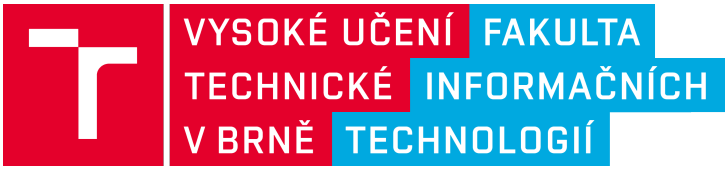


IEL – protokol k projektu

Vadim, Goncearenco
xgonce00

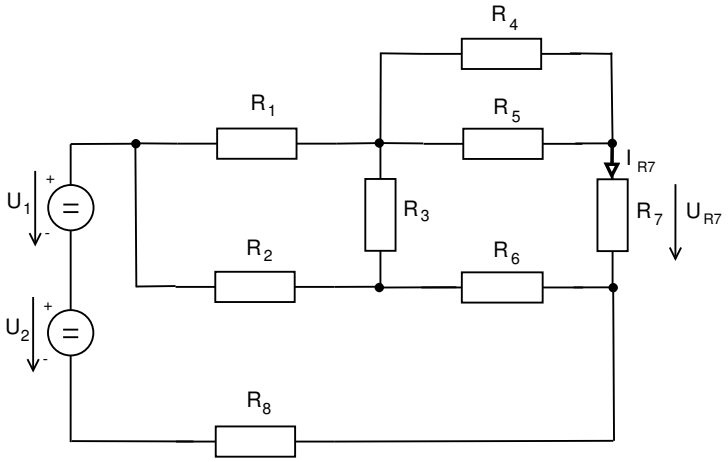
30. dubna 2022

Obsah

Příklad 1

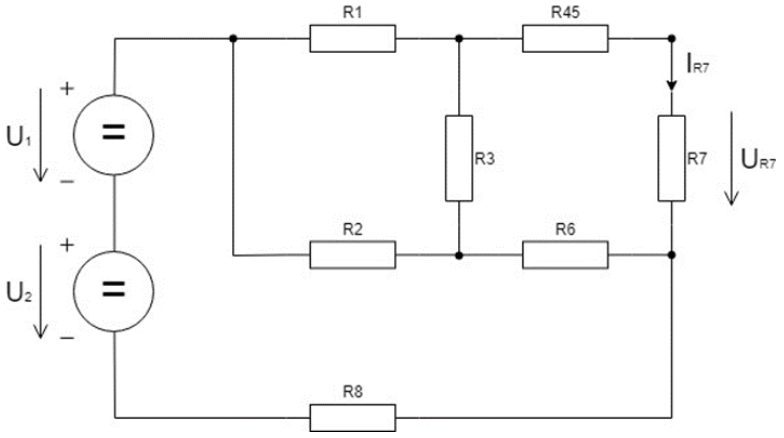
Stanovte napětí U_{R7} a proud I_{R7}. Použijte metodu postupného zjednodušování obvodu.

sk.	U_1 [V]	U_2 [V]	R_1 [Ω]	R_2 [Ω]	R_3 [Ω]	R_4 [Ω]	R_5 [Ω]	R_6 [Ω]	R_7 [Ω]	R_8 [Ω]
H	135	80	680	600	260	310	575	870	355	265



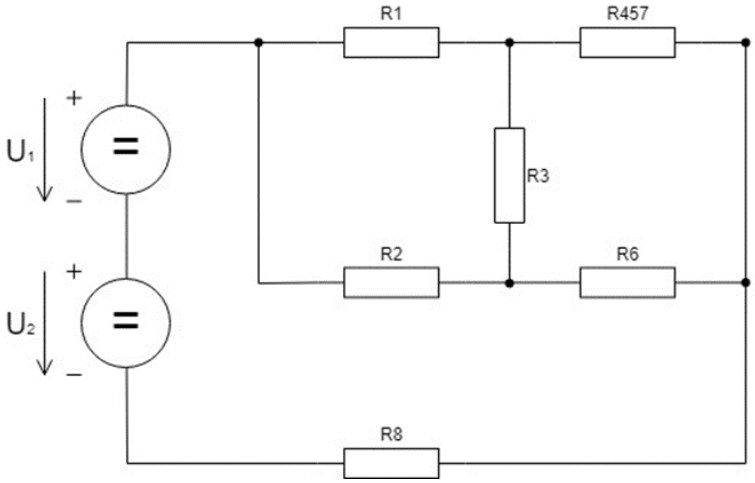
Vypočítáme R_{ekv} postupným zjednodušováním obvodu

