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Optimality and Efficiency in Auctions through Deep Learning

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Contents

1 Introduction

During the last decade the world witnessed a dawn of Machine Learning. Methods of this science allowed humans to solve seemingly unsolvable problems. Language models, generation images from text and vice versa, self-driving cars, Go-gaming algorithms etc. And, among others, methods of automatic auction design were created. Let's step aside for a moment and briefly discuss auctions.

1.1 Auctions

Auction is basically an economic mechanism which determines how an item, or a bundle of items, or several items and so on should be distributed among one or many bidders (also we will call them players or agents), in exchange for some amount of money.

Consider the situation, when the auctioneer is willing to distribute $M = \{1, 2, \dots m\}$ items between $N = \{1, 2, \dots n\}$ players. Each player i has a valuation function $v_i : 2^M \to \mathbb{R}^+$, such that $v_i(S)$ defines the value i-th player has for a subset S of M. Each bidder's valuation function is drawn from a distribution F_i over possible valuation functions V_i. Thus, by $v = (v_1, v_2, \dots v_n)$ we denote a profile of valuations and, respectively $V = \prod_{i=1}^{n} V_i$. The auctioneer knows all the distributions F_i but has no idea about which are realized valuations of bidders.

Agents should submit their valuations (maybe truthfully, but maybe not). Afterwards, the auctioneer decides how to distribute items, and how to charge payments for them according to some rules (g, p), called auction, where $g_i : V \to 2^M$, $p_i : V \to \mathbb{R}^+$, which can be randomized, and are computed with b as an argument.

Agent i receives a utility $u_i(v_i, b) = v_i(g_i(b)) - p_i(b)$. Let $V\_i = \prod_i V_j$ and $v_{-i} = (v_1, v_2, \ldots, v_{i-1}, v_{i+1}, \ldots v_n)$. And auction is DSIC and IR if, respectively, $u_i(v_i, (v_i, b_{-i})) = v_i(g_i(v_i, b_{-i})) - p_i(v_i, b_{-i}) \geq v_i(g_i(b)) - p_i(b) = u_i(v_i, b)$ and $u_i(v_i, (v_i, b_{-i})) \geq 0$, $\forall i \in N, v_i \in V_i$ and $b_{-i} \in V_{-i}$.

Optimal auction design seeks to identify a DSIC auction that maximizes revenue Rev, and the goal of effective auction design is to find a DSIC auction that maximizes social welfare, W.

In case when $M > 1$, i.e. when the auctioneer has to distribute many items she faces a challenge:

$$v_i(S_1 \cup S_2) = v(S_1) + v(S_2) \tag{1}$$

should not necessarily hold. E.g. v_i might be equal to $\max(S1, S2)$. However, in our research we will focus on the simplest case: when valuation is additive, and equation (1) holds. Now we can turn back to the topic.

Previously mentioned optimal auction problem was partially resolved by R. Myerson 1981 in his seminal work. Myerson developed an approach allowing to set an optimal auction in setting with N bidders and 1 item to distribute. Another major contribution was made almost after forty years by Dütting et al. 2017. They propose a framework based on deep-learning techniques for learning far more complex auction-rules, in $N \times M$ (n-bidders, m-items) settings.

Inspired by their astonishing success we decided to investigate more deeply properties of those auctions that were not discovered earlier. The main object of interest of our research was Pareto Frontier for optimality and efficiency for a given setting V. Pareto Frontier is basically how much the social welfare would increase if an auctioneer agreed to give up a share of her optimal revenue and vice versa. Or, more formally: we seek to identify a set of alpha-beta approximations where each pair (α_i, β_i) satisfies the following equation.

$$\alpha = \frac{\max\{W|Rev = \beta R_{opt}\}}{W_{opt}} \tag{2}$$

This part of our survey is mainly motivated by a paper of Daskalakis and Pierrakos 2011 who proved that

Theorem 1. *In every single-item setting with n bidders whose values are distributed according to independent (possibly non-identical) regular distributions and for every $p \in [0,1]$, there exists a Vickrey auction with (generally non-anonymous) reserve prices that simultaneously achieves a $p-$fraction of the optimal social welfare and a $\frac{1}{1-p}-$fraction of the optimal revenue.*

Another direction of study is the difficulty of a given setting: what is the maximum achievable revenue (or social welfare). Our main motivation in this part of study was a pretty anecdotal situation that happened in New Zealand. Government decided to distribute the cellular rights, and obviously they chose a classic Second-Price auction[1]. It turned out that the highest bid was about \$100,000 whilst the second one was equal to \$16. Though, undoubtedly, second price auction is an effective one, because the only agent who has been able to create jobs got the rights, the government might wish to achieve more pleasant results.

