text{Definition of definite integral}$$

$$= \left[\frac{x^3}{3} - x\right]_1^2 \qquad \text{Find antiderivative.}$$

$$= \left(\frac{2^3}{3} - 2\right) - \left(\frac{1^3}{3} - 1\right) \qquad \text{Apply Fundamental Theorem.}$$

$$= \frac{2}{3} - \left(-\frac{2}{3}\right)$$

$$= \frac{4}{3} \qquad \text{Simplify.}$$