$$R_{45} = \frac{R4 \times R5}{R4 + R5} = 201,4124\Omega$$



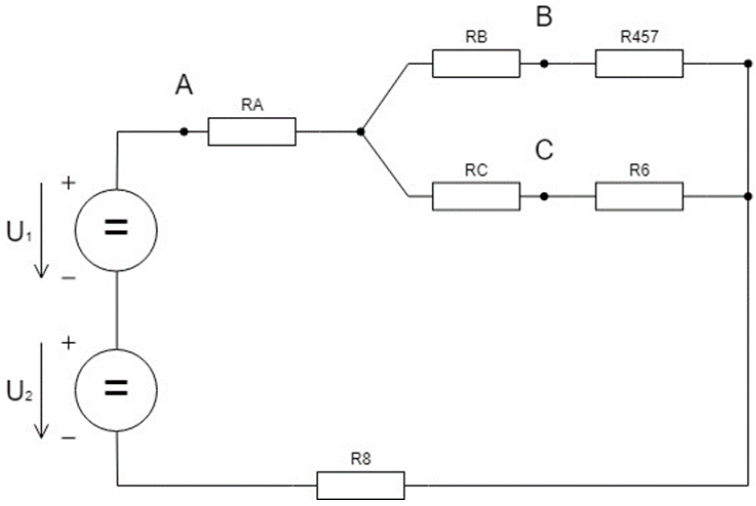
Obrázek 1: $R_{45} \sim R_4, R_5$ paralelně

$$R_{457} = R_{45} + R_7 = 201,4124\Omega + 335\Omega = 536,4124\Omega$$



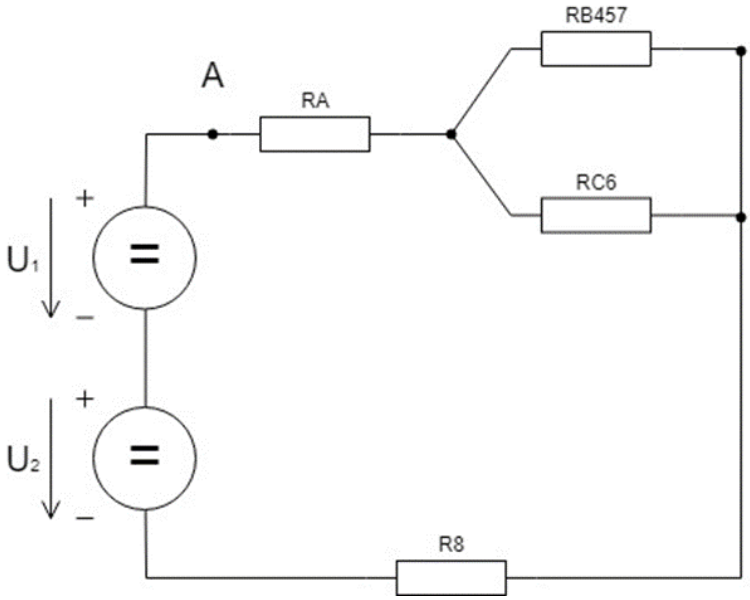
Obrázek 2: $R_{457} \sim R_{45}, R_7$ sériově

$$R_A = \frac{R_1 \times R_2}{R_1 + R_2 + R_3} = \frac{408000\Omega}{1540\Omega} = 264,9350\Omega$$
$$R_B = \frac{R_1 \times R_3}{R_1 + R_2 + R_3} = \frac{176800\Omega}{1540\Omega} = 114,8051\Omega$$
$$R_C = \frac{R_2 \times R_3}{R_1 + R_2 + R_3} = \frac{156000\Omega}{1540\Omega} = 101,2987\Omega$$