[1]The source of this information is the most reliable – E. Ianovski himself.

2 Study

2.1 Single item, Multi bidders auction

2.1.1 Methodology

In this part of our research we will rely on findings of above-mentioned Dütting et al. 2017. They proposed a framework named MyersonNet. Let's denote expected revenue of the auction, conditioned on truthful bids by

$$Rev(g, p) = \mathbb{E}_{v \sim F}[\sum_{i=1}^{n} g_i(v) t_i(v)] \tag{3}$$

Myerson 1981 proved the following theorem:

Theorem 2. *There exist a collection of monotonically non-decreasing functions, $\bar{\phi}_i : \mathbb{R}^+ \to \mathbb{R}$ called the ironed virtual valuation functions such that the optimal BIC auction for selling a single item is the DSIC auction that assigns the item to the buyer with the highest ironed virtual value $\bar{\phi}_i(v_i)$ provided that this value is non-negative, with ties broken in an arbitrary value-independent manner, and charges the bidders according to $p_i(v_i) = v_i g_i(v_i) - \int_0^{v_i} g_i(t)dt$*

Motivated by the above theorem, one may wish to find this set of ϕ_i. The most straightforward way is to approximate them using a neural network described by the following equation:

$$\phi_i(\bar{b_i}) = \min_{k \in [K]} \max_{j \in [J]} w^i_{kj} b_i + \beta^i_{kj} \tag{4}$$

It also useful to set each w^i_{kj} equal to $e^{\alpha^i_{kj}}$, since in this case one can easily perform an inverse transformation:

$$\phi_i^{-1}(b_i) = \max_{j\in[J]} \min_{k\in[K]} e^{-\alpha^i_{kj}}(\overline{\phi}_i(b_i) - \beta^i_{kj}) \tag{5}$$

According to Sill 1998, if we choose large enough K and J we'll be able to approximate any continuous, bounded monotone function pretty accurately. Next we might wish to perform a second price auction over virtual values of a bidders according to this rules:

$$g_i^0 = \frac{exp(\kappa\phi_i(b_i))}{\sum_{j=1}^{n+1} exp(\kappa\phi_j(b_j))} \tag{6}$$

where $b_{n+1} = 0$ is dummy variable, and

$$t_i^0 = \max \widetilde{b}_{-i} \tag{7}$$

where $\widetilde{b}_{-i} = (b_1, b_2, ...b_{i-1}, b_{i+1}, ...b_n, 0)$ Finally, we want to compute payments which might be calculated as $p_i = \phi_i^{-1}(t_i^0)$

Having an optimal auction is enough to find a Pareto Frontier. It is known from the literature that a Myerson Auction might be considered as a Vickrey auction with reserve price[2]. So, if we iteratively adjust payments, solving the following optimization problem at each step:

$$W_{iter} = \max_{w,b} \Big[\sum_{i=1}^{n} u_i | Rev_{iter} \geq \alpha \cdot Rev_{opt},\, \alpha \in (0,1) \Big] \tag{8}$$

we will eventually converge to an effective mechanism. The record of every solution of the above mentioned optimization problem can serve us as an approximation of Pareto Frontier. The only concern here is that how can we be sure that the mechanism, obtained in each iteration, is truthful (if the opposite is true, we can not guarantee that the needed fraction

[2] Sergio Parreiras. "Expected revenue obtained by the Vickery auction with reserve price 1/2". stackexchange.

of revenue will be achieved). Luckily, we can state that each mechanism we need will be truthful (at least approximately) – due to the fact that MyersonNet models a monotone function.

Another issue we dealt with is the instability of MyersonNet. The quality of an achieved result using this framework varies drastically, depending on starting values of MyersonNet parameters, and chosen configuration (hyperparameters). And this happens even in the case of only one setting. But the situation turns even worse if we want to estimate the results of many auctions.