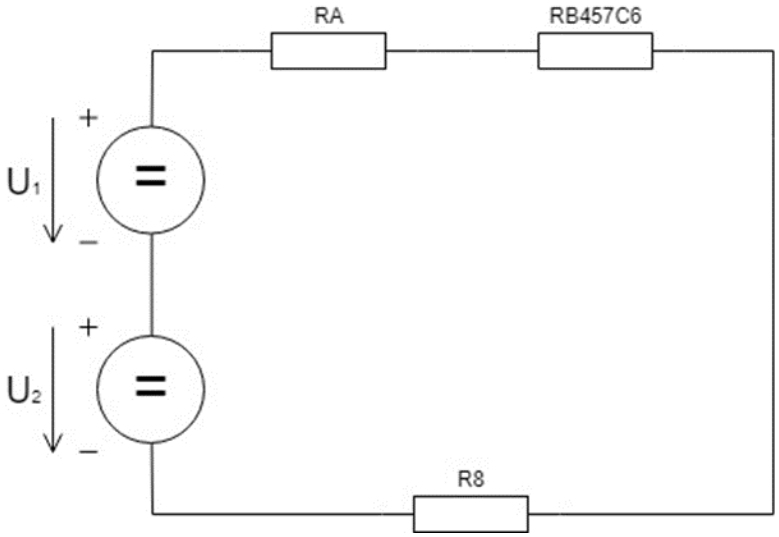
Obrázek 3: Přechod trojúhelník → hvězda

$$R_{B457} = R_B + R_{457} = 114,8051\Omega + 536,4124\Omega = 651,2175\Omega$$
$$R_{C6} = R_C + R_6 = 101,2987\Omega + 870\Omega = 971,2987\Omega$$



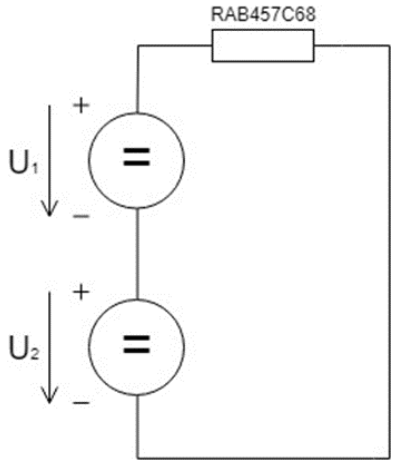
Obrázek 4: $R_{B457} \sim R_B, R_{457}$ sériově, $R_{C6} \sim R_C, R_6$ sériově

$$R_{B457C6} = \frac{R_{B457} \times R_{C6}}{R_{B457} + R_{C6}} = \frac{632526,7111\Omega}{1622,5162\Omega} = 389,8430\Omega$$



Obrázek 5: $R_{B457C6} \sim R_{B457}, R_{C6}$ paralelně

$$R_{AB457C6} = R_A + R_{B457C6} = 264,9350\Omega + 389,8430\Omega = 654,778\Omega$$
$$R_{AB457C68} = R_{AB457C6} + R_8 = 654,778\Omega + 265\Omega = 919,778\Omega$$
$$R_{ekv} = R_{AB457C68} = 919,778\Omega$$



Obrázek 6: $R_{AB457C6} \sim R_A, R_{B457C6}$ sériově, $R_{ekv} \sim R_{AB457C6}, R_8$ sériově

Zpětným chodem vypočítáme $U_{R7},\ I_{R7}$

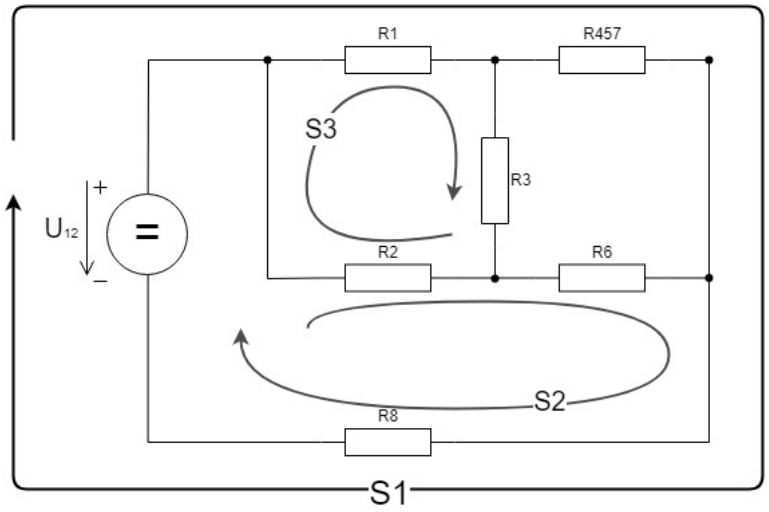
$$U = U_1 + U_2 = 135V + 80V = 215V$$
$$I = \frac{U}{R_{ekv}} = \frac{215V}{919,778\Omega} = 0,2337A$$

$$U_{RAB457C6} = R_{AB457C6} \times I = 654,778\Omega \times 0,2337A = 153,0216V$$
$$U_{R8} = R_8 \times I = 265\Omega \times 0,2337A = 61,9305V$$

$$U_{RA} = R_A \times I = 264,9350\Omega \times 0,2337A = 61,9153V$$
$$U_{RB457C6} = R_{B457C6} \times I = 389,8430\Omega \times 0,2337A = 91,1063V$$

$$I_{RB457} = \frac{U_{RB457C6}}{R_{B457}} = \frac{91,1063V}{651,2175\Omega} = 0,1399A$$
$$I_{RC6} = \frac{U_{RB457C6}}{R_{C6}} = \frac{91,1063V}{971,2987\Omega} = 0,0937A$$

$$U_{RB} = R_B \times I_{RB457} = 114,8051\Omega \times 0,1399A = 16,0612V$$
$$U_{R457} = R_{457} \times I_{RB457} = 536,4124\Omega \times 0,1399A = 75,0440V$$
$$U_{RC} = R_C \times I_{RC6} = 101,2987\Omega \times 0,0937A = 9,4916V$$
$$U_{R6} = R_6 \times I_{RC6} = 870\Omega \times 0,0937A = 81,519V$$



Obrázek 7: Smyčka S1, S2, S3

$$S1 : U_{R1} + U_{R457} + U_{R8} - U = 0 \textit{ (II Kir. Z)}$$
$$S2 : U_{R2} + U_{R6} + U_{R8} - U = 0 \textit{ (II Kir. Z)}$$
$$S3 : U_{R2} - U_{R1} - U_{R3} = 0 \textit{ (II Kir. Z)}$$

$$U_{R1} = U - U_{R457} - U_{R8} = 215V - 75,0440V - 61,9305V = 78,0255V$$
$$U_{R2} = U - U_{R6} - U_{R8} = 215V - 81,519V - 61,9305V = 71,5505V$$
$$U_{R3} = U_{R1} - U_{R2} = 78,0255V - 71,5505V = 6,4750V$$

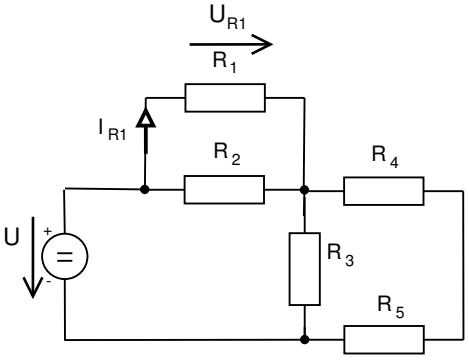
$$I_{R457} = \frac{U_{R457}}{R_{457}} = \frac{75,0440V}{536,4124\Omega} = 0,1398A$$
$$I_{R457} = \frac{U_{R45}}{R_{45}} = \frac{U_{R7}}{R_7}$$
$$U_{R45} = I_{R457} \times R_{45} = 0,1398A \times 201,4124\Omega = 28,1574V$$

$$\boldsymbol{U_{R7} = I_{R457} \times R_7 = 0,1398A \times 335\Omega = 46,8330V}$$
$$\boldsymbol{I_{R7} = I_{R457} = 0,1398A}$$

Příklad 2

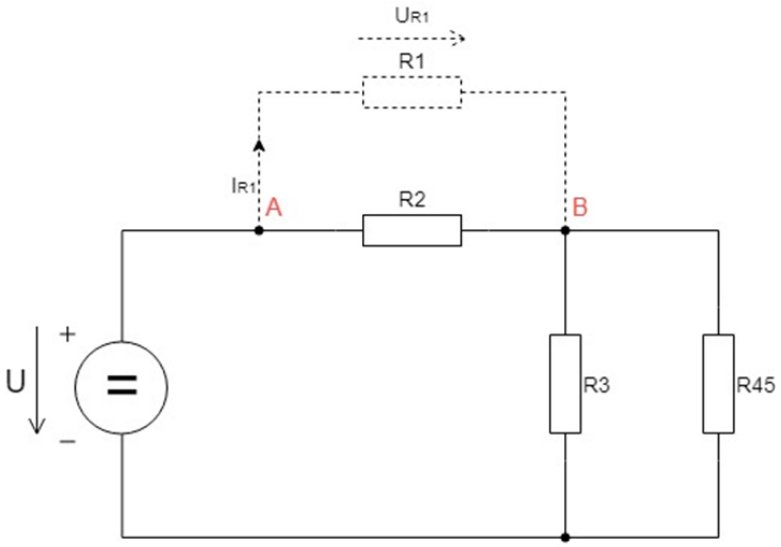
Stanovte napětí U_{R1} a proud I_{R1}. Použijte metodu Théveninovy věty.