We tried to solve this issue in two separate ways:

- Use Optuna framework for optimisation of hyperparameters in neural networks, commonly known and widely used in the ML community.
- Augment MyersonNet with some fully-connected layers, and train this network on mini-batches from several distributions. Now matrices w and b are calculated as $w = \gamma_1(batch)$, $b = \gamma_2(batch)$. It allows for more robust results. The intuition behind this idea is that we may wish to not only maximize revenue, but also to be conditioned on which distribution a sample came from.

2.1.2 Results

In order to understand the instability of MyersonNet one can look at the picture below.

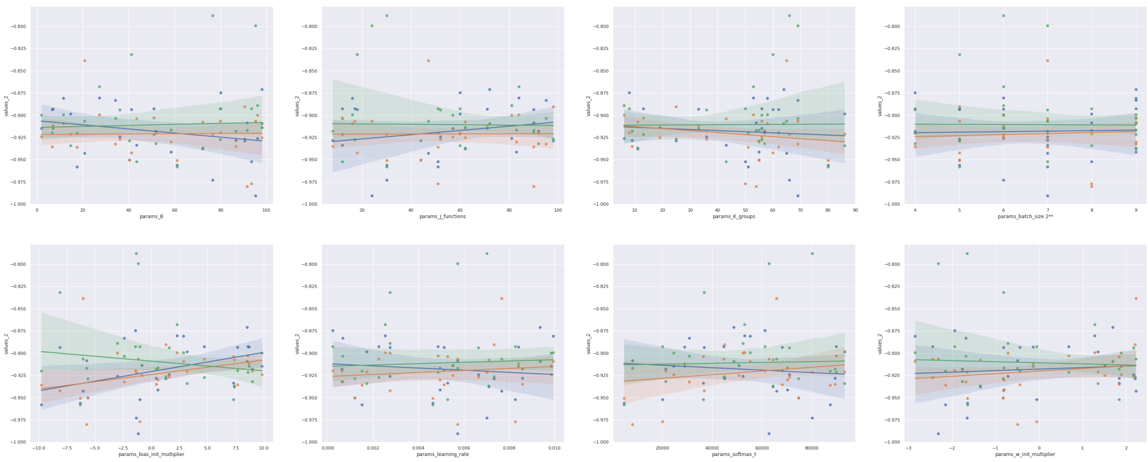

Fig. 1: Parameters effects.
We used a Statsmodels package to plot this graph. After 20 iterations of Optuna we recorded achieved results and then regressed the result on used parameters. It's clearly non-clear. We can not find the dependence between a quality of a result and a combination of parameters used.

Using version MyersonNet we tried to compare different auctions on different distributions. Before we start, we need to design a metric that would capture the difficulty of distributions. The most straightforward idea is to divide achieved optimal revenue by the mean value of valuations sample.

$$\mathcal{D} = \frac{\overline{Rev}}{\overline{F}} \tag{9}$$

Indeed, thus we can get rid of the effect of scale (optimal revenue of $V \sim U[0,1]$ will be much less than $V \sim U[5,6]$). And, this metric will be useful in case of our interest: if distribution is irregular (common interpretation of such distributions: if we have agents with different purchasing ability), we want to investigate how badly the strength of one type compared to another one affects the properties of auction. Thus, if we move the right border in a positive direction, the mean value of a sample from such a distribution will also increase. For simplicity, further we will name this metric difficulty.

We plotted the results on three graphs. First one (Fig. 2) describes the mean value of achieved optimal revenue by a naive MyersonNet. But those results are not very credible. Because, as it can be seen from the second picture (Fig. 3), the maximum value achieved while initializing MyersonNet with different hyper- and starting parameters, varies almost unpredictable (though one can notice a downward slope). The problem here is that each value of revenue on this graph were obtained by estimating a mean value on a batch with a size of 128 * 3. Much more than usually prescribed by a rule of thumb size of a sample for estimating a mean, equal to 30. Nevertheless, we still can say that it is much harder to find a proper combination of hyperparameters and starting parameters. Whilst almost each combination allows for achieving an optimal value in case of regular distribution, the stronger the second type of bidders, the harder it is to initialize a MyersonNet properly.

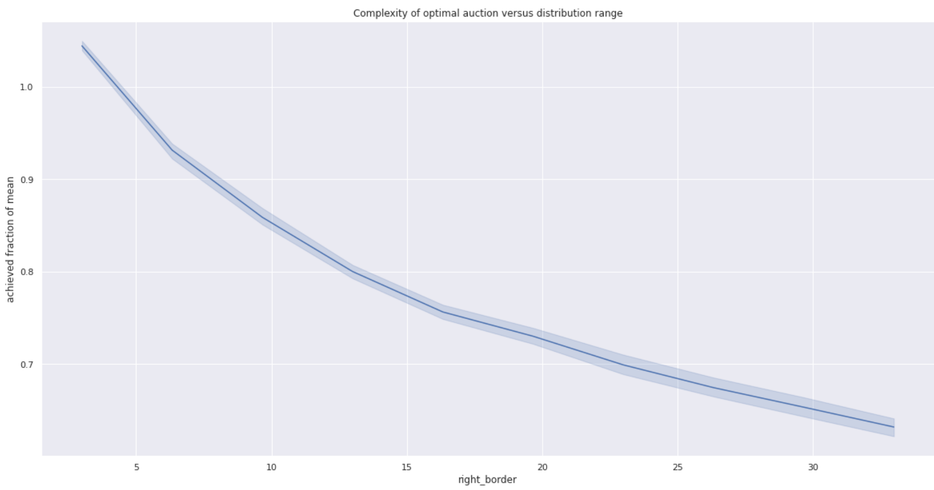

Fig. 2: Difficulty versus right border of a distribution (MyersonNet with Optuna)

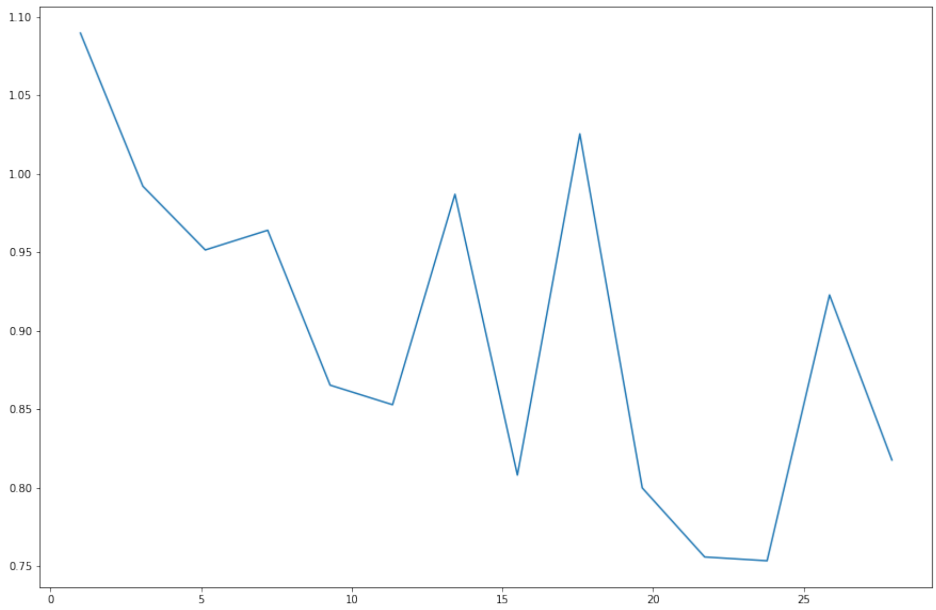

Fig. 3: Difficulty versus right border of a distribution (only best revenues used, MyersonNet with Optuna)

The latter approach seems to be inconsistent, despite the fact that it, however, enlightens the picture. E.g. we can not undoubtedly determine what is the exact problem with a strong second type. Maybe it is possible to tune a MyersonNet so we'll achieve a constant factor of mean no matter what the right border of a second type is, or maybe that large size of validation sample is not enough for effective estimation of an optimal revenue. In order to distinguish between these two situations we proposed an augmentation for a basic MyersonNet architecture. It allows to make this approach more general, and applicable to several distributions. Instead of tuning each MyersonNet separately we aim to achieve an optimal point in each case of interest at once. And as we are able to use a backpropagation technique for such an optimization we can rely on a strong mathematical background related to this technique to state a consistency of the results. Indeed, the picture (Fig. 4) obtained via augmented MyersonNet seems to be more clear.