sk.	U [V]	R_1 [Ω]	R_2 [Ω]	R_3 [Ω]	R_4 [Ω]	R_5 [Ω]
C	200	70	220	630	240	450



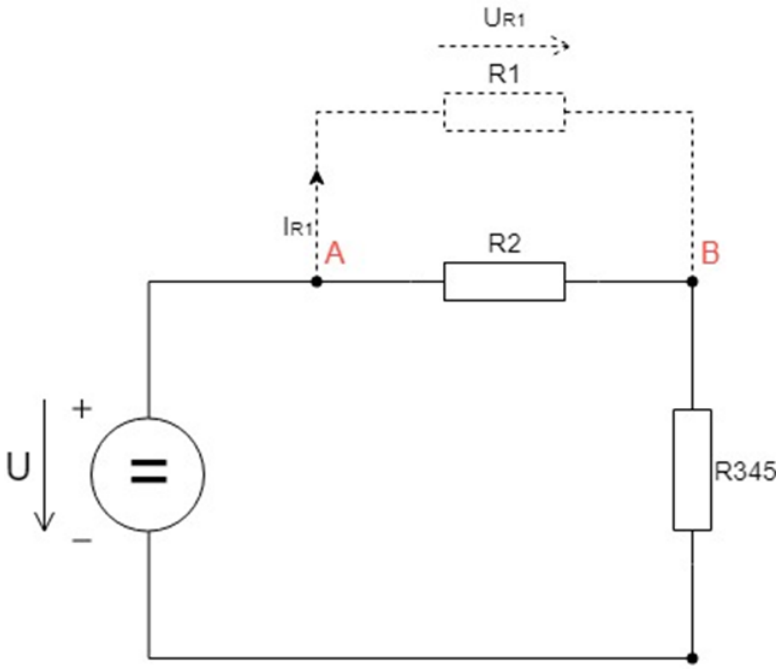
Zjednodušovaní obvodu

$$R_{45} = R_4 + R_5 = 240\Omega + 450\Omega = 690$$



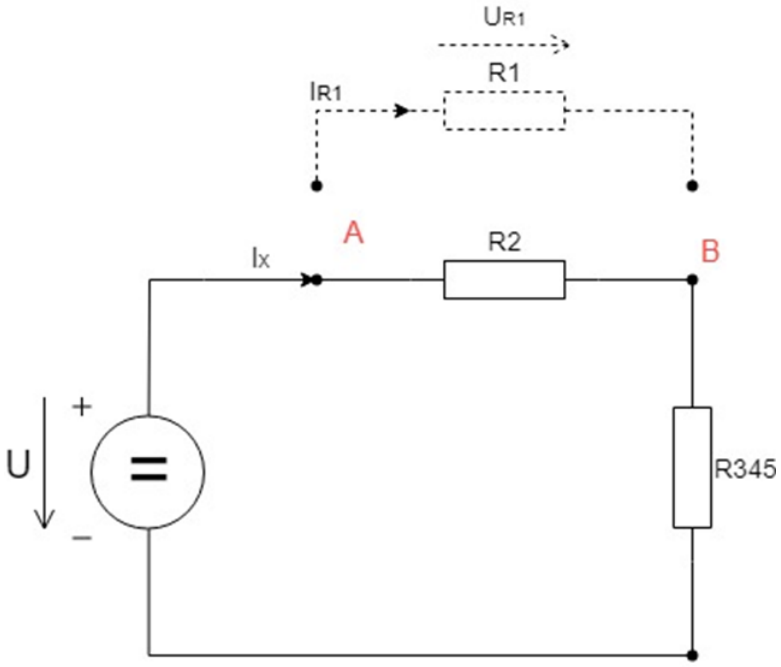
Obrázek 8: $R_{45} \sim R_4, R_5$ seriove

$$R_{345} = \frac{R_3 \times R_{45}}{R3 + R45} = \frac{630\Omega \times 690\Omega}{630\Omega + 690\Omega} = \frac{434700\Omega}{1320\Omega} = 329,3181\Omega$$