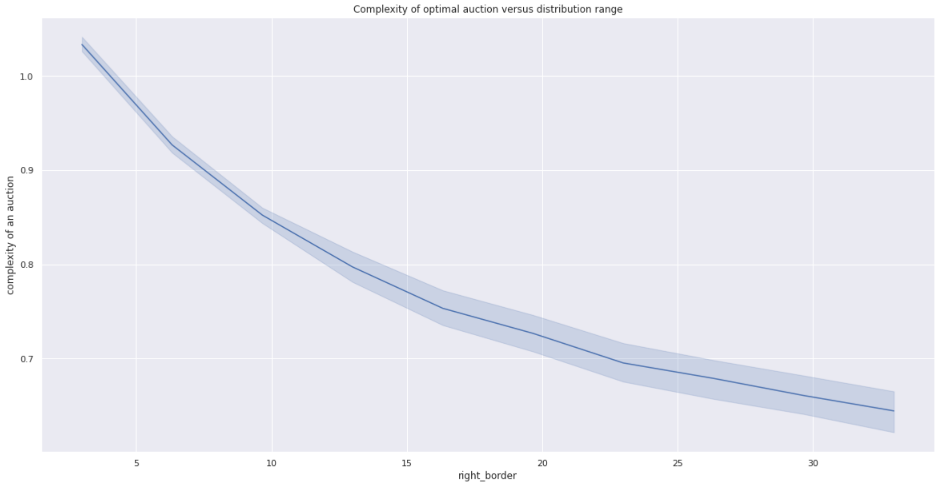

Fig. 4: Difficulty versus right border of a distribution (Augmented MyersonNet)

While the results deeply reminds of a previous attempt: downward slope, seemingly convex function, one can note that trust interval envidens as the right border increases. So we can happily conclude that the larger a right border of a type is, the more difficult it is to achieve a large amount of revenue. And also, the stronger second type is, the more random the result will be.

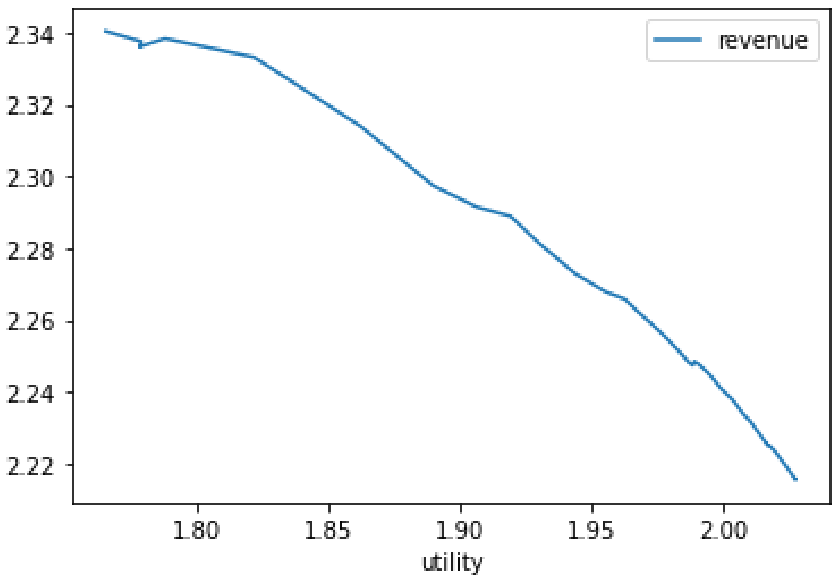

Fig. 5: Pareto Frontier, via non-augmented MyersonNet

And the last thing: Pareto Frontier (Fig. 5). One might see that the change of utility in the process of convergence is much notable than one of revenue. And large deviations of optimal revenue in case of non-augmented MyersonNet can deeply affect the results. We hope in the future we can expand our research with application of augmented version of MyersonNet, that luckily appeared to be useful.

2.2 Many items, Multi bidders auction

2.2.1 Methodology

The most useful way to deal with such settings is a regret-based approach. In short, the main idea behind that is that they seek to find such a pair of functions (g, p), that they want to solve the following optimization task:

$$\begin{aligned} \min_{w \in \mathcal{N}} \quad & \mathbb{E}_{v \sim F}\Big[-\sum_{i \in N} p_i^w(v)\Big] \\ \text{s.t.} \quad & rgt_i(w) = 0, \forall i \in N \end{aligned} \tag{10}$$

Where

$$rgt_i(w) = \mathbb{E}\Big[\max_{{v_i}' \in V_i} u_i^w(v_i; ({v_i}', v_i)) - u_i^w(v_i; (v_i, v_i))\Big] \tag{11}$$

In order to repeat the results obtained by Dütting et al. 2017, and to move on exploiting their findings, we decided to reproduce their pipeline. But we faced an insurmountable obstacle: RegretNet is so sensitive to values of starting parameters, and implies so many hyperparameters, that it is clearly impossible to reproduce the results. Driven by the same reasons, Rahme, Jelassi, and Weinberg 2020, and later Ivanov et al. 2022, provided some other architectures that are proved to be more robust, effective and include much less hyperparameters to tune. In the latter work two independent modifications were introduced: a new model architecture based on the attention mechanism, called RegretFormer, and an alternative loss function. We want to express our gratitude to D. Ivanov who gave us access to their implementation, so we didn't have to build their models from scratch.

2.2.2 The consistency of regret-based approaches

Now it is time to discuss the consistency of regret-based approaches. There are three main issues:

1. The lack of domain knowledge in this characterisation, apparently leads to poor convergence of every regret-based algorithm. Nine hours of learning is a pretty large number.

2. Validation. Though one can agree that this regret-based characterization of the task is useful for learning an auction rule, the convergence of an algorithm doesn't prove that the task is solved. We'd like to provide a quick example: if we transform a time series to a stationary one to train a model, before we evaluate the results we need to perform an inverse transformation on obtained prediction. In other words, we'd like to evaluate trained RegretNet (and derivatives like ...) on real bids and record an achieved revenue, rather than estimate "possible" revenue on true valuation. One could say that they do use real bids (and call them "misreports") in the process of learning, so it's the easiest thing to make such an evaluation. However, we'd like to say that it would be better for everyone if they named "misreports" as "quasi-misreports", in order to avoid misinterpretations. So-called misreports are not matrices of real bids that bidders would submit in real auction. What actually happens: we allow only one of n bidders, bidder i to adjust her bids, whereas all others should submit their true valuations. And stacking such matrices for each i-th bidder we'll get a 3-d "misreports" tensor. So, those adjusted bids of i-th agent should not always be the best response.

3. Misinterpretation of the results. It is common to think about the value of Regret as a mean amount of revenue which the auctioneer didn't gain. However, it is not actually true. The term Regret is widely used in connection with repeated games. And this construct

has a useful property: when Nash equilibrium is achieved, the Regret of each player is equal to zero. But, in our case, the only suitable interpretation of regret is a measure of stability of a learned auction. And, as soon as there are no results with regret equal to zero, and as regret is non convex function, no one can guarantee nor non-existence of points (matrices of misreports), where bidders can get more utility than if they bid truthfully, neither a stability of a solution.

2.3 Results

Yet to be done.

3 Conclusion

We reproduced some results related to the topic of automated auction design. Having access to the modern techniques we managed to investigate some of the statistical properties of optimal and efficient auctions. Also, some of the bottlenecks of such methods were discovered and some possible ways to get rid of those limitations were proposed.

Now we can manifest possible ways to further research:

1. Further discovery of statistical properties of $\alpha - \beta$ efficient auctions:
   - E.g. Asymptotic case of irregular distribution. E.g. what happens with revenue/welfare if we draw valuations from a distributions $U[a_i, a_{i+1}]$, $\forall i \in (1, 2,..n)$ with a given probabilities, if $n \to \inf$, and $a_{i+1} - a_i \leq \frac{1}{n}$
2. How to change the regret-based approach according to earlier mentioned notes?
   - The main idea is to try to enforce convexity of a model. Thus, even if we will have regret non equal to zero, the real bids will be somewhere close to truthful values. Of course, there is a need for more credible validation. As a spoiler, we'd made some steps in this direction but decided to postpone the discussion till the better times.

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