Obrázek 9: $R_{345} \sim R_3, R_{45}$ parallelne

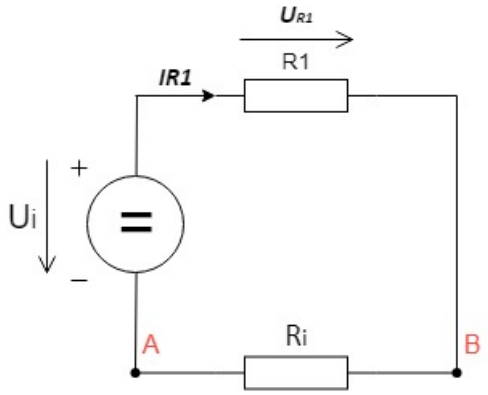
Přechod na ekvivalentní obvod



Obrázek 10: Odpojíme větev s R1

$$U_i = I_X \times R_2 + I_X \times R_{345} \textit{ (II Kir. Z.)}$$
$$I_X = \frac{U}{R_2 + R_{345}} = \frac{200V}{220\Omega + 329,3181\Omega} = \frac{200V}{549,3181\Omega} = 0,3640A$$
$$U_{AB} = U_{R2} = I_X \times R_2 = 0,3640A \times 220\Omega = 80,0800V$$
$$R_i = \frac{R_2 \times R_{345}}{R_2 + R_{345}} = \frac{220\Omega \times 329,3181\Omega}{220\Omega + 329,3181\Omega} = \frac{72449,9820\Omega}{549,3181\Omega} = 131,8907\Omega$$

$$\boldsymbol{I_{R1} = \frac{U_{AB}}{R_1 + R_i} = \frac{80,0800V}{70\Omega + 131,8907\Omega} = \frac{80,0800V}{201,8907\Omega} = 0,3966A}$$
$$\boldsymbol{U_{R1} = I_{R1} \times R_1 = 0,3966A \times 70\Omega = 27,7620V}$$

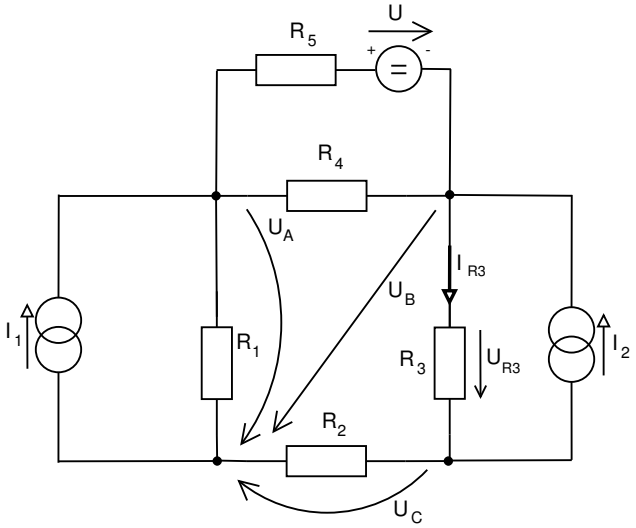


Obrázek 11: Ekvivalentní obvod

Příklad 3

Stanovte napětí U_{R3} a proud I_{R3}. Použijte metodu uzlových napětí (U_A, U_B, U_C).

sk.	U [V]	I_1 [A]	I_2 [A]	R_1 [Ω]	R_2 [Ω]	R_3 [Ω]	R_4 [Ω]	R_5 [Ω]
C	110	0.85	0.75	44	31	56	20	30



Zvolíme uzel D jako referenční

$$U_D = 0V$$

Vzorce pro jednotlivé proudy

$$I_{R1} = \frac{U_A - U_D}{R_1} = \frac{U_A}{R_1}$$
$$I_{R2} = \frac{U_C - U_D}{R_2} = \frac{U_C}{R_2}$$
$$I_{R3} = \frac{U_B - U_C}{R_3}$$
$$I_{R4} = \frac{U_A - U_B}{R_4}$$
$$I_{R5} = \frac{U_B - U_A + U}{R_5}$$

I Kir. Z. pro proudy v uzlech

$$I_1 + I_{R5} - I_{R1} - I_{R4} = 0 \ (UzelA.)$$
$$I_{R4} + I_2 - I_{R5} - I_{R3} = 0 \ (UzelB.)$$
$$I_{R3} - I_2 - I_{R2} = 0 \qquad (UzelC.)$$

$$
\begin{aligned}
I_1 + \frac{U_B - U_A + U}{R_5} - \frac{U_A}{R_1} - \frac{U_A - U_B}{R_4} &= 0 \\
\frac{U_A - U_B}{R_4} + I_2 - \frac{U_B - U_A + U}{R_5} - \frac{U_B - U_C}{R_3} &= 0 \\
\frac{U_B - U_C}{R_3} - I_2 - \frac{U_C}{R_2} &= 0
\end{aligned}
$$

$$
\begin{aligned}
-U \times G_5 - I_1 &= U_B \times (G_5 + G_4) - U_A \times (G_5 + G_1 + G_4) \\
U \times G_5 - I_2 &= U_A \times (G_4 + G_5) - U_B \times (G_4 + G_5 + G_3) + U_C \times G_3 \\
I_2 &= U_B \times G_3 - U_C \times (G_3 + G_2)
\end{aligned}
$$

Dosadíme hodnoty a vypočítáme napětí na uzlech

$$
\begin{aligned}
-4.5166A &= U_B \times \frac{1}{12\Omega} - U_A \times \frac{7}{66\Omega} \\
2.9166A &= U_A \times \frac{1}{12\Omega} - U_B \times \frac{17}{168\Omega} + U_C \times \frac{1}{56\Omega} \\
0.75A &= U_B \times \frac{1}{56\Omega} - U_C \times \frac{87}{1736\Omega}
\end{aligned}
$$

$$
\begin{aligned}
U_A &= 52.3536V \\
U_B &= 12.4327V \\
U_C &= -10.5355V
\end{aligned}
$$

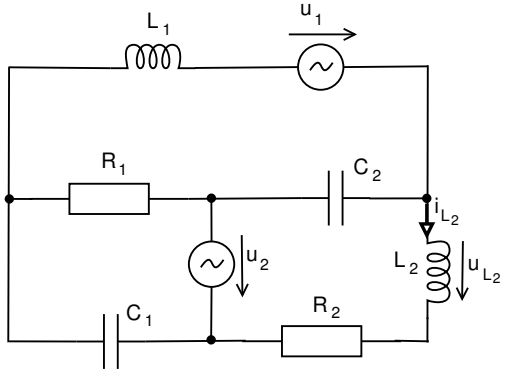
Vypočítáme napětí a proud na rezistoru R_3

$$
\begin{aligned}
U_{R3} &= U_B - U_C = 22.9656V \\
I_{R3} &= \frac{U_{R3}}{R_3} = 0.4101A
\end{aligned}
$$

Příklad 4

Pro napájecí napětí platí: $u_1 = U_1 \cdot \sin(2\pi f t)$, $u_2 = U_2 \cdot \sin(2\pi f t)$.
Ve vztahu pro napětí $u_{L_2} = U_{L_2} \cdot \sin(2\pi f t + \varphi_{L_2})$ určete $|U_{L_2}|$ a φ_{L_2}. Použijte metodu smyčkových proudů.
Pozn: Pomocné směry šipek napájecích zdrojů platí pro speciální časový okamžik ($t = \frac{\pi}{2\omega}$).

sk.	U_1 [V]	U_2 [V]	R_1 [Ω]	R_2 [Ω]	L_1 [mH]	L_2 [mH]	C_1 [µF]	C_2 [µF]	f [Hz]
H	5	6	10	10	160	75	155	70	95



Vzorce pro impedanci a uhlovou frekvenci

$$\omega = 2\pi f$$
$$Z_C = \frac{-j}{\omega C}$$
$$Z_L = j\omega L$$

II Kir. Z. pro rovnice proudu ve smyčkách

$$I_A : U_{L1} + U_1 + U_{C2} + U_{R1} = 0$$
$$I_B : U_{R1} + U_2 + U_{C1} = 0$$
$$I_C : U_{C2} + U_{L2} + U_{R2} - U_2 = 0$$

$$I_A : I_A(Z_{L1} + Z_{C2} + R_1) - I_B R_1 - I_C Z_{C2}$$
$$I_B : I_B(R_1 + Z_{C1}) - I_A R_1 = -U_2$$
$$I_C : I_C(Z_{C2} + Z_{L2} + R_2) - I_A Z_{C2} = U_2$$

Výpočet proudu pomoci matic

$$\begin{pmatrix} Z_{L1} + Z_{C2} + R_1 & -R_1 & -Z_{C2} \\ -R_1 & R_1 + Z_{C1} & 0 \\ -Z_{C2} & 0 & Z_{C2} + Z_{L2} + R_2 \end{pmatrix} \times \begin{pmatrix} I_A \\ I_B \\ I_C \end{pmatrix} = \begin{pmatrix} -U_1 \\ -U_2 \\ U_2 \end{pmatrix}$$

$$\begin{pmatrix} 10 + 71.5713j & -10 & 23.9391 \\ -10 & 10 - 10.8085j & 0 \\ 23.9331j & 0 & 10 + 20.8346j \end{pmatrix} \times \begin{pmatrix} I_A \\ I_B \\ I_C \end{pmatrix} = \begin{pmatrix} -5 \\ -6 \\ 6 \end{pmatrix}$$

Násobením inverzní matici zleva dostáváme hodnoty proudu

$$I_A = (-0.2105 + 0.2255j)A$$
$$I_B = (-0.4862 - 0.3j)A$$
$$I_C = (0.4099 - 0.3502j)A$$

Formuli pro napětí a fázový posun L2

$$U_{L2} = I_C \times Z_{L2}$$
$$|U_{L2}| = \sqrt{re(U_{L2})^2 + im(U_{L2})^2}$$

$$\varphi = atan(\frac{im(U_{L2})}{re(U_{L2})})$$

Dosadíme hodnoty do vzorců

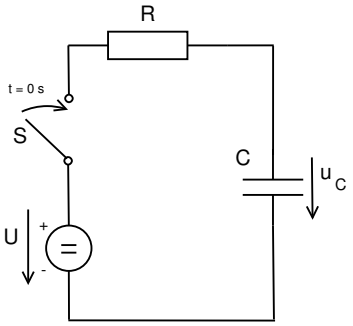
$$I_{L2} = I_C = (0.4099 - 0.3502j)A$$
$$U_{L2} = I_{L2} \times Z_{L2} = (0.4099 - 0.3502j)A \times (0 + 44.7676j)A = (15.6810 + 18.3519j)V$$

$$\boldsymbol{\varphi = atan(\frac{im(U_{L2})}{re(U_{L2})}) = atan(\frac{18.3519}{15.6810}) = 0.8637\ ^\circ}$$
$$\boldsymbol{|U_{L2}| = \sqrt{re(U_{L2})^2 + im(U_{L2})^2} = \sqrt{15.6810^2 + 18.3519^2} = 24.1389V}$$

Příklad 5

V obvodu na obrázku níže v čase $t = 0$ [s] sepne spínač S. Sestavte diferenciální rovnici popisující chování obvodu na obrázku, dále ji upravte dosazením hodnot parametrů. Vypočítejte analytické řešení $u_C = f(t)$. Proveďte kontrolu výpočtu dosazením do sestavené diferenciální rovnice.

sk.	U [V]	R [Ω]	C [F]	$u_C(0)$ [V]
C	45	5	30	12



Vzorec pro napětí na kondenzátoru

$$U_C' = \frac{I}{C}$$

II Kir. Z. pro obvod

$$U_R + U_C - U = 0$$

Algebraické úpravy

$$U_C' = \frac{U_R}{RC}$$
$$U_R = U - U_C$$
$$U_C' = \frac{U - U_C}{RC}$$
$$\boldsymbol{RC \times U_C' + U_C = U}$$
$$150 \times U_C' + U_C = 45$$

Očekávané řešeni

$$\boldsymbol{U_C(t) = k(t) \times e^{\lambda t}}$$

Najdeme λ

$$(U_C{}' = \lambda; U_C = 1)$$
$$0 = 150 \times \lambda + 1$$
$$\lambda = -\frac{1}{150}$$

Dosadíme λ do vzorce a najdeme $k'(t)$

$$U_C(t) = k(t) \times e^{-\frac{t}{150}}$$

Derivujeme $U_C(t)$ a najdeme $k'(t)$

$$\begin{aligned} U_C'(t) &= k'(t) \times e^{-\frac{t}{150}} - \frac{1}{150} \times k(t) \times e^{-\frac{t}{150}} \\ 45 &= 150k'(t) \times e^{-\frac{t}{150}} \\ k'(t) &= \frac{3}{10} \times e^{\frac{t}{150}} \end{aligned}$$

Integrujeme $k'(t)$ a dosadíme do vzorce pro $U_C(t)$

$$\begin{aligned} k(t) = \int \frac{3}{10} \times e^{\frac{t}{150}} dt = \frac{3}{10} \times 150 \times e^{\frac{t}{150}} &= 45 \times e^{\frac{t}{150}} + C \\ U_C(t) &= (45 \times e^{\frac{t}{150}} + C) \times e^{-\frac{t}{150}} \\ \boldsymbol{U_C(t)} &= \boldsymbol{45 + C \times e^{-\frac{t}{150}}} \end{aligned}$$

Najdeme C

$$\begin{aligned} U_C(0) &= 45 + C \\ 12 &= 45 + C \\ C &= -33 \end{aligned}$$

$$\boldsymbol{U_C(t) = 45 - 33 \times e^{-\frac{t}{150}}}$$

Shrnutí výsledků

Příklad	Skupina	Výsledky	
1	H	$U_{R7} = 46,8330V$	$I_{R7} = 0,1398A$
2	C	$U_{R1} = 27,7620V$	$I_{R1} = 0,3966A$
3	C	$U_{R3} = 22.9656V$	$I_{R3} = 0.4101A$
4	H	$\lvert U_{L_2} \rvert = 24.1389V$	$\varphi_{L_2} = 0.8637°$
5	C	$u_C = 45 - 33 \times e^{-\frac{t}{150}}